THE COUNTER-CYCLICAL MOVEMENT OF THE EXPECTED EQUITYPREMIUM AND CAPITAL ADJUSTMENT COSTS

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ABSTRACT

Much of financial economics literature provides the evidence of counter-cyclical movement of the expected equity premium. The *weak* correlation between real aggregate quantities and the equity premium, however, have led the debates over the forecastability of asset returns. Using the empirical methodology of permanent and transitory decomposition, I observe the strong negative correlation between the equity premium and the transitory component of the growth rates of GDP, consumption, and investment in the U.S. quarterly data. This paper analyzes whether real business cycles (RBC) models help to understand these empirical observations. With the assumption that a transitory shock originates from the capital accumulation technology, it suggests that moderate capital adjustment costs model reproduce the observed counter-cyclical dynamics of the expected equity premium as well as the comovement of the real aggregate quantities.

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I. INTRODUCTION

This paper studies the role of capital adjustment costs to understand the business cycle characteristics of the expected equity premium.

The negative correlation of the expected equity premium with business conditions is recognized as well-known evidence in much of financial economics literature (e.g. Fama and French (1989) and Chen (1991)). The counter-cyclical property, however, is not robust when we add the strongly positively correlated data in the 1990s. The overall business cycle implication is the weak correlation between real and financial economy rather than their negative or positive correlation. Although the weak relationship is comforting to the advocates of the *efficient market hypothesis*, there have been tremendous efforts to forecast future assets returns. This paper participates in these efforts by decomposing the real aggregate quantities into permanent and transitory components and looking at their correlation with the expected equity premium.

By the empirical methodology of the permanent and transitory decomposition, I observe the negative relationship of the expected equity premium with the transitory variations in real aggregate quantities and its positive correlation with the permanent ones. The vector autoregression (VAR) of real and financial variables with the long-run restriction (e.g. Blanchard and Quah (1989), King, Plosser, Stock and Watson (1991), and Rotemberg and Woodford (1996)) shows that less than twenty percent variations in the expected equity premium are accounted for by a single permanent shock over all horizon. These findings suggest that there exist important sources of transitory variations in the economy that generate the equity premium to move in the reverse direction with the transitory component of the growth rates of GDP, consumption, and investment.

Based on the observations, this paper addresses the two questions. First, what are transitory shocks? Second, in what model do the transitory shocks generate the observed counter-cyclical dynamics of the expected equity premium? This paper attempts to answer to the questions by taking real business cycles (RBC) models into account. Previous papers by Jermann (1994), Rouwenhorst (1995), Boldrin, Christiano and Fisher (1995), Lettau and Uhlig (1995), and Tallarini (1996) have tried to explain the *equity premium puzzle* in the RBC models. In contrast, this paper puts an emphasis on correlation of the expected equity premium with real aggregate quantities in order to examine the business cycle relationship.

A crucial assumption in this paper is the existence of a transitory shock on capital accumulation technology. The premise of the shock of this kind is based on the possibility that purely financial events cause the short-run variations in the real side of economy (e.g. Bernanke (1983) and Fisher and Merton (1984)). In the neo-classical stochastic growth model, this shock may refer to be *financial* because it represents varying conditions in investment opportunities. It may be also interpreted as a news shock for the ground that the shock is known but not realized at the time of investment decision: though news about productivity on future capital stock being available, current period investment does not change the predetermined capital stock used in current period goods production.

The shock on capital accumulation technology is an alternative representation of investment-specific technology shock proposed by Greenwood, Hurcowitz and Krusell (1997). They use the investment-specific shock to disentangle its long-run effects from the traditional Hicks-neutral form of technology progress. In this paper, I assume the shock to be transitory as an engine of temporary fluctuations rather than growth.

The findings of model simulations suggest that capital adjustment costs (e.g. Lucas and Prescott (1971)) mainly account for the empirical observations. They generate not only the counter-cyclical movement of the equity premium but also the comovement of real aggregate quantities in response to the transitory shock. The ability of the model to generate this dynamics is the result of variable labor supply. With no restrictions on labor, people tend to adjust labor supply instead of consumption under the existence of high capital adjustment costs. Then intertemporal substitution effect triggers persistent growth of real aggregate variables. The subsequently higher interest rate, together with less high return on equities due to the capital adjustment costs, will lower the equity premium. The dynamics is consistent with the explanation of Fama and French (1989), when one refers the source of limited supply of capital-investment opportunities to capital adjustment costs.

One of the important issues in this paper is the choice of computation methods. The projection methods studied by Judd (1992) are used to solve the model. It is found that log-linearization is quite precise in calculating quantity variables and the first moment of asset returns, but it may mislead the variations in the expected equity premium. Thus, one should be cautious to apply log-linear approximation for measuring the time-varying equity premium.

Chapter II reports the empirical observations. Financial factor models are used to form the time series of the expected equity premium. Then, I scrutinize the business cycle characteristics of the equity premium in terms of various statistical measures including vector autoregression (VAR) models with the long-run restriction. Chapter III provides detailed arguments about a shock on capital accumulation function. After the shock is defined, I explain the motivation of the RBC models with capital adjustment costs and habit persistence in consumption. Chapter IV describes stationary equilibrium of the models. In Chapter V, I argue that projection methods are one of the best approximation methods in the context of this paper. The projection methods and model solution algorithm are elucidated afterwards. In Chapter VI, I choose parameter values and report the numerical results of the model. Then, I evaluate the accuracy of the projection methods and compare them with log-linear approximation. Concluding remarks are provided in Chapter VII.

II. EMPIRICAL OBSERVATIONS

A number of financial economists have paid considerable attention to the joint cyclical characteristics of expected asset returns and real aggregate quantities. One of the most prominent observations in the financial markets is that expected equity premium varies inversely to current business conditions. For example, Fama and French (1989) report that the variation in expected asset returns, including bonds and stocks, is negatively correlated with business cycles, where it is stronger for stocks than for bonds. Chen (1991) finds that the market excess returns are negatively correlated with recent economic growth.

The objective of this chapter is to reinterpret this relationship with various permanent and transitory decomposition methods by addressing two following questions. First, do we find stronger relationship between decomposed real aggregate quantities and the equity premium? Second, how much of cyclical variations in those variables are attributable to permanent and transitory shocks? To the first question, the answer is yes. The transitory components of the GDP, consumption, and investment growth are strongly negatively correlated with the equity premium and their permanent components are positively correlated. To the second question, the permanent shock explains much of the forecast errors in real aggregate quantities, but only small fraction in the equity premium. The transitory shock plays an important role in generating the counter-cyclical movement of the equity premium.

2.1. THE DATA AND BASIC STATISTICS

The data are quarterly U.S. observations on real aggregate income account flow variables and on aggregate asset markets. The aggregate real flow variables are the logarithm of per capita real gross domestic product (y), per capita real consumption expenditure (c), and per capita real gross domestic fixed investment (i). The financial variables are log-transformed *ex post* equity premium (ep) measured by the difference between equity return (r^{VW}) and short term Treasury bill return (r^{TB}).[1]

The expected equity premium is forecast by financial factor models. Among others, I choose term premium (*TERM*) and dividend price ratio (*D/P*) as factors.[2] Many researchers find that default premium (*DEF*) also forecasts asset returns. I do not use default premium as factors here, because it has strong positive correlation with dividend price ratio. In fact, both of the factors capture similar variations in asset returns: long-term business conditions. Further, there is much evidence that dividend price ratio is the single best factor in forecasting asset returns (e.g. Fama and French (1988) and

[1] This paper employs CITIBASE data for GDP (GDPQ), consumption (GCNQ+GCSQ), and investment (GIFQ). These series are divided by population (POP) to get per capita quantities. The equity return and risk-free asset return are NYSE value-weighted returns (VWRETD) and three month T-Bill returns (FYGN3). They are divided by the inflation rates from Consumer Price Index (PUNEW) to get real returns and then log-transformed. The data are drawn quarterly from 1960:1 to 1996:4.

[2] This paper uses the factors suggested by Fama and French (1989). Chen, Roll and Ross (1986) use term premium, default premium, industrial production, expected and unexpected inflation rates to explain the cross-section of average returns on NYSE stocks. Fama and French (1993) identify three factors for individual stocks and two factors for individual bonds. The stock market factors are an overall market factor, factors related to firm size, and book-to-market equity. The bond market factors are term and default premium.

Cochrane (1994)). In contrast, term premium is known to forecast different components of asset returns: short-term business cycles.

Table 1 summarizes basic statistics of real aggregate quantity and financial variables. Table 2 provides the results of financial factor models to forecast expected equity premium. Two facts are worthwhile to note. First, although these factor models are not successful in forecasting future equity premium (only seven percent of the variations of the equity premium is traced by term premium and dividend price ratio), the factors do price equity premium significantly (p-value is less than 0.5 percent). Second, consistent with previous findings, the slope coefficients of the factors are positive. This explains the characteristics of the expected equity premium over business conditions. The dividend yield is high when the economy is in recession due to the low price of equities. Then, the positive regression coefficient of the dividend yield means that the expected equity premium is high when business conditions are persistently weak and low when the conditions are strong. The term premium is known to relate short-term business conditions to financial markets, since the long-term bond yields do not vary as much as the short-term bond yields. The term premium is high when the business conditions is temporarily weak with the optimistic expectation on the future economy. At that time, the equity premium is high.

2.2. CROSS CORRELATION

As it reveals the business cycle characteristics, the cross correlation of the equity premium with the real aggregate variables attracts keen interests. Table 3 shows various cross correlation of the expected equity premium with real aggregate GDP, consumption and investment growth. Notice, in Panel A of Table 3, that the correlation of the real aggregate quantities is weakly negative for all the leads of the expected equity premium, whereas it is quite strongly positive for its lags. The contemporaneous correlation is slightly positive. In the previous literature by Fama and French (1989) and Chen (1991), the contemporaneous correlation was weak in the time periods from the early 1950s to the mid 1980s. The difference comes from the different sample periods: this paper subtracts the data in the 1950s and adds them in the 1990s. The main fact this paper focuses on is that the correlation is negligible.

Before scrutinizing the analysis of vector autoregression, it is helpful to examine the Hodrick-Prescott (HP) filtered time series. Since the HP filter extracts a complicated nonlinear trend from each time series data, it breaks the trend stationary time series into growth and cyclical components. Panel B of Table 3 shows the cross-correlation between cyclical components of real variables from the HP filter and the expected equity premium. It makes us confirm the temporary counter-cyclical movement of the equity premium.

Table 4 reports the same statistics for the *ex post* equity premium. The overall results are similar to the *ex ante* case, but the contemporaneous variables are more or less strongly negatively correlated in Panel A of Table 4. For the lagged *ex post* equity premium, the growth rate of GDP and investment is positive with the peak at the two quarter lags. This means that current equity premium is six-month ahead leading indicator of the economy. The *ex post* equity premium is also negatively correlated with the cyclical components of the real aggregate quantities from the HP filter.

2.3. VECTOR AUTOREGRESSION WITH THE LONG-RUN RESTRICTION

To look at the interaction between real and financial economy, I consider vector autoregression (VAR) model of the variables from both sectors. Based on the premises that (i) there are multiple orthogonal shocks in the economy, and (ii) the shocks may be either transitory or permanent, the VAR answers to the following question: how and how much do the permanent and transitory shocks affect the cyclical variations in financial variables along with those in real aggregate ones?

There have been extensive studies about the nature of the permanent shock in the context of real business cycles theory, which is usually identified as the permanent technology shock affecting the balanced growth path. The transitory shocks in the VAR, here, are particularly interesting for the reason that business cycle fluctuations of financial variables seem to be subject to the temporary disturbances. Thus, the identification of transitory disturbances by the VAR of this kind will provide underpinnings on which one theorizes a model with shocks other than technology shocks.

I study the two-variable and four-variable VAR's with the long-run restriction: the permanent shock affect the level of real aggregate variables in the long-run, but not that of equity premium, and the transitory shocks do not affect either the level of real aggregate variables nor that of the equity premium. In a formal expression, the bivariate regression is constructed as:

(2.1.1) bivariate regression : $\begin{bmatrix} \Delta y_t \\ f_t \end{bmatrix} = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} + A(L)\begin{bmatrix} \Delta y_{t-1} \\ f_{t-1} \end{bmatrix} + u_t$,

(2.1.2) long-run restriction : $u_t = \Gamma_0 \varepsilon_t$, $\left[(I - A(1))^{-1}\Gamma_0\right]_{1,2} = 0$, and

$$Var(\varepsilon_t) = \Sigma_\varepsilon ,$$

where Γ_0 is a (2×2) long-run restriction matrix, ε_t is a vector of the permanent and transitory disturbances, $(\varepsilon_t^P, \varepsilon_t^T)'$, and Σ_ε is a diagonal matrix. f_t stands for the *ex post* or expected equity premium. The long-run restriction of (2.1.2) is the same as that of Blanchard and Quah (1989). Similarly, four-variable VAR is constructed as:

(2.2.1) four-variable VAR : $\begin{bmatrix} \Delta y_t \\ c_t - y_t \\ i_t - y_t \\ f_t \end{bmatrix} = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} + A(L)\begin{bmatrix} \Delta y_{t-1} \\ c_{t-1} - y_{t-1} \\ i_{t-1} - y_{t-1} \\ f_{t-1} \end{bmatrix} + u_t$,

(2.2.2) long-run restriction : $u_t = \Gamma_0 \varepsilon_t$, $\left[(I - A(1))^{-1}\Gamma_0\right]_{1,j} = 0$ for $j \neq 1$, and

$$Var(\varepsilon_t) = \Sigma_\varepsilon .$$

Note that cointegrating relation among aggregate variables are embedded in (2.2.1). The objective of four-variable VAR is to see the effects of the permanent shock on GDP, consumption, investment, and the equity premium. The four-variable VAR cannot identify transitory shocks with the long-run restriction unless further restrictions are assumed for each transitory shock.[3] Therefore, the bivariate VAR is run to exactly identify both permanent and transitory shock series.

Panel C's of Table 3 and Table 4 present the cross correlation of the expected and *ex post* equity premia with aggregate quantities from each bivariate VAR. The main results are as follows: first and most importantly, for every transitory component of real aggregate growth, both the *ex post* and expected equity premia have negative correlation. Second, the correlation is very strong in the *ex post* equity premium (over 0.6 for permanent components of the real variables and over -0.6 for their transitory components). Third, the permanent components of real aggregate variables are weakly positively correlated to the expected equity premium. Figure 1 displays the estimated impulse response functions of GDP growth and expected equity premium bivariate regressions. They illustrate the above explanation graphically.

Forecast-error variance decomposition of the four-variable VAR in Table 5 implies that only small fraction of variations in the equity premium is attributable to the permanent shock: about thirty three percent movement of the *ex post* equity premium and about less than twenty percent variations in the expected one. That is, transitory shocks account for more substantial portion of equity premium fluctuations.

Finally, it is worthwhile to mention the warning against the long-run restriction. As Faust and Leeper (1996) point out, the VAR with the long-run restriction might be slippery, since we calculate the permanent component at a spectral density zero. Their argument is highlighted in Table 6 and Table 7. In the four-variable VAR of the expected and *ex post* equity premium, the correlation is much weaker. Further, the results can be different when we run regressions of different lag periods. The empirical results about the correlation of the equity premium with the transitory components of the real aggregate quantities, however, have two valid implications: first, the permanent technology shock is not enough to account for the variations in the financial markets. Second, they provide a reference to modeling transitory shocks for financial economy in the business cycle theory.

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[3] The long-run restriction of (2.2.2) is the same as that of Rotemberg and Woodford (1996). King, Plosser, Stock, and Watson (1991) use the similar restriction on vector error correction model (VECM) of difference stationary variables. The VAR of (2.2.1) is run for the reason that equity premium is known to be stationary, but asset returns are not. The dynamic multiplier associated with the shock is used to calculate forecast-error variance decomposition.

III. OVERVIEW OF THE MODEL

The first-order Euler equation from which the asset pricing formulae are derived consists of three parts: (i) conditional expectations that depend on the underlying uncertainties, (ii) stochastic discount factors that reflect preference, and (iii) return functions that summarize production technology. The objective of this chapter is to propose the specifications to each component that may possibly account for the observed counter-cyclical movement of the equity premium as well as the comovement of GDP, consumption and investment growth rates.

3.1. SHOCK ON CAPITAL ACCUMULATION

The vector autoregressions of the real aggregate quantity variables and the equity premium in the previous section take into account the interaction between real and financial economy. The permanent shock decomposed by the long-run restriction is usually understood as a factor capturing permanent productivity change (King, Plosser, Stock and Watson (1991) among others). The transitory shocks are not exactly identified by the long-run restriction alone.[4]

This paper attempts to set the transitory shock to be one on the level of capital accumulation. This assumption is motivated by the attention that purely financial events may drive short-run variations in real aggregate quantities. Although the causation of financial economy to real economy has not drawn extensive consensus, many financial economists mention its possibility. For example, Bernanke (1983) argues that, in the period of Great Depression, financial crises may have affected output on top of the excessive contraction of money supply. Fisher and Merton (1984) illustrate a case in which exogenous events that primarily affect stock prices also have influence on investment.

The identification of transitory shocks of this kind is tractable to deal with financial disturbances under the neo-classical stochastic growth model. In a formal way, a transitory shock, $\{\varepsilon_t^A\}$, is defined in a capital accumulation function as:

[4] Blanchard and Quah (1989) call them demand-disturbances in GNP growth and unemployment regressions, but there is no firm ground that it stems from the demand side. Further, if we consider three and more variable VAR, the transitory shocks are multiple. In this case, long-run restriction alone does not provide sufficient criteria for identifying shocks.

(3.1) $$K_{t+1} = f(K_t, I_t; \varepsilon_t^A),$$

where K_t is a predetermined capital stock (in period t-1), I_t gross investment in period t. f(.,.) represents capital accumulation technology, where f(.,.) is increasing in both capital and investment and homogeneous of degree one. This is a generalized form of the investment-specific technology shock studied by Greenwood, Hercowitz, and Krusell (1997).[5] In this paper, $\{\varepsilon_t^A\}$ is not confined to investment. The assumption is that not only new investment but also preexisting capital stock is subject to uncertainties.

This shock, $\{\varepsilon_t^A\}$, may be interpreted as a news shock. To see this, we need to look at the timing of realization of the shocks, production, investment and capital accumulation. At the beginning of each period, $\{\varepsilon_t^A\}$ and $\{\varepsilon_t^\gamma\}$ (specified in section 4) are observed and then single consumption or investment goods are produced by employing predetermined capital stock and labor. After production, people either consume or invest goods. At the end of period, the next period capital is produced with depreciated current period capital stock and new investment. Note that capital stock is predetermined and that the transitory shock is known at the beginning of the period and realized at the end of period. Thus, $\{\varepsilon_t^A\}$ does not directly affect current period output. It rather affects goods-production by way of changing labor supply and then reallocating consumption and investment. Yet realized $\{\varepsilon_t^A\}$ plays a role as a news shock in the stage of production, consumption and investment. As a matter of fact, many disturbances in the financial sectors seem to be news ones. Equity and bond prices fluctuate at the arrival of new information about the quality, efficiency, or utilization of future capital stock due to the change of future corporate tax rate, financial transaction costs, monetary policies on interest rates, etc. Consumption and labor decisions also reflect the news simultaneously.

I assume further that the news shock is transitory with persistence. Namely,

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[5] Christiano and Fisher (1995) develop the investment-specific technology shock in a slightly different context. They assume capital accumulation under no uncertainty, but separate out an investment goods sector and embed an investment-specific technology shock in that sector. This paper considers $\{\varepsilon_t^A\}$ directly in capital accumulation function in the flavor of financial disturbances.

(3.2.1) $$K_{t+1} = A_t g(K_t, I_t)$$

(3.2.2) $$A_{t+1} = A_t^{\rho} \exp(\sigma_A \varepsilon_{t+1}^A), \qquad 0 < \rho < 1,$$

where $\{\varepsilon_t^A\}$ are normally distributed random variables with mean zero and unit variance. Then, the formula (3.2.1) has the following interpretation: A_t is the ratio of the future capital stock to the capital stock under no uncertainty.[6] The movement of this ratio is assumed to be stationary.

3.2. CAPITAL ADJUSTMENT COSTS

Given the transitory shock in the previous subsection, I adopt adjustment costs in the capital evolution function as in Lucas and Prescott (1971). The role of capital adjustment costs is three-dimensional. They generate the counter-cyclical movement of the equity premium and make the real aggregate quantities comove in response to the assumed transitory shock. They also help to jack up the equity premium to some degree. Therefore, it is capital adjustment costs that mainly account for the empirical observations on the equity premium over business cycles.

The mechanical reason for capital adjustment costs model to be attractive is as follows. In order for the equity premium to be counter-cyclical, equity returns must either decrease more or increase less than the risk-free rate in a booming period. Consumption growth rate is high so that the risk-free rate or interest rate gets higher. Thus, equity returns increase necessarily less than the risk-free rate or decrease. The latter is not the case in a boom. Hence, the only possible combination of equity returns and the risk-free rate is that equity returns do not vary as much as the interest rate. This

[6] Christiano and Fisher (1995) assume that permanent shifts in the technology for producing consumption goods are embodied in capital. Then, the investment-specific technology shock is posited to be a random walk with a positive drift. The economy-wide shock, however, is transitory with the notion that only disturbances affecting the consumption goods-production are shocks to weather, natural disasters, or labor disputes. The description of shocks in this paper may seem to be reverse to theirs, but it turns out to be just different. The news shock is not an investment-specific technology shock. More generally, it is not a shock in capital embodiment. What it implies is the uncertainty about future capital formation for whatever reasons. This paper hopes to match it with financial disturbances by comparing

is a necessary condition for a model to be consistent with the data. A capital adjustment costs model is a good candidate, because their existence keeps the return on equities from increasing excessively in response to favorable investment opportunities.

As seen in the previous subsection, a salient feature of $\{\varepsilon_t^A\}$ is a news shock. News shocks in general do not produce the comovement of aggregate quantity variables. For example, the positive news about forming capital stock triggers current investment at the cost of foregone consumption, because we expect to enjoy more future consumption by doing so. This implies the inverse movement of consumption and investment, which is not consistent to the empirical findings.

One of the ways for the real aggregate quantities to comove vis-à-vis news shocks is imposing adjustment costs in capital production technology. The existence of convex costs of new investment necessarily makes consumption more favorable in response to the positive shock, since it is not beneficial from the intertemporal perspective to excessively exploit good investment opportunities. Thus, people increase both consumption and investment by rather working more.

Another advantage of capital adjustment costs lies in the financial market implications. As Boldrin, Christiano and Fisher (1995) point out, the condition for generating high equity premium is the technology restriction that frustrates the desire of the asset demand when marginal utility of consumption is low and the desire of the asset supply when marginal utility of consumption is high. Capital adjustment costs inherit less elastic supply of capital stock so that foregone consumption would not be invested unless high returns are anticipated.

The parametric form of capital adjustment costs used in this paper is as follows:

$$(3.3) \qquad g(K_t, I_t) = \eta[(1-\theta)\{(1-\delta)K_t\}^{-\xi} + \theta I_t^{-\xi}]^{-1/\xi}, \qquad -1 < \xi \neq 0 .$$

The choice of this functional form is rather arbitrary. Obviously, this is an example of the capital adjustment costs by Lucas and Prescott (1971). One of the nice features of this CES function is that the curvature of capital adjustment costs can be expressed as a

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the model with the empirical observations. The validity of the argument requires further research on the nature of financial disturbances.

single parameter, or ξ: the marginal rate of substitution between capital and investment (ε_{KI}) is $1/(1+\xi)$. Another advantage of the function is that, since θ is a share parameter of investment relative to depreciated capital stock, consumption share and marginal rate of substitution between capital and investment sufficiently specify the model.[7] Therefore, it is handy to make a quantitative analysis.

3.3. HABIT PERSISTENCE IN CONSUMPTION

Even though capital adjustment costs carry the main dynamics in this paper, their asset market implications are limited due to the low level of equity returns and premium, especially in variable labor models. Recently, extensive literature pays a special attention to habit formation in consumption as a source of high equity premium under the low relative risk aversion (e.g. Campbell and Cochrane (1997), Boldrin, et al. (1995)). In the most related work with this paper, Jermann (1994) reports the performance of capital adjustment costs in the general equilibrium asset pricing model, but the equity premium is not satisfactorily large with capital adjustment costs alone. He is able to find high equity premium by imposing habit formation in consumption as in Constantinides (1990).

The habit persistence deserves additional attention from the business cycle perspective (e.g. Chen (1991)). In the model, past consumption plays a similar role as subsistence level. If the current consumption is low relative to past consumption, relative risk aversion is so high that people ask for high risk premium to forego consumption and to make investment. Then, consumption growth and the equity premium may be negatively correlated. In the general equilibrium asset pricing models, it is a testable hypothesis whether the habit persistence in consumption explains the counter-cyclical dynamics of the expected equity premium.

The parametric form of habit persistence used in this paper is as follows:

$$U(C_t + \zeta C_{t-1}, 1 - H_t) = \log(C_t + \zeta C_{t-1}) + \phi H_t, \text{ for } \zeta < 0 \text{ and } \phi < 0. \tag{3.4}$$

[7] For the detailed argument about parameters in the CES function, see Section 1 in Appendix.

Indivisible labor is assumed as in Hansen (1985) and only one-period lagged consumption constitutes habit formation of consumption. The choice of this parametrization is solely computational purpose. This is the simplest form to implement numerical solution methods without loss of the characteristics of habit persistence models. For matching the data better, Campbell and Cochrane (1997) use the state-of-the-art habit formation that asks for more state variables.

Here is a remark for using a habit persistence model in production economy. Boldrin, et al. (1995) report no equity premium in production economy unlike in endowment economy. This is true for extreme parameter values of ζ like -0.8. Depending on other parameters, high degree of habit may produce even negative equity premium. It is consistent with consumption volatility anomaly proposed by Lettau and Uhlig (1995). In a production economy with strong habit persistence in consumption, flexible labor responds to shocks more than consumption. With no restrictions on labor market participation, consumption would vary too little to resolve the *equity premium puzzle*. Jermann (1994) depends on the fixed labor to generate relatively high equity premium. Boldrin, et al. (1995) make a partial success to get high equity premium, assuming limited intersectoral mobility of factors of production. Therefore, it is very important to take labor market conditions into account when one builds an asset pricing models with habit persistence of consumption in a production economy. Assuming flexible labor, this paper sets the low value of the habit persistence parameter, $\zeta = -0.1$, so as to achieve large equity premium.

IV. MODEL AND EQUILIBRIUM

Based on the overview of the model, we consider the standard RBC model by King, Plosser and Rebelo (1988 b) as a baseline framework. The model provides two different shocks, but of one kind: technology shocks. In this paper I keep the labor productivity shock as a permanent source of technology change in production, but substitute the shock on capital accumulation for the total factor productivity shock as an engine for transitory variations. Capital adjustment costs and habit persistence are modeled so as to be compatible with the existence of stationary steady state.

4.1. ECONOMIC ENVIRONMENT

Preferences

The identical individual households have two roles in this economy: consumers of commodities and suppliers of labor. The representative individuals are assumed to be infinitely lived and have preferences over goods and leisure by

$$(4.1) \qquad U = E_0[\sum_{t=0}^{\infty}\beta^t u(C_t, L_t)], \qquad L_t = 1 - H_t, \qquad 0 < \beta < 1,$$

where C_t is consumption, L_t is leisure, H_t is labor hour supplied in period t and the individual's total amount of time is normalized to be one. We restrict preferences so that intertemporal elasticity of substitution in consumption is invariant to the scale of consumption and that marginal utility of leisure is constant in the efficient steady state. These are necessary conditions, in the preference side, for the feasible steady state dictated by the production technology to be an optimal outcome.

Production technology

For each individual, one final good is the result of constant-returns-to-scale production technology given by

$$(4.2) \qquad Y_t = F(K_t, Z_t H_t)$$

where K_t is the predetermined (in t-1) capital stock of period t. $F(\cdot)$ is assumed to be a concave, increasing and twice continuously differentiable function of the capital stock and labor effort. Labor productivity changes, Z_t, captures permanent technology variations. Then, the steady state growth path is interpreted as the situation where the level of certain key variables grows at constant rates. For the computational convenience I fix production technology to be constant-returns-to-scale Cobb-Douglas function and assume the labor productivity shock to follow random walk property:

$$(4.2.1) \qquad F(K_t, Z_t H_t) = K_t^{1-\alpha}(Z_t H_t)^{\alpha},$$

(4.2.2) $\gamma^{Z}_{t+1} = \gamma^{Z} \exp(\sigma_{\gamma} \varepsilon^{\gamma}_{t+1})$,

where $\gamma^{z}_{t} \equiv Z_t / Z_{t-1}$. $\{\varepsilon^{\gamma}_{t}\}$ are normally distributed random variables with mean zero and unit variance. The evolution of capital is described as in (3.2.1)-(3.2.2) and (3.3).

Individual resource constraint

The individuals are constrained in the choice of consumption and leisure by the total endowment of goods and time:

(4.3) $C_t + I_t \leq Y_t$, $C_t > 0$, $0 < H_t < 1$.

4.2. OPTIMAL STATIONARY EQUILIBRIUM

The restrictions for the existence of a steady state growth path of the system have been imposed in Section 4.1. It is convenient to transform the economy into a stationary one where preferences and technology are expressed in terms of variables that will be constant in the steady state. In the steady state equilibrium, output, consumption, investment, and wage grow at the same rate of Z and marginal utility of wealth at Z^{-1}. Thus, the variables are transformed in the following manner:

$$y_t = \frac{Y_t}{Z_t}, \qquad i_t = \frac{I_t}{Z_t}, \qquad c_t = \frac{C_t}{Z_t}, \qquad w_t = \frac{W_t}{Z_t}, \qquad \lambda_t = \Lambda_t Z_t \quad ,$$

where Λ_t is marginal utility of wealth of untransformed economy, W_t is real wage on labor. Since capital is predetermined, capital used for the current production grows at the previous labor productivity ($k_t = K_t / Z_{t-1}$). Then, the optimal stationary equilibrium conditions for the transformed economy are described as $\{\lambda_t, c_t, i_t, H_t, r_t, w_t, k_{t+1}\}^{\infty}_{t=0}$, for $\{k_t, A_t, \gamma^{z}_{t}\}^{\infty}_{t=0}$ given, that satisfies:

(4.4.1) $$u_C(c_t, L_t) = \lambda_t,$$

(4.4.2) $$u_L(c_t, L_t) = \lambda_t w_t,$$

(4.4.3) $$r_t = F_K(k_t / \gamma_t^Z, H_t),$$

(4.4.4) $$w_t = F_L(k_t / \gamma_t^Z, H_t),$$

(4.4.5) $$c_t + i_t = F(k_t / \gamma_t^Z, H_t),$$

(4.4.6) $$k_{t+1} = A_t g(k_t / \gamma_t^Z, i_t),$$

(4.4.7) $$\lambda_t = \beta E_t \left[\frac{\lambda_{t+1}}{\gamma_{t+1}^Z} \left\{ r_{t+1} + \frac{g_K}{g_I}\left(\frac{k_{t+1}}{\gamma_{t+1}^Z}, i_{t+1}\right) \right\} A_t g_I\left(\frac{k_t}{\gamma_t^Z}, i_t\right) \right], \text{ and}$$

(4.4.8) $$\lim_{t \to \infty} \beta^t \lambda_t k_{t+1} = 0,$$

where $L_t = 1 - H_t$.

The expected equity return and risk-free rate are defined as:

(4.5.1) $$E_t[R_{t+1}^e] = E_t \left[\left\{ r_{t+1} + \frac{g_K}{g_I}\left(\frac{k_{t+1}}{\gamma_{t+1}^Z}, i_{t+1}\right) \right\} A_t\, g_I\left(\frac{k_t}{\gamma_t^Z}, i_t\right) \right], \text{ and}$$

(4.5.2) $$R_{t+1}^f = E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t \gamma_{t+1}^Z} \right]^{-1}.$$

To make the data and model comparable, I log-transform the returns as:

(4.6.1) $$r_{t+1}^e \equiv \log(R_{t+1}^e), \text{ and}$$

(4.6.2) $$r^f_{t+1} \equiv \log(R^f_{t+1}) .$$

The expected equity premium is defined as:

(4.7.1) $$E_t[ep_{t+1}] \equiv E_t[r^e_{t+1}] - r^f_{t+1} .$$

V. NUMERICAL APPROXIMATION METHODS

The purpose of this chapter is to discuss numerical approximation methods suitable for asset pricing models.

It is known that even a simple stochastic growth model with non-zero depreciation of capital stock does not have a closed-form solution. A host of macro-finance and RBC literature relies heavily on log-linear (LL) approximation and linear-quadratic (LQ) approximation methods to illustrate the quantitative performance of models. There are some grounds, however, that both LL and LQ approximation methods are not adequate for explaining the movement of the expected equity premium over the business cycles. First, they do not measure the expected equity premium properly. The LL approximation of first-order equilibrium conditions does not yield non-zero equity premium by eliminating second and higher order terms. The LQ approximation captures the time-varying equity premium, not because the risk premium in risky asset prices is time-varying, but because asset returns are nonlinear functions of state variables. Second, they are local approximation methods around the steady state. Since the aggregate quantity variables are smooth functions of state variables, the errors are not large enough for both methods to be invalid. In contrast, the errors of asset returns and premium are not negligible, especially far away from the steady state. Third, linearizing the capital evolution function oversimplifies the dynamics of the capital adjustment costs model. The significant roles of capital adjustment costs in the cyclical movement of the equity premium and aggregate quantity variables call for keeping their nonlinearity intact.

Over the last decade, there have been substantial developments in numerical solution methods for nonlinear rational-expectations models along with innovations in computer technology.[8] Section 5.1 discusses projection methods as one of the appropriate approximation methods in the context of this paper. In Section 5.2, I display the agenda on their application for the comparison of the model with the empirical observations.

5.1. PROJECTION METHODS

Now that they are problem-specific, numerical methods should be deliberately chosen in line with particular problems of interest. The criteria of selecting appropriate methods[9] for this paper are as follows; first, it is important to be accurate with respect to first-order conditions, for asset-pricing formulae are directly derived from it. Second, we need global approximation methods for the purpose of simulating the models with empirically observed data. Third, it is desirable to start with a small number of initial guesses for the sake of computation.

Projection methods studied by Judd (1992) satisfy these criteria. In short, the methods approximate consumption policy function with polynomials in the Euler equation. They are accurate with respect to the first-order conditions. The number of the initial guesses is the number of the state variables times the order of the polynomials of each state variable. In comparison with discretization methods,[10] projection methods need relatively fewer initial guesses. They are global approximation methods in the sense that the range of the state variables covers from minimum to maximum.[11] In the

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[8] Taylor and Uhlig (1990) summarize a variety of nonlinear solution methods and compare their performances. Judd (1992) introduces projection methods for solving aggregate stochastic growth models. McGrattan (1993) applies to the problems finite element methods that are widely used in engineering applications. Gaspar and Judd (1997) propose perturbation methods to handle large-scale rational-expectations models with many state variables due to heterogeneous agents, multiple assets, sectors, and shocks, etc. Christiano and Fisher (1997) suggest Chebyshev parameterized expectations algorithms for solving dynamic models with occasionally binding constraints.

[9] The general criteria of a good numerical approximation methods consist of minimal errors, minimal programming and computing time, stability, etc.

[10] The examples of discretization are value function iteration methods as in Tauchen and Hussey (1991) and Santo (1994) or finite element methods by McGrattan (1993). These methods discretize the realizations of each state variables. The number of initial guesses is the product of all realizations of each state variables. Since accuracy needs finer grids, we must guess exorbitantly many values. In practice, it may be difficult without knowing the overall shape of the functional values we want to approximate.

[11] This paper considers an aggregate model of the small number of state variables so that global approximation methods are tractable. In the case of large-scale models like multi-industries, multi-agents, multi-uncertainties, or their combined models, global approximation methods are neither tractable nor accurate. Then, local approximation methods like perturbation methods studied by Gaspar and Judd (1997) are one of the alternative choices.

context of this paper they have another advantage by approximating policy function directly. To see this, I briefly explain the projection methods. More formal description is presented in Appendix D.

The solution of a stochastic growth model is a consumption policy function. Once we identify the functional form of consumption, we can analytically express other variables using equilibrium conditions of Chapter IV. At first, suppose that consumption is given by a function of state variables, x_t. The residual function of Euler equation is defined as:

$$\Re(x_t, x_{t+1}; \vec{a}) \equiv u_C(x_t; \vec{a}) - E_t[u_C(x_t, x_{t+1}; \vec{a}) R^e(x_t, x_{t+1}; \vec{a})], \tag{5.1}$$

where $\vec{a}$ is a vector of coefficients associated with the state variables, and $u_C(.)$and $R^e(.)$ stand for the marginal utility of consumption and investment return, respectively.

Representing the residual function numerically in (5.1), we should determine the approximate functional form and the method of numerical integration. One of the simplest functional forms is the polynomial. Weierstrass Theorem says that for any continuous function there exists a polynomial that converges to it. However, the ordinary polynomials, $\{1, x, x^2, x^3, \cdots\}$, are by no means the best.[12] This paper applies Chebyshev polynomials for an approximate consumption policy function. The next choice is a specific numerical integration over the expectation. Gauss-Hermite quadrature weights and abscissas are used for normally distributed shocks.

A projection method adjusts $\vec{a}$ in (5.1) until it finds a “good” $\vec{a}$ which makes the residual function “nearly” the zero function. The final question is what is criterion for the closeness to zero. Among many projections,[13] the Galerkin method is to average out

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[12] Judd (1992) points out that the ordinary polynomials are monotonically increasing and positive on $\Re^+$. They are not necessarily orthogonal in any natural inner product on $\Re^+$. Furthermore, since they vary in size, they involve scaling problem. Christiano and Fisher (1997) illustrate a textbook example in which Chebyshev polynomials are better choice than ordinary polynomials.

[13] According to the criterion of closeness, we classify the projection methods into several branches. The least squares method minimizes the L^2 norm of the residual function zero over the given time horizon. The subdomain method averages the residual function to be zero over the time horizon. Obviously, this is a less direct way than the least squares method to find a good-fitting approximation. The collocation method, in somewhat different sense, proceed by choosing $\vec{a}$ so that the residual function is zero at a particular set of points. The Galerkin method is one of collocation method, using Chebyshev collocation points.

the error in the residual function. More specifically, for each value of the state variables dictated by Chebyshev polynomials, or Chebyshev collocation points, one constructs a projection of the residual function and solves a set of nonlinear simultaneous equations. Note that the number of projections is the same as the dimension of $\vec{a}$. Then, the solution of the Galerkin method is expressed as:

$$\text{(5.2)} \qquad c_t \cong \vec{a}' x_t ,$$

where x_t is a vector of Chebyshev collocation points of state variables. Now we can express the functional form of output, investment, asset returns, etc., as nonlinear functions of state variables. These explicit expressions are the advantage of projection methods to simulate the model with empirically observed data series.

5.2. ALGORITHM

In this subsection I set up the algorithm of the entire project to find the simulated predictions comparable to the empirical observations in Chapter II. In particular, I calculate the time-series of model variables generated by the decomposed shock series and then compare their statistics with those of the actual data. The algorithm is as follows:

(1) to calculate deterministic steady state capital stock and consumption,

(2) to set initial guess for $\vec{a}$ from log-linear approximation,

(3) to solve the Euler equation by the projection methods,

(4) to generate simulated data by feeding the shock series, and

(5) to calculate impulse response functions and statistics of the models.

By the shock series in (4), I mean the shock series from the bivariate vector autoregressions of the real aggregate quantity growth and *ex post* equity premium (e.g. $X_t = [\Delta y_t, ep_{t+1}]'$) with the long-run restriction. Since the decomposed shocks are constructed to follow standard normal distribution, the randomly generated artificial shock series may be used.[14]

Accuracy of the projection methods depends on the choice of an initial guess and the range of state variables. An initial guess for the coefficient of consumption policy function plays a primal role in accuracy because of multiplicity of local solutions. Since it offers a good solution in a small neighborhood of the steady state, log-linearization is a useful reference to start with. Actually, the solution of the spectral methods with the initial guess from log-linearization is not very different from the initial guess itself in a simple model. Precision of the projection methods also relies on the range of state variables. As the range gets larger, a solution is required to be global. The range is set here large enough to encompass the variability of the permanent and transitory shocks and their span of other state variables like capital and lagged consumption. Again, if the range gets smaller, log-linearization provides a similar solution as the projection methods. We will discuss the comparison between the two approximation methods in Chapter VI.

VI. NUMERICAL RESULTS

6.1. PARAMETERS

Table 8 summarizes the parameter values chosen in the paper. Time-discount rate (0.9872) is set to equate the return on capital in each model with the average annual equity returns of 7.44 percent in the sample period. The steady state growth rate of trend ($\gamma^Z = 1.0049$) is the measured as the mean of GDP growth. The leisure preference parameter ($\phi = -2.65$) is associated with one third of time devoted to labor. The value of labor share ($\alpha = 0.62$) is a time series average of ratio of compensation of employee to output. The capital stock is depreciated after production ($\delta = 0.025$). The transitory shock persistence ($\rho = 0.6$) is chosen to achieve high correlation between real aggregate

[14] Table 2 displays the histograms of the permanent and transitory shocks from the bivariate regression of GDP growth and the *ex post* equity premium. The graphs suggest that the shock series do not follow normal distribution. The distribution has left-fat and right-thin tails. That means that more bad realizations of the shocks occur in the actual data.

quantities and the two- or three-quarter lead of expected equity premium. The standard deviation of transitory and permanent shocks is assigned for experimental purposes ($\sigma_A = 0.01$ and $\sigma_\gamma = 0.01$). The parameter values for capital adjustment costs and habit persistence are free to choose. This paper considers moderate ($\varepsilon_{KI} = 1.5$) and high capital adjustment costs ($\varepsilon_{KI} = 0.9$) because the models of low capital adjustment costs do not generate the comovement of real aggregate quantity variables. As discussed in Chapter III, the low value of the habit persistence ($\zeta = 0.1$) is used to achieve high equity premium under the flexible labor.

6.2. NUMERICAL RESULTS

The main result of the simulations is that a moderate capital adjustment costs model ($\varepsilon_{KI} = 1.5$) with the shock on capital accumulation is consistent with the empirical observation that the expected equity premium moves inversely to consumption, output, and investment growth. It is explained for the following dynamics. In response to a favorable transitory shock about future capital formation, people want to invest more to exploit the favorable investment opportunities by increasing labor hours worked and reducing consumption. The existence of capital adjustment costs, however, effectively prevents excessive investment. At some investment level, they are not better off investing more at the expense of current consumption. If the adjustment costs are sufficiently high, or they do not find proper assets that will compensate enough for current foregone consumption, they would rather work more to make investment without reducing consumption. At the end of the period, the realization of the shock and new investment will increase the amount of capital stock that triggers more production in the future, and in turn higher consumption and investment. The persistent effect of the transitory shock makes the responses of output, consumption and investment hump-shaped. Thus, the risk free rate becomes high vis-à-vis increasing consumption growth rate, while equity returns are necessarily high due to the shock. If the consumption growth is so dramatic to dominate the increase of the equity returns, then expected equity premium will be lower.

Figure 3 illustrates the above verbal explanation. The simulated impulse response functions (percentage deviation from the steady state) due to unit positive transitory shock in Model 1 ($\varepsilon_{KI} = 1.5$) are reported. But note that the measured variation of the equity premium is extremely low in this model. It is one ten thousandth of the actual variation. Panel A of Table 9 shows that the level of the expected equity premium in the capital adjustment costs model are too low to even mention resolving the *equity premium puzzle*. Figure 4 displays the estimated impulse response functions of the bivariate regressions of output growth rate and the expected equity premium in the model, using permanent and transitory shock series from the actual data. We observe the qualitatively similar pattern of impulse responses to both permanent and transitory shocks. The cross correlation in Panel B of Table 10 reveals that the moderate capital adjustment costs model ($\varepsilon_{KI} = 1.5$) is compatible with the counter-cyclical variations of equity premium. With the chosen persistence of transitory shock ($\rho = 0.6$) the growth rate of real aggregate quantities has peak correlation with the two- or three-quarter lead of the expected equity premium.

In the case of high capital adjustment costs ($\varepsilon_{KI} = 0.9$), the contemporary negative correlation of the equity premium with the transitory component of GDP growth rate is strong and the correlation is negative for all leads and lags (Panel A of Table 10). However, its correlation with the permanent component is weakly negative. This is because the existence of high capital adjustment costs lowers the variability of the equity returns relative to the risk-free rate. Figure 6 shows that this model qualitatively matches the impulse response functions to the transitory shock but not those to the permanent one.

To obtain high level of the expected equity premium, I simulate the models of habit persistence in consumption (Model 2). As seen in other literature like Jermann (1994) and Boldrin, et al. (1995), the level of unconditional expected equity premium is large (0.45 percent per quarter) in the habit persistence model (Panel B of Table 9). It is one third of the actual expected equity premium. This is a fairly good result for the level of the equity premium, because this paper considers log-utility function.

Adding habit persistence in consumption to the capital adjustment costs models, however, I observe the different cross correlation dynamics (Panel A and B of Table 11). The sign of simulated cross correlation of the real aggregate quantities with the expected equity premium is opposite to the numbers in Panel C of Table 3. The permanent component of the real aggregate quantities is negatively correlated with the expected equity premium and the transitory component is positively correlated. The estimated impulse response functions of the bivariate regressions of output growth rate and the expected equity premium in Model 2 also display the reverse features with the actual data (Figure 8 and 10). Due to the transitory shock, both output and the expected equity premium increase. Vis-à-vis the permanent shock output increases whereas the expected equity premium decreases. This phenomenon is accounted for by the low variability of consumption growth rate in the habit persistence model. As argued in Lettau and Uhlig (1995), consumption is very smooth under flexible labor in this model. Then the risk-free rate does not vary much relative to equity returns due to shocks, notwithstanding the low variability of equity returns under the existence of capital adjustment costs. The simulated impulse response functions (percentage deviation from the steady state) due to unit positive transitory shock show the dynamics graphically in Figure 7 and 9.

6.3. DISCUSSIONS ON NUMERICAL APPROXIMATION METHODS

Some might wonder how accurate the numerical methods are. Some might raise questions how different non-linear approximation solutions are from those of log-linearization, and prediction of which variables are severely affected by the choice of approximation methods. This section answers to the questions.

Accuracy for the projection methods is difficult to check due to the lack of analytic solutions of the models. A natural initial comparison goes through steady state values of real quantity variables. Table 12 presents the level of output, consumption, and investment in both stationary steady state and numerical solutions. In short, the projection methods bring very precise solutions in reference to the steady state values.

In business cycle literature, log-linearization is the most widely used solution methods. Danthine, Donaldson, and Lance (1987) argue that linear approximation is adequate for macroeconomic purposes. Judd (1990) documents, however, that the adequacy of the linear approximation is much less likely when we take risk premia and term structure of interest rates into account. Table 9, 13 and 14 answer to the arguments. Table 9 presents the comparison of unconditional moments from both approximation methods. The means of returns and premium are not very much different from each other, but the difference between the standard deviations of measured equity premium is quite large so that we end up with misleading conclusions. Table 13 and 14 show that log-linearization dictates high contemporaneous correlation between the real variables and equity premium, but it is spurious. The histograms of the simulated equity premium in Figure 11 account for the reason: since log-linearization is a local solution method, equity premium does not move very widely. Concentration around the steady state makes the contemporaneous correlation high. Thus, when we are interested only in the mean of the equity returns or financial premia, the log-linearization is effective without getting involved in additional time consuming calculations. On the other hand, analyzing time-varying properties of those variables necessarily asks for adopting non-linear approximation methods.

VII. CONCLUSION

I investigate the time-varying expected equity premium of a stochastic growth model to find its link to real aggregate flow variables. The empirical evidence suggests that the transitory component of the growth rate of GDP, consumption, and investment is negatively correlated with the expected equity premium and the negative correlation persists for several quarters. I explain these phenomena with capital adjustment costs model under a news shock on capital accumulation. The quantitative results show that the model reproduces the observed cross correlation between the expected equity premium and transitory component of real aggregate quantity growth rates.

The RBC model is a just starting point, for it lacks too many features in asset markets. As King, et al. (1991) argue, accelerations and decelerations in money growth and inflation may explain the variability of real flow variables. Since the early 1970s, we have observed the strong negative relation between inflation rate and asset returns. These facts lead us to take into account the inflation disturbances as omitted sources of the RBC models, which possibly accounts for the variability of expected equity premium as well as real aggregate quantity variables. Jensen, Mercer, and Johnson (1996) survey the literature about this issue and report the evidence that the behavior of the business-conditions proxies and their influence on expected security returns is significantly affected by the monetary sector.

As Fama and French (1990) point out, the measured expected asset returns and premia from the data are not fully explained by financial factors. In fact, any proxy for expected returns and expected return shocks, and other macro variables hardly have the explanatory power of variance of stock market returns over fifty percent. This is why the forecastibility of assets returns is so protracted an argument in relation to the so called *efficient market hypothesis*.

Since the seminal paper by Kydland and Prescott (1982), computational methods are one of the important issues of the RBC research. As new computational techniques are introduced and the straitjacket of computation is attenuated, we are freer to consider more complicated models in high accuracy. The projection methods used here enables one to obtain quite precise and robust solutions of asset pricing models. To the contrary, log-linearization removes too much valuable information about asset returns, for the otherwise existing second moment has very important implications. Therefore, the choice of approximation methods is one of the relevant issues to those who think highly of the precision of quantitative solutions and who are willing to devote more time and efforts to computation.

APPENDIX: Computation Algorithm

This appendix describes the algorithm to solve the models numerically. The main part of the solution methods is the application of projection methods to approximate a consumption policy function. Log-linearization is a preliminary step to set the initial guesses for the solution of projection methods. After the models are solved, simulations with the decomposed shock series or random numbers are run to evaluate them statistically. In this appendix, I present the capital adjustment costs model as an example. The modification of the habit persistence model with capital adjustment costs is straightforward with some complications.

D.1. DETERMINISTIC STEADY STATE

The capital adjustment costs model does not have a closed form solution for a deterministic steady state equilibrium due to the non-linearity of the capital accumulation function. In the deterministic steady state, the equilibrium conditions in the Section 4 are reduced to the following two equations with two unknowns, {y, k}:

(A.1) $$\frac{\gamma^{Z}}{\beta} = \eta\left[(1-\alpha)w + \frac{(1-\delta)(1-\theta)}{\theta}x^{\xi+1}\right]\left[(1-\theta)+\theta x^{-\zeta}\right]^{-\frac{1}{\xi}-1}x^{-\xi-1},$$ and

(A.2) $$S_{C}y = -\frac{\alpha}{\phi}w^{\frac{\alpha-1}{\alpha}},$$ [1]

where $w \equiv \gamma^{Z}y/k$ and $x \equiv [\gamma^{Z}(1-S_{C})y]/[(1-\delta)k]$, for given parameter values of $\{\beta,\gamma^{Z},\alpha,\theta,\delta,\phi,\eta,\xi,S_{C}\}$. Once these simultaneous equations are solved numerically, one can solve the steady state values of other variables, using equilibrium conditions. Since θ is set to be investment share to depreciated capital stock, $\theta = x$ in the deterministic steady state. The level parameter value is set to be

[1] In the habit persistence model, $S_{C}y = -\frac{\alpha}{\phi(\gamma^{Z}+\zeta)/(\gamma^{Z}+\beta\zeta)}w^{\frac{\alpha-1}{\alpha}}$.

$\eta \equiv \gamma^{Z} / [(1-\delta)\{(1-\theta)+\theta^{1-\xi}\}^{1/\xi}]$ so that capital accumulation function always holds with equality.

D.2. LOG-LINEARIZATION

The purpose of log-linearization is to find a set of proper initial guesses with which to apply projection methods in the next step. The multiple solutions of the projection methods make this step crucial for accuracy.

The initial guesses are the coefficients of the consumption policy function. Note that, since log-transformed Chebyshev polynomials are used for its approximate function, the values of the initial guesses should also be transformed correspondingly.

D.3. PROJECTION METHODS

The projection methods comprise four components: approximate function, numerical integration, projection conditions, and nonlinear solution methods. This paper uses the popular recipe of Chebyshev ploynomial for approximate policy function, Gauss-Hermite quadrature weights and abscissas for numerical integration, Gelerkin method for projection conditions, and Newton's method for nonlinear simultaneous equations.

Step One: Construction of Residual Function

Since all of the components center around the residual function of the first-order Euler equation, it is the most important step to construct the approximate residual function with state variables only, given an approximate policy function and equilibrium conditions. Then, the solution is the coefficients of the consumption policy function that make the residual function as close as zero.

I transform all of the variables by taking logarithm, or $\hat{k}_t \equiv \log(k_t)$, etc. This log-transformation is useful to compare the projection method with log-linearization. Since the current period capital stock is determined in the previous period, the consumption policy function depends on the predetermined capital stock and exogenous shocks. Namely, (A.3) $\quad \hat{c}_t = h(\hat{k}_t, \hat{A}_t, \hat{\gamma}_t^Z)$,[2]

In this paper, I approximate $h(\cdot,\cdot,\cdot)$ by

$$\text{(A.4)} \quad \bar{c}_t \cong \bar{h}(\tilde{k}, \tilde{A}, \tilde{\gamma}^Z; \bar{a}) = \sum_{i=1}^{n_k} \sum_{j=1}^{n_A} \sum_{k=1}^{n_\gamma} a_{ijk} \phi_i(\tilde{k}) \phi_j(\tilde{A}) \phi_k(\tilde{\gamma}^Z) \,,$$

where $\phi_i(\tilde{k})$ is a Chebyshev collocation point of $\tilde{k}$ in the order of i.[3] I set n_i to be two for $\hat{k}$, $\hat{A}$, and $\hat{\gamma}^Z$. Then, the approximating consumption policy function, $\hat{h}$, is linear in $\hat{k}$, $\hat{A}$, and $\hat{\gamma}^Z$.

The residual function is defined as:

$$\text{(A.5)} \quad \Re(\tilde{k}_t, \tilde{A}_t, \tilde{\gamma}_t^Z; \bar{h}) \equiv \frac{1}{\exp[\bar{c}_t]} - \beta E_t \left[\frac{\exp[\bar{r}(t+1)] + \bar{g}_K(t+1) / \bar{g}_I(t+1)}{\exp[\bar{c}_{t+1}]} \right] \bar{g}_I(t)$$

where $\bar{r}$ is an approximate function of real rental for capital and $\bar{g}_K$ and $\bar{g}_I$ are approximate first derivatives of capital accumulation function with respect to capital and investment, respectively.

Step Two: Numerical Integration

To resolve the uncertainties of the residual function, I exploit the distributional assumption of the underlying shocks in section 4.1. Since $\{\varepsilon_A\}$ and $\{\varepsilon_\gamma\}$ follow i.i.d. normal distribution with mean zero and unit variance, approximating residual function of (A.5) becomes

[2] In the habit Persistence model, $\tilde{c}_t = h(\tilde{k}_t, \tilde{c}_{t-1} \tilde{A}_t, \tilde{\gamma}_t^Z)$.

[3] Chebyshev collocation points are defined as $\phi_1(\tilde{k}) \equiv 1$ and $\phi_2(\tilde{k}) \equiv -1 + \dfrac{2(\tilde{k} - \tilde{k}_{min})}{\tilde{k}_{max} - \tilde{k}_{min}}$.

$$\text{(A.6)}\quad \Re = \frac{1}{\exp[\hat{c}_t]} - \beta\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(\tilde{k},\tilde{A},\tilde{\gamma}^Z,\varepsilon_A,\varepsilon_\gamma;\vec{a})\frac{\exp[-(\varepsilon_A^2+\varepsilon_\gamma^2)/2]}{2\pi}d\varepsilon_A d\varepsilon_\gamma\right]\hat{g}_I(t)$$

where

$$\text{(A.7)}\quad I(\tilde{k},\tilde{A},\tilde{\gamma}^Z,\varepsilon_A,\varepsilon_\gamma;\vec{a})$$

$$\equiv \frac{\exp[\hat{r}(\tilde{k},\tilde{A},\tilde{\gamma}^Z,\varepsilon_A,\varepsilon_\gamma;\vec{a})]+\hat{g}_K(\tilde{k},\tilde{A},\tilde{\gamma}^Z,\varepsilon_A,\varepsilon_\gamma;\vec{a})/\hat{g}_I(\tilde{k},\tilde{A},\tilde{\gamma}^Z,\varepsilon_A,\varepsilon_\gamma;\vec{a})}{\exp[\hat{h}(\tilde{k},\tilde{A},\tilde{\gamma}^Z,\varepsilon_A,\varepsilon_\gamma;\vec{a})](\sigma_\gamma\varepsilon_\gamma)\tilde{\gamma}^{Z^*}}$$

I use Gauss-Hermite quadrature weights and abscissas for both transitory and permanent shocks. Then, the approximation will be to approximate the integral in (A.B.4), with a finite sum,

$$\text{(A.8)}\quad \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(\tilde{k},\tilde{A},\tilde{\gamma}^Z,\varepsilon_A,\varepsilon_\gamma;\vec{a})\frac{\exp[-(\varepsilon_A^2+\varepsilon_\gamma^2)/2]}{2}d\varepsilon_A d\varepsilon_\gamma$$

$$\cong \sum_{i=1}^{Z_A}\sum_{j=1}^{Z_\gamma} I(\tilde{k},\tilde{A},\tilde{\gamma}^Z,\sqrt{2}\varepsilon_{Ai},\sqrt{2}\varepsilon_{\gamma j};\vec{a})w_i w_j\ ,$$

where $\{w_i, w_j\}$ and $\{\varepsilon_{Ai,}\ \varepsilon_{\gamma j}\}$ are Gauss-Hermite quadrature weights and abscissas. The approximating residual function is formed by

$$\text{(A.9)}\quad \hat{\Re}(\tilde{k},\tilde{A},\tilde{\gamma}^Z;\vec{a})$$

$$= \frac{1}{\hat{h}(\tilde{k},\tilde{A},\tilde{\gamma}^Z;\vec{a})} - \beta\left[\sum_{i=1}^{Z_A}\sum_{j=1}^{Z_\gamma} I(\tilde{k},\tilde{A},\tilde{\gamma}^Z,\sqrt{2}\varepsilon_{Ai},\sqrt{2}\varepsilon_{\gamma j};\vec{a})w_i w_j\right]\hat{g}_I(\tilde{k},\tilde{A},\tilde{\gamma}^Z;\vec{a})\,.$$

Step Three: Construction of Projections and Solution Method

I construct projections over the coefficients of consumption policy functions as:

$$\text{(A.10)}\quad P_{ijk}(\vec{a}) \equiv \int_{k_m}^{k_M}\int_{A_m}^{A_M}\int_{\gamma_m}^{\gamma_M} \Re(\tilde{k},\tilde{A},\tilde{\gamma}^Z;\vec{a})\phi_i(\tilde{k})\phi_j(\tilde{A})\phi_k(\tilde{\gamma}^Z)d\tilde{\gamma}^Z d\tilde{A}d\tilde{k} \qquad .$$

The Galerkin method finds $\{a_{ijk}\}$ such that $P_{ijk}(\vec{a}) = 0$ for all i, j and k. To calculate (A.10) numerically, approximating projections are set up over the cubic of $[k_m, k_M] \times [A_m, A_M] \times [\gamma^Z_m, \gamma^Z_M]$, where subscript m is minimum and M is maximum:

$$\text{(A.11)} \quad \bar{P}_{ijk}(\vec{a}) \equiv \sum_{h_k=1}^{m_k} \sum_{h_A=1}^{m_A} \sum_{h_\gamma=1}^{m_\gamma} \Re(\tilde{k}_i, \tilde{A}_j, \tilde{\gamma}^Z_k; \vec{a}) \phi_i(\tilde{k}_{h_k}) \phi_j(\tilde{A}_{h_A}) \phi_k(\tilde{\gamma}^Z_{h_\gamma}) = 0,$$

for all i, j, and k,

where $\{\tilde{k}_{h_k}, \tilde{A}_{h_A}, \tilde{\gamma}^Z_{h_\gamma}\}$ Gauss-Chebyshev quadrature points. Since we have eight equations with eight unknown coefficients, the simultaneous equations are solved by Newton's method.

D.4. SIMULATION

The projection methods yield an approximate consumption policy function. It is then straightforward to simulate models by feeding shock series. I use the estimated permanent and transitory shock series from the bivariate regression of real aggregate quantity and the *ex post* equity premium (five lags) with the long-run restriction. We can also simulate the models with random numbers from standard normal distribution.

The expected equity and risk-free returns are calculated by numerical integration. To compare the performance between projection methods and log-linearization, this paper uses the consumption policy function from log-linear approximation and applies numerical integration again. This procedure is used to distinguish the effects of log-linearization from those of numerical integration.

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TABLE 1

Sample Moments of Real Aggregate Quantity and Financial Variables: 1960:1 to 1996:4 with 148 Quarters

Variables	Description	Mean (%)	Std (%)
Δy	per capita GDP growth	0.49	0.95
Δc	per capita consumption growth	0.50	0.49
Δi	per capita investment growth	0.65	2.46
r^{VW}	NYSE value-weighted quarterly returns	1.79	8.13
r^{TB}	short-term T-Bill quarterly return	0.33	0.69
ep	ex post equity premium (= $r^{VW} - r^{TB}$)	1.47	7.98
$E[ep]$	expected equity premium (constructed in Table 2)	1.47	2.26

For the expected equity premium, this paper uses the factor models of the term premium and dividend price ratio presented in Panel A of Table 2.

TABLE 2

Financial Factor Models for Expected Equity Premium: 1960:1 to 1996:4 with 148 Quarters

A. Factors of Term Premium and Dividend Price Ratio

$ep_{t+1} = a + bTERM_t + c(D_t / P_t) + \varepsilon_t$		$R^2 = 8.1\%$	$\overline{R}^2 = 6.8\%$
	a	b	c
slope	-0.076	12.487	2.9648
Std	0.031	4.253	0.095
t-statistic	-2.46	2.93	3.28
p-value	0.015	0.004	0.001

B. Factors of Term Premium and Default Premium

$ep_{t+1} = a + bTERM_t + cDEF_t + \varepsilon_t$		$R^2 = 6.0\%$	$\overline{R}^2 = 4.7\%$
	a	b	c
slope	-0.017	9.415	18.990
Std	0.017	3.988	7.000
t-statistic	-1.03	2.36	2.71
p-value	0.307	0.020	0.007

TERM = 10 Year Government Bond Returns – Federal Funds Rate
DEF = BAA Corporate Bond Returns – AAA Corporate Bond Returns
D/P = Dividend Price Ratio of NYSE Value Weighted Portfolio
TERM and DEF are quarterly and D/P is annual.

TABLE 3

Estimated Unconditional Cross Correlation between Real Quantity Variables and the Expected Equity Premium: Actual Data from 1960:1 to 1996:4 with 148 Quarters

	$E_t[ep_{t+j+1}]$								
j	-4	-3	-2	-1	0	1	2	3	4
Panel A. Raw Data									
Δy_t	0.22	0.23	0.22	0.14	0.03	-0.08	-0.12	-0.14	-0.12
Δc_t	0.06	0.13	0.14	0.12	0.02	-0.05	-0.08	-0.17	-0.24
Δi_t	0.33	0.33	0.26	0.19	0.10	-0.04	-0.05	-0.08	-0.09
Panel B. Hodrick-Prescott Filtered Data									
y_t^c	0.13	-0.01	-0.16	-0.30	-0.40	-0.42	-0.41	-0.37	-0.33
c_t^c	0.17	0.08	-0.05	-0.18	-0.30	-0.37	-0.40	-0.43	-0.41
i_t^c	0.16	0.00	-0.15	-0.27	-0.36	-0.41	-0.40	-0.40	-0.38
Panel C. Bivariate Regressions									
Δy_t^P	0.00	0.00	0.00	0.00	0.18	0.11	0.05	0.04	-0.10
Δy_t^T	0.37	0.37	0.35	0.21	-0.17	-0.25	-0.31	-0.28	-0.25
Δc_t^P	0.00	0.00	0.00	0.00	0.07	0.04	0.09	0.04	-0.04
Δc_t^T	0.12	0.23	0.22	0.19	-0.06	-0.15	-0.27	-0.34	-0.38
Δi_t^P	0.00	0.00	0.00	0.00	0.31	0.22	0.22	0.14	0.10
Δi_t^T	0.41	0.40	0.31	0.23	-0.08	-0.18	-0.20	-0.20	-0.17

$E_t[ep_{t+j}]$: j-period ahead expected equity premium forecast at t

Δy_t : GDP growth rate

y_t^c : HP-filtered cyclical component of GDP

Δy_t^P : Permanent component of GDP growth rate from VAR with the long-run restriction

Δy_t^T : Transitory component of GDP growth rate from VAR with the long-run restriction

TABLE 4

Estimated Unconditional Cross Correlation between Real Quantity Variables and the *Ex Post* Equity Premium: Actual Data from 1960:1 to 1996:4 with 148 Quarters

	ep_{t+j+1}								
j	-4	-3	-2	-1	0	1	2	3	4
Panel A. Raw Data									
Δy_t	0.14	0.32	0.22	-0.03	-0.10	-0.17	-0.16	-0.09	-0.10
Δc_t	0.01	0.05	0.22	0.13	-0.12	-0.06	-0.04	-0.16	-0.04
Δi_t	0.21	0.37	0.22	-0.05	-0.17	-0.18	-0.12	-0.09	-0.04
Panel B. Hodrick-Prescott Filtered Data									
y_t^c	0.10	0.03	-0.15	-0.27	-0.25	-0.17	-0.09	-0.02	0.03
c_t^c	0.04	0.03	0.00	-0.12	-0.20	-0.13	-0.11	-0.11	-0.03
i_t^c	0.10	0.01	-0.16	-0.25	-0.22	-0.13	-0.05	-0.01	0.03
Panel C. Bivariate Regressions									
Δy_t^P	0.00	0.00	0.00	0.00	0.63	-0.02	-0.18	-0.05	-0.09
Δy_t^T	0.20	0.45	0.27	-0.02	-0.68	-0.19	-0.01	-0.04	-0.08
Δc_t^P	0.00	0.00	0.00	0.00	0.36	0.08	-0.06	-0.12	0.02
Δc_t^T	0.04	0.08	0.31	0.20	-0.55	-0.16	-0.04	-0.06	-0.06
Δi_t^P	0.00	0.00	0.00	0.00	0.64	0.01	-0.16	-0.05	-0.05
Δi_t^T	0.25	0.45	0.26	-0.06	-0.62	-0.19	-0.01	-0.07	-0.04

ep_{t+j} : *ex post* (t+j)-period equity premium

Δy_t : GDP growth rate

y_t^c : HP-filtered cyclical component of GDP

Δy_t^P : Permanent component of GDP growth rate from VAR with the long-run restriction

Δy_t^T : Transitory component of GDP growth rate from VAR with the long-run restriction

TABLE 5

Forecast-Error Variance Decomposition: Four-Variable VAR with Five Lags

Fraction of the Forecast-Error Variance Attributed to Permanent Shock				
Panel A. VAR of the Expected Equity Premium				
quarters	*GDP*	*Consumption*	*investment*	$E[ep]$
1	48.0	69.1	45.1	13.0
4	70.4	75.3	63.0	5.9
8	83.9	85.0	78.3	8.7
12	89.9	89.4	82.1	15.0
16	91.1	91.4	83.6	16.1
20	92.4	92.6	84.3	16.1
Panel A. VAR of the *Ex Post* Equity Premium				
Quarters	*GDP*	*Consumption*	*investment*	*ep*
1	23.2	42.7	31.6	33.9
4	71.0	77.7	70.1	32.7
8	90.2	91.4	87.8	33.7
12	93.8	93.9	85.8	33.5
16	95.4	95.4	87.6	33.5
20	96.4	96.3	89.4	33.5

TABLE 6

Estimated Unconditional Cross Correlation between Real Quantity Variables and the Expected Equity Premium from the Four-Variable VAR with the Long-Run Restriction: Actual Data from 1960:1 to 1996:4 with 148 Quarters

	$E_t[ep_{t+j+1}]$								
j	-4	-3	-2	-1	0	1	2	3	4
Panel A. Four-Variable VAR with Five Lags									
Δy_t^P	0.00	0.00	0.00	0.00	-0.01	-0.11	-0.07	0.02	-0.03
Δc_t^P	0.00	0.00	0.00	0.00	-0.04	-0.14	-0.05	-0.02	-0.02
Δi_t^P	0.00	0.00	0.00	0.00	-0.02	-0.16	-0.02	-0.09	0.00
Δy_t^T	0.05	-0.07	-0.05	-0.01	0.00	0.10	0.07	-0.02	0.03
Δc_t^T	0.03	-0.10	-0.03	0.03	0.04	0.14	0.05	0.01	0.01
Δi_t^T	0.01	-0.05	-0.03	0.01	0.01	0.14	0.01	0.08	-0.01
Panel B. Four-Variable VAR with Eight Lags									
Δy_t^P	0.00	0.00	0.00	0.00	0.14	-0.01	-0.08	-0.15	-0.17
Δc_t^P	0.00	0.00	0.00	0.00	0.18	-0.05	-0.09	-0.12	-0.10
Δi_t^P	0.00	0.00	0.00	0.00	-0.13	-0.14	-0.03	-0.13	-0.14
Δy_t^T	-0.09	-0.13	-0.09	0.02	-0.15	0.01	0.08	0.15	0.17
Δc_t^T	0.03	-0.05	-0.08	-0.11	-0.19	0.05	0.09	0.12	0.06
Δi_t^T	-0.16	-0.23	-0.10	-0.00	0.13	0.13	0.03	0.13	0.13

Δy_t^P : Permanent component of GDP growth rate from VAR with the long-run restriction

Δy_t^T : Transitory component of GDP growth rate from VAR with the long-run restriction

TABLE 7

Estimated Unconditional Cross Correlation between Real Quantity Variables and the *Ex Post* Equity Premium from the Four-Variable VAR with the Long-Run Restriction: Actual Data from 1960:1 to 1996:4 with 148 Quarters

	ep_{t+j+1}								
j	-4	-3	-2	-1	0	1	2	3	4
Panel A. Four-Variable VAR with Five Lags									
Δy_t^P	0.00	0.00	0.00	0.00	0.27	-0.02	-0.09	-0.03	-0.09
Δc_t^P	0.00	0.00	0.00	0.00	0.21	-0.12	-0.12	-0.03	-0.02
Δi_t^P	0.00	0.00	0.00	0.00	0.06	-0.01	0.06	-0.16	-0.10
Δy_t^T	-0.01	-0.05	0.02	0.05	-0.28	0.02	0.09	0.03	0.08
Δc_t^T	0.10	-0.06	-0.04	-0.07	-0.22	0.12	0.12	0.02	0.01
Δi_t^T	-0.06	-0.18	0.12	0.08	-0.08	-0.01	-0.07	0.15	0.09
Panel B. Four-Variable VAR with Eight Lags									
Δy_t^P	0.00	0.00	0.00	0.00	0.04	-0.02	-0.05	-0.07	-0.12
Δc_t^P	0.00	0.00	0.00	0.00	0.02	-0.03	-0.05	-0.09	-0.16
Δi_t^P	0.00	0.00	0.00	0.00	0.07	0.05	-0.00	-0.09	-0.16
Δy_t^T	0.02	0.03	-0.03	0.01	-0.04	0.01	0.05	0.07	0.12
Δc_t^T	0.07	0.06	0.05	0.03	-0.02	0.03	0.05	0.09	0.16
Δi_t^T	-0.03	-0.01	0.02	-0.03	-0.06	-0.06	-0.00	-0.08	0.15

Δy_t^P : Permanent component of GDP growth rate from VAR with the long-run restriction

Δy_t^T : Transitory component of GDP growth rate from VAR with the long-run restriction

TABLE 8

Parameter Values used for Model Simulations

	Description	Model 1	Model 2
β	Time Discount Rate	0.9872	0.9872
ϕ	Leisure Preference	-2.65	-2.65
ζ	Degree of Habit Persistence	N/A	-0.1
α	Labor Share	0.62	0.62
δ	Depreciation Rate	0.025	0.025
S_C	Consumption Share	0.7	0.7
ε_{KI}	Elasticity of Substitution (K and I)	free	free
ρ	Persistence of Transitory Shock	0.6	0.6
γ^Z	Growth Rate of Trend	1.0049	1.0049
σ_A	Std of Transitory Shock	0.01	0.01
σ_γ	Std of Permanent Shock	0.01	0.01

Model 1: Capital Adjustment Costs
Model 2: Capital Adjustment Costs and Habit Persistence in Consumption

TABLE 9

Simulated Unconditional Mean and Standard Deviation of the Expected Asset Returns and Equity Premium of the Models by Projection Methods and Log-Linearization

A. Model 1				
	Projection Methods		Log-Linearization	
	$\varepsilon_{KI} = 0.9$	$\varepsilon_{KI} = 1.5$	$\varepsilon_{KI} = 0.9$	$\varepsilon_{KI} = 1.5$
$E[r^e_{t+1}]$	1.8225 (0.4436)	1.8215 (0.4941)	1.8226 (0.4437)	1.8214 (0.4949)
$E[r^f_{t+1}]$	1.8206 (0.4436)	1.8202 (0.4941)	1.8206 (0.4436)	1.8203 (0.4939)
$E[ep_{t+1}]$	0.0019 (0.00003)	0.0013 (0.00026)	0.0020 (0.00021)	0.0011 (0.00193)
B. Model 2				
	Projection Methods		Log-Linearization	
	$\varepsilon_{KI} = 0.9$ $\zeta = -0.1$	$\varepsilon_{KI} = 1.5$ $\zeta = -0.1$	$\varepsilon_{KI} = 0.9$ $\zeta = -0.1$	$\varepsilon_{KI} = 1.5$ $\zeta = -0.1$
$E[r^e_{t+1}]$	1.8332 (0.4560)	1.8199 (0.5055)	1.8226 (0.4551)	1.8218 (0.5064)
$E[r^f_{t+1}]$	1.3849 (0.4541)	1.3847 (0.5048)	1.3848 (0.4523)	1.3844 (0.5028)
$E[ep_{t+1}]$	0.4484 (0.01909)	0.4353 (0.01311)	0.4378 (0.00481)	0.4374 (0.00497)

The returns are quarterly in percent. The numbers in the parenthesis are standard deviation in percent.
Model 1: Capital Adjustment Costs
Model 2: Capital Adjustment Costs and Habit Persistence in Consumption

TABLE 10

Estimated Unconditional Cross Correlation between Real Quantity Variables and the Expected Equity Premium: Simulated Data from the Bivariate Regressions in the Model of Capital Adjustment Costs (Model 1)

	$E_t[ep_{t+j+1}]$								
j	-4	-3	-2	-1	0	1	2	3	4
Panel A. Model 1 ($\varepsilon_{KI} = 0.9$)									
Δy_t	-0.03	-0.03	-0.07	-0.20	-0.29	-0.26	-0.16	-0.20	-0.09
Δy_t^P	0.00	0.00	0.00	0.00	-0.09	-0.10	-0.01	-0.09	-0.03
Δy_t^T	-0.08	-0.11	-0.16	-0.42	-0.35	-0.29	-0.27	-0.19	-0.10
Δc_t	0.07	0.02	0.03	-0.03	-0.19	-0.22	-0.14	-0.16	-0.10
Δc_t^P	0.00	0.00	0.00	0.00	0.06	-0.06	-0.04	-0.06	-0.04
Δc_t^T	0.09	-0.01	0.03	-0.07	-0.42	-0.27	-0.18	-0.18	-0.12
Δi_t	-0.13	-0.14	-0.17	-0.28	-0.33	-0.22	-0.19	-0.17	-0.05
Δi_t^P	0.00	0.00	0.00	0.00	-0.15	-0.04	-0.04	-0.08	-0.02
Δi_t^T	-0.23	-0.25	-0.30	-0.48	-0.27	-0.27	-0.22	-0.13	-0.05
Panel B. Model 1 ($\varepsilon_{KI} = 1.5$)									
Δy_t	0.06	0.05	0.06	0.08	0.02	-0.11	-0.12	-0.08	-0.09
Δy_t^P	0.00	0.00	0.00	0.00	0.12	0.03	0.03	0.07	0.04
Δy_t^T	0.10	0.09	0.11	0.15	-0.11	-0.20	-0.21	-0.20	-0.17
Δc_t	0.13	0.11	0.13	0.10	-0.02	-0.09	-0.13	-0.14	-0.13
Δc_t^P	0.00	0.00	0.00	0.00	0.08	0.09	0.05	0.07	0.02
Δc_t^T	0.20	0.17	0.20	0.17	-0.09	-0.18	-0.21	-0.23	-0.19
Δi_t	0.01	-0.01	0.00	0.03	0.07	-0.02	-0.04	-0.01	0.03
Δi_t^P	0.00	0.00	0.00	0.00	0.13	0.05	0.03	0.05	0.08
Δi_t^T	0.02	-0.03	-0.01	0.06	-0.10	-0.13	-0.13	-0.12	-0.10

$E_t[ep_{t+j}]$: j-period ahead expected equity premium forecast at t

Δy_t : GDP growth rate

Δy_t^P : Permanent component of GDP growth rate from VAR with the long-run restriction

Δy_t^T : Transitory component of GDP growth rate from VAR with the long-run restriction

TABLE 11

Estimated Unconditional Cross Correlation between Real Quantity Variables and the Expected Equity Premium: Simulated Data from the Bivariate Regressions in the Model of Capital Adjustment Costs and Habit Persistence (Model 2)

	$E_t[ep_{t+j+1}]$								
j	-4	-3	-2	-1	0	1	2	3	4
Panel A. Model 2 ($\varepsilon_{KI} = 0.9$, $\zeta = -0.1$)									
Δy_t	-0.03	0.02	0.06	0.06	0.01	-0.05	-0.03	-0.01	-0.10
Δy_t^P	0.00	0.00	0.00	0.00	-0.16	-0.17	-0.10	-0.03	-0.07
Δy_t^T	-0.05	0.06	0.12	0.12	0.34	0.24	0.14	0.05	-0.08
Δc_t	-0.10	-0.03	-0.07	-0.11	-0.09	-0.10	-0.04	0.00	-0.09
Δc_t^P	0.00	0.00	0.00	0.00	-0.31	-0.27	-0.18	-0.08	-0.08
Δc_t^T	-0.21	-0.09	-0.15	-0.21	0.28	0.21	0.18	0.11	-0.05
Δi_t	0.13	0.15	0.30	0.24	0.17	0.05	0.06	0.02	-0.07
Δi_t^P	0.00	0.00	0.00	0.00	0.21	0.09	0.11	0.10	0.02
Δi_t^T	0.17	0.21	0.30	0.39	-0.01	-0.06	-0.06	-0.11	-0.14
Panel B. Model 2 ($\varepsilon_{KI} = 1.5$, $\zeta = -0.1$)									
Δy_t	-0.12	-0.05	-0.00	0.03	0.04	0.08	0.15	0.17	0.09
Δy_t^P	0.00	0.00	0.00	0.00	-0.21	-0.14	-0.01	0.08	0.07
Δy_t^T	-0.20	-0.09	-0.01	0.02	0.41	0.35	0.27	0.17	0.04
Δc_t	-0.14	-0.08	-0.13	-0.15	-0.10	0.03	0.13	0.18	0.08
Δc_t^P	0.00	0.00	0.00	0.00	-0.31	-0.17	-0.04	0.08	0.08
Δc_t^T	-0.28	-0.15	-0.23	-0.26	0.29	0.30	0.28	0.19	0.03
Δi_t	-0.07	-0.01	0.07	0.18	0.29	0.16	0.21	0.20	0.14
Δi_t^P	0.00	0.00	0.00	0.00	0.02	-0.07	0.03	0.09	0.12
Δi_t^T	-0.13	-0.03	0.08	0.27	0.48	0.38	0.31	0.21	0.08

$E_t[ep_{t+j}]$: j-period ahead expected equity premium forecast at t

Δy_t : GDP growth rate

Δy_t^P : Permanent component of GDP growth rate from VAR with the long-run restriction

Δy_t^T : Transitory component of GDP growth rate from VAR with the long-run restriction

TABLE 12

Accuracy of Projection Methods around the Steady States

A. Model 1				
	$\varepsilon_{KI} = 0.9$		$\varepsilon_{KI} = 1.5$	
	Steady State	Projection Methods	Steady State	Projection Methods
Y	1.291703	1.291707	0.668173	0.668188
C	0.904192	0.904190	0.467721	0.467715
I	0.387511	0.387517	0.200452	0.200473
B. Model 2				
	$\varepsilon_{KI} = 0.9,\ \zeta = -0.1$		$\varepsilon_{KI} = 1.5,\ \zeta = -0.1$	
	steady state	Projection method	Steady state	projection method
Y	1.293531	1.293125	0.669118	0.669040
C	0.905472	0.905626	0.468383	0.468412
I	0.388059	0.387499	0.200735	0.200628

Capital letters of Y, C and I represent output, consumption and investment in level before log-transformation, respectively.
Model 1: Capital Adjustment Costs
Model 2: Capital Adjustment Costs and Habit Persistence in Consumption

TABLE 13

Estimated Unconditional Cross Correlation between Output Growth and the Expected Equity Premium: Simulated Data from Projection Methods

	$E_t[ep_{t+j+1}]$								
j	-4	-3	-2	-1	0	1	2	3	4
Panel A. Model 1 ($\varepsilon_{KI} = 0.9$)									
Δy_t	-0.03	-0.03	-0.07	-0.20	-0.29	-0.26	-0.16	-0.20	-0.09
Δy_t^P	0.00	0.00	0.00	0.00	-0.09	-0.10	-0.01	-0.09	-0.03
Δy_t^T	-0.08	-0.11	-0.16	-0.42	-0.35	-0.29	-0.27	-0.19	-0.10
Panel B. Model 1 ($\varepsilon_{KI} = 1.5$)									
Δy_t	0.06	0.05	0.06	0.08	0.02	-0.11	-0.12	-0.08	-0.09
Δy_t^P	0.00	0.00	0.00	0.00	0.12	0.03	0.03	0.07	0.04
Δy_t^T	0.10	0.09	0.11	0.15	-0.11	-0.20	-0.21	-0.20	-0.17
Panel C. Model 2 ($\varepsilon_{KI} = 0.9$, $\zeta = -0.1$)									
Δy_t	-0.03	0.02	0.06	0.06	0.01	-0.05	-0.03	-0.01	-0.10
Δy_t^P	0.00	0.00	0.00	0.00	-0.16	-0.17	-0.10	-0.03	-0.07
Δy_t^T	-0.05	0.06	0.12	0.12	0.34	0.24	0.14	0.05	-0.08
Panel D. Model 2 ($\varepsilon_{KI} = 1.5$, $\zeta = -0.1$)									
Δy_t	-0.12	-0.05	-0.00	0.03	0.04	0.07	0.15	0.17	0.09
Δy_t^P	0.00	0.00	0.00	0.00	-0.21	-0.14	-0.01	0.08	0.07
Δy_t^T	-0.20	-0.09	-0.01	0.02	0.41	0.35	0.27	0.17	0.04

Δy_t^P : permanent component of GDP growth of the model

Δy_t^T : transitory component of GDP growth of the model

Model 1: Capital Adjustment Costs

Model 2: Capital Adjustment Costs and Habit Persistence in Consumption

TABLE 14

Estimated Unconditional Cross Correlation between Output Growth and the Expected Equity Premium: Simulated Data from Log-Linearization

	$E_t[ep_{t+j+1}]$								
J	-4	-3	-2	-1	0	1	2	3	4
Panel A. Model 1 ($\varepsilon_{KI} = 0.9$)									
Δy_t	0.12	0.10	0.24	0.50	0.33	0.07	0.01	0.06	0.02
Δy_t^P	0.00	0.00	0.00	0.00	0.65	0.35	0.03	0.07	0.02
Δy_t^T	0.13	0.13	0.29	0.63	-0.12	-0.20	-0.01	0.01	0.01
Panel B. Model 1 ($\varepsilon_{KI} = 1.5$)									
Δy_t	0.12	0.07	0.17	0.41	0.38	0.10	0.01	0.08	0.03
Δy_t^P	0.00	0.00	0.00	0.00	0.61	0.30	-0.01	0.10	0.04
Δy_t^T	0.17	0.11	0.25	0.62	-0.15	-0.22	0.03	-0.00	0.00
Panel C. Model 2 ($\varepsilon_{KI} = 0.9$, $\zeta = -0.1$)									
Δy_t	0.15	0.01	0.23	0.42	0.17	0.11	-0.13	0.05	-0.16
Δy_t^P	0.00	0.00	0.00	0.00	0.61	0.09	-0.03	0.07	-0.15
Δy_t^T	0.22	0.03	0.33	0.60	-0.35	0.06	-0.22	0.00	-0.07
Panel D. Model 2 ($\varepsilon_{KI} = 1.5$, $\zeta = -0.1$)									
Δy_t	0.17	0.08	0.25	0.52	0.36	0.20	-0.06	0.05	-0.12
Δy_t^P	0.00	0.00	0.00	0.00	0.71	0.27	0.03	0.11	-0.07
Δy_t^T	0.22	0.11	0.34	0.67	-0.14	0.02	-0.11	-0.03	-0.10

Δy_t^P : permanent component of GDP growth of the model

Δy_t^T : transitory component of GDP growth of the model

Model 1: Capital Adjustment Costs

Model 2: Capital Adjustment Costs and Habit Persistence in Consumption

FIGURE 1

Estimated Impulse Response Functions of the Bivariate Regressions (Five Lags) of GDP Growth and the Expected Equity Premium with the Long-Run Restriction: 1960:1 to 1996:4 with 148 Quarters

A. Permanent Shock B. Transitory Shock

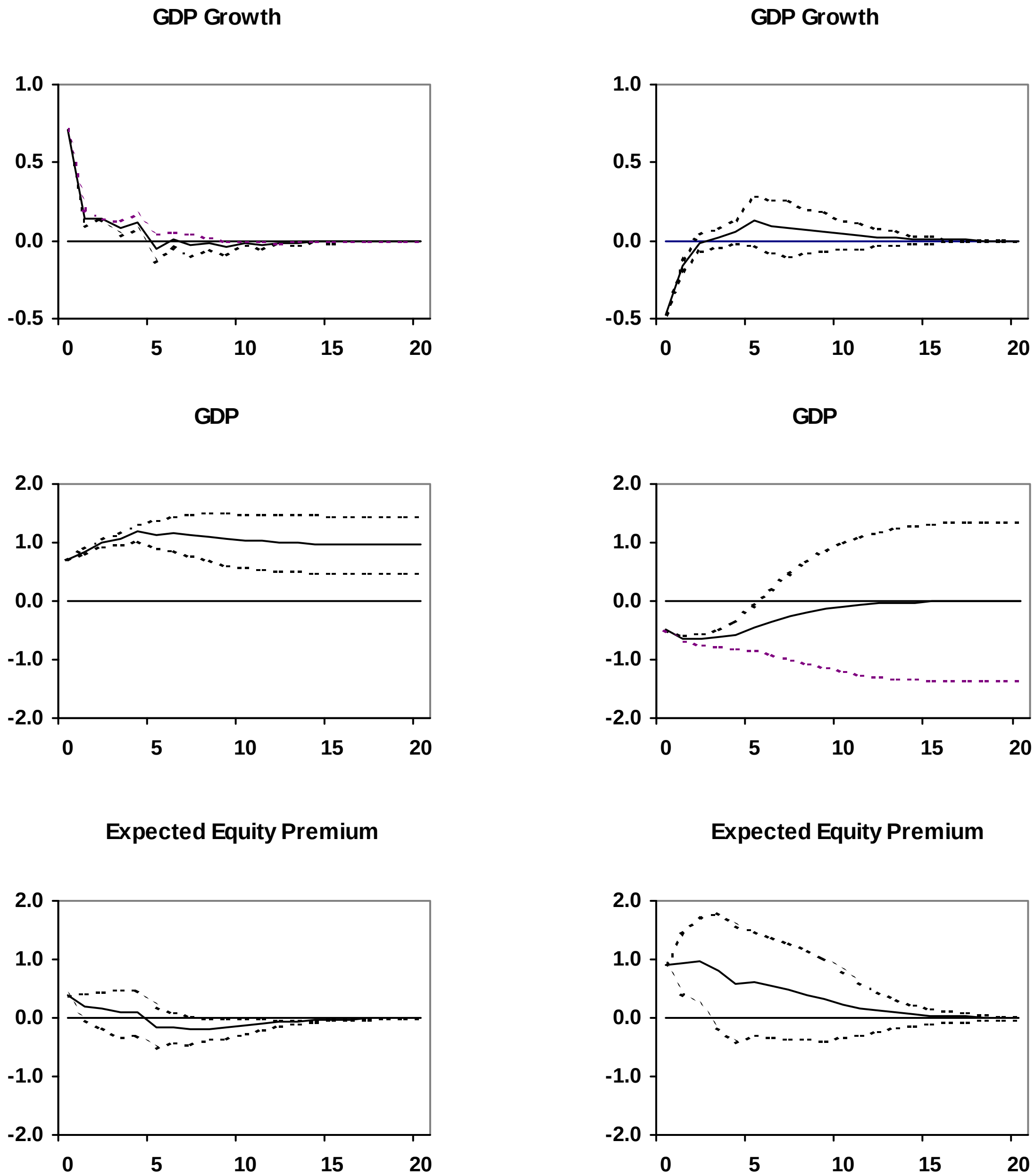
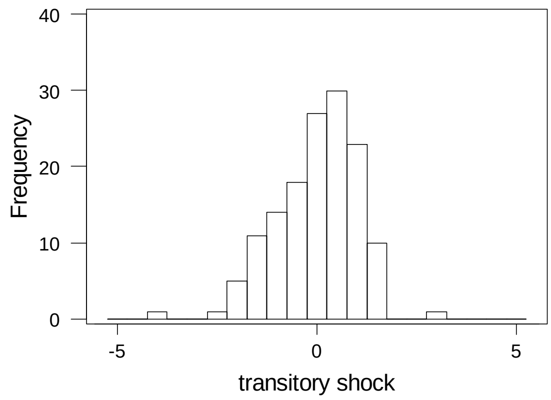


FIGURE 2

Histograms of Transitory and Permanent Shocks from the Bivariate Regression (Five Lags) of GDP Growth and the *Ex Post* Equity Premium with the Long-Run Restriction: 1961:3 to 1996:3 with 141 Observations



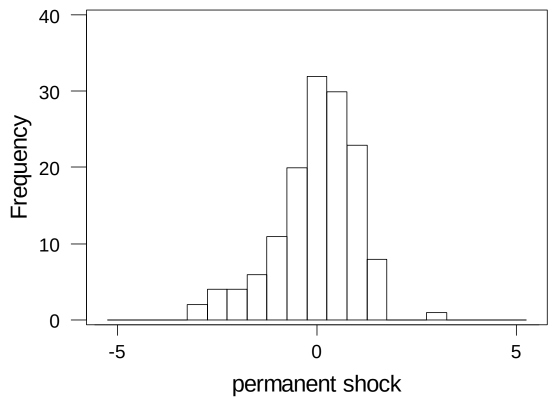


FIGURE 3

Simulated Impulse Response Functions to Unit Positive Transitory Shock in Model 1: Capital Adjustment Costs ($\varepsilon_{KI} = 1.5$)

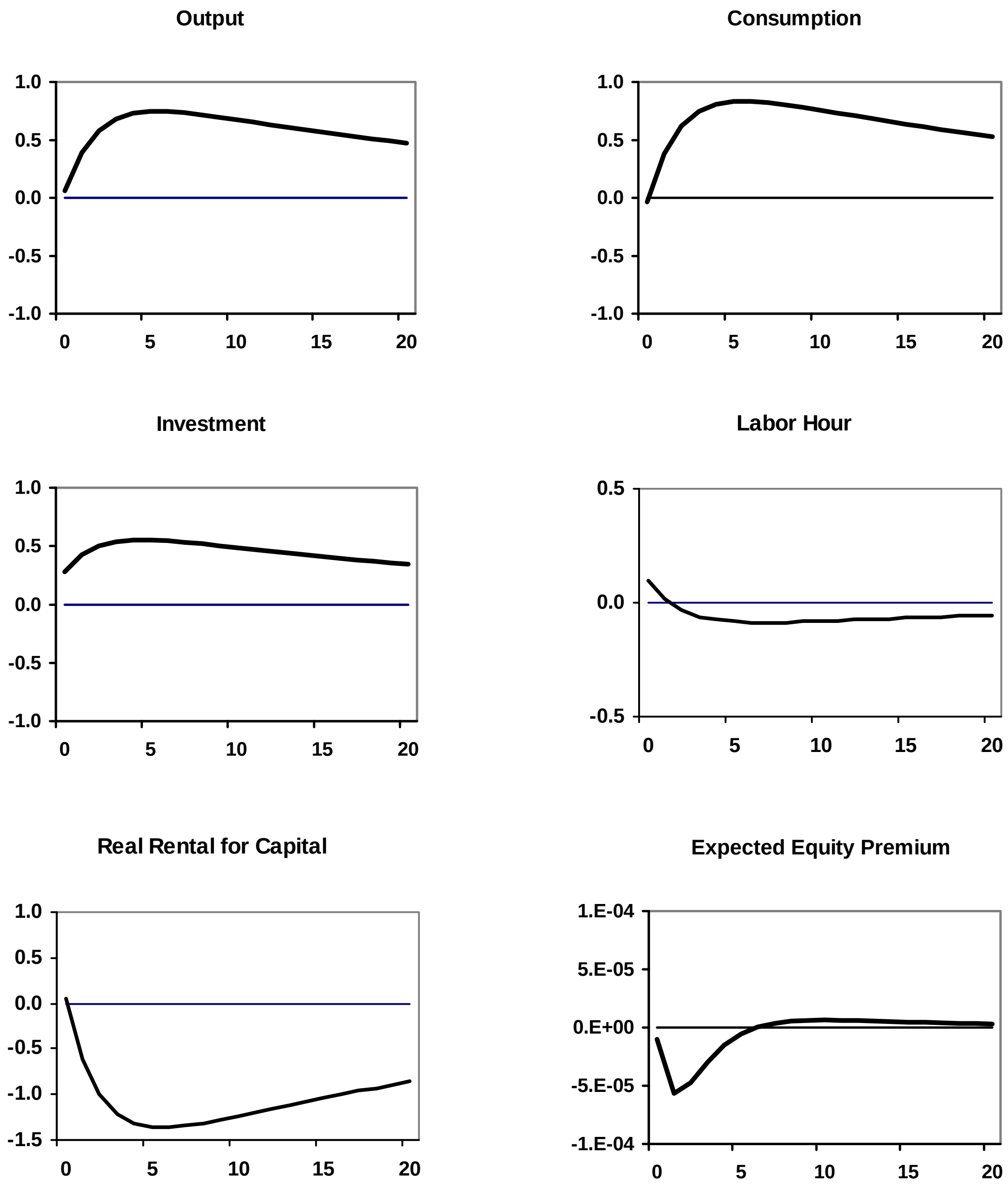


FIGURE 4

Estimated Impulse Response Functions of the Bivariate Regression (Five Lags) with the Long-Run Restriction in Model 1: Capital Adjustment Costs ($\varepsilon_{KI} = 1.5$)

A. Permanent Shock B. Transitory Shock

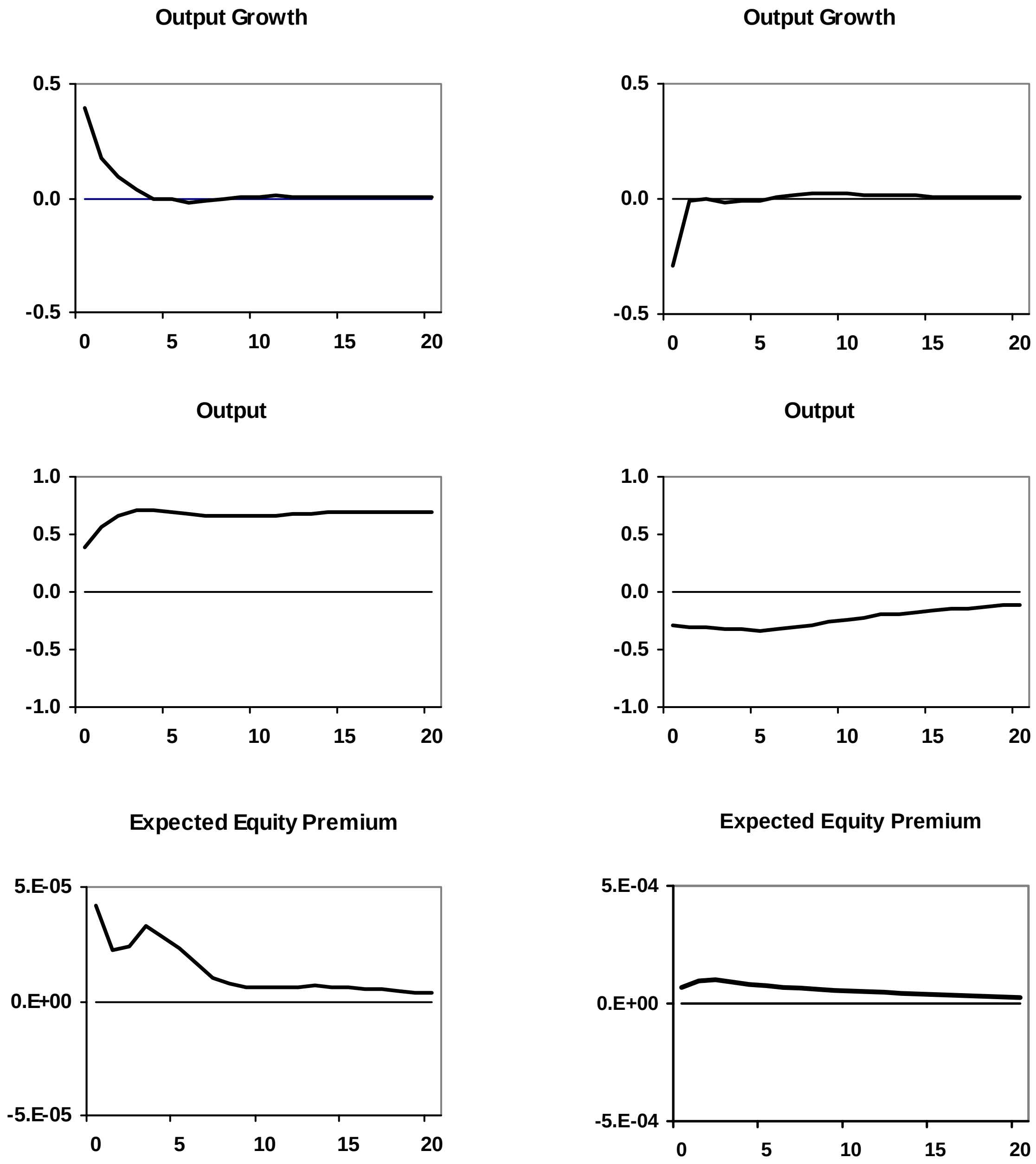


FIGURE 5

Simulated Impulse Response Functions to Unit Positive Transitory Shock in Model 1: Capital Adjustment Costs ($\varepsilon_{KI} = 0.9$)

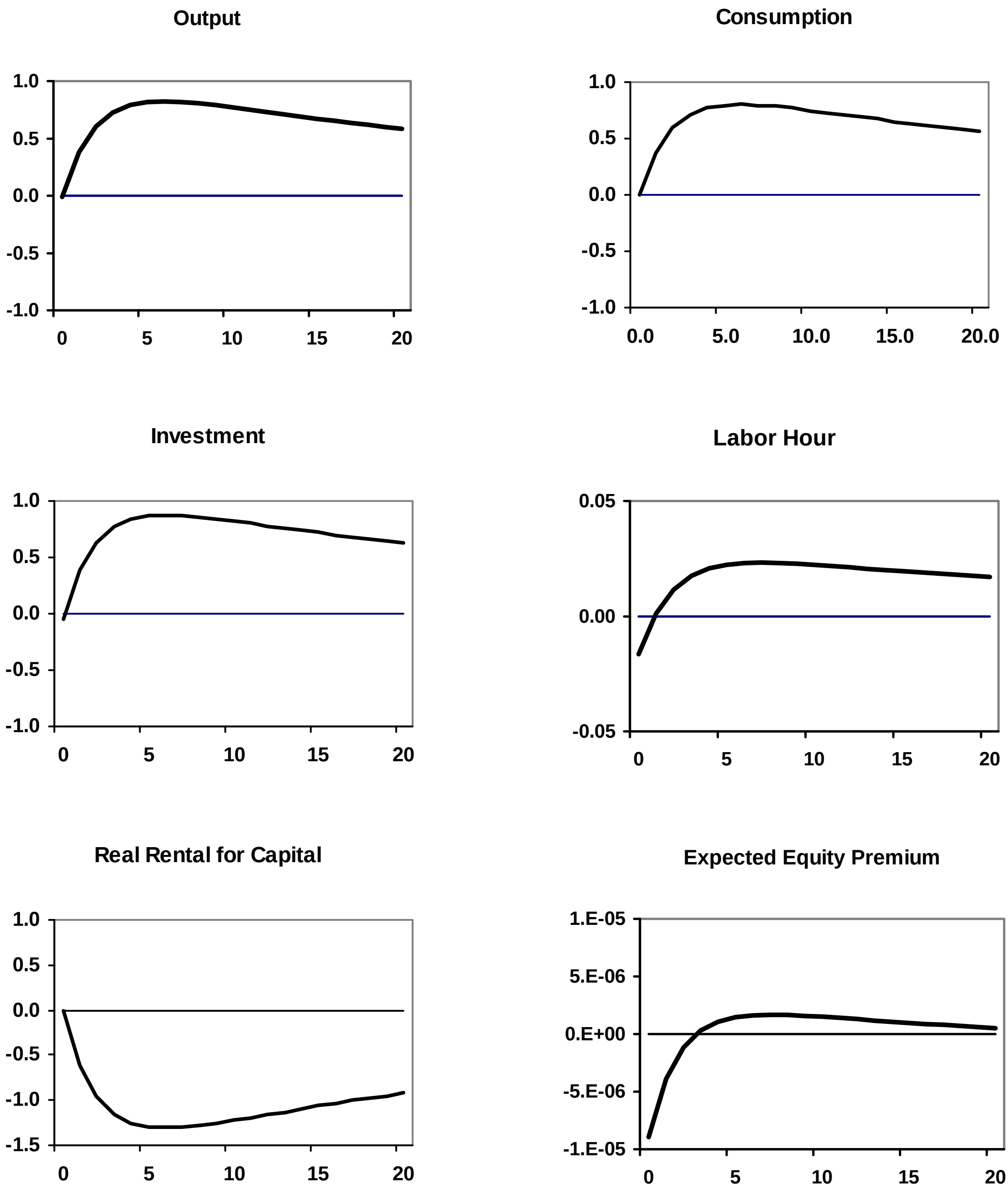


FIGURE 6

Estimated Impulse Response Functions of the Bivariate Regression (Five Lags) with the Long-Run Restriction in Model 1: Capital Adjustment Costs ($\varepsilon_{KI} = 0.9$)

A. Permanent Shock

B. Transitory Shock

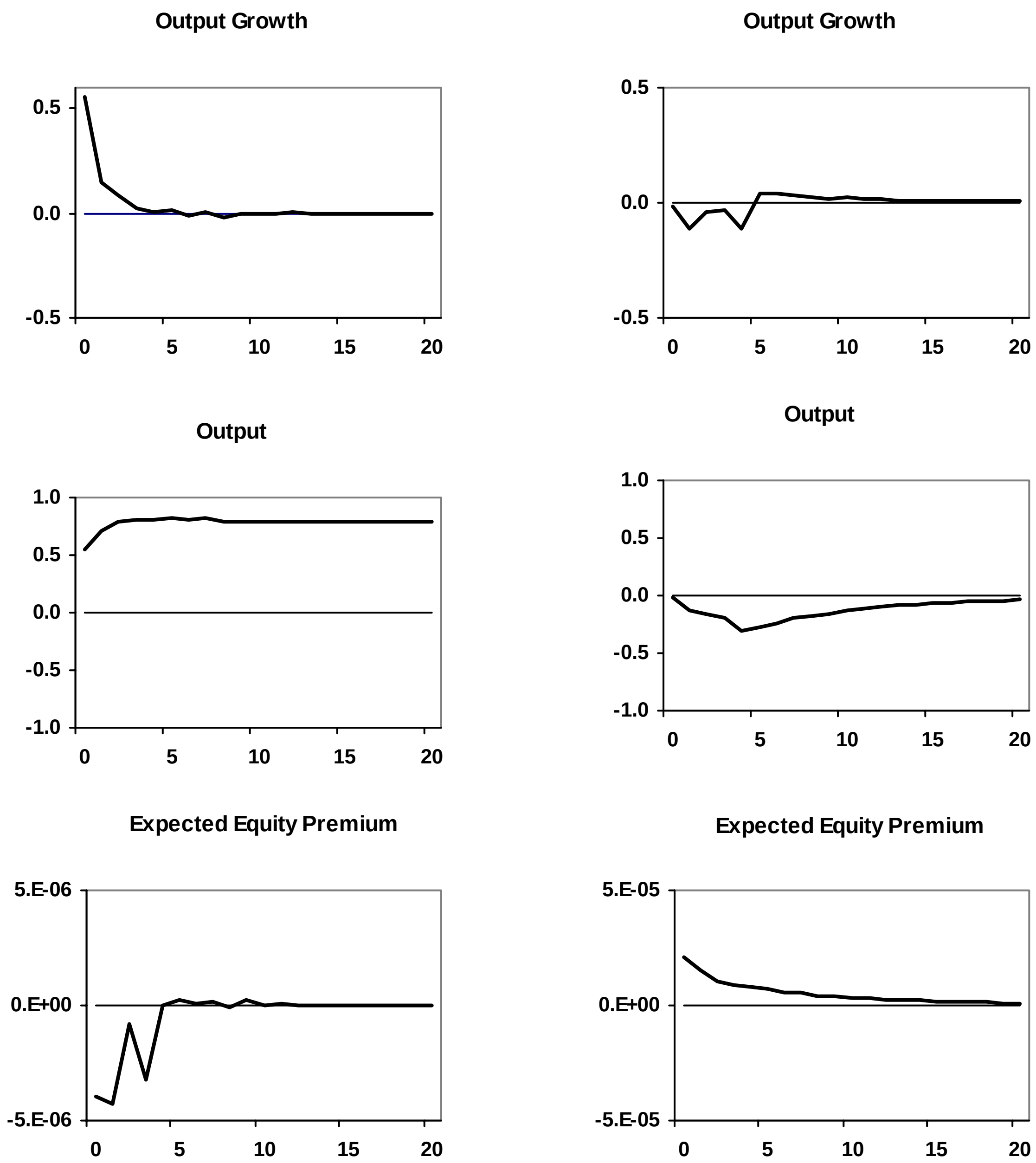


FIGURE 7

Simulated Impulse Response Functions to Unit Positive Transitory Shock in Model 2: Capital Adjustment Costs ($\varepsilon_{KI} = 1.5$) and Habit Persistence ($\zeta = -0.1$)

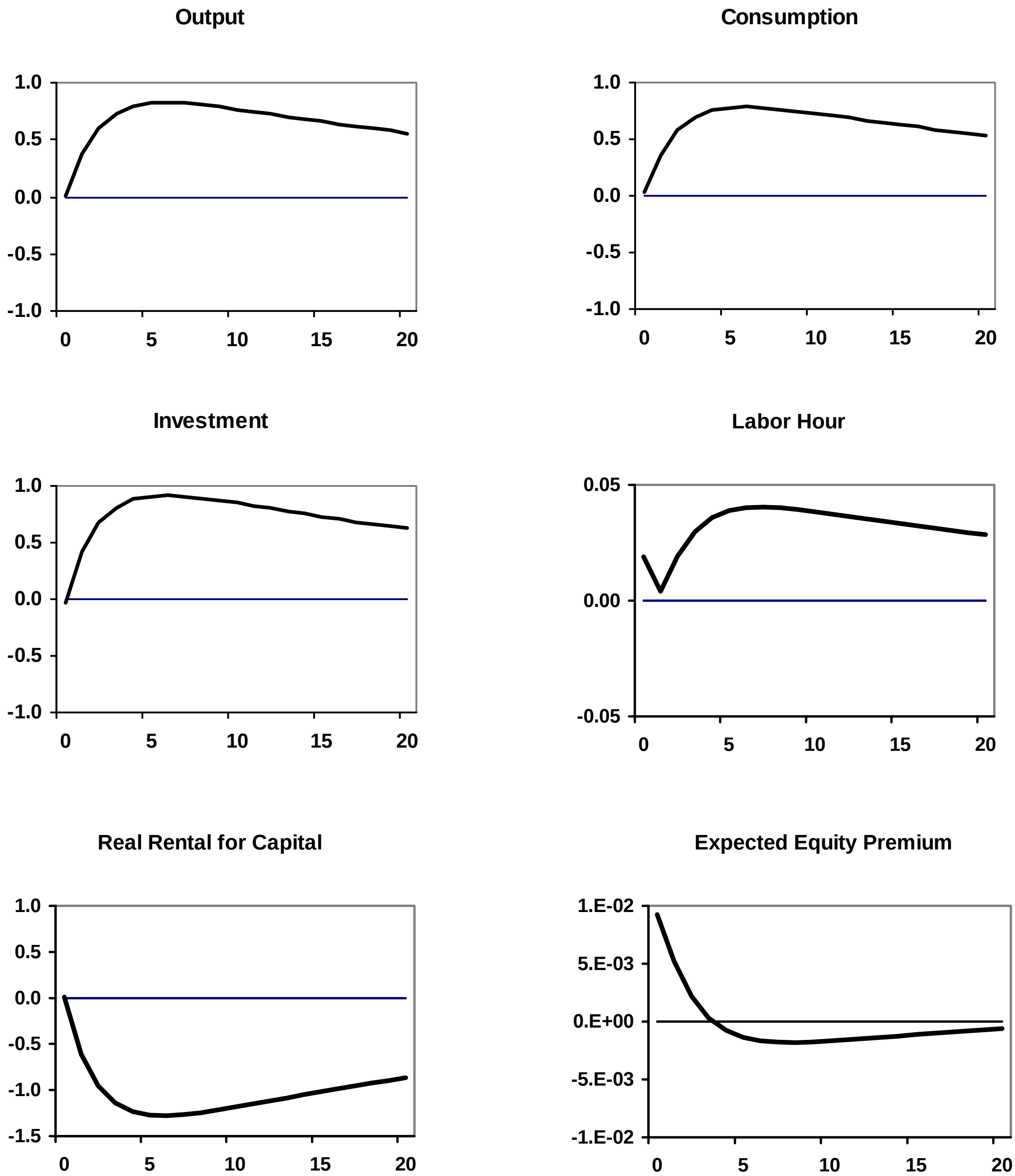


FIGURE 8

Estimated Impulse Response Functions of the Bivariate Regression (Five Lags) with the Long-Run Restriction in Model 2: Capital Adjustment Costs ($\varepsilon_{KI} = 1.5$) and Habit Persistence ($\zeta = -0.1$)

A. Permanent Shock

B. Transitory Shock

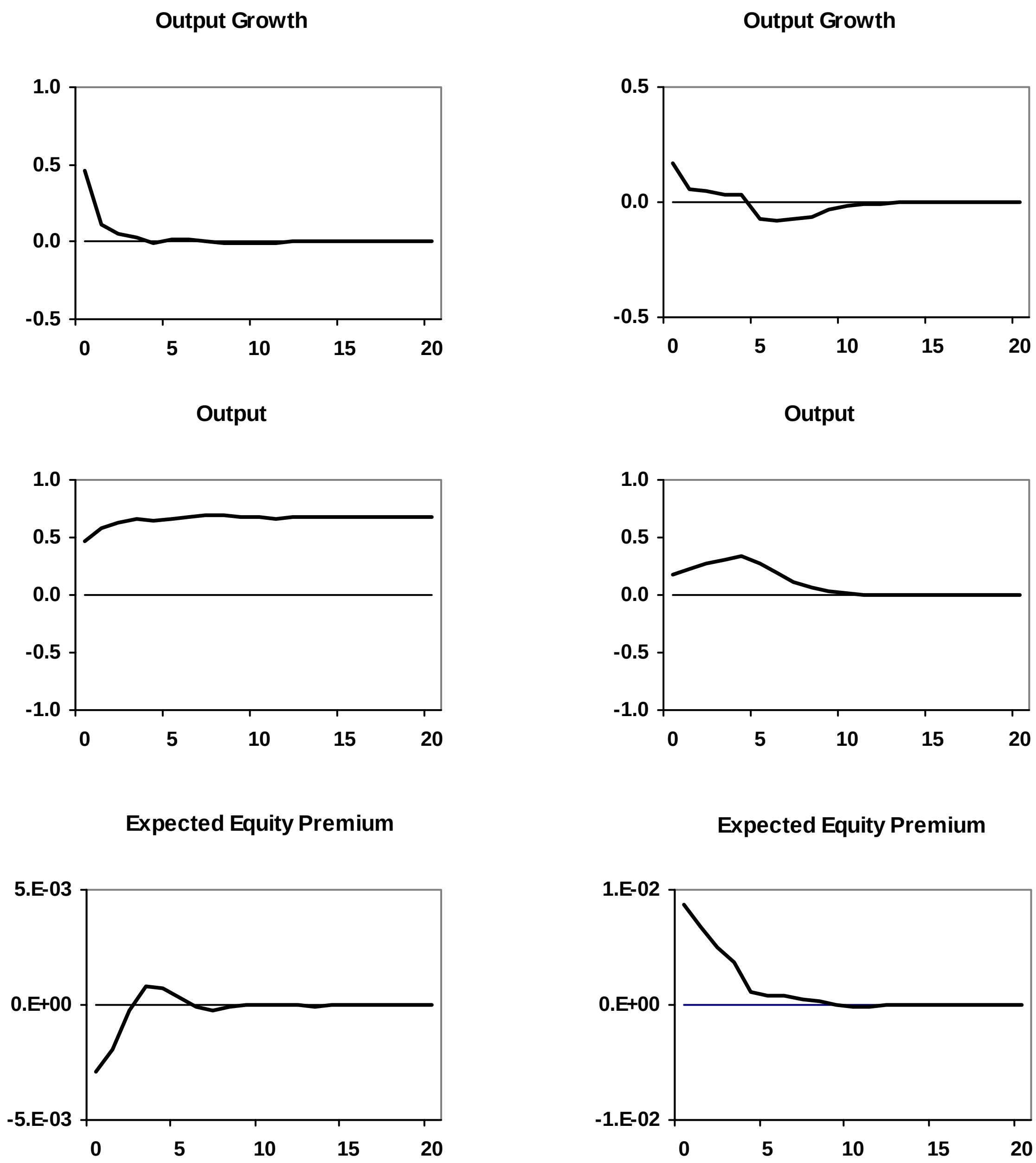


FIGURE 9

Simulated Impulse Response Functions to Unit Positive Transitory Shock in Model 2: Capital Adjustment Costs ($\varepsilon_{KI} = 0.9$) and Habit Persistence ($\zeta = -0.1$)

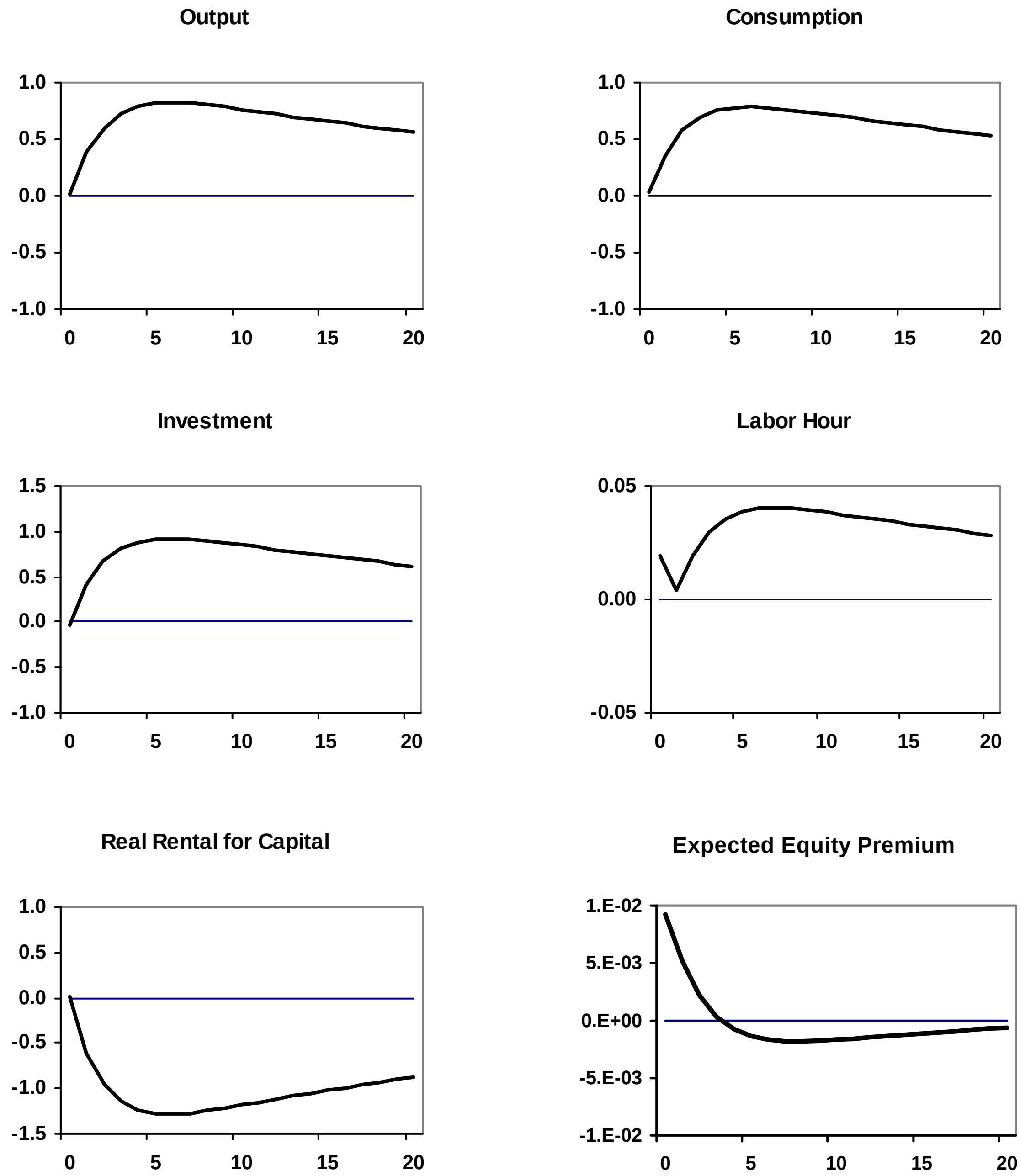


FIGURE 10

Estimated Impulse Response Functions of the Bivariate Regression (Five Lags) with the Long-Run Restriction in Model 2: Capital Adjustment Costs ($\varepsilon_{KI} = 0.9$) and Habit Persistence ($\zeta = -0.1$)

A. Permanent Shock B. Transitory Shock

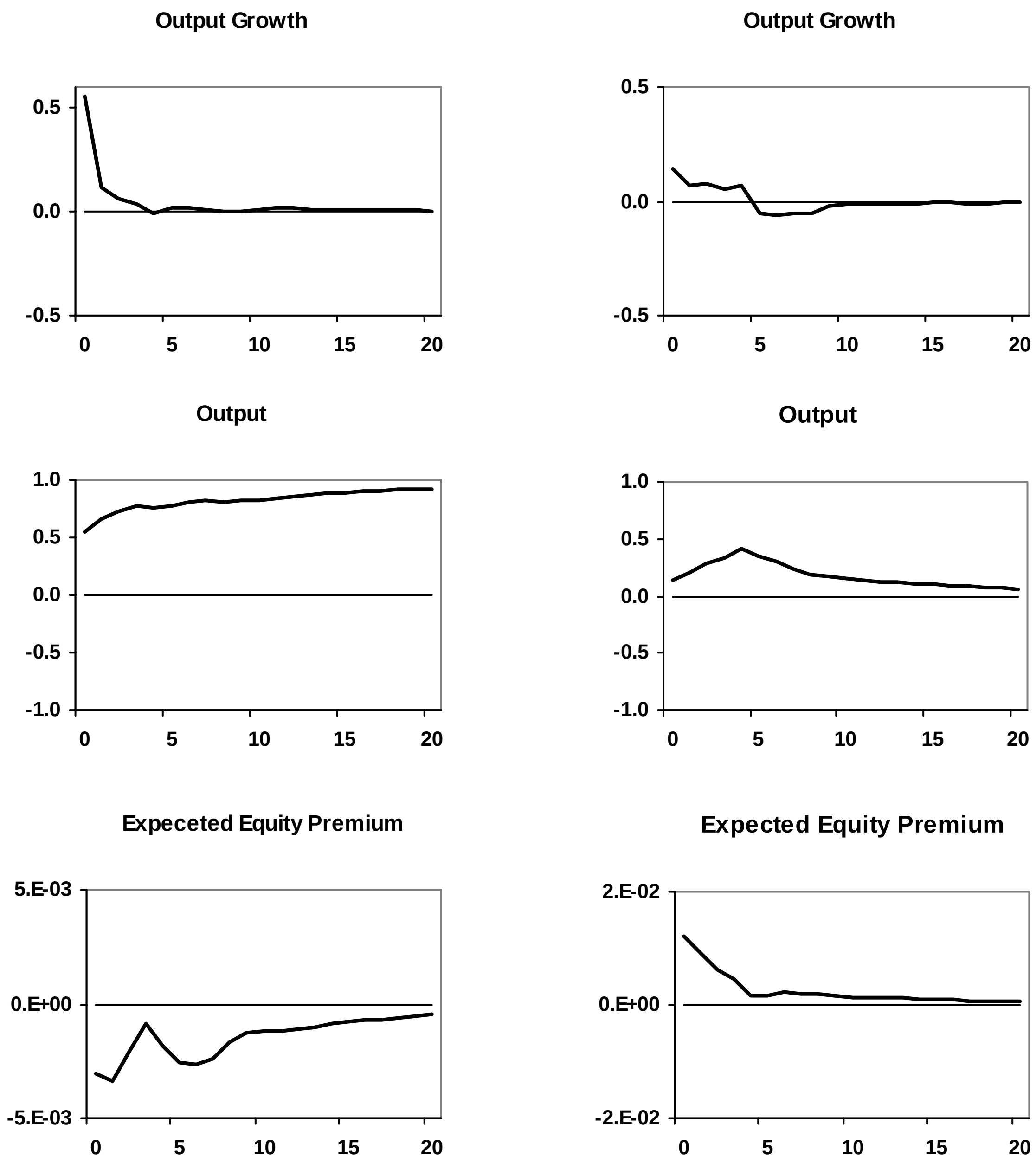
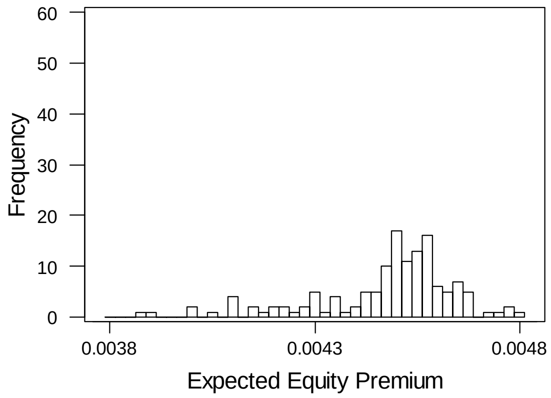


FIGURE 11

Histograms of the Simulated Expected Equity Premium in Model 2: Capital Adjustment Costs ($\varepsilon_{KI} = 0.9$) and Habit Persistence ($\zeta = -0.1$)

A. Projection Methods



B. Log-Linearization