Inventory, Factor-Hoarding and the Dynamic Response to Monetary Shock

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June 3, 2008
(First Draft: December 2007)

Abstract

This paper proposes a new model accounting for the delayed effect of monetary shocks on output. The key feature of the model is to distinguish a variety of margins (i.e., inventory adjustments, hours per worker, efforts and employments) on which firms adjust output in response to macroeconomic shock. When these multiple margins are properly introduced to an otherwise standard modern monetary business cycles model, the interplay between inventory adjustments and the one-period lag in adjusting employment can produce the hump-shaped response of output to monetary shock. More importantly, this can be done even without relying upon the habit persistence model that has been decisively rejected in recent papers by Dynan (2000) and Flavin and Nakagawa (2004).

Keywords: Monetary Shock, Inventory, Sticky Prices, Factor-Hoarding, Habit Formation

JEL Classification: E52, E32

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1 Introduction

This paper integrates inventory adjustments and labor-related decisions (hours per worker, work efforts and employments) through which firms adjust production into an otherwise conventional modern monetary business cycles model\(^1\). As shown in Bresnahan and Ramey (1994), these margins differ in their adjustment costs and their variable costs. Thus, differentiating among these multiple margins implies changing the shape of the underlying cost function. This in turn will affect the nature of how aggregate shocks are propagated through the economy. Considering the importance of distinguishing multiple margins in understanding business cycles, there have been surprisingly few efforts to introduce multiple margins to a New Keynesian model. The New Keynesian models proposed in the literature either lump labor decisions together or ignore inventory adjustments\(^2\).

To my knowledge, this paper is the first to examine the consequence of considering all important margins together in a New Keynesian model. I show that investigating the interaction among a variety of margins along which firms adjust production sheds a new light on understanding the delayed effect of monetary shock on output\(^3\).

My findings stand in sharp contrast with the previous results in the literature. The leading explanation for the delayed effect of monetary shock on output thus far is that the sluggish adjustment of aggregate consumption induced by “habit formation” gives rise to the hump-shaped response of output to monetary shocks. When habit formation is introduced to a standard New Keynesian model, monetary shocks can generate a one or two quarter-delayed effect on output depending upon the magnitude of the habit persistence parameter. However, recent papers by Dynan (2000) and Flavin and Nakagawa (2004) find that there is very little evidence of habit persistence at the household level. Hence, it appears that accounting for the delayed effect of monetary shocks on output based on habit persistence has lost its validity.

This paper seeks a new explanation for the delayed effect from the observation that a

\(^1\)Henceforth, the term “conventional modern monetary business cycles model” will be used interchangeably with “New Keynesian model”.

\(^2\)The standard New Keynesian model, such as in Ireland (2001), lumps labor decisions and neglects inventory adjustment. Recently, there have been attempts to modify this standard New Keynesian model so as to differentiate between intensive and extensive margins of labor (e.g., Barnichon (2007), Dotsey and King (2006) and Trigari (2004 and 2006)). However, these papers have not incorporated inventory adjustment. On the other hand, several authors (e.g., Boileau and Letendre (2004) and Jung and Yun (2005)) introduce inventories to a New Keynesian model, but their models lump labor decisions.

\(^3\)Typically, after a positive monetary shock, output rises over several quarters and then declines.
variety of margins differ in their adjustment costs and variable costs. Obviously, one could instead proceed to investigate a different class of utility functions besides habit formation\textsuperscript{4} that rationalize the sluggish response of aggregate consumption. However, the purpose of this paper is to show that the sluggish response of consumption is not a prerequisite for explaining the delayed effect of monetary shocks on output. Once the model becomes rich enough to incorporate all important margins considered in the literature that firms can utilize in accommodating changes in demand, it can account for the delayed effect of monetary policy on output even without modelling the sluggish response of aggregate demand.

Following Bresnahan and Ramey (1994) and Ramey and Vine (2006), I divide a variety of margins into two types. The first are ‘intensive margins’, which can be characterized as ‘high marginal cost-low adjustment cost margins’. Bresnahan and Ramey (1994) and Ramey and Vine (2006) show in studying an auto industry that inventory adjustment and overtime hours belong to this intensive margin classification. Motivated by this observation, I allow firms to change inventory and hours per worker instantaneously to meet the increased demand due to a positive monetary shock. Another important intensive margin considered in the literature (e.g., Burnside et al. (1993)) is varying the degree of work efforts. I thus also assume that firms can make workers exert more efforts immediately in response to monetary shocks. The second type of margins are ‘extensive margins’, which can be characterized as ‘low marginal cost-high adjustment cost margins’. The size of employment is an example of this type, because firms and workers have to spend time and resources to vary it, due to the search and matching friction in the labor market.

In the model, I simplify the fact that the size of employment is a low marginal cost-high adjustment cost margin relative to the intensive margins in the following manner. I assume that making current-quarter adjustments to employment is infinitely costly, but in the subsequent quarter employment adjustments are relatively less costly than varying inventories\textsuperscript{5}, hours per worker and work efforts.

Given the assumptions about the structures of the various margins on which firms make adjustments, the mechanism through which the model can generate the hump-shaped response of

\textsuperscript{4}One promising approach might be to replace habit formation with housing consumption with adjustment cost, as Flavin and Nakagawa (2004) suggest. This is because housing consumption with adjustment cost can be structurally interpreted as habit persistence (See Flavin and Nakagawa (2004) for further discussions).

\textsuperscript{5}This implies that in the model, firms choose to make all current-quarter adjustments on the intensive margin. This assumption has been widely used in Burnside et al. (1993) and Burnside and Eichenbaum (1996).
output to monetary shocks is quite simple. It can be understood best by considering the case first where one differentiates among the intensive and extensive margins of labor\(^6\) but does not introduce inventory adjustments. In the absence of inventory adjustments, output is demand-determined. Without habit formation, consumption, the lion’s share of aggregate demand, does not display the hump-shaped response. In this case, firms have to vary their hours per worker and work efforts substantially to meet the largest change in aggregate demands that occurs in the first period. In the subsequent period when it is feasible to adjust employment, the changes in employment will be relatively small since firms see their demands decline in the second period. Thus, without taking account of inventory adjustments, even the model assuming a one-period lag in adjusting employments cannot generate a one-period delay in the response of labor input.

In contrast, introducing inventory adjustments reduces the relative importance of varying the hours per worker and work efforts in accommodating the initial increases in demand due to expansionary monetary shocks. This is because firms can meet some portion of the changes in demand instantaneously through inventory adjustments. This makes the response of employment in the subsequent period quite large relative to the initial responses of hours per worker and work efforts. Hence, integrating both inventory adjustments and the intensive and extensive margins of labor introduces a one-period delay in adjusting labor input in response to monetary shocks. This in turn helps to generate the hump-shaped response of output to monetary shocks.

However, I should also point out that introducing inventory adjustment alone cannot generate the delayed response of output to monetary shocks. When it is assumed that employment can be adjusted instantaneously as opposed to being predetermined, the introduction of inventory adjustments does not lead to a hump-shaped response of output to monetary shocks. Therefore, only when inventory adjustments, hours per worker, work efforts and employment are properly introduced, can one obtain the delayed effect of monetary shocks on output.

The findings in this paper have some further important implications for the recent attempts to introduce the search and matching friction explicitly to an otherwise standard New Keynesian model. In particular, recent papers by Trigari (2004 and 2006) and Barnichon (2007) also allow for changes in the labor input at the intensive margins when they incorporate search and matching frictions into a New Keynesian model. Their models do not consider inventory adjustments,

\(^6\)For labor, the intensive margins are hours per worker and work efforts, and the extensive margin is employment.
however. The one-period lag in employment adjustments assumed in this paper can be interpreted as the reduced form of a fully articulated search and matching model. Thus, it is not surprising that Trigari’s and Barnichon’s models without inventory adjustment predict that labor input varies mostly at the intensive margins and fail to generate the hump-shaped response of output. The results here suggest that introducing inventory adjustments improves the search and matching model in generating a more realistic dynamics of output in response to monetary shocks.

Unsurprisingly, the model presented in this paper is not the only one to generate the hump-shaped response of output to monetary shocks. Without assuming habit formation, Álvarez-Lois (2006) also presents an alternative model by introducing “putty-clay” technology and idiosyncratic demand uncertainty. In his model, goods are produced by combining capital stock and employment with “putty-clay” technology. This means that both factors of production are substitutes \textit{ex ante} (i.e., before investment decisions are made) but complement \textit{ex post} (i.e., once equipment is installed). Each firm chooses an amount of capital and the maximum level of work stations (employment capacity) that can be used with the level of capital chosen. In addition, he introduces the idiosyncratic demand uncertainty for the goods each firm produces at the time of capacity choices. In equilibrium, a portion of firms face demand shortages and have idle capacity, while others are at full capacity. After a monetary shock, only firms with excess capacity are initially able to expand their levels of production while firms at full capacity are unable to serve extra demand. With this mechanism, his model can generate the delayed effect of monetary shocks on output. However, it seems unlikely that his model captures the characteristic of industries that are considered the important source of cyclical variability in the economy. Much of the cyclical variation in output stems from variability in the manufacturing sector such as the automobile industry and capital goods-producing sectors. While the scope of substitutability between capital and employment is quite limited, Shapiro (1996) shows that variations in the workweek of capital are an important source of output fluctuations in the manufacturing sector. In his model, there are no variations in the fraction of hours per period over which capital is operated. In contrast, I allow for variations in the workweek of capital by assuming that if workers work for a longer number of hours each week, the capital stock is used for more hours as well.

The remainder of this paper is organized as follows. Section 2 presents the model economy, and Section 3 describes its calibration. Section 4 shows how introducing the variety of margins
leads to a hump-shaped response. Section 4 concludes.

2 The Model Economy

The economy consists of households, a central bank in charge of the conduct of monetary policy and two productive sectors: a competitive sector producing a final good and a monopolistic sector providing intermediate goods. These intermediate goods are the only input necessary for the production of the final good, which can be used for consumption or investment. Intermediate goods are produced by combining capital services, labor inputs and inventories. Following Kydland and Prescott (1982), Christiano (1988) and Ramey (1989), inventories are treated as a factor of production. As emphasized by these authors, the inclusion of inventories to the production function is well warranted. They reduce the equipment downtime associated with shifting from producing one type of good to another. Furthermore, the fixed cost involved in shipping finished goods implies that capital and labor input can be conserved by shipping their products in batches and holding inventories. In other words, fewer truck and man-hours are required by holding inventories.

Firms in the intermediate goods sector must choose the size of employment before observing monetary shocks, because it is by assumption infinitely costly to make current-quarter adjustment on the employment. Even though firms in the intermediate goods sector cannot change employment in response to monetary shocks, they can make adjustments instantaneously through varying inventories, hours per worker and work efforts.

Finally, the model incorporates nominal rigidities in the form of the adjustment cost of changing prices à la Rotemberg (1982). This may reflect the costs of advertising or the fact that erratic pricing causes consumer dissatisfaction and erodes the reputations of firms.

2.1 Households

The economy is populated by a continuum of identical households of unit measure. Their momentary utility function is given by

\[ u(C_t, M_t/P_t, N_{t-1}, h_t, e_t) = \left( \frac{\gamma}{\gamma - 1} \right) \ln \left( \frac{C_t^{\gamma-1}}{\gamma - 1} + \left( \frac{M_t}{P_t} \right)^{\frac{\gamma-1}{\gamma}} \right) - V(N_{t-1}, h_t, e_t) \] (1)
where \( C_t, N_{t-1}, h_t, e_t, \) and \( M_t/P_t \) are consumption, the number of workers, hours per worker, effort per hour of work and real balances, respectively. \( V(N_{t-1}, h_t, e_t) \) describes the disutility of providing labor services. Following Bils and Cho (1994), we specify that

\[
V(N_{t-1}, h_t, e_t) = \left[ \theta_1 \frac{N_{t-1}}{1+\nu} + \theta_2 N_{t-1} \frac{h_t^{1+\chi}}{1+\chi} + N_{t-1} h_t \frac{e_t^{1+\zeta}}{1+\zeta} \right]
\]

The first component of \( V(N_{t-1}, h_t, e_t) \) represents the cost of sending \( N_{t-1} \) members of households to work in a period \( t \), even if the hours worked are arbitrarily small. It may be interpreted as costs for commuting or costs incurred due to having fewer people available for home production. Notice that the subscript \( t-1 \) in \( N_{t-1} \) is due to the assumption that the size of employment is predetermined. The second component reflects the disutility of working \( h_t \) hours per period associated with reduced leisure and longer work during nonstandard hours. Finally, the third term reflects disutility from exerting effort.

Next, I describe the sources of funds that can be used to purchase consumption goods and assets. Households enter each period holding an \( M_{t-1} \) amount of money stock and amount \( B_{t-1} \) of a risk free discount bond. Households receive a lump-sum nominal transfer \( T_t \) from the monetary authority and an amount \( D_t \) corresponding to intermediate firms’ profits. Finally, households receive a (real) total wage payment by providing labor services from intermediate goods firms. We assume that the equilibrium wage bill is determined as Bils and Cho (1994) suggest: households present their employer with a wage bill that takes the form of \( V(N_{t-1}, h_t, e_t) \) and allow firms to freely choose the size of employment, hours per worker and effort. Hence, the equilibrium (real) total wage, \( W_t \), takes the following form:

\[
W_t = \left[ \theta_1 \frac{N_{t-1}}{1+\nu} + \theta_2 N_{t-1} \frac{h_t^{1+\chi}}{1+\chi} + N_{t-1} h_t \frac{e_t^{1+\zeta}}{1+\zeta} \right]
\]

Households use their funds to purchase an amount \( C_t \) of the finished good at a nominal price \( P_t \). Households purchase \( B_t \) risk-free bonds at an unitary cost of \( 1/R_t \), where \( R_t \) is the gross nominal rate of return between periods \( t \) and \( t+1 \). The following relation, which represents households’ budget constraint, must hold at every period:
\[ C_t + \frac{B_t}{P_t} - \frac{B_{t-1}}{P_t} + \frac{M_t}{P_t} - \frac{M_{t-1}}{P_t} = W_t + \frac{T_t}{P_t} + \frac{D_t}{P_t} \] (2)

This states that consumption expenditures plus asset accumulation must equal disposable income.

Household’s preferences are given by the life-time utility function \( U_0 \). This function represents the expectation of the discounted sum of the monetary utility function conditional on the information set at date \( t = 0 \):
\[ U_0 = E_0 \sum_{t=0}^{\infty} \beta^t u (C_t, M_t/P_t, N_{t-1}, h_t, e_t) \] (3)
where \( \beta \) denotes households’ discount factor.

Household’s optimal behavior involves choosing a sequence of \( \{C_t, M_t, B_t\} \) that maximizes their life-time utility function (3) subject to the budget constraint (2).

### 2.2 Final Goods Firms

The representative final good-producing firm uses \( S_t(i) \) units of each intermediate good \( i \in [0, 1] \) to produce \( G_t \) units of the final good using the technology
\[ G_t = \left[ \int_0^1 S_t(i) \frac{i^{\epsilon - 1}}{\epsilon} \, di \right]^{\frac{1}{\epsilon}} \] (4)

Given that the price of intermediate good \( i \) is \( P_t(i) \), the finished good sells at the nominal price \( P_t \). The finished goods-producing firm chooses \( G_t \) and \( S_t(i) \) to maximize its profits,
\[ P_t G_t - \int_0^1 P_t(i) S_t(i) \, di \] (5)
subject to the constraint imposed by (4). The first-order conditions for this problem imply that the optimal level of demand for an intermediate good \( i \) is given by
\[ S_t(i) = [P_t(i)/P_t]^{-\epsilon} G_t \] (6)
Since the firm is operating in a competitive market, the zero-profit condition determines $P_t$ as a Dixit-Stiglitz aggregator given by

$$P_t = \left[ \int_0^1 P_t(i)^{1-\varepsilon} \, di \right]^{\frac{1}{1-\varepsilon}} \quad (7)$$

### 2.3 Intermediate goods firms

There is a continuum of monopolistically competitive firms, each producing an intermediate good. The representative intermediate goods firm produces its output from effective units of labor $L_t(i)$, effective units of capital $K'_t(i)$ and inventories stock $X_{t-1}(i)$. I use the production function studied in Kydland and Prescott (1982) and Christiano (1988); namely:

$$Y_t(i) = A \left[ (1 - \sigma)(K'_t(i))^{-\psi} + \sigma X_{t-1}(i)^{-\psi} \right]^{(1-\alpha)/\psi} [L_t(i)]^\alpha \quad (8)$$

where $0 < \alpha < 1$, $0 < \sigma < 1$, $0 < \psi < \infty$, and $A$ represents an aggregate productivity parameter.

The production function results in a share $\alpha$ of labor inputs in the steady state. The elasticity of substitution between capital services and inventory is $1/(1 + \psi)$. This elasticity is probably less than one, which is why it is required that $\nu$ be positive.

In contrast to Kydland and Prescott (1982) and Christiano (1988), I allow three dimensions of effective labor units: employment, $N_t$, hours per worker, $h_t$, and effort per hour of work, $e_t$. However, I assume that it is infinitely costly to make current-quarter adjustment on the employment. This means that intermediate firms start each period $t$ with a predetermined size of employment. $L_t(i)$ is therefore given by

$$L_t(i) = h_t(i)e_t(i)N_{t-1}(i)$$

Furthermore, I also relax the assumption of a given short-run quantity of capital made in Kydland and Prescott (1982) and Christiano (1988). Following Bils and Cho (1994), I assume instead that if a worker works longer hours or works at a more rapid physical pace, the utilization of the capital he operates will increase proportionately. $K'_t$ is therefore given by

$$K'_t(i) = h_t(i)e_t(i)K_{t-1}(i)$$
where $K_{t-1}$ denotes the capital stock at the end of period $t - 1$.

A representative intermediate goods firm chooses a sequence of \{\(K_t(i), I_t(i), N_t(i), h_t(i), e_t(i), P_t(i), X_t(i)\)\} that maximizes the discounted stream of expected nominal profits $D_t$:

$$E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t D_t(i) / P_t$$

subject to

$$S_t(i) \leq Y_t(i) - (X_t(i) - X_{t-1}(i))$$  \hspace{1cm} (10)

$$S_t(i) = \left[ P_t(i) / P_t \right]^{-\varepsilon} G_t$$  \hspace{1cm} (11)

$$Y_t(i) = A \left[ (1 - \sigma) (h_t(i) e_t(i) K_{t-1}(i))^{\psi} + \sigma X_{t-1}(i)^{\psi} \right]^{-(1-\alpha)/\psi} [h_t(i) e_t(i) N_{t-1}(i)]^\alpha$$  \hspace{1cm} (12)

The constraint (10) imposes the requirement that a representative intermediate goods firm satisfies the representative final goods firm’s demand through adjusting production and inventories. Equations (11) and (12) are the representative final goods firm’s demand and the technology available to an intermediate goods firm, respectively. In (9), $\lambda$ is the Lagrange multiplier on the budget constraint from the representative household’s problem.

The real profits of a typical intermediate goods firm at the beginning of any period $t$, $\frac{D_t(i)}{P_t}$, are defined as

$$\frac{D_t(i)}{P_t} = \frac{P_t(i) S_t(i)}{P_t} - \left( \theta_1 N_{t-1}^{1+\nu}(i) + \theta_2 N_t(i) \frac{h_t^{1+\chi}(i)}{1+\chi} + N_{t-1}(i) h_t(i) \frac{e_t^{1+\varsigma}(i)}{1+\varsigma} \right)$$  \hspace{1cm} (13)

$$- I_t(i) - AC_{k,t}(i) - AC_{p,t}(i)$$

where

$$I_t(i) = K_t(i) - (1 - \delta) K_{t-1}(i)$$  \hspace{1cm} (14)

is investment, with $\delta$ being the rate of depreciation. The terms $AC_{k,t}$ and $AC_{p,t}$ in (13) represent a capital adjustment cost and a cost of changing the nominal price of the goods it produces, measured
in terms of the finished goods:

\[ AC_{k,t}(i) = \frac{\phi_k}{2} \left( \frac{I_t(i)}{K_{t-1}(i)} - \delta \right)^2 K_{t-1}(i) \]  \hspace{1cm} (15) \]

\[ AC_{p,t}(i) = \frac{\phi_p}{2} \left( \frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right)^2 Y_t \]  \hspace{1cm} (16) \]

where \( \pi \) is the steady-state rate of inflation.

### 2.4 Monetary Authority

At each period of time, the monetary authority supplies the money stock, which is growing at the rate

\[ \mu_t = \frac{M_t}{M_{t-1}} \]  \hspace{1cm} (17) \]

It is assumed that the monetary authority follows an exogenous policy rule:

\[ \mu_t = (1 - \rho_\mu) \mu + \rho_\mu \mu_{t-1} + \varepsilon_{\mu,t} \]  \hspace{1cm} (18) \]

where \( \rho_\mu \) is the persistence parameter, and \( \varepsilon_{\mu,t} \) the serially uncorrelated policy shock normally distributed with mean zero and standard deviations \( \sigma_\mu \).

### 2.5 Symmetric Equilibrium

I assume a symmetric monopolistic competition equilibrium. In a symmetric equilibrium, all intermediate goods firms make identical decisions, so that \( P_t(i) = P_t \), \( Y_t(i) = Y_t \), \( h_t(i) = h_t \), \( N_t(i) = N_t \), \( e_t(i) = e_t \), \( K_t(i) = K_t \), \( X_t(i) = X_t \), and \( S_t(i) = S_t \) for all \( i \in [0,1] \) and \( t \). In addition, the market-clearing condition for bonds, \( B_t = B_{t-1} = 0 \), must hold for all \( t \). These equilibrium conditions, together with the first-order conditions for the households and the intermediate-goods firms and the process for monetary supply shocks, characterize the symmetric equilibrium. Among these, equilibrium condition in the final goods market deserves some comment, due to the presence of inventory adjustment. It requires

\[ C_t + I_t + \frac{\phi_k}{2} \left( \frac{K_t}{K_{t-1}} - 1 \right)^2 K_{t-1} + \frac{\phi_p}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 Y_t = Y_t - (X_t - X_{t-1}) \]  \hspace{1cm} (19) \]
where

\[ Y_t = A_t \left[ (1 - \sigma) (h_t e_t K_{t-1})^{-\psi} + \sigma X_{t-1}^{-\psi} \right]^{-(1-\alpha)/\psi} [h_t e_t N_{t-1}]^\alpha \]

Note that equation (19) is not the concept of national income account identity. In this model, output is not counted \textit{ex post} as a mechanical sum of consumption and investment (including inventory investment). It is determined simultaneously by firms’ optimal choices of factors of production. Thus, the proper interpretation of equation (19) is that firms should meet the change in demand through varying production and inventories. A complete description of the symmetric equilibrium is summarized in Appendix A.

3 Calibration

Table (1) summarizes all of the values assigned to the parameters in the model. In the utility function specification (1), the parameter values dictating the responsiveness of employment, hours per worker and work efforts per hour are taken from Bils and Cho (1994). The values for \( \nu \), \( \chi \) and \( \varsigma \) are 1.57, 2 and 3, respectively. Notice that the parameter governing the disutility of employment (\( \nu = 1.57 \)) is smaller than those governing the disutilities of hours per worker and work efforts (\( \chi = 2, \varsigma = 3 \)). This implies that when firms can change the size of employment, they find it less costly to do so than to vary the hours per worker and work efforts. Hence, parameterizing the values for \( \nu \), \( \chi \) and \( \varsigma \) this way, together with the assumption that employment adjustments are predetermined, embodies the idea that employment is a high adjustment-low marginal cost margin whereas capital hours and work efforts are low adjustment-high marginal cost margins. Following the estimates of Ireland (2001), I set the elasticity of money demand to the nominal interest rate \( \gamma \) to 0.1184. Finally, the discount factor \( \beta \) is set to 0.99, so that the steady-state real interest rate is 3%.

Labor’s share of aggregate income, \( \alpha \), is set to 0.662. The remaining parameters of the production function of the intermediate goods firms are the parameters \( \sigma \) and \( \psi \), which determine the shares of and substitution between inventories and capital services. Kydland and Prescott (1982) argue that the substitution opportunities between capital and inventory are small, suggesting that \( \nu \) should be considerably larger than zero. Following their guideline, I set \( \psi = 4 \). The parameter \( \sigma \) is set to ensure that the steady-state ratio of inventories to output is about 0.33,
approximately consistent with the data. The resulting value of $\sigma$ is $0.476 \times 10^{-9}$. Along with $\psi = 4$ and the steady-state values for the factors of production given below, this value implies that in the steady state only 0.2% of output is attributable to inventories. Hence, inventories play only a small direct role in production. The demand elasticity of intermediate goods, $\varepsilon$, is set to 6 so that the steady-state markup of the intermediate-goods producing firms is 1.2. The depreciation rate $\delta$ is set to 0.018, implying that capital stock depreciates approximately at an annual rate of 7%.

The steady-state value of the fraction of hours beyond a standard 40-hour workweek is set to 0.26, taken from Ramey and Shapiro (1998). This implies a 50.4-hour workweek of capital in the steady state. Normalizing a 40-hour workweek to unity, I set the steady-state value of the workweek of capital, $h$, to be 1.26. The steady-state value of employment, $N$, is set to ensure that the steady-state ratio of total hours worked to the total time endowment of the households is 0.24. The resulting value of $N$ is 0.56. Finally, the scale coefficients $\theta_1$ and $\theta_2$ in the utility function and the remaining steady-state values for factors of production are obtained from solving the equilibrium conditions satisfied in the steady-state. The procedure for finding these values is given in Appendix B.

Using the estimates of Ireland (2001), I set the parameters of the price adjustment costs function $\phi_p$ to be 77.1. As for the capital adjustment cost parameter, it is set at $\phi_k = 4.2$, so that the peak response of investment to a monetary shock is (approximately) three times that of output in the model. In the alternative cases where I shut down inventory adjustments, I will vary $\phi_k$ to have the same relative volatility of investment.

Finally, the exogenous process for the monetary growth rate is parameterized, following Christiano et al. (2005), by setting the persistence parameter at $\rho_\mu = 0.5$ and the mean growth rate of the money stock at $\mu = 1.016$, which is equal to the steady state rate of inflation, $\pi$. The standard deviation of the monetary policy shock is set at $\sigma_\mu = 0.003$.

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7 According to Kydland and Prescott (1982), this ratio is about 0.25.
8 The procedure for obtaining this figure is given in Appendix B.
9 The 0.2% figure for inventories is the product of the marginal product of inventories and the stock of inventories, divided by output, evaluated in a non-stochastic steady-state.
10 The time endowment available to households is normalized to 2.63.
11 A different monetary policy rule such as a Taylor-type rule does not affect the results.
Table 1: Parameter Values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu$</td>
<td>Parameter governing disutility of employment</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Parameter governing disutility of hours per worker</td>
</tr>
<tr>
<td>$\varsigma$</td>
<td>Parameter governing disutility of efforts per worker</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Elasticity of money demand</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Intertemporal discount rate</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Elasticity of substitution between capital and inventory</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Weight on inventories in the production function</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Elasticity of intermediate good</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
</tr>
<tr>
<td>$h$</td>
<td>Steady-state hours per worker</td>
</tr>
<tr>
<td>$n$</td>
<td>Steady-state participation rate</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>Scale coefficient in the utility function</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>Scale coefficient in the utility function</td>
</tr>
<tr>
<td>$\phi_p$</td>
<td>Price adjustment costs</td>
</tr>
<tr>
<td>$\phi_k$</td>
<td>Capital adjustment costs</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Steady-state inflation rate</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Mean money growth rate</td>
</tr>
<tr>
<td>$\rho_\mu$</td>
<td>Persistence of money growth</td>
</tr>
<tr>
<td>$\sigma_\mu$</td>
<td>Standard deviation of money growth</td>
</tr>
</tbody>
</table>

4 Multiple Margins and Monetary Shocks

This section shows that integrating inventory adjustments and labor decisions properly induces the hump-shaped response of output to monetary shocks. To facilitate understanding of the model, I will begin by considering the case where inventory adjustments is shut down. That is, firms can only instantaneously vary the hours per worker and the work efforts in response to monetary shocks. I then introduce inventory adjustments as another option that firms can utilize to accommodate the initial change in demand. Finally, I drop the assumption that firms are unable to make current-quarter employment adjustment in order to highlight that inventory adjustments alone cannot generate the hump-shaped response of output.
4.1 When inventory adjustments are absent

Figure (1) displays the response of key macroeconomic variables to a positive monetary shock when inventory adjustments are not incorporated\(^{12}\). This is the case where the parameter determining the share of inventories in production, \(\sigma\), is set to 0. Since the model does not introduce a habit formation, it cannot generate the hump-shaped response in consumption. That is, the largest change in aggregate demand occurs in the first period, when employment cannot adjust. Given that output is demand-determined in this case, hours per worker and work efforts need to increase substantially in the first period to allow firms to meet the higher demand due to an expansionary monetary shock. In the subsequent period, firms substitute away from hours per worker and work efforts toward employment, since the model is parameterized in such a way that employment is a high adjustment-low marginal cost margin relative to hours per worker and work efforts. Due to the decline in demand in the second period, however, the increase in employment in the second period is not large enough compared to the rise in hours per worker and work efforts in the first period. Hence, the friction in adjusting employment in itself cannot generate the hump-shaped response in effective labor input and thus output to a monetary shock. This result stands in sharp contrast to the productivity shock case. Burnside and Eichenbaum (1996) shows that one-period lag in employment adjustments can lead to a hump-shaped response of output to productivity shocks.

4.2 When inventory adjustments are introduced

The previous exercise reveals that it is crucial to include certain features in order to generate a hump-shaped response of output to monetary shocks. The model has to either incorporate a mechanism that could lead to a hump-shaped response of consumption to a monetary shock or be modified in such a way that output is not demand-determined. The conventional way to achieve the hump-shaped response of consumption is through introducing habit formation to the utility function. Dynan (2000) and Flavin and Nakagawa (2004) show that very little evidence of habit persistence is found at the micro-level data. Instead of developing a new utility function that can replace habit formation, I explore a second avenue by incorporating inventory adjustments.

Inventories can insulate production from swings in demands. Without inventory adjust-

\(^{12}\)The capital adjustment coefficient \(\phi_k\) is changed to 10, so that the peak response of investment to a monetary shock is (approximately) three times that of output.
ments, as shown in the previous exercise, firms have to vary their hours per worker and work efforts significantly in the first period. In contrast, when inventories can be used to serve some fraction of the increased demand$^{13}$, firms rely less on hours per worker and work efforts, so that output does not need to be adjusted that much in the first period. Thus, incorporation of inventories opens up the possibility that the initial responses of hours per worker and work efforts are quite small relative to the response of employment, which could produce the hump-shaped response of effective labor input and output. This pattern is consistent with the empirical findings of Trigari (2004). He estimates the response of a set of labor market variables to a monetary shock and finds that the response of employment to monetary shock is significantly larger and more persistent than that of hours per worker.

Figure (2) confirms this possibility. It portrays the response of key macroeconomic variables to a positive monetary shock when inventory adjustments are incorporated. Compared to the previous exercise, the initial responses of hours per worker and work efforts decline substantially due to the introduction of inventory adjustments and the employment adjustments in the subsequent period thus becomes relatively large. This in turn generates a one-period lag in the adjustment of labor input, and the peak response of output occurs in the second period rather than in the first period.

One may think that the hump-shaped response of output in this model is not remarkable. It is due to the fact that the ratio of a compensated employment supply elasticity to a compensated hours-per-worker supply elasticity is probably set too low, so that the response of employment is not that significantly larger than that of hours per worker. Given $\nu = 1.57$ and $\chi = 2$, the implied ratio of the employment elasticity to hours-per-worker elasticity is 1.27$^{14}$. However, Dotsey and King (2006) provide a different guidance on how to calibrate this ratio. They calibrate their model so that the ratio takes a substantially larger value than 1.27. They set it at 2.33. Given $\chi = 2$, this implies that $\nu$ should be set to 0.85 in order for the ratio of employment elasticity to hours-per-worker elasticity to be 2.33. Therefore, once the model is calibrated following Dotsey and King (2006), the response of employment becomes much larger than that of hours per worker. As a result, the model generates a more noticeable hump-shaped response of output to monetary

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$^{13}$In this model, the cost of decreasing current inventories to serve some of the increase in demand is the decrease in inventories available for next period production.

$^{14}$The compensated employment and hours-per-worker supply elasticities in this model is $1/\nu$ and $1/\chi$. 

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shocks. Figure (3) shows this.

Other than the response of output, the model cannot yield hump-shaped responses of aggregate demand (i.e., consumption and investment) to monetary shocks. This is not a drawback of the model. Rather, it is deliberately intended. The results here highlight the fact that contrary to conventional wisdom, even the model which does not include sluggish adjustment in aggregate demand can produce the hump-shaped response of output to monetary shocks.

How robust is the result to varying parameter values? Given the important role of inventories, one might think that the result could be sensitive to different values of $\sigma$ and $\psi$, which determine the shares of and substitution between capital services and inventory. To check this, I consider a variety of values for $\sigma$ and $\psi$ while maintaining the assumption that the ratio of inventories to output is about 0.3. As Figure (4) clearly shows, the result is insensitive to changing the values of $\sigma$ and $\psi$. The responses of output to a positive monetary shock are virtually identical with various different values\textsuperscript{15}. A parameter that turns out to important is the magnitude of price stickiness. I use the estimate of Ireland (2001) for the coefficient of the price adjustment cost, $\phi_p = 77.1$. As the real effects of monetary shock die out more quickly, however, the response of employment in the second period relative to the initial response of hours per worker and work efforts might get smaller. In other words, firms will not hire new workers that much in the second period when they see their demands decline more quickly. Hence, the less sticky the price is, the less likely is it that the model will produce the hump-shaped response of output to monetary shocks\textsuperscript{16}. Figure (5) displays the response of output to a positive monetary shock with different values for the price adjustment coefficient, $\phi_p$. Over a wide range of $\phi_p$, the peak response of output occurs in the second period. Once $\phi_p$ takes the values below 32.5, however, the model is unable to generate the hump-shaped response of output.

\textsuperscript{15}Following Kydland and Prescott (1982), I restrict the values of $\psi$ to be no less than two. It should also be noted that once $\psi$ takes values greater than 4.1, the model becomes unstable, so I did not consider the case where $\psi > 4$.

\textsuperscript{16}As mentioned, `Alvarez-Lois (2006) shows that the model with ‘putty-clay’ technology and idiosyncratic demand uncertainty can generate the hump-shaped response in output to monetary shock. He also uses the same value of price adjustment coefficient as this paper, $\phi_p = 77.1$. However, he does not conduct sensitivity analysis whether his result is robust to varying the values of the price adjustment coefficient. His result also seems to depend upon how sticky the price is: The extent to which firms that are capacity-constrained in the first period increase their employment capacity in the second period will decline as the price gets less sticky.
4.3 When employment adjustments are not predetermined

Finally, I investigate the case where firms can adjust the size of employment freely and instantaneously in response to monetary shocks. This permits one to determine whether introducing inventories without assuming a one-period lag in employment adjustments is sufficient for generating the hump-shaped response of output to monetary shocks. Figure (6) portrays the response of key macroeconomic variables to a positive monetary shock when employment can be freely and instantaneously adjusted. It clearly shows that introducing inventory adjustments in itself cannot produce the hump-shaped response of effective labor input and output. Thus, the exercise here illustrates that inventories themselves are not the source of the delayed effect of monetary shocks on output. Rather, it shows that they play a prominent role in enabling the friction involved in adjustment of employment to generate the hump-shaped response of output to monetary shocks.

5 Conclusion

Firms can accommodate changes in demand due to monetary shocks by utilizing a variety of margins. They can vary inventories, hours per worker, work efforts and employments. These margins differ in their variable costs and adjustments costs. However, the conventional New-Keynesian models often simplify these multiple margins that firms can use in response to monetary shocks. They either lump labor decisions or exclude inventory adjustments.

To fill this gap, this paper examines the consequences that stem from including these multiple margins in an otherwise conventional New-Keynesian model. The striking result here is that the interplay between inventory adjustments and the one-period lag in adjusting employment can produce the hump-shaped response of output to a positive monetary shock, without the need to assume habit formation.

When inventory adjustments are not introduced, labor input varies mostly at the intensive margins of labor (i.e., hours per worker and work efforts), while changes at the extensive margin are relatively small. Thus, even though the model assumes a one-period lag in adjusting employment, it cannot produce the hump-shaped response of labor input. In contrast, when coupled with inventories adjustment, the friction in adjusting employments produces the hump-shaped response of labor input. Since firms can serve some of the initial increase in demand through inventories,
the presence of inventories counteracts the initial responses of hours per worker and work efforts. This in turn makes employment adjustments in the subsequent period relatively large. As a result, the peak response of labor input occurs in the second period and the model can generate the hump-shaped response of output without relying on habit formation.
References


Figure 1: Dynamic Response to a Positive Monetary Shock Without Inventory Adjustments
Figure 2: Dynamic Response to a Positive Monetary Shock With Inventory Adjustments
Figure 3: Changing the Ratio of Employment Elasticity to Hours-Per-Worker Elasticity

\[ \nu = 1.57 \text{ and } \chi = 2 \]

\[ \nu = 0.85 \text{ and } \chi = 2 \]
Figure 4: Sensitivity Analysis: Different Values of $\psi$ and $\sigma$

\[
\begin{align*}
\psi &= 4, \quad \sigma = 0.476 \times 10^{-9}, \quad X/Y = 0.33 \\
\psi &= 2, \quad \sigma = 0.28 \times 10^{-5}, \quad X/Y = 0.35 \\
\psi &= 2.8, \quad \sigma = 0.28 \times 10^{-7}, \quad X/Y = 0.26 \\
\psi &= 3.5, \quad \sigma = 0.28 \times 10^{-8}, \quad X/Y = 0.30
\end{align*}
\]

$X/Y$ denotes the steady-state ratio of inventories to output.

Figure 5: Sensitivity Analysis: Different Magnitudes of Price Stickiness
Figure 6: Dynamic Response to a Positive Monetary Shock With Employment Not Predetermined
Appendices

A Characterizing Equilibrium

Let $\tau$ denote the Lagrange multiplier associated with the constraint (10). Note also that the optimality conditions for a representative intermediate goods firm are obtained after I substitute the final goods firm’s demand (11), the production function (12) and investment (14) into an intermediate goods firm’s profit maximization problem. The symmetric equilibrium is characterized by the following system of equations:

- **Aggregate production**
  \[ Y_t = A_t \left[ (1 - \sigma) (h_t e_t K_{t-1})^{-\psi} + \sigma X_{t-1}^{-\psi} \right]^{- (1 - \alpha)/\psi} \left[ h_t e_t N_{t-1} \right]^\alpha \] (20)

- **Marginal utility of consumption**
  \[ \frac{C_t^{\frac{1}{\gamma}}}{\left[ C_t^{\frac{\gamma-1}{\gamma}} + m_t^{\frac{\gamma-1}{\gamma}} \right]} = \lambda_t \] (21)

- **Money demand**
  \[ m_t = \left( 1 - \frac{1}{R_t} \right)^{-\gamma} C_t \] (22)

- **Bond holding**
  \[ \lambda_t = \beta R_t E_t \lambda_{t+1}^{\gamma}/\pi_{t+1} \] (23)

- **Investment decision**
\[
\lambda_t \left\{ 1 + \phi_k \left( \frac{K_t}{K_{t-1}} - 1 \right) \right\} = \beta E_t \left[ \lambda_{t+1} \left\{ (1 - \delta) + \phi_k \left( \frac{K_{t+1}}{K_t} - 1 \right) \frac{K_{t+1}}{K_t} - \frac{\phi_k}{2} \left( \frac{K_{t+1}}{K_t} - 1 \right)^2 \right\} \right] + \beta E_t \left[ \tau_{t+1} (1 - \alpha)(1 - \sigma) \frac{Y_{t+1}}{K_t} \right] \]

+ \beta E_t \left[ \tau_{t+1} (1 - \alpha)(1 - \sigma) \frac{Y_{t+1}}{K_t} \right] \frac{(h_{t+1}e_{t+1}K_t)^{-\psi}}{(1 - \sigma)(h_{t+1}e_{t+1}K_t)^{-\psi} + \sigma X_{t-1}^{\psi}} \]

(24)

- Employment decision

\[
E_t \left[ \lambda_{t+1} \left( \theta_1 N_t^\nu + \theta_2 \frac{h_{t+1}^{1+\chi}}{1 + \chi} + h_{t+1} \frac{e_{t+1}^{1+\varsigma}}{1 + \varsigma} \right) \right] = E_t \left[ \tau_{t+1} \alpha \frac{Y_{t+1}}{N_t} \right] \frac{\partial Y_{t+1}}{\partial N_t} \]

(25)

- Hours per worker decision

\[
\lambda_t \left[ \theta_2 N_{t-1} h_t^\chi + N_{t-1} \frac{e_{t-1}^{1+\varsigma}}{1 + \varsigma} \right] = \tau_t \frac{Y_t}{h_t} \left( 1 - \alpha \right)(1 - \sigma) \frac{(h_te_tK_{t-1})^{-\psi}}{(1 - \sigma)(h_te_tK_{t-1})^{-\psi} + \sigma X_{t-1}^{\psi} + \alpha} \]

(26)

- Work efforts decision

\[
\lambda_t \left[ \theta_2 N_{t-1} h_t^\chi + N_{t-1} \frac{e_{t-1}^{1+\varsigma}}{1 + \varsigma} \right] = \tau_t \frac{Y_t}{h_t} \left( 1 - \alpha \right)(1 - \sigma) \frac{(h_te_tK_{t-1})^{-\psi}}{(1 - \sigma)(h_te_tK_{t-1})^{-\psi} + \sigma X_{t-1}^{\psi} + \alpha} \]

(27)

- Inventory adjustment decision

\[
\tau_t = \beta E_t \left[ \tau_{t+1} \left( 1 - \alpha \right) \frac{Y_{t+1}}{K_t} \frac{X_t^{\psi}}{(1 - \sigma)(h_{t+1}e_{t+1}K_t)^{-\psi} + \sigma X_{t-1}^{\psi} + 1} \right] \frac{\partial Y_{t+1}}{\partial X_t} \]

(28)

- Price adjustment decision

\[
\lambda_t \left\{ (1 - \epsilon)S_t - \phi_p \left( \frac{\pi_t}{\omega} - 1 \right) \frac{\pi_t S_t}{\omega} \right\} + \epsilon \tau_t S_t + \beta E_t \left[ \lambda_{t+1} \left[ \phi_p \left( \frac{\pi_{t+1}}{\omega} - 1 \right) \frac{\pi_{t+1} S_{t+1}}{\omega} \right] \right] = 0 \]

(29)
• Capital accumulation

\[ K_t = (1 - \delta)K_{t-1} + I_t \]  

(30)

• Equilibrium in the final goods market

\[ C_t + I_t + \frac{\phi_k}{2} \left( \frac{K_t}{K_{t-1}} - 1 \right)^2 K_{t-1} + \frac{\phi_p}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 Y_t = Y_t - (X_t - X_{t-1}) \]  

(31)

• Money growth

\[ \mu_t = \frac{m_t}{m_{t-1}} \pi_t \]  

(32)

• Money supply shock

\[ \mu_t = (1 - \rho)\mu + \rho \mu_{t-1} + \varepsilon_{\mu t} \]  

(33)

B Calculating the Steady-State Values

1. Given the values of \( \pi \) and \( \beta \), the steady-state level of the nominal interest rate, \( R \), can be obtained using equation (23).

\[ R = \frac{\pi}{\beta} \]  

(34)

2. Given the value of \( \epsilon \), \( \frac{\lambda}{\tau} \) can be derived from equation (29).

\[ \frac{\lambda}{\tau} = \left( \frac{\varepsilon}{\varepsilon - 1} \right) \]  

(35)

3. Given the assigned values of \( N \), \( h \), \( \alpha \), \( \psi \), \( X/Y \) and \( \frac{\lambda}{\tau} = \frac{\varepsilon}{\varepsilon - 1} \), the steady state values of \( e \), \( Y \), \( K \) and \( X \), and the value of \( \sigma \) can be obtained by solving equations (36)-(39).

\[ \frac{1}{\beta} \frac{\lambda}{\tau} = (1 - \delta)\frac{\lambda}{\tau} + (1 - \alpha)(1 - \sigma) \left( \frac{Y}{K} \right) \left( \frac{(hek)^{-\psi}}{(1 - \sigma)(hek)^{-\psi} + \sigma X^{-\psi}} \right) \]  

(36)

\[ \frac{1}{\beta} = (1 - \alpha)\sigma \left( \frac{Y}{X} \right) \left( \frac{X^{-\psi}}{(1 - \sigma)(hek)^{-\psi} + \sigma X^{-\psi}} \right) + 1 \]  

(37)
\[ Y = A \left[ (1 - \sigma)(heK)^{-\psi} + \sigma X^{-\psi} \right]^{-\frac{(1 - \alpha)\psi}{\psi} (heN)\alpha} \]  
\[ \lambda Ne^\varsigma = \tau \left( \frac{Y}{e} \right) \left[ (1 - \alpha)(1 - \sigma) \frac{(hek)^{-\psi}}{(1 - \sigma)(heK)^{-\psi} + \sigma X^{-\psi} + \alpha} \right] \]  

Equations (36), (37), (38) and (39) stem from equations (24), (28), (20) and (27), respectively.

4. Once the values for \( e, Y, K \) and \( X \) are given, \( \theta_2 \) and \( \theta_1 \) can be obtained by using equation (26) and (25), respectively.

\[ \theta_2 = \frac{1}{Nh^\chi} \left[ \frac{\tau}{\chi} \left( \frac{Y}{h} \right) \left( 1 - \alpha \right)(1 - \sigma) \frac{(hek)^{-\psi}}{(1 - \sigma)(heK)^{-\psi} + \sigma X^{-\psi} + \alpha} \right] - N e^{\frac{1+\varsigma}{1+\varsigma}} \]  
\[ \theta_1 = \frac{1}{N^\nu} \left( \frac{\tau}{\chi} \alpha \frac{Y}{N} - \theta_2 \frac{h^{1+\chi}}{1+\chi} \frac{e^{1+\varsigma}}{1+\varsigma} \right) \]  

5. Given the values for \( \delta \) and \( K \), one can obtain \( I \) using equation (30).

\[ I = \delta K \]  

6. Given the values for \( \delta, K \) and \( Y, C \) can be obtained by using equation (31).

\[ C = Y - \delta K \]  

7. Given the values for \( R, \gamma \) and \( C \), one can obtain \( m \) using equation (22).

\[ m = \left( 1 - \frac{1}{R} \right)^{-\gamma} C \]  

8. Given the values for \( \gamma, C \) and \( m \), \( \lambda \) can be obtained by using equation (21).

\[ \lambda = \frac{C^{-\frac{1}{\gamma}}}{\left[ C^{-\frac{1}{\gamma}} + m^{-\frac{1}{\gamma}} \right]} \]