Improved Measures of Financial Risk for Hedge Funds

Abstract

During the current financial crisis, several US and foreign banks and investment firms have failed due to excessive losses in some of their investments. Many of these financial institutions relied on a widely-used risk model known as the Value-at-Risk (VaR) to gauge the risks taken by their businesses, and several authors have pointed to some key flaws in the VaR measure that tend to understate the risks that these firms actually faced. To overcome the subadditivity flaw of VaR, researchers in financial economics have proposed an alternative measure, the Conditional Value-at-Risk (CVaR), which is defined as the expected losses that are strictly larger than the VaR. The purpose of this paper is to evaluate two recent innovations in the financial economics literature that may help banks and investment firms to properly assess the risks they face. First, we employ Extreme Value Theory (EVT) to estimate non-normal models of the return distribution tails. In particular, we use the peaks-over-threshold (POT) method in which extremes are defined as excesses over a threshold, and we estimate the marginal (univariate) return distributions. The POT observations are used to estimate the Generalized Pareto (GP) model of the upper and lower tail areas of the return distributions. Second, we use the estimated GP models to compare the relative performance of the VaR and CVaR for assessing forward-looking risk in observed hedge fund returns. The main objective of this analysis is to evaluate competing claims from the financial economics literature about the relative importance of the VaR flaws (e.g., subadditivity) and probability model specification errors in risk measurement.

JEL Classification: C22, G22, G12

Keywords: Hedge fund, VaR, CVaR, Extreme Value Theory, POT

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1 Introduction

This study of risk management in the hedge fund industry is concerned with how new methodologies to manage risks compare to traditional methods such as Value-at-Risk (VaR). Improved techniques are potentially important from a risk management standpoint because they provide an opportunity for financial institutions and investment firms to better manage various risks.

Historical experience has shown abundant evidence of the potential risks, and these may be illustrated with the list of large-scale financial crises. For example, extensive U.S. bank failures occurred later in the Great Depression of 1929 and did harm the entire economy. In the summer of 1997, the Asian financial crisis broke out. In August 1998, the Russian government defaulted on their debt, and Russian ruble faced difficulties. The dot-com bubble burst in 2000 and the trouble extended into 2001. In the summer of 2007, big losses on U.S. subprime mortgage loans sparked financial stress, which was widespread across other sectors.

Faced with the possibility of large financial panics and smaller adverse events, every market participant is exposed to persistent and potential risks of loss associated with unexpected changes in asset values. From the standpoint of risk management, better measures of risk are strongly required to avoid substantial losses. The most widely used risk model is VaR. In words, VaR is defined as the maximum value that may be lost over a certain period (e.g., one month) with a given probability or confidence level (a technical definition is provided later). The measure is commonly used to determine how much capital should be set aside to control the risk facing financial institutions or investment firms at the stated confidence level. The VaR measure has three main drawbacks. First, VaR is generally not subadditive in the sense that the risk of a combined portfolio may be larger than the sum of the individual risk portfolios. Non-subadditivity of VaR may lead to bias when assessing the value of diversification of a portfolio. Second, VaR is not convex, which may hamper the search for risk optimized portfolios. Third, the most common VaR applications are based on the normality assumption of asset returns, which is not appropriate for most assets and can lead to understated risk estimates.

Due to the presence of the drawbacks contained in VaR, researchers have developed Conditional Value-at-Risk (CVaR), which is defined as the expected losses that are strictly larger than VaR (a formal definition is provided later). The CVaR measure may more appropriately represent the potential losses associated with holding two or more underlying assets because it is subadditive. The improved risk measure may be useful to enhance financial institutions' risk management operations. Here the meaning of "improved" is based on an axiomatic approach that characterizes a coherent risk measure. Unlike VaR, CVaR satisfies the axioms and is a coherent risk measure. It provides potentially more valid estimates of the capital requirement for corporate and financial institutions relative to VaR.

Efforts to build probability models that capture the behaviors of extreme risk factors have been made over a long period of time. The extreme events such as financial panics are rare and thus it is hard to draw reliable inferences based only on historical data. Conceptually, the extreme events are represented by the tail areas in the return. For example, the fat-tailed character of the distribution implies that the extremely rare events occur with higher probability than under the normal distribution. As a result, the normal distribution model of risk might understate the true risk.

To deal with those problems, researchers developed Extreme Value Theory (EVT) as described by Embrechts, Klüppelberg and Mikosch (1997). EVT is used to model the extreme quantiles or tails of the probability distribution for some random variable. As an application of the proposed methods in this paper, hedge fund returns are used to compare the relative performance of VaR and CVaR for assessing forward-looking risk. The estimated risk measures are based on the Generalized Pareto (GP) model, which is a prominent EVT tool. For instance, given an upper threshold, we fit the observed extreme values of asset returns over the threshold to the distribution, which is called peak-over-threshold (POT) approach. Then VaR is estimated from the fitted GP model, and CVaR is calculated by the expected value of losses over VaR.

There are several reasons that we focus on the hedge fund industry for this application. Since the 1990s, the hedge fund industry has grown rapidly and this is partly due to financial globalization. The primary motivation is to find an opportunity for higher profits by moving beyond traditional investment vehicles. Hedge funds are limited to wealthy individuals and institutional investors who meet a minimum standard of wealth (e.g., 5 million dollars). Due to the impressive profits, the performance fee charged by hedge fund managers is relatively high and may be more than 20 percent, which is in addition to a management fee of 2 percent of the net asset value. The outstanding performance (i.e., commonly double-digit annual returns) of hedge funds stems from their use of long/short[1] trading strategies and investment in illiquid assets. While traditional investment funds gain value from asset appreciation, hedge fund investments may be based on long or short positions and may gain value from either rises or declines in asset prices.

Traditionally, a long investor will profit if the price of the financial instruments increases and a short investor will profit from a decrease in price. To take advantage of an expected lower price in the future, hedge fund managers can borrow shares from their brokerage firms or banks and short sell them at the current price. At some point in the future, the fund managers must return the shares and the interest to the brokerage firms or banks. If the price declines, the managers will take a profit. However, they will take a loss if the price rises. Thus, hedge funds can generate higher returns than traditional long-only investment funds by taking advantage of both asset value declines and increases. At this point, it is important to note that the use of the term “hedge” to describe these funds may be distinct from the practice of hedging risks with forward or futures contracts. Although hedge funds can use traditional hedging strategies to reduce risk, this activity is not their main objective.

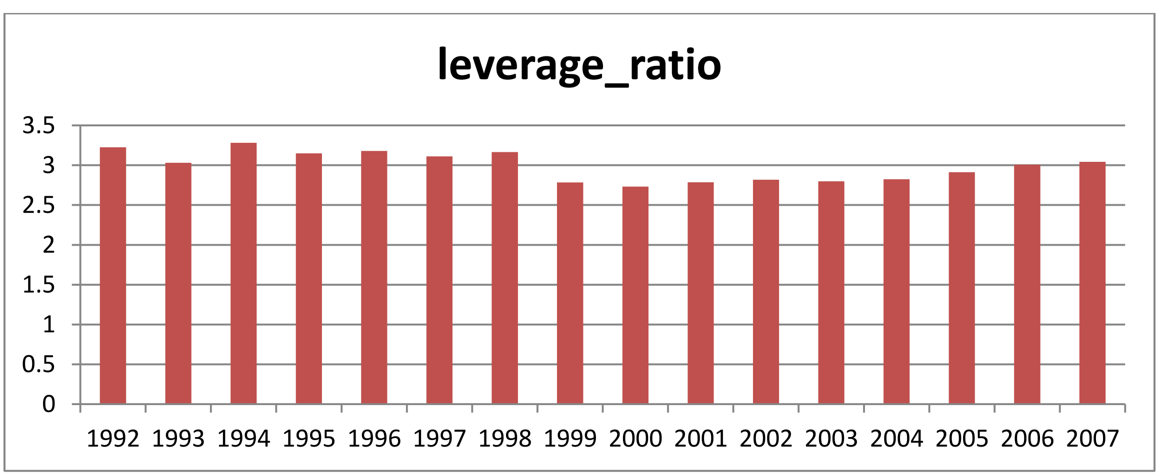
More broadly, the difference between a traditional long-only investing strategy and a hedge fund strategy is that hedge funds can use flexible tools such as short-selling and derivatives to reduce risk and generate returns. For example, the equity long/short strategy engages in short positions to hedge market exposure. This means that hedge funds have more diverse tools relative to long-only investing. However, hedge fund managers may take on more risk by using leverage, which may result in the expansion of risk and return.

[1] An investor is referred to as taking a short position if he or she sells futures contracts and a long position if he or she purchases futures contracts.

Hedge funds borrow in order to increase the size of the investment portfolio and gain better performance when the underlying asset value rises. The practice of borrowing is called leverage. For example, borrowing to buy a stock is called a margin loan. In general hedge fund leverage is known as being above investor capital. Unlike mutual funds that operate under the Securities and Exchange Commission (SEC) regulations, hedge funds have great exemptions from the usual regulations. Although short selling is theoretically useful because it increases market efficiency (Renshaw 1977), it can cause financial distress. Some hedge fund strategies simultaneously buy and sell a security or other financial instruments in two different markets to profit from the price anomalies in the two different markets, which is called arbitrage. For example, in bond arbitrage, hedge funds buy illiquid bonds that may be mispriced and capture arbitrage profits by short selling derivatives based on more liquid bonds that are accurately priced.

Figure 1-1 shows the leverage ratios of hedge funds over time. The leverage ratios are calculated as the ratio of average gross leverage plus average assets under management (AUM) to average assets under management. According to this result, the weighted average hedge fund has constant leverage ratios of 2.5 times over the average assets under management. The ratio of three to one leverage means that a hedge fund manager controls $3 million in the investment portfolio with $1 million in capital. As the ratio increases, an increase in asset values leads to an expanded gain. However, in the opposite case, the losses would be severely magnified. We can see the gradual increasing pattern since 1999, and the leverage ratio reached three times over the AUM in 2007.

Figure 1-1 Hedge Funds’ Leverage Ratio with CISDM database



The aggressive investment practices of hedge funds bring up big concerns because of their impacts on the global financial system which may offset their crucial role in liquidity supply and price discovery. There are debates about sensible regulations for hedge funds due to the spillover effects to other sectors. Some people insist that hedge funds be regulated due to concerns about their opaqueness. On the other hand, advocates assert that hedge funds make financial markets more efficient (Ackerman, McEnally and Ravenscraft, 1999; Danielsson and Zigrand, 2007). At this time when the U.S. financial crisis of 2007-2009 recedes, it seems that there is no general agreement on regulations of hedge funds.

It is essential to understand the source of the returns embedded in each hedge fund strategy to improve the risk management of the hedge funds. Hedge fund returns have two main components. The first part can come from diversified risk exposures such as credit risk, liquidity risk, exchange rate risk, or event risk. Hedge funds that have different risk profiles are compensated for the risks in the form of risk premiums. For example, a merger arbitrage manager can obtain an event (e.g., merger and acquisition) risk premium since there is a possibility that a merger would not occur after the merger's announcement. The second component is the manager's ability to exploit the excess market return caused by market inefficiencies such as price anomalies which lead to arbitrage opportunities. For example, currency based macro strategies would borrow a currency with a low interest rate and then purchase currencies with a high interest rate (i.e., carry trade). The strategy works fine with good market conditions but it causes trouble when exchange rates shift.

Hedge fund investors or managers consider risk management seriously. As shown in Lo (2008, p.238), considering risk management with a financial gain through an investment strategy at the same time might be profitable. For a simple illustration, he assumes that there is a fund that provides an annual expected return $E[R]$ of 10% and an annual volatility of 75%. As a risk management practice, suppose a fund manager can avoid losses below -20%. His new return is

$$R^* = \max[R, -20\%] \quad (1\text{-}1)$$

Based on the log-normality assumption of returns, the expected value of R^* is 20.9% by truncating the left tail of the return distribution (see Table 9.1, p.240). The resulting expected return doubles. It is very crucial to have the ability to manage risks in a hedge fund operation.

There are common risks embedded in hedge fund strategies as described by the U.S. Securities and Exchange Commission (2003). According to the classification, the risk can be divided into five branches: market, credit, leverage, liquidity, and operational risk. Here we focus on some common risks to several hedge fund strategies. First, hedge fund managers are exposed to leverage risk. They need to borrow some money from their partners or banks to increase their performance. When investments go bad, big losses may occur. Furthermore, those who use leverage will take on the risk that one has to pay back when the lender requests the loan.

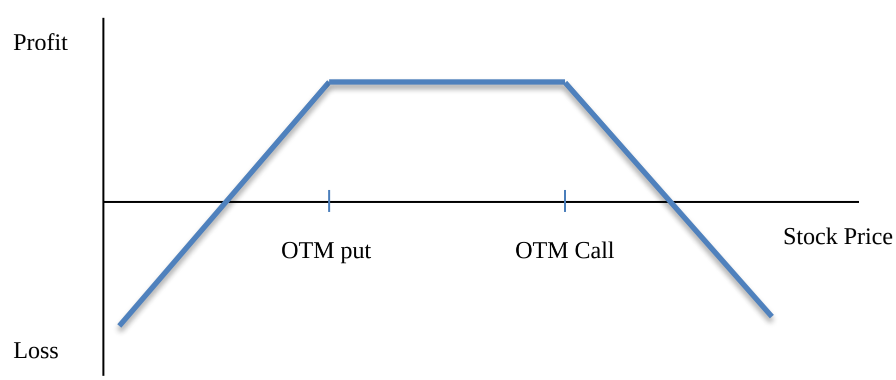
Second, there is liquidity risk. When an economic disturbance occurs, there may be many investors who wish to exit a hedge fund. Faced with this situation, hedge funds may be forced to sell positions at prices lower than their real value. Subprime mortgages and related mortgage-backed securities suffered large losses during 2007. As a result, one of the five largest investment bank in the United States, Lehman Brothers, announced bankruptcy on September 14, 2008. This triggered a liquidity risk in the hedge fund sector because their prime brokers were reluctant to fund these positions.

Third, the failure of Long Term Capital Management (LTCM) in 1998 is often described as an example of credit risk. A default by Russia on its debt cast serious doubt on the quality of the fund. LTCM responded by going long in illiquid assets (i.e., bonds) and short in the corresponding liquid bonds. After the Russian default, the spread between the two underlying assets widened severely. LTCM was highly leveraged and suffered from huge losses after receiving margin calls on its positions. Unfortunately, LTCM was not able to meet those calls. Note that even though the liquidity and the credit risk are separate exposures, they are interconnected as in the case of the LTCM collapse.

Fourth, hedge fund managers may be exposed to counter party risk. In the financial affair of LTCM, Bear Sterns, LTCM's prime brokerage firm, faced a large margin call as the value of the portfolios deteriorated. It was at risk if LTCM did not meet further margin calls.

Finally, specific event risks are the possibility that individual events that have not happened before would occur. The use of derivative securities such as options makes it possible to trade volatility. For example, when the hedge fund managers do not know how the underlying stock price changes up or down, the short strangle strategy may be pursued by some hedge fund managers. The strangle strategy in options trading is the simultaneous selling of an out-of-the-money (OTM) put and an out-of-the-money call with the same expiration but with different strike prices. It can be used when we expect the market volatility to stay in a bounded range before expiration. In Figure 1-2, the blue solid line denotes the profits of a short strangle strategy. The put option is said to be out-of-the-money (OTM) when its strike price is below the current stock price and a call option is said to be OTM when its strike price is above the current stock price. Writing (or selling) an OTM call option can be profitable when the underlying stock at expiration is below a call's strike price. Similarly writing an OTM put option can be profitable when the underlying stock at expiration is above a put's strike price. Hence, only if the underlying stock stays in between the two strike prices, we can profit. In the case of high volatility, the stock can rise or decline quite far from the option strike prices, and we need to close the short strangle position by buying the in-the-money option. However, it may be very expensive to buy back, and we may be exposed to unlimited risk. The hedge fund managers can use this short strangle strategy, but when the market is volatile, large loss can occur.

Figure 1-2 Short Strangle



Many studies such as Brooks and Kat (2002) and Geman and Kharoubi (2003) find that hedge fund returns are not normally distributed. Most hedge fund indices show not only high negative skewness, which is a measure for the asymmetry around the mean, but also high kurtosis, which is a measure of tail thickness. The latter implies that there are more observations on the tails than a normal distribution. Thus, both statistics indicate that those returns are not normally distributed.

The purpose of this paper is to evaluate two recent innovations in the financial economics literature that may help hedge fund investors and managers to properly assess the risks they might confront. We will use CVaR risk measure and EVT to estimate the risk of hedge fund returns under the constituent strategies. The study will directly examine competing claims about the relative importance of VaR flaws and probability model specification errors in risk measurement.

The paper is organized as follows: The next section provides a detailed literature review. The subsequent three sections briefly discuss risk measures and then those risk measures apply to two different parametric specifications on extreme values. The sixth section describes the data and presents empirical results from various methods. Finally, concluding remarks are presented.

2 Literature Review

Value-at-Risk (VaR) was developed by JP Morgan. In October 1994, J.P. Morgan and Reuters (1996) made its RiskMetrics freely available on the internet, which contributes to the wide use of VaR. There are a few other significant merits. VaR can be applied to any asset class, including equities, bonds and derivatives. Also, VaR provides a means to aggregate the component risks of a given trading desk. Furthermore, VaR is expressed by potential lost money, which is easy to interpret.

Despite these advantages, there are criticisms as well. Taleb (1997) claims that the method may not accurately measure tail probabilities. While arguing that forecasts based only on past observations is naïve, he writes “Nothing predictable can be truly harmful and nothing truly harmful can be predictable.” Ju and Pearson (1999) show that the use of

VaR in controlling risks at the level of individual traders or trading desks leads to large biases.

Due to the vulnerability of the financial system stemming from potential financial crises, the international banking forum (i.e., Bank for International Settlements (BIS)) has built guidelines and standards for banking supervisory issues. The committee strengthens the 2001 Basel II capital framework, which is based mainly on VaR. However, there are some arguments by Danielsson, Embrechts, et al. (2001) against contents suggested by the Basel II proposal. They point out that VaR has limitations as a risk measure and the statistical model can generate inconsistent and biased forecasts of risk. These criticisms highlight the significance of a better understanding of extreme risks in order to reduce them.

In terms of risk measurement, a seminal paper by Artzner, et al. (1999) is a turning point. Since this work, the concept of risk measures becomes refined and stands on more solid ground. There are four axioms that risk measures must satisfy to be called coherent measures. The axioms of coherence are translation invariance, subadditivity, positive homogeneity, and monotonicity. According to the proposed standards, the widely used VaR turns out not to be a coherent risk measure because it violates subadditivity. In contrast, CVaR satisfies all properties required for coherency.

The application of EVT in the finance and insurance literature begins from Embrechts, Klüppelberg and Mikosch (1997) and Reiss and Thomas (1997). The two books explain the theoretical background and show how the EVT methods can be used to estimate the risk measures in applications. For example, Longin (1996) uses the maxima of daily returns for U.S. stock indices, which is modeled with the Fréchet distribution. The Fréchet distribution is one of the standard generalized extreme value (GEV) distributions.

Danielsson and Vries (2000) analyze the extreme returns of the S&P 500 index using unconditional extreme value theory along with the Generalized Autoregressive Conditional Heteroskedastic (GARCH) model. Use of the unconditional[2] distribution of asset returns is criticized by McNeil and Frey (2000), who conclude that the method

[2] The probability that an event will occur can be characterized without any information obtained from past and related events. On the other hand, the conditional approach utilizes past events to model the distribution of future outcomes.

based on the conditional return distribution provides better estimates of VaR and CVaR than unconditional EVT. However, in cases where the conditional distributions taking into account all values such as the GARCH model is employed, there is still the possibility for large unexpected events to be ignored as in Longin (2000). Danielsson and Vries (2000) argue that for the portfolio constructed from a number of assets, the huge conditional variance matrices can make the conditional approach infeasible. They suggest that there are cases where an unconditional EVT method is suited for some financial low-frequency (i.e., daily or weekly etc.) data. In contrast, they propose that the conditional volatility models may perform better over short horizons, such as intra-day data. Christoffersen and Diebold (2000) present findings that agree with Danielsson and Vries (2000).

Regarding applications to hedge fund returns, VaR has been used with EVT to control the market risk associated with hedge funds. Gupta and Liang (2005) use VaR to determine the equity capital requirement for hedge funds based on EVT. Lhabitant (2003) estimates VaR and CVaR using two funds of hedge funds on a monthly basis with an EVT model. Blum, Dacorogna and Jaeger (2004) calculate VaR to capture the tail risk measure from a Generalized Pareto (GP) model.

In this chapter, we rely on the unconditional volatility approach to compute the standard deviation required for VaR and CVaR. The data for this application is low-frequency (i.e., weekly or monthly) hedge fund returns. With regard to applications to risk management, McNeil and Frey (2000) show that the GP distribution of EVT works better for CVaR than the Gaussian model. In the same vein, Fernandez (2003) verifies that EVT outperforms a GARCH model with normal innovations using the Chilean daily stock index conditional on restriction of the shape parameter (e.g., $\xi > -0.25$). However, these findings depend on the data under study and may not be generalized.

Hedge fund operations are differentiated by the investment vehicles that they can provide to investors. Boudt, Peterson and Croux (2008/2009) document that modified expected shortfall (a variant of CVaR) based on portfolios that outperform the fund-of-fund index. Fung and Hsieh (2001) show that hedge funds differ from mutual funds, which usually keep long-only buy-and-hold strategy. Agarwal and Naik (2000) report that the hedge fund strategies outperform the traditionally diversified investment method by more than 6 percent. It suggests that hedge funds provide better opportunities for diversification by their low correlation with different indices and even different asset returns, such as stocks and bonds.

Some studies deal with hedge fund characteristics and management styles. Compared to the performance of mutual funds, hedge fund returns generate less correlation with those of standard asset classes (Fung and Hsieh 1997). Specifically while hedge fund returns have low and occasionally negative correlation with other asset returns (e.g., stock market index), mutual fund returns are highly and positively correlated with asset class returns. In relation to the impact on crises, Fung and Hsieh (2000) conclude that hedge funds seem not to have had a major role.

Attempts to estimate historical returns for hedge funds are recognized in the literature to have several biases such as survivorship bias or backfill bias. Hedge fund managers do not disclose their performance to the public. They have an incentive to report performance to the data vendors only when they have relatively good results. Thus, persistently successful funds tend to be contained, causing survivorship bias. There is also instant history bias. In the case where a new fund manager starts reporting, it is more likely for recent and good performance record to be included in the database. Accordingly, they tend to highly exaggerate the returns. This observation is backed by several studies including Ackerman, McEnally and Ravenscraft (1999), Brown, Goetzmann and Ibbotson (1999), and Schneeweis, Sputgin and McCarthy (1996).

Regarding the distributional characteristics of hedge fund returns, several studies (Brooks and Kat 2002, Bacmann and Gawron 2005, Agarwal and Naik 2004) find that hedge fund index returns are often not normally distributed as skewness and kurtosis are significantly identified and are significantly correlated with each other. Amin and Kat (2003) find that skewness decreases and kurtosis rises with portfolios containing hedge funds. Due to the non-normality of hedge funds returns, Gupta and Liang (2005) cast doubt on the validity of the normal VaR, which is usually used to evaluate the validity of the capital adequacy of hedge fund operations (Jorion 2000). An interesting point is that despite the claim that hedge funds are uncorrelated with market indices, some studies (Brooks and Kat 2002, and Agarwal and Naik 2004) show the existence of high correlation with the equity indices.

3 Risk Measures

In general, risk is explained by its exposure and uncertainty (Holton 2004). In financial management, exposure is characterized by financial loss and uncertainty that occurs because we are not aware of future events. So uncertainty can be described by a probability distribution of future values (i.e., prices or returns) of the financial assets, which are represented by random variables. Risk, which is subjective, is related to uncertainty[3] but they are not identical (Rachev, Stoyanov and Fabozzi 2008). A classical uncertainty measure is the standard deviation which is the square root of the variance. Despite such difficulties we can attempt to formalize it by relying on probabilistic models denoted by the portfolio loss distribution. To quantify a risk of a financial position that is regarded as a random variable, the risk measure is defined as a mapping of portfolio losses into real numbers. Let X be a random variable on the probability space $(S, \mathcal{F}, P)$. In other words, risk measures are real-valued functions

$$\rho(X): S \to \mathbb{R} \tag{3-1}$$

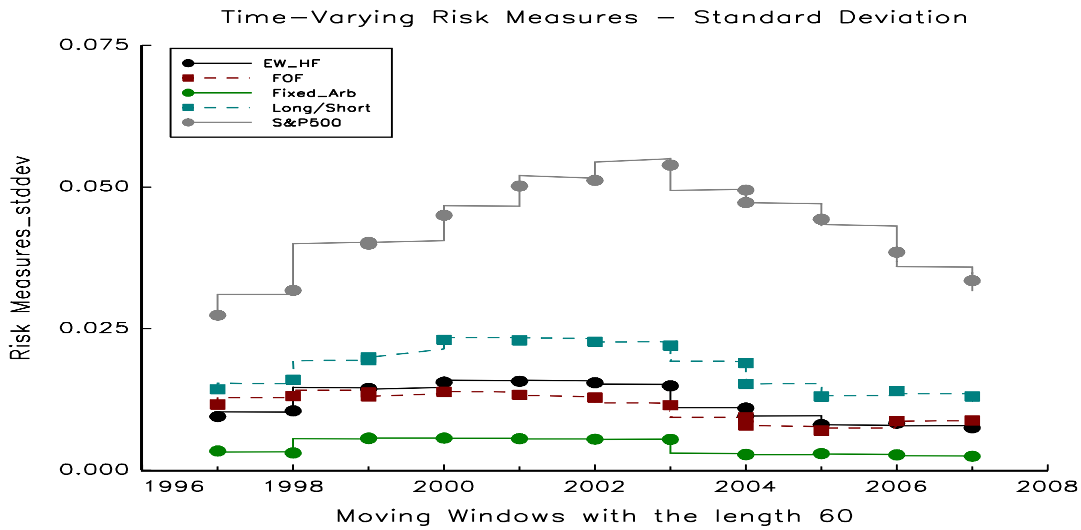
defined on S, the random loss from financial positions.

3.1 Standard Deviation

The standard deviation is used to quantify the amount of volatility. In a statistical term, the standard deviation is a measure of dispersion around the mean. As the standard deviation gets higher, the volatility becomes larger, and vice versa. The standard deviation can be used to measure the risk of hedge fund returns over a specified time period. However, it is very sensitive to extreme values, which may be more likely when hedge fund returns are not normally distributed. When we incorrectly use the normal distribution, we can underestimate the possibility of large losses.

[3] Uncertainty stems from the inability to predict the future values of assets of interest. Therefore it can be formed by a probability distribution of future prices or returns.

Figure 3-1 Volatility with Selected Hedge Fund and S&P500 Returns



In Figure 3-1, we compute the time-varying volatility. The lowest curve is obtained from the returns of fixed arbitrage strategy. The next two lines correspond to fund of fund (FOF) index and equal weighted hedge fund portfolio returns. The fourth curve from the bottom represents the standard deviation of an equity long/short returns. In contrast to the hedge fund returns, the volatility of S&P 500 returns show a different pattern. It stays above the contrasting hedge funds indices' volatilities. It increases steadily over the next eleven years to peak in August, 2003. From then until July, 2007 the values fall consistently. The implications are that through the diversification the hedge fund managers seem to achieve superior returns and relatively lower volatility.

3.2 Value at Risk

To define VaR we need three components: a cumulative distribution function, a fixed time horizon and a given confidence level. Let X be the random variable (the loss) of risky asset returns. By convention, X is a random variable corresponding to loss and negative values of X corresponds to profits. We assume that F_X represents the cumulative distribution function for the loss random variable. Given a confidence level α where $\alpha \in (0,1)$, VaR at the specified horizon time τ is defined as

$$VaR_\alpha(X) = inf\{x|F_X(x) \geq \alpha\}. \tag{3-2}$$

In words, it means that VaR is the smallest number x such that the probability that the loss X exceeds x is no larger than $1-\alpha$. In probabilistic terms, VaR is simply an upper α −quantile of the loss distribution. For example, the VaR for $\alpha = 0.99$ indicates that there is a one percent probability that losses exceed VaR in the given time interval.

3.3 Conditional Value at Risk

To cope with the well-known drawbacks of VaR such as non-subadditivity and non-convexity, CVaR was proposed by Artzner, et al. (1999) and satisfies the four coherence properties: translation invariance, subadditivity, positive homogeneity, and monotonicity. Now we formally introduce those axioms in more detail based on Equation 1-2. Here X denotes the loss of a position.

Property 1 (translation invariance) states that for every possible loss,

$$\rho(X+l) = \rho(X) + l \tag{3-3}$$

The additional loss l (i.e., a constant) should be taken into account by the same quantity when we update the existing risk measure. Property 2 (subadditivity) is summarized by the following equation.

$$\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2) \tag{3-4}$$

This tells us that the risk of the combined portfolio is less than or equal to the sum of the individual risks. However, the idea behind this property is that risk may be reduced by diversification. When this property is violated, for example, a financial organization would have an incentive to have various small-sized subsidiaries to reduce regulatory capital requirements. Property 3 (positive homogeneity) is

$$\rho(hX) = h\rho(X), \text{ for } h > 0 \tag{3-5}$$

This implies that the risk of a financial position is proportional to its size. Note that both subadditivity and positive homogeneity imply that the risk measure is convex. Property 4 (monotonicity) means that positions exposed to higher losses need more capital requirements. For $X < Y$,

$$\rho(X) < \rho(Y) \tag{3-6}$$

where X and Y stand for potential losses.

CVaR is defined as follows:

$$CVaR_{\alpha}(X) = E[X|X \geq VaR_{\alpha}(X)] \tag{3-7}$$

The CVaR risk measure is the expected size of a loss that exceeds the VaR level. For a given loss distribution the CVaR is worse than or equal to VaR at the specified quantile. By the first and fourth properties, the standard deviation is ruled out as a coherent measure.

4 Normal Distribution and Risk Measures

If the mean (μ) and the standard deviation (σ) in the normal loss distribution are known, the VaR at a given confidence level, α, (say $\alpha = 0.99$) is simply obtained by

$$\mu + \sigma\Phi^{-1}(\alpha) \tag{4-1}$$

where Φ denotes the standard normal distribution function and $\Phi^{-1}(\alpha)$ is the α −quantile of the standard normal distribution. We assume $\mu = 0$. For CVaR,

$$CVaR = E[X|X > VaR_{\alpha}] = \mu + \sigma E\left(\frac{X-\mu}{\sigma} \,|\, \frac{X-\mu}{\sigma} \geq q_{\alpha}\left(\frac{X-\mu}{\sigma}\right)\right)$$

$$= \sigma\frac{\phi\left(\Phi^{-1}(\alpha)\right)}{1-\alpha}. \tag{4-2}$$

where $q_{\alpha}(X)$ is the α −quantile of the standard normal distribution. The last equality in Equation 4-2 can be obtained by applying integration by parts to $E\left(\frac{X-\mu}{\sigma} \,|\, \frac{X-\mu}{\sigma} \geq q_{\alpha}\left(\frac{X-\mu}{\sigma}\right)\right)$.

$$E\left(\frac{X-\mu}{\sigma} \,|\, \frac{X-\mu}{\sigma} \geq q_{\alpha}\left(\frac{X-\mu}{\sigma}\right)\right) = \frac{1}{1-\alpha}\int_{\Phi^{-1}(\alpha)}^{\infty} l\phi(l)dl$$

$$= \frac{1}{1-\alpha}\left[-\phi(l)\right]_{\Phi^{-1}(\alpha)}^{\infty} = \frac{\phi\left(\Phi^{-1}(\alpha)\right)}{1-\alpha} \tag{4-3}$$

where the second equation from the right is based on the property of the normal distribution, $\int \Phi(l)dl = l\Phi(l) - \int l\phi(l)dl$. In practice, we need to calculate the mean of the truncated distribution, $X \geq VaR_{\alpha}$. Following Maddala (1983, p.365), the mean of the truncated distribution is given by

$$E(X \geq VaR_\alpha) = \frac{\phi(VaR_\alpha)}{1-\Phi(VaR_\alpha)} \tag{4-4}$$

5 Extreme Value Theory and Risk Measures

In the EVT literature, there have been efforts to link real data about extremal events to probability models. There are two parametric approaches: generalized extreme value (GEV) and generalized Pareto (GP). The former is central for analysis of maxima or minima while the latter deals with exceedances over a specified threshold. The GP model is the primary topic of this paper. The GP distribution was introduced by Pickands (1975).

To estimate the parameters of the GP model, the commonly used methods are the method of moments (MOM) and maximum likelihood (ML) techniques. Monte Carlo experiments conducted in Ashkar and Tatsambon (2007) show that the maximum likelihood method is preferable in terms of reducing the inconsistency rate, which is the percentage of times that each method produces an estimate of the GP upper bound that is inconsistent with the simulated data. As an alternative method, Hosking and Wallis (1987) use the MOM for estimating parameters of the GP distribution. It turns out that both estimators are consistent and efficient.

The GEV model depends on the shape parameter. When the parameter is greater than zero the distribution is a Fréchet distribution. Longin (2000) and McNeil (1997) use estimation techniques based on limit theorems for block maxima. However, there is a tradeoff between the number and size of the blocks. As the size of the blocks increases, we can obtain more accurate estimation results, which mean a low bias in the parameter estimates. When many blocks are involved, the number of observations in each block given the fixed data points shrink. It leads to lower variance of the parameter estimates.

In general there are two different approaches to identify extreme values. Rather than using block maxima method based on a pre-specified period, we model the tails of the loss distribution using the peaks over the threshold (POT) method which focuses on the realizations exceeding an upper tail threshold. Let u be an upper tail threshold. Suppose that $F_u(x)$ is the distribution function of exceedances of X above a certain threshold u. Let $y = X - u$ be the excess, and the conditional excess distribution function is defined by

$$F_u(y) = P(X - u \leq y | X > u), \tag{5-1}$$

where $0 \leq y \leq x_F - u$ and $x_F \leq \infty$ is the right end point. Using the conditional probability property, F_u can be rewritten as

$$F_u(y) = \frac{F(u+y) - F(u)}{1 - F(u)} = \frac{F(x) - F(u)}{1 - F(u)} \tag{5-2}$$

The purpose of EVT is to model the probability of those exceedances beyond a given threshold.

Let $G_{\xi,\sigma}$ be a Generalized Pareto (GP) distribution with the shape parameter ξ and the scale parameter σ. As u gets large, the distribution of exceedances is approximated by the Generalized Pareto distribution:

$$F_u(y) \approx G_{\xi,\sigma}(y) = \begin{cases} 1 - \left(1 + \frac{\xi}{\sigma} y\right)^{-\frac{1}{\xi}}, & \xi \neq 0 \\ 1 - e^{-\frac{y}{\sigma}}, & \xi = 0 \end{cases} \quad \text{with}$$

$$y \in \begin{cases} [0, x_F], & \xi \geq 0 \\ \left[0, -\frac{\sigma}{\xi}\right], & \xi < 0 \end{cases} \tag{5-3}$$

The shape parameter (ξ) may take real values, whereas the scale (σ) is restricted to a positive value. The shape parameter is a good indicator of the extent of heaviness in the tail. A negative value implies very thin tailed distributions with less probability assigned to extreme outcomes than the normal distribution. A zero value means it has tail weight comparable to a normal distribution. Any positive value represents a heavy tailed character for the tail area. Typically, the shape parameter is greater than or equal to zero for financial return data.

The parameters of the GP model can be estimated in various ways. We rely on the method of moments (MOM) following Hosking and Wallis (1987). Ashkar and Tatsambon (2007) provide simulation results for estimating GP quantiles with a small sample size (e.g.,$T = 10$) and show that there are no significant differences between the various estimation methods. For monthly hedge fund returns, we take exceedances that are in the top 20 percent of observations, which amounts to 12 data points. Furthermore, the estimated shape parameters over 122 windows have averages of 0.198 and 0.116 for the equal weighted portfolio and the FOF index, respectively. These results are consistent with the recommendations provided by Hosking and Wallis (1987) that moment estimators are preferable for $\xi > 0$ and ξ near 0. In contrast, maximum likelihood estimation can be recommended for very large samples when $\xi > 0.2$.

When $\xi > 0$, it may be shown that the moments of the GP distribution are $E(X^k) = \infty$ for $\xi \leq -1/k$ (Embrechts et al., 1997, p.568). When $\xi > -0.5$, the mean and the variance of the GP distribution is defined as

$$E(X) = \frac{\sigma}{1+\xi}, \quad \text{and } VAR(X) = \frac{\sigma^2}{(1+\xi)^2(1+2\xi)} \tag{5-4}$$

Following Hosking and Wallis (1987), if the moments of the generalized Pareto distribution exist[4], the equations for estimating parameters of the GP distribution using the method of moment are

$$\hat{\xi} = \frac{1}{2}\left(\frac{\bar{x}}{s^2} - 1\right) \tag{5-5}$$

$$\hat{\sigma} = \frac{1}{2}\bar{x}\left(\frac{\bar{x}}{s^2} + 1\right) \tag{5-6}$$

where $\bar{x}$ and s^2 are the sample mean and variance. For the hedge fund applications, those exceedances in every window have values which are larger than -0.5 as estimates of the shape parameters, so that the first and second moments of the GP distribution exist. In the case where $\xi > -0.25$, the shape and scale parameter estimators are asymptotically normal. When $\xi = 0$, the MOM and MLE methods are both asymptotically efficient.

We compute the parameter estimates over a rolling window of a fixed length. In this case parameters in the model are not constant over time. The method proceeds as follows: First the historical data are split into two parts where one part is reserved for prediction. Then the model is fit under the fixed length of sub-samples and a one-step ahead prediction is made. Within the sample period, the window rolls ahead and produces the estimation results.

[4] The rth moments of X exists when $\xi > -1/r$. The mean exists if $\xi > -1$ and the variance exists if $\xi > -1/2$. The moment equation is defined as $E(1 - \xi X/\sigma)^r = 1/(1 + r\xi)$ if $1 + r\xi > 0$.

As pointed out by Castillo and Hadi (1997) and Dupuis (1996), although the MOM estimates exist, there could be a potential problem if the estimated parameters do not fall inside the feasible range. The range of the exceedances (e.g., x) that are consistent with the MOM estimates is $x > 0$ for $\xi \leq 0$, and $0 < x < \hat{\sigma}/\hat{\xi}$ for $\xi > 0$. Therefore, we must check whether $x_{n:n} < \hat{\sigma}/\hat{\xi}$ for $\xi > 0$, where $x_{n:n}$ is the largest order statistic or observation in a sample size of n. For the equal weighted hedge fund returns, most estimates from the rolling windows satisfy the conditions for consistency of the MOM estimates. In only 10 rolling windows, the conditions were violated in the sense that $x_{n:n} > \hat{\sigma}/\hat{\xi}$ for $\xi > 0$. In contrast, for the FOF returns, all rolling windows satisfy the associated conditions without exception.

Among the 10 rolling windows with exceedances that are not consistent with the MOM estimates for the equal weighted hedge fund returns, eight outcomes occur in untroubled times. In contrast, the two windows on November and December 1997 are in the middle of the Asian financial crisis, and the ratio of the scale to the shape estimates (i.e., 0.0158) is less than the largest value (i.e., 0.0165) in the associated windows. In this case, the MOM estimates are not meaningful. However, it is important to note that although the MOM estimates are not consistent with the sample values in this case, the largest order statistic exceeds the estimated ratio by some small amount, which may be caused by sampling errors.

Given the estimated GP parameters, $F(x)$ is approximated by

$$F(x) = \big(1 - F(u)\big)F_u(y) + F(u), \tag{5-7}$$

where $x = u + y$ for a sufficiently high threshold u. From a practical standpoint, it is hard to determine an appropriate threshold, and several authors deal with the issue on the selection of the threshold (e.g., Embrechts, Klüppelberg and Mikosch 1997). However, there is not a clear answer for this issue, and a graphical tool may be used as in Gilli and Këllezi (2006). In our work, we used the largest 20 percent of the subsample for every rolling window as the threshold level following Harmantzis, Miao and Chien (2006).

In equation (5-7) above, $F_u(y)$ is replaced by $G_{\hat{\xi},\hat{\sigma}}(y)$ and $F(u)$ can be approximated non-parametically as $(n - N_u)/n$ where n is the total sample size and N_u

is the number of exceedances over the threshold u. We get the estimate of $F(x)$ as follows:

$$\hat{F}(x) = 1 - \frac{N_u}{n}\left(1 + \frac{\hat{\xi}}{\hat{\sigma}}(x-u)\right)^{-1/\hat{\xi}} \tag{5-8}$$

where $\hat{\xi}$ and $\hat{\sigma}$ are the MOM estimates of the corresponding parameters.

To calculate estimates of VaR and CVaR, the above equation for $\hat{F}(x)$ may be inverted to get a quantile of the underlying distribution because $VaR_\alpha = F^{-1}(\alpha)$, where F^{-1} is the general inverse function of F. For $\alpha > F(u)$, solving $\hat{F}(x)$ for x yields

$$\widehat{VaR}_\alpha = u + \frac{\hat{\sigma}}{\hat{\xi}}\left(\left(\frac{n}{N_u}(1-\alpha)\right)^{-\hat{\xi}} - 1\right) \tag{5-9}$$

Let us rewrite CVaR as

$$CVaR_\alpha = E[X|X > VaR_\alpha] = VaR_\alpha + E[X - VaR_\alpha|X > VaR_\alpha], \tag{5-10}$$

where the second term is the average loss above the threshold VaR_α. Recall that when the exceedances over a threshold follows a GP distribution, the mean excess function, $e(\cdot)$, is denoted by

$$e(z) = E[X - z|X > z] = \frac{\sigma + \xi z}{1-\xi}, \tag{5-11}$$

where $z \geq u$ and $\xi < 1$. (McNeil, Frey, and Embrechts, 2005) The mean excess function is identical to the second term on the right hand side of Equation (5-11). As a result, for $z = VaR_\alpha - u$ and X representing the excesses y over u we can get the expression for estimated CVaR.

$$\widehat{CVaR}_\alpha = \widehat{VaR}_\alpha + \frac{\hat{\sigma} + \hat{\xi}(\widehat{VaR}_\alpha - u)}{1-\hat{\xi}} = \frac{\widehat{VaR}_\alpha}{1-\hat{\xi}} + \frac{\hat{\sigma} - \hat{\xi}u}{1-\hat{\xi}} \tag{5-12}$$

6 Application

The data used in this study are hedge fund indices provided by the Center for International Securities and Derivatives Markets (CISDM) for the monthly returns and Hedge Fund Research Inc. (HFR) for the weekly returns. The CISDM hedge fund indices are median performance indices of the fund strategies in the database. We select 10 major hedge fund strategy indices as well as the fund-of-fund (FOF) index that reflects the performance of all hedge funds reporting to the database. We construct an equal-weighted hedge fund return that is composed of the 10 strategies. In Table 1-1, the different hedge funds indices used in this study are listed.

Returns are constructed by the instantaneous changes in the index values, $r_t = lnP_t - lnP_{t-1}$, where P_t denotes the value at time t. This is called the continuously compounded return or log return. We include the FOF index because it is less sensitive to the various biases inherent in the individual hedge fund strategies. Fung and Hsieh (2000) argue that the FOF returns more accurately represent overall returns on hedge funds than a hedge fund index because a collection of diversified hedge funds is more likely to survive than the individual hedge fund. Thus, the FOF may be more appropriate to reflect an investor's losses.

Table 1-1 CISDM Hedge Fund Classifications and Definitions

Hedge Fund Style	Definition
Distressed Securities	Invest in companies in financial distresses and bankruptcy
Emerging Markets	Invest in equity or fixed income in emerging markets
Equity Long/Short	Investing on both the long and short sides of the market
Equity Market Neutral	Trade long equity positions and an approximately equal dollar-amount of offsetting short positions in order to achieve an approximately zero net equity market exposure
Event Driven Strategy	Capture price movements caused by a merger, corporate restructuring, reorganization, and bankruptcy
Fixed Income MBS	Attempts to take advantage of mispricing opportunities among different types of mortgage backed fixed income securities while neutralizing exposure to interest rate and/or credit risk
Fixed Income Arbitrage	Benefit from price anomalies between related interest rate securities while neutralizing exposure to interest rate risk
Global Macro	Employs long and short strategies in anywhere a value opportunity exists. Manages use leverage and derivatives to enhance positions
Merger Arbitrage	Capture the price spread between current market prices of securities and their value upon successful completion of a takeover, mergers, spin-off or similar transaction
Sector	Specializing in securities from particular industries or economic sectors
FOF index	Reflects the median performance of all hedge fund of funds managers

Table 1-2 presents the descriptive statistics for the monthly and weekly returns of the indices used in the study. The results in the table show that the empirical distributions are a bit negatively skewed except for two styles, Fixed Income MBS and Global Macro, which means that data are left skewed. Each hedge fund index exhibits a fat tail since excess kurtosis is greater than zero. The positive excess Kurtosis (Leptokurtic) has a higher peak and heavier tails than the normal distribution. Ignoring kurtosis tends to understate the risk of the variables with heavy tails. The AD test for normality evaluates departures from the normal distribution. The AD test statistics for all cases except for Equity Market Neutral yield p-values smaller than 0.05 (i.e., at the 5% significance level) which are significant. Therefore, the AD tests generally reject the null hypothesis of normality.

Table 1-2 Descriptive Statistics of Monthly hedge fund returns by strategy class

Variable	Mean	Std.dev	Skewness	Kurtosis*	ADtest**
Panel A: CISDM Hedge Fund Indices: Monthly Returns					
Distressed Securities	0.0101	0.0126	-1.2813	7.6614	1.9997 (0.0000)
Emerging Markets	0.0097	0.0348	-2.7491	19.7382	4.0667 (0.0000)
Equity Long/Short	0.0105	0.0172	-0.1528	2.5962	1.1302 (0.0058)
Equity Market Neutral	0.0067	0.0059	-0.0092	0.7495	0.4991 (0.2098)
Event Driven Strategy	0.0106	0.0133	-1.3146	6.9203	1.2989 (0.0022)
Fixed Income MBS	0.0088	0.0062	2.2472	16.1653	2.6272 (0.0000)
Fixed Income Arbitrage	0.0072	0.0043	-1.5073	8.0541	1.9002 (0.0000)
Global Macro	0.0077	0.0117	0.6037	2.0096	0.9848 (0.0134)
Merger Arbitrage	0.0074	0.0075	-1.0025	3.2003	1.2767 (0.0025)
Sector	0.0128	0.0287	-0.0553	6.3568	1.8887 (0.0000)
FOF index	0.0075	0.0111	-0.1727	2.1814	1.2379 (0.0031)
EW Portfolio	0.0092	0.0114	-1.4285	9.2145	1.2454 (0.0032)
Panel B: HFRX Global Hedge Fund Index: Weekly Returns					
HFR index	0.0512	0.7221	-2.3519	12.9135	11.3847(0.0000)

Note: Data source: CISDM for the monthly returns and HFR for the weekly returns. *Excess kurtosis relative to a normal distribution. **Anderson-Darling test for normality. P-values appear in parentheses by AD test statistics. Panel A in the table presents descriptive statistics of monthly returns of 10 hedge fund indexes, the fund of fund index and an equally weighted portfolio comprising the given 10 indexes from June 1992 to July 2007. Panel B reports descriptive statistics of weekly returns of constituent hedge fund indexes with 405 observations from 4/11/2003 to 12/31/2010.

The preceding results implicitly require the unconditional moments of the data to exist. For our purposes, we assume those processes under consideration are ergodic for the first four moments. For example, a process is said to be ergodic (p.46-47, Hamilton 1994) for the mean if the autocovariance γ_j goes to zero as j increases

$$\sum_{j=0}^{\infty}|\gamma_j| < \infty \tag{6-1}$$

A process is said to be ergodic for the second moments if

$$\frac{1}{T-j}\sum_{t=j+1}^{T}(Y_t - \mu)(Y_{t-j} - \mu) \xrightarrow{p} \gamma_j \text{ for all } j \tag{6-2}$$

where T is a sample of size, Y_t is some random variable, and μ is a constant. As a special case, when a Gaussian process is stationary, the condition in Equation (1-26) ensures ergodicity for all moments. By assuming that the third and fourth moments are ergodic, we can use those summary statistics reported in Table 1-2. If the population moments are infinite or undefined, the sample moments still provide useful information.

A moving window of fixed length (e.g., 60 months or 200 weeks) can be used to estimate the model's stability. For each month or week, the prediction is updated by adding one observation forward and dropping the first observation. So we can keep the number of observations in each window the same while updating the sample as new data becomes available.

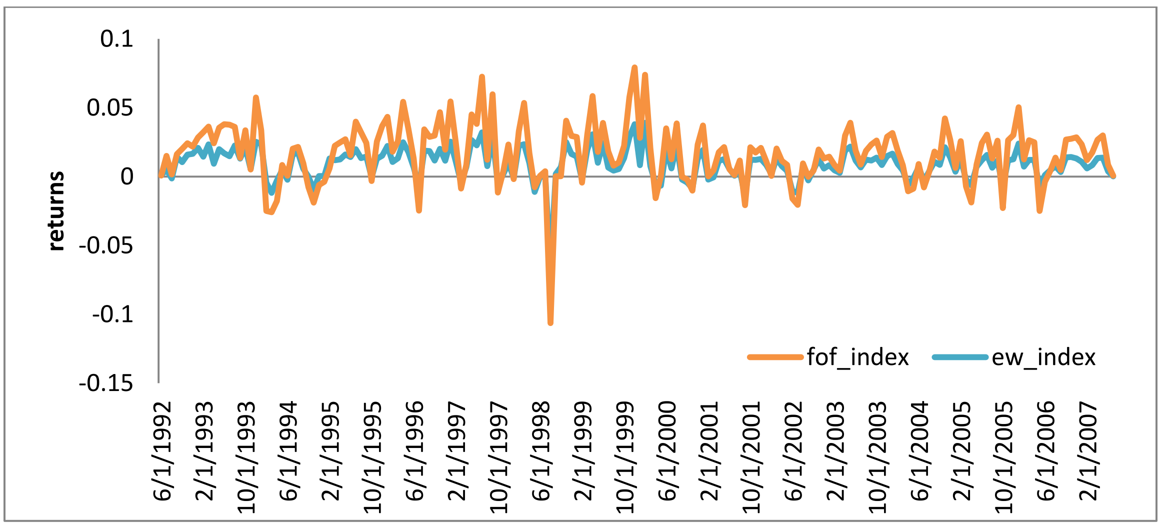
For the monthly returns, we have 182 monthly data points that range from June 1992 to July 2007, which are used to estimate the rolling one-month VaR and CVaR. For the weekly returns, only Net Asset Value (NAV) is available in this case. The NAV data can be used as a proxy to determine the price at which investors enter or exit a hedge fund. The weekly data is collected from Bloomberg, and the HFRX Global hedge fund index[5] is used for the weekly returns. The data ranges from April 11, 2003 to December 31, 2010, which is 405 observations. The HFRX global hedge fund index is designed to be representative of the overall composition of the hedge fund universe. It consists of eligible hedge fund strategies, including convertible arbitrage, distressed securities, equity hedge, equity market neutral, event driven, macro, merger arbitrage, and relative value arbitrage. The fixed length for every moving window is 200 weeks, which is equivalent to about four years.

[5] The index NAV is 1000 at inception.

Figure 6-1 plots the observed (n=182) monthly returns of an equally-weighted hedge fund portfolio comprising 10 hedge fund strategies and a FOF index. Both returns move in a similar way and appear to exhibit dependence in the volatility. The changing environment is closely related to the hedge fund performance. The return series are affected by the financial crises in the past. For example, the negative returns around 1994 may be due to the collapse of the Mexican financial markets. Thereafter, there is the 1997 Asian crisis followed by the Russian crisis in August 1998.

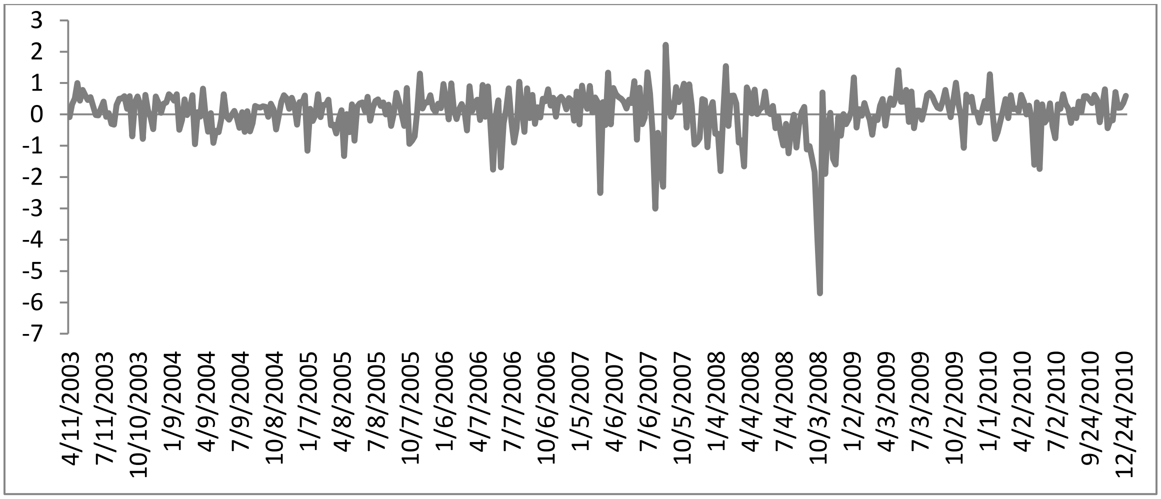
During such economic turmoils, hedge funds may suffer big losses from the unusual market events. One of the most tragic outcomes was the collapse of Long Term Capital Management (LCTM) in 1998, and the sharp drops in returns happen in June 1998. When the US stock market experienced the Dot-Com crash in January 2001, the hedge fund returns plummet, as well.

Figure 6-1 Monthly returns of equally-weighted hedge fund and Fund of Fund index



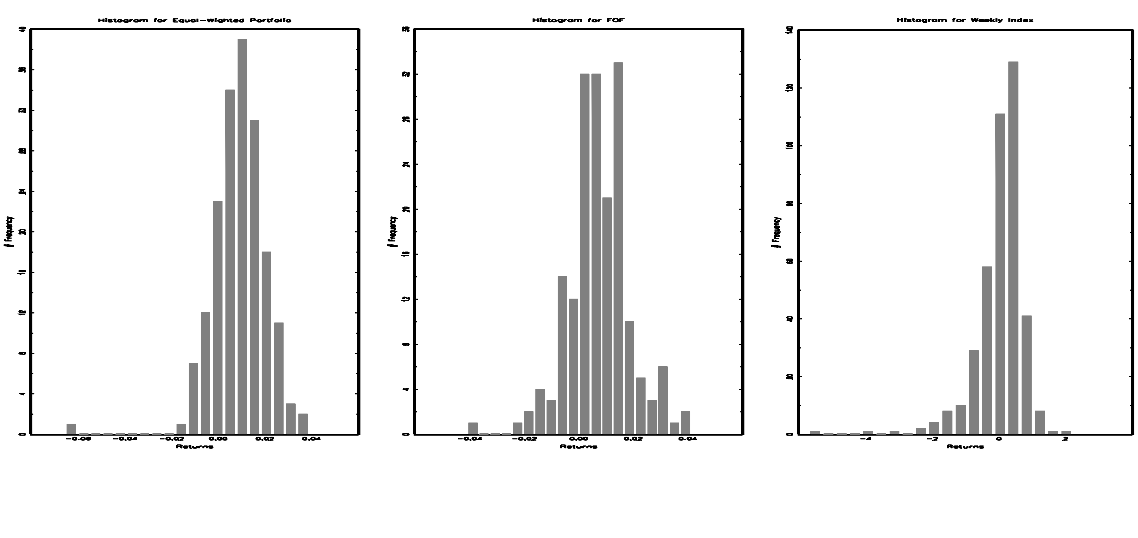
The returns for the weekly data are plotted in Figure 6-2, and several negative returns are realized in 2006 and 2007. Over the period, housing prices start falling and they may affect hedge funds' performance if the hedge funds are involved with subprime mortgages-backed securities. Also, the investment bank Lehman Brothers, which is one of the largest U.S. firms in the industry, declared bankruptcy on September 15, 2008 due to large losses on mortgage-backed securities.

Figure 6-2 Weekly returns of HFRX Global hedge fund index



In Figure 6-3, the histograms of the Equally-Weighted monthly hedge fund portfolio, the FOF index, and the weekly hedge fund returns are presented. They are useful for visualizing how the data are distributed. All three histograms indicate that the data are perhaps not normally distributed because the distributions are peaked in the center and there are a few extreme data points on the left of the distribution, which might suggest fat tails on at least the left-hand side.

Figure 6-3 Histograms of Equally-Weighted, FOF and Weekly Returns



6.1 Estimation Results for Monthly Returns

In this section, the time varying estimates of the risk measures are reported. For the monthly returns, 60 observations are selected as the fixed window length. For visual illustration, we take absolute values of the original risk measures and scale them. The results from both the equal- weighted hedge fund portfolio and the FOF index are shown in Figure 6-4 and Figure 6-5. The upper panel in both figures is associated with the equal-weighted hedge fund portfolio, and the lower panel is the FOF index. The estimates based on the normal distribution assumption are in Figure 6-4 and the outcome under the EVT-GP model is presented in Figure 6-5. For every graph in both figures, the CVaR measures are above the VaR estimates, which are expected since the CVaR measure is the average of exceedances above VaR.

Figure 6-4 Normal distribution based time varying risk measures

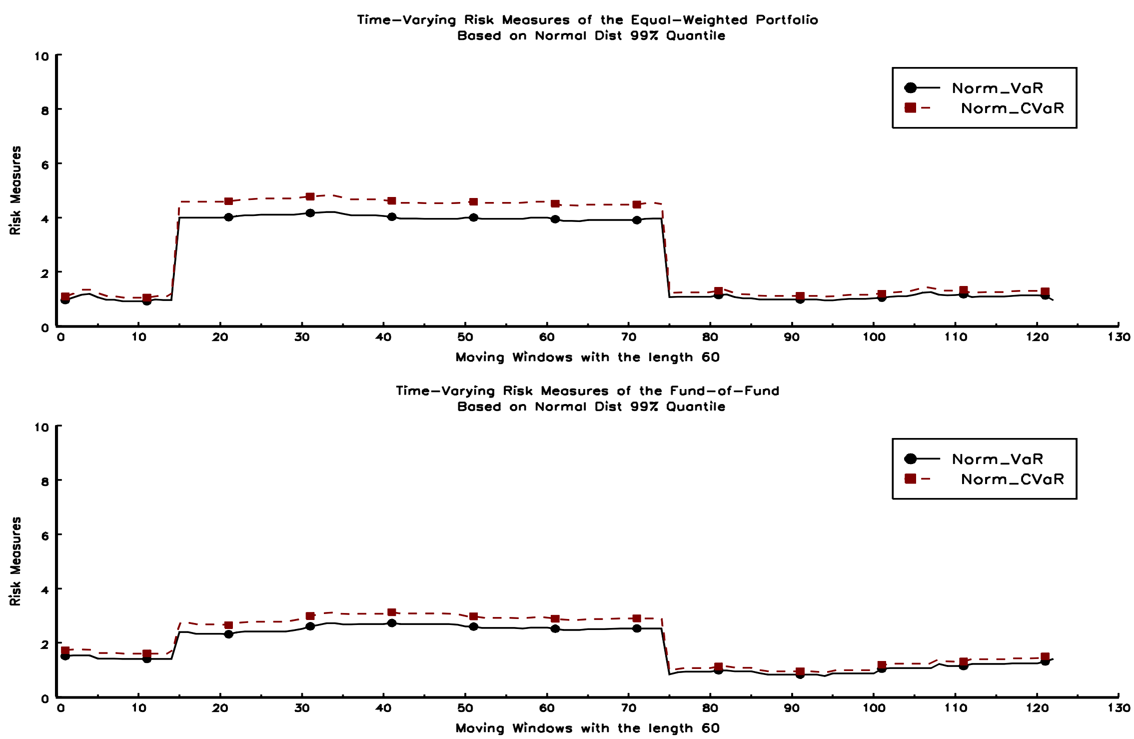
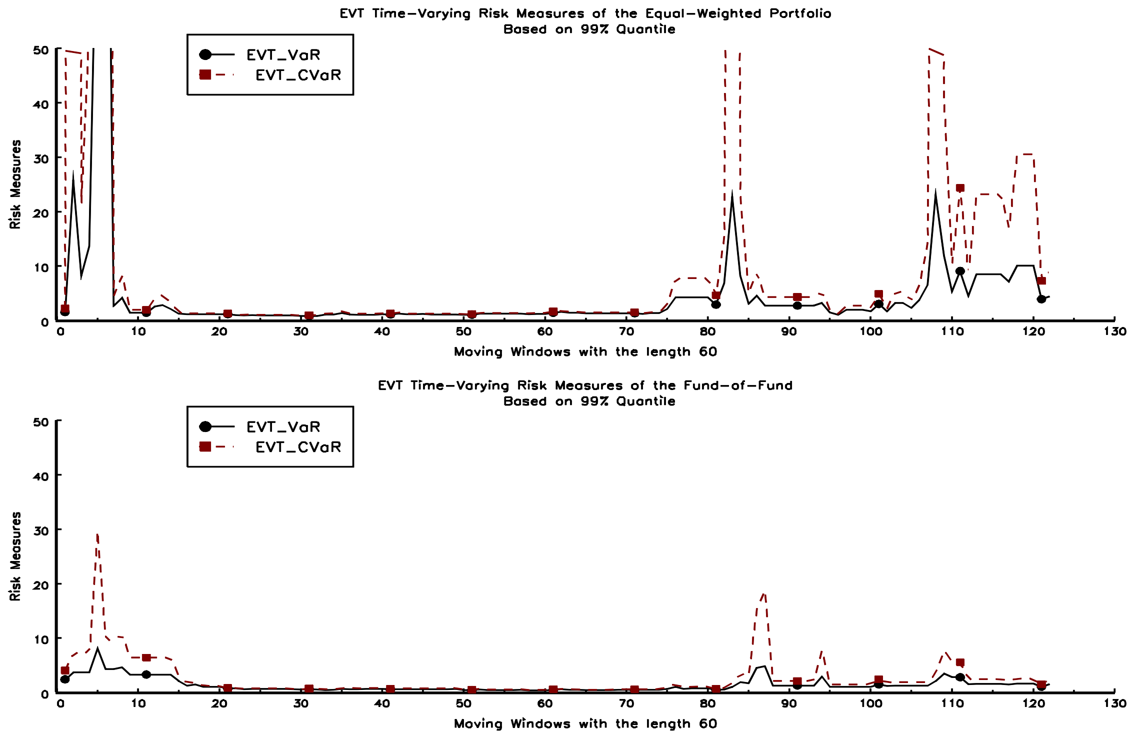


Figure 6-5 EVT-GP based time varying risk measures



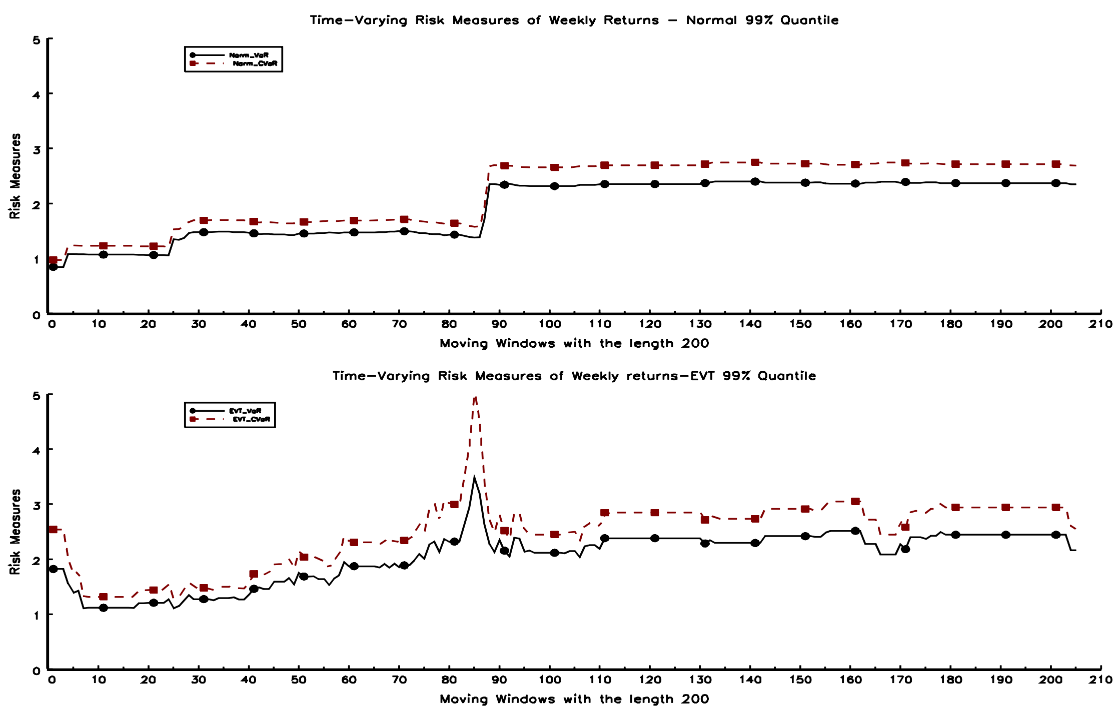
Under the normality assumption about exceedances, the estimates of the risk measures demonstrate persistent risk until July 31, 1998, when the estimates suddenly increase and then remain at the same level until July 31, 2003. After that period, the estimates for risk measures return to the original level. The results for both return series under the normality assumption only capture one standout event, the collapse of Long Term Capital Management that occurred in June 1998. In other words, this approach fails to capture several economic distresses that happened at different points in time.

In contrast, the time varying risk measures based on the EVT-GP model capture the bad performance that scatters at several points in time. For the first 15 periods, the estimated risk measures show the Asian crisis in 1997 and the LTCM debacle and Russian default in 1998. Then there are two incidents where the risk estimates are high in June 2004 and May 2006. Whereas it is not possible to detect the sharp increase in risk under the normality assumption, the EVT based dynamic risk measures are more sensitive to events in the observed sample.

6.2 Estimation Results for Weekly Returns

In Figure 6-6, the upper panel shows the estimates of VaR and CVaR based on the normal distribution. The lower panel represents the estimates based on the GP model. Both risk measures move together while the estimate of CVaR is consistently above VaR estimate, as expected. Over time those estimates are steadily increasing in both panels. However, the pattern is a little bit different. For the normal distribution case, the estimated risk measures exhibit three break points in time. For example, the third break around September 2008 reflects the collapse of Lehman Brothers. The GP based results show a slightly different time varying pattern in the estimates of VaR and CVaR. For the first few time periods, the short and quick decreases in the risk measure estimates are followed by a persistently increasing pattern over time that peaks in September 2008.

Figure 6-6 Time varying risk measures with the weekly returns



7 Conclusion

Financial risk management relies mainly on the ability to accurately compute the magnitudes and probabilities of large losses due to extreme events, including various economic distresses. In the literature, it is well known that traditional parametric and non-parametric methodologies have limitations to capture such extreme events. In this paper, we have illustrated the methods of extreme value theory by modeling the tail behavior of a loss distribution while considering tail related risk measures such as VaR and CVaR. The results show that the improved EVT-GP risk model more easily identifies the past global financial crises than the normal model. Although VaR is still widely used under the normal assumption, new tools such as the EVT/CVaR may become more important as investment firms look for better ways to gauge and manage the market risk.

References

Ackerman, Carl, Richard McEnally, and David Ravenscraft. 1999. The Performance of Hedge Funds. *Journal of Finance* 54, no. 3 (July): 833-874.

Agarwal, Vikas, and Narayan Y. Naik. 2000. Multi-Period Performance Persistence Analysis of Hedge Funds. *Journal of Financial and Quantitative Analysis* 35, no. 3 (September): 327-342.

Agarwal, Vikas, and Narayan Y. Naik. 2004. Risks and Portfolio Decisions Involving Hedge Funds. *Review of Financial Studies* 17, no. 1 (Spring): 63-98.

Amin, Gaurav S., and Harry M. Kat. 2003. Hedge Fund Performance 1999-2000: Do the Money Machines Really Add Value? *Journal of Financial and Quantitative Analysis* 38, no. 2 (June): 251-274.

Artzner, Philippe, Freddy Delbaen, Jean-Marc Eber, and David Heath. 1999. Coherent Measures of Risk. *Mathematical Finance* 9, no. 3 (July): 203-228.

Ashkar, F., and C. Nwentsa Tatsambon. 2007. Revisiting some estimation methods for the generalized Pareto distribution. *Journal of Hydrology* 346, no. 3-4 (November): 136-143.

Bacmann, Jean-François, and Gregor Gawron. 2005. Fat-Tail Risk in Portfolios of Hedge Funds and Traditional Investments. In *Hedge Funds: Insights in Performance Measurement, Risk Analysis, and Porfolio Allocation*, by Greg N. Fregoriou, Georges Hubner and and Fabrice Rouah Nicolas Papageorgiou. New Jersey: John Wiley & Sons.

Blum, Peter, Michel M. Dacorogna, and Lars Jaeger. 2004. Performance and Risk Measurement Challenges for Hedge Funds: Empirical Considerations. In *The New Generation of Risk Management for Hedge Funds and Provacy Equity Investments*, by Lars Jaeger. New York: Euromoney Trading Ltd.

Boudt, Kris, Brian Peterson, and Christophe Croux. 2008/2009. Estimation and Decomposition of Downside Risk for Portfolios with Non-normal Returns. *Journal of Risk* 11, no. 2 (Winter): 79-103.

Brooks, Chris, and Harry M. Kat. 2002. The Statistical Properties of Hedge Fund Index Returns and Their Implications for Investors. *Journal of Alternative Investments* 5, no. 2 (Fall): 26-44.

Brown, Stephen J., William N. Goetzmann, and Roger G. Ibbotson. 1999. Offshore Hedge Funds: Survival and Performance, 1989-95. *Journal of Business* 72, no. 1 (January): 91-117.

Castillo, Enrique, and Ali S. Hadi. 1997. Fitting the Generalized Pareto Distribution to Data. *Journal of the American Statistical Association* 92, no.440 (Dec): 1609-1620.

Christoffersen, Peter F., and Francis X. Diebold. 2000. How Relevant is Volatility Forecasting for Financial Risk Management. *Review of Economics and Statistics* 82, no. 1 (February): 1-11.

Danielsson, Jon, and Casper G. de Vries. 2000. Value-at-Risk and Extreme Returns. *Annales D'econonie et de Statistique* 60: 239-270.

Danielsson, Jon, and Jean-Pierre Zigrand. 2007. Regulating Hedge Funds. *Financial Stability Review*, no. 10 (April): 29-36.

Danielsson, Jon, Paul Embrechts, Charles Goodhart, Con Keating, Felix Muennich, Olivier Renault, and Hyun Song Shin. 2001. *An Academic Response to Basel II*. LSE Financial Markets Group.

Dupuis, D.J. 1996. Estimating the Probability of Obtaining Nonfeasible Parameter Estimates of the Generalized Pareto Distribution. *Journal of Statistical Computation and Simulation* 54: 197-209.

Embrechts, Paul, Claudia Klüppelberg, and Thomas Mikosch. 1997. *Modelling Extremal Events for Insurance and Finance*. Berlin: Springer Verlag.

Fernandez, Vivian. 2003. Extreme Value Theory and Value at Risk. *Revista de Analisis Economico* 18, no. 1 (June): 57-85.

Fung, William, and David A. Hsieh. 1997. Empirical Characteristics of Dynamic Trading Strategies: The Case of Hedge Funds. *Review of Financial Studies* 10, no. 2: 275-302.

Fung, William, and David A. Hsieh. 2000. Performance Characteristics of Hedge Funds and Commodity Funds: Natural vs. Spurious Biases. *Journal of Quantitative and Financial Analysis* 35, no. 3: 291-307.

Fung, William, and David A. Hsieh. 2001. The Risk in Hedge Fund Strategies: Theory and Evidence from Trend Followers. *Review of Financial Studies* 14, no. 2 (Summer): 313-341.

Geman, Hélyette, and Cécile Kharoubi. 2003. Hedge funds revisited: distributional characteristics, dependence structure and diversification. *Journal of Risk* 5, no. 4 (Summer): 55-73.

Gilli, Manfred, and Evis Këllezi. 2006. An Application of Extreme Value Theory for Measuring Financial Risk. *Computational Economics* 27, no. 2-3 (May): 207-228.

Gupta, Anurag, and Bing Liang. 2005. Do hedge funds have enough capital? A Value-at-Risk approach. *Journal of Financial Economics* 77, no. 1 (July): 219-253.

Hamilton, James D. 1994. *Time Series Analysis*. New Jersey: Princeton University Press.

Harmantzis, Fotios C., Linyan Miao, and Yifan Chien. 2006. Empirical Study of Value-at-Risk and Expected Shortfall Models with Heavy Tails. *Journal of Risk Finance* 7, no. 2: 117-135.

Holton, Glyn A. 2004. Defining Risk. *Financial Analysts Journal* 60, no. 6 (Nov/Dec): 19-25.

Hosking, J.R.M, and J.R. Wallis. 1987. Parameter and Quantile Estimation for the Generalized Pareto Distribution. *Technometrics* 29, no. 3 (August): 339-349.

J.P. Morgan and Reuters. 1996. *RiskMetrics-Technical Document*. New York: Morgan Guaranty Trust Company.

Jorion, Philippe. 2000. Risk Management Lessons from Long-Term Capital Management. *European Financial Management* 6, no. 3: 277-300.

Ju, Xionwei, and Neil D. Pearson. 1999. Using Value-at-Risk to Control Risk Taking: How Wrong can you be? *Journal of Risk* 1, no. 2 (Winter): 5-36.

Lhabitant, François-Serge. 2003. Hedge Funds: A Look beyond the Sample. In *Hedge Funds: Strategies, Risk Assessment, and Returns*, by Greg N. Gregoriou and and Vassilios N. Karavas Fabrice Rouah, 209-234. Washington D.C.: Beard Books.

Liang, Bing. 2000. Hedge Funds: The Living and the Dead. *Journal of Financial and Quantitative Analysis* 35, no. 3 (September): 309-326.

Lo, Andrew W. 2009. Regulatory Reform in the Wake of the Financial Crisis of 2007-2008. *Journal of Financial Economic Policy* 1, no.1: 4-43.

Lo, Andrew W. 2008. *Hedge Funds: An Analytic Perspective*. Princeton: Princeton University Press.

Longin, François M. 2000. From value at risk to stress testing: The extreme value approach. *Journal of Banking & Finance* 24, no. 7 (July): 1097-1130.

Longin, François M. 1996. The Asymptotic Distribution of Extreme Stock Market Returns. *Journal of Business* 69, no. 3 (July): 383-408.

Maddala, G.S. 1983. *Limited-Dependent and Qualitative Variables in Econometrics*. Cambridge: Cambridge University Press.

McNeil, Alexander J. 1997. Estimating the tails of loss severity distributions using extreme value theroy. *ASTIN Bulletin* 27, no. 1 (May): 117-138.

McNeil, Alexander J., and Rudiger Frey. 2000. Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. *Journal of Empirical Finance* 7, no. 3-4 (November): 271-300.

McNeil, Alexander J., Rudiger Frey, and Paul Embrechts. 2005. *Quantitative Risk Management: Concepts, Techniques and Tools.* New Jersey: Princeton University Press.

Pickands, James III. 1975. Statistical Inference Using Extreme Value Order Statistics. *Annals of Statistics* 3, no. 1 (January): 119-131.

Rachev, Svetlozar T., Stoyan V. Stoyanov, and Frank J. Fabozzi. 2008. Risk and Uncertainty. In *Advanced Stochastic Models, Risk Assessment, and Portfolio Optimization*, 171-205. New Jersey: John Wiley & Sons.

Reiss, R.-D., and M. Thomas. 1997. *Statistical Analysis of Extreme Values with Applications to Insurance, Finance, Hydrology and Other Fields*. Basel: Birkhäuser Verlag.

Renshaw, Edward F. 1977. Short Selling and Financial Arbitrage. *Financial Analysts Journal* 33, no. 1 (Jan-Feb): 58-65.

Schneeweis, Thomas, Richard Sputgin, and David McCarthy. 1996. Survivor Bias in Commodity Trading Advisor Performance. *Journal of Futures Markets* 16, no. 7 (October): 757-772.

Taleb, Nassim. 1997. The Jorion-Taleb Debate. *Derivatives Strategy* 2, no.4 (April).

U.S. Securities and Exchange Commission. 2003. Implications of the Growth of the Hedge Funds. Washington, D.C..