

# Haircut, Interest Rate, and Collateral Quality in the Repo Market: Evidence and Theory\*

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## Abstract

Leveraging the universe of transaction-level data from the Korean tri-party repo market, we examine the determinants of a haircut and an interest rate in a repo contract with particular attention to their joint determination. First, both the haircut and the interest rate increase with the riskiness of collateral, resulting in a seemingly positive relationship between the two. Second, when fully controlling for collateral risk, the negative relationship between the haircut and the interest rate is revealed. A 10bp increase in the interest rate is associated with a 0.13pp reduction in the haircut. Third, this elasticity of the haircut decreases in market uncertainty, suggesting that the relative importance of the haircut in the negotiation of repo terms increases when a concern for default risk is larger. We develop a theory of collateralized debt featuring collateral risk, incentives to acquire information about collateral, and opportunistic default, which can jointly account for the three main findings.

**JEL Classification:** D53, D8, E44, G23

**Keywords:** Repo market, haircut, interest rate, collateralized debt, collateral quality, costly information acquisition

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# 1 Introduction

Repurchase agreements (repos) serve as vital instruments in short-term financing, utilizing collateral to mitigate risks and facilitate transactions. However, the global financial crisis illustrated the potential vulnerabilities within the repo market as a form of a sudden increase in haircuts and a run on repo, emphasizing its systemic importance and the associated risks to the broader financial system (Gorton and Metrick (2012), Krishnamurthy et al. (2014), Martin et al. (2014)). Despite its significance, the functioning of repo markets remains opaque because of the OTC (over-the-counter) nature of the market, hindering comprehensive and timely empirical investigations. This is in sharp contrast to a large theoretical literature on the understanding of contract terms in collateralized loans.<sup>1</sup> Moreover, most empirical studies using data on repo contract terms are focused on the U.S. repo market and to a lesser extent the European repo market, which calls for more studies on other markets to obtain external validity.<sup>2</sup>

This study deepens our understanding of the repo market both empirically and theoretically. Empirically, we leverage a dataset comprising seven million contract-level observations from the Korean tri-party repo market, providing insights into how haircuts, interest rates, and collateral quality interact with each other.<sup>3</sup> Other than documenting the determinants of haircuts and interest rates in the Korean repo market, we pay particular attention to how they are jointly determined. Despite the growing literature on what determines haircuts in the repo market, most studies have not considered a repo rate as a determinant of the size of a haircut (e.g., Gottardi et al. (2019), Hu et al. (2021), Julliard et al. (2022)). This is surprising given that borrowers and lenders negotiate both the haircut and the interest rate given various types of risk (Dang et al. (2013), Barsky et al. (2016),

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<sup>1</sup>See Bester (1985), Dang et al. (2013), Simsek (2013), Antinolfi et al. (2015), Fostel and Geanakoplos (2015), Barsky et al. (2016), and Gottardi et al. (2019) for example.

<sup>2</sup>Notable exceptions are Julliard et al. (2022) for the United Kingdom and Suzuki and Sasamoto (2022) for Japan.

<sup>3</sup>Here, haircuts are the percentage difference between the collateral value and loan size. For example, suppose an agent borrows \$80 million and pledges \$100 million of bonds, then, the haircut is 20% [= (100-80)/100].

and [Liu and Xie \(2023\)](#)).

Moreover, even the limited set of studies has not reached a consensus on the relationship between haircuts and interest rates. For example, using comprehensive transaction data in the Japanese tri-repo market, [Suzuki and Sasamoto \(2022\)](#) found a positive relationship between the two. Similarly, using bilateral repo transactions of 47 major European banks, [Barbiero et al. \(2024\)](#) found that a repo contract with a positive haircut tends to have a higher interest rate than a contract without a haircut, although this result is not statistically significant. In contrast, [Baklanova et al. \(2019\)](#) and [Auh and Landoni \(2022\)](#) documented some preliminary evidence on the negative relationship between the haircut and the interest rate using rather limited data on the U.S. bilateral repo market.<sup>4</sup> Importantly, none of these studies has investigated how the joint determination of the haircut and the interest rate varies over time.

Compared to the existing studies on the relationship between the haircut and the interest rate in a repo contract, our analysis is much more comprehensive in both cross-sectional and time-series dimensions. We leverage the universe of the daily Korean repo contracts from 2015 to 2020. The dataset contains detailed information about contract terms, including the loan rate, haircut, loan sizes, maturity, and types of borrowers and lenders. Especially, our data also contains a unique identifier (International Securities Identification Number, ISIN) for collateral in every contract, which can be further merged into an independent security-level database. This unique information turns out to be the key to revealing the negative relationship between the haircut and the interest rate, as will be explained later.

We conduct a series of regression analyses to shed light on the equilibrium relationship between the haircut and the interest rate. First, we regress the transaction-level haircut and repo spread on standard determinants found in the literature, such as a proxy for

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<sup>4</sup>The results in [Baklanova et al. \(2019\)](#) are obtained from only three-day snaps of repo data from nine U.S. bank holding companies and the findings of [Auh and Landoni \(2022\)](#) are limited to proprietary data from a single borrower.

collateral risk, counterparty risk, and market risk. The estimation results largely confirm the previous findings in the literature that both haircuts and spreads increase in the proxies for collateral risk. We then regress the haircut on the repo spread (the spread between the repo rate and the inter-bank rate of the same maturity) in addition to a set of standard control variables. To sharpen identification and understand the source of variation in this equilibrium relationship, we flexibly introduce a different set of fixed effects as a proxy for various types of risk.<sup>5</sup>

Our main empirical findings are threefold. First, there is a seemingly positive relationship between the haircut and the interest rate when the quality of the underlying collateral is not properly controlled. In other words, a repo contract with riskier collateral tends to have both a higher haircut and a higher interest rate, which is consistent with the findings in [Suzuki and Sasamoto \(2022\)](#) and [Barbiero et al. \(2024\)](#). The positive relationship continues to hold even after absorbing various fixed effects, including maturity, borrower type, lender type, collateral type, and time.

Second, once collateral risk is fully absorbed by exploiting data on unique collateral identifiers (i.e., controlling for collateral-fixed effects), a negative association between the haircut and the interest rate emerges. The negative relationship we revealed is indeed consistent with [Baklanova et al. \(2019\)](#) and [Auh and Landoni \(2022\)](#) who exploited the collateral-level information (CUSIP number) as we did. However, our data is much more comprehensive, and therefore, provides clearer evidence on the negative relationship between the haircut and the interest rate. For example, we find that this relationship is not only statistically significant but economically meaningful, unlike the economically insignificant relationship found in [Baklanova et al. \(2019\)](#). A 10bp increase in the spread (corresponding to about one standard deviation of the repo spread in our sample) is related to a 0.13pp decline in the haircut. Moreover, a sequential absorption of various fixed

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<sup>5</sup>Throughout the paper, we use interest rates and spreads interchangeably for two reasons. First, all our empirical results are nearly identical whether using interest rates or spreads given the presence of time-fixed effects. Second, interest rates and spreads are the same in the model.

effects sheds light on why previous studies found mixed results. Failing to account for collateral risk in a repo contract due to the absence of precise security-level information resulted in a misguided relationship between the haircut and the interest rate.

Third, we find that market uncertainty or economic downturn has significant influence on the relationship between the haircut and the interest rate. Once we interact the spread with the implied stock market volatility in our preferred baseline regression conditioning on the same collateral, this interaction term turns out to be positive and highly statistically significant, dampening the substitution effect of the interest rate on the haircut. Similar results are obtained when measures of economic activity are used instead. Intuitively, as market uncertainty increases or economic activity falls, the overall default risk of the contract also increases. This, in turn, raises the relative importance of the haircut compared to the interest rate (i.e., changes in the haircut become less sensitive to changes in the interest rate) since the haircut is a direct instrument to insure lenders against a default state, thereby reducing the size of the coefficient of our interest.

To provide theoretical explanations for the three empirical findings, we build a model of collateralized debt characterized by two periods and two risk-neutral agents (a borrower and a lender). The borrower has liquidity needs in the first period and borrows goods from the lender, using assets as collateral. The asset provides stochastic dividends at the end of the second period, and information about the dividend state is revealed at the beginning of the second period. Both agents can acquire private information about the future value of assets in the first period at a cost before trading with each other.<sup>6</sup>

In the model, the borrower never acquires private information about the asset's quality and collateralized debt can be one of three types depending on the extent of the lender's incentives for information acquisition. First, when the information acquisition cost is sufficiently high, the lender has no incentive to acquire information about the future collat-

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<sup>6</sup>For example, an agent can procure analytical reports that assess a company's prospects with greater precision than conventional perceptions. This information can inform decisions regarding the purchase or sale of equity shares, or their use as collateral in secured loan contracts.

eral value, and thus collateralized debt is called fully information insensitive (fully IIS hereafter). Second, when the information acquisition cost is in the intermediate range, the lender does not produce information, but the incentive constraint that prevents information acquisition binds. Thus, collateralized debt is termed partially information insensitive (partially IIS hereafter). Third, if the acquisition cost is sufficiently low, then it is optimal to allow the lender to produce information, so collateralized debt is called information sensitive (IS hereafter).

Our model offers insights into the economic rationale behind the three empirical findings. First, in the model, as the collateral risk—represented by the mean-preserving spread of the distribution of the tree’s terminal values—increases, both interest rates and haircuts rise, thereby accounting for their unconditional positive relationship. Next, an important property of partially IIS debt is that interest rates and haircuts are simultaneously determined given the size of loans. Therefore, a rise in interest rates reduces haircuts, and vice versa. Thus, given the characteristics of agents and collateral assets, the negative relationship still emerges even when agents’ characteristics are allowed to change, as long as the risk of collateral assets is kept constant. This feature of collateralized loans provides a theoretical explanation for the second empirical finding.<sup>7</sup> Finally, an increase in the risk of collateral assets reduces the sensitivity of changes in the haircut to changes in the interest rate of partially IIS debt contracts, aligning with the third empirical finding.

The rest of the paper is organized as follows. Section 2 overviews the Korean repo market and presents the main empirical findings about how the haircut and the interest rate are jointly determined. A battery of robustness checks is also provided. Section 3

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<sup>7</sup>Bester (1985) and Kang (2021) show that interest rates and haircuts may respond differently in response to a change in a certain parameter in their models. For example, Bester (1985) shows that safe firms are willing to pledge more collateral (i.e., a higher haircut given the fixed loan size) to signal their quality and obtain lower interest rates, driving a negative correlation between interest rates and haircuts. Kang (2021) shows that the interest rate increases and the haircut falls as the cost of faking the quality of collateral assets increases. However, in these models, the type of borrowers or the property of collateral assets is not controlled. However, in contrast to our model, within a group of borrowers with the same default risk using the same collateral assets, there is no negative correlation between the interest rate and the haircut in their models. One of the main contributions of our study is to obtain the substitution effect between interest rates and haircuts, even after controlling for all other confounding factors.

builds a theoretical model of collateralized debt to explain the main empirical findings. All omitted proofs are provided in Appendix B. Section 4 concludes the paper.

## 2 Empirical Evidence

In this section, we present a series of empirical findings pertaining to repo contracts, drawing upon the universe of transaction-level data from the Korean tri-party repo market. The dataset encompasses detailed information on each repo contract, including the loan rate, haircut, loan size, maturity, a unique identifier for collateral (International Securities Identification Numbers), and type of cash borrowers (security sellers) and cash lenders (security buyers). This extensive dataset, available on a daily basis, enables us to effectively control for various characteristics in loan contracts, thereby shedding light on the determinants of haircuts and interest rates and the relationship between the two. In particular, information about a unique identifier for all collateral used in repo contracts creates an optimal environment where collateral risk can be fully controlled.

### 2.1 Data

#### 2.1.1 Market dynamics and regulatory framework

The Korean repo market comprises three primary categories: customer repo, institutional repo, and Bank of Korea (BOK) repo—designed for the conduct of monetary policy.<sup>8</sup> Our empirical analysis focuses on the tri-party institutional repo market in which the Korea Securities Depository (for over-the-counter trades) retains collateral until the maturity date. Over the years, the list of entities eligible to participate in the tri-party repo market steadily expanded, accompanied by a progressive relaxation of regulatory restrictions on

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<sup>8</sup>Customer repos involve transactions between financial institutions and general customers, while BOK repos are executed between the Bank of Korea and prime dealers as part of open market operations.

participants and trade terms in the repo market.<sup>9</sup>

A noteworthy regulatory development occurred in May 2015 when non-bank financial institutions were restricted from accessing the call market (Suh (2016)), which led to a significant inflow of short-term funds into the repo market, predominantly from security companies. Consequently, the money market underwent a restructuring with the repo market at its core, resulting in a consistent increase in both the number of repo contracts and the size of loans, which governs the beginning of our sample period.

In an effort to enhance market efficiency and risk management, the government implemented a policy in July 2020 mandating borrowers to retain a portion of repo trades in cash-equivalent assets. This requirement is contingent upon transaction maturity, with the proportion decreasing as the maturity extends. More importantly, the government issued guidelines for setting haircuts, taking into account collateral risk and borrower profiles. This notable structural break driven by government policy changes ends our sample under study. Thus, our baseline analysis spans from January 2015 to June 2020. For further details of institutional features of the Korean repo market, see Yun and Heijmans (2013)).

There are certain practices in the Korean tri-party repo market. For example, overnight repos continue to dominate the market, constituting over 95% of total transactions, which is higher than the share of overnight contracts (80%) in the U.S. tri-party repo market (Paddrik et al. (2021)). The haircut, representing the percentage by which the value of the collateral exceeds the loan value, is heavily clustered at 5%, irrespective of counterparties and collateral types. Our analysis focuses on the domestic currency (Won)-denominated repos only, as they account for over 99.99% of the tri-party institutional repo contracts.

Figure 1 illustrates the daily total loan size computed from the final sample used in regression analysis and the official daily trading volume obtained from the Korea Securities Depository. The monthly average is reported for both series. It is apparent that the trad-

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<sup>9</sup>For example, in December 2007, all restrictions on the types of underlying assets accepted as collateral in institutional repos were lifted.

ing volume computed from our final sample used in estimation closely tracks that in the official statistics. Consistent with the aforementioned narrative, the Korean repo market has experienced substantial growth since 2015, which governs the starting period of our sample. By the end of our sample period, the daily average loan size reached 90 trillion Won (about 70 billion USD). The corresponding figure for the U.S. tri-party repo market is about 2.3 trillion USD.

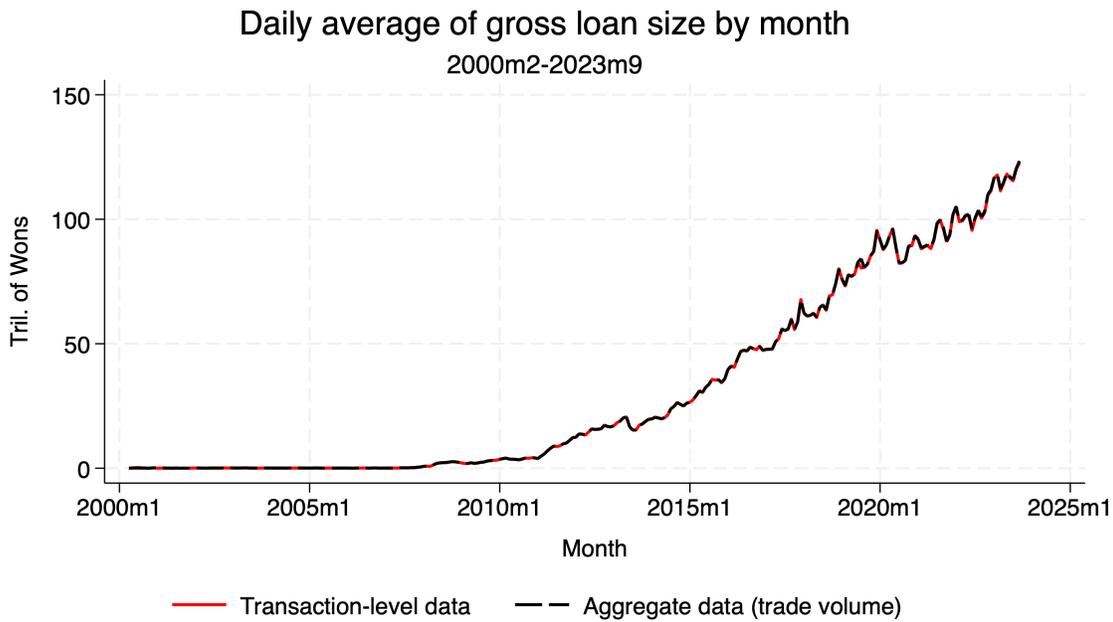


Figure 1: Size of the Korean repo market over time

### 2.1.2 Description of data

Table 1 provides a comprehensive summary of median values for a haircut, spread, loan size, and maturity, along with the number of observations and unique collateral identifiers for each collateral class. In our baseline analysis, we use spreads rather than loan rates to account for the level effect driven by confounding factors, such as monetary policy.<sup>10</sup> However, as will be shown later, both specifications deliver nearly identical results

<sup>10</sup>Loan spread is defined as the difference between the repo rate and the risk-free rate corresponding to the same maturity. Specifically, the repo rate is subtracted by the call rate, 1-week KORIBOR, 1-month

given the presence of time-fixed effects. Collateral classes are arranged in descending order based on the number of observations.

Table 1: Collateral class and loan terms

Collateral class	Haircut (%)	Spread (%)	Principal (bil. of wons)	Maturity (days)	Obs.	Unique ISINs
Government Bond	5	.05	23.6	1	3,035,368 (41.05%)	539
Bank Bond	5	.05	39.5	1	1,830,212 (24.75%)	3,784
Monetary Stabilization Bond	5	.04	28.5	1	694,515 (9.39%)	536
Special Bond	5	.04	37.2	1	693,176 (9.38%)	2,076
Financial Bond	5	.1	18.4	1	584,935 (7.91%)	4,806
Municipal Bond	5	.04	5.9	1	323,812 (4.38%)	1,537
Corporate Bond	5	.11	17	1	223,251 (3.02%)	4,993
ETF Security (else)	30	.38	47.8	7	4,045 (0.05%)	50
Equity	45	1.19	.98	30	3,598 (0.05%)	335
Unknown	5	.13	9.3	1	555 (0.01%)	27
All Collateral	5	.05	26	1	7,393,467	15,895

The sample period is 2015m1-2020m6. The table shows the median value of haircut, spread, principal, and maturity, as well as the number of observations and the unique identifier for collateral assets across various collateral classes. The percentages in the *obs* column represent the share of each collateral class in relation to the total number of observations.

Some observations merit attention. First, within the Korean tri-party repo market, a majority of contracts involve relatively safe assets as collateral, including government bonds, bank bonds, monetary stabilization bonds, and special bonds. These safe collateral classes collectively constitute 85% of all observations. Second, positive haircuts are prevalent across asset classes, even for government bonds. This is in sharp contrast to

KORIBOR, and 3-month KORIBOR for 1-day, 1-week, 1-month, and 3-month maturity, respectively. For other maturities (1-year and over-1year), the spread is constructed by subtracting the repo rate from the 1-year KORIBOR.

scarce haircuts in the European, U.K., and Japanese repo markets during normal times (see, for example, Barbiero et al. (2024), Julliard et al. (2022), and Suzuki and Sasamoto (2022), respectively) and makes the Korean repo market an ideal place to study the joint determination of the haircut and the interest rate in repo contract terms. Overall, both haircuts and spreads in our sample remain low with little variation, suggesting that this segment of money markets has been stable without much material risk during our sample period. This is consistent with the relative stability of tri-party repo markets compared to bilateral markets documented in the United States (Copeland et al. (2014) and Krishnamurthy et al. (2014)) and Europe (Mancini et al. (2016)). Table A-1 in Appendix A presents the same information only for the overnight maturity.

For a granular description of the Korean tri-party repo market, Table 2 and Table 3 present distinct facets of the market's dynamics. In Table 2, the distribution of the type of cash borrowers and cash lenders is delineated. Notably, security companies constitute the largest share of contracts (47.40%) as a cash borrower, followed by collective investment schemes. They are involved in over 80% of all observations as borrowers. On the cash lender side, banks (trusts) emerge prominently, accounting for 41.13%, with collective investment schemes as the second most frequent buyers at 26.07%. These two entities collectively contribute to approximately 67% of total contracts. Remarkably, ten out of the 14 types of investors assume dual roles as both a borrower and a lender.<sup>11</sup>

Turning to Table 3, an analysis of the top 10 pairs of investors based on loan size reveals heterogeneity in various aspects. There is substantial dispersion in the median spread and principal across pairs. Additionally, the most-used collateral type for contracts within each pair varies from 35% to 73%. It is also noteworthy that the most-used collateral types for all top 10 pairs are considered safe assets. An interesting observation from the table is that specialized credit finance companies (e.g., credit card companies, hire-purchase finance companies) borrow larger amounts of money, often with a negative spread and

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<sup>11</sup>See Tables A-2 and A-3 in Appendix A for the same statistics based on the number of contracts not the size of loans.

Table 2: Types of cash borrowers and cash lenders

Type of investors	Borrower type (%)	Lender type (%)
Securities company	47.40	3.10
Collective investment scheme	34.44	26.07
Specialized credit finance company	10.03	4.92
Securities company (trust)	3.20	10.07
Government	2.49	0.37
Domestic bank	2.24	7.24
Foreign bank	0.15	0.49
Insurance company	0.03	2.54
Foreigner	0.00	0.00
Pension funds and guarantee	0.00	4.00
Bank (trust)	.	41.13
ETC (trust)	.	0.07
Credit union and thrift institution	.	0.00
Other financial institutions	.	0.00

The sample period is 2015m1-2020m6. The table separately shows the share of each borrower type and lender type in all observations. The share is calculated based on loan size.

longer maturity. This distinctive behavior illustrates the unique financing characteristics of these entities within the repo market.

## 2.2 Empirical analysis

### 2.2.1 Regression framework

An individual repo contract as a unit of observation, the main regression specification takes the following form:

$$Haircut_{B,L,M,c,t} = \beta Spread_{B,L,M,c,t} + \gamma Size_{B,L,M,c,t} + \Upsilon_{B,L,M,c,t} + \epsilon_{B,L,M,c,t}, \quad (1)$$

where a tuple  $\{B, L, M, c, t\}$  consists of borrower type  $B$ , lender type  $L$ , maturity type  $M$ , unique collateral id  $c$ , and date of transaction  $t$ . Thus,  $Haircut_{B,L,M,c,t}$  is the haircut applied to the repo contract on date  $t$  between borrower type  $B$  and lender type  $L$  over maturity type  $M$  using specific collateral  $c$ .  $Spread_{B,L,M,c,t}$  denotes a spread for this contract defined above and  $Size_{B,L,M,c,t}$  is the log of loan sizes. Importantly,  $\Upsilon_{B,L,M,c,t}$  is a

Table 3: Top 10 investor pairs and loan terms

Investor pair		Most-used collateral type	Spread (%)	Haircut (%)	Principal (bil. of wons)	Maturity (days)	Share (%)
Borrower type	Lender type						
Securities company	Bank (trust)	Bank bond (49.93%)	.04	5	100	1	21.33
Collective investment scheme	Bank (trust)	Bank bond (48.96%)	.07	5	50	1	13.20
Securities company	Collective investment scheme	Government bond (54.98%)	.03	5	15	1	12.41
Collective investment scheme	Collective investment scheme	Government bond (36.95%)	.06	5	12.5	1	11.21
Specialized credit finance company	Securities company (trust)	Special bond (35.09%)	-.19	5	200	360	5.03
Specialized credit finance company	Bank (trust)	Bank bond (70.26%)	-.16	5	250	360	4.22
Securities company	Domestic bank	Government bond (72.55%)	.07	5	60	1	3.52
Collective investment scheme	Domestic bank	Government bond (57.48%)	.09	5	36.7	1	3.13
Securities company	Pension funds and guarantee	Government bond (39.79%)	.04	5	100	1	2.87
Securities company	Securities company (trust)	Government bond (34.74%)	.06	5	34.5	1	2.75

The sample period is 2015m1-2020m6. The table shows the median value of haircut, spread, principal, and maturity by investor pairs. The percentages in the *most-used collateral type* column are the share of repo contracts that the corresponding pair of investors agree to set a collateral of the corresponding most-used collateral type. All share in the table is calculated based on loan size.

vector of fixed effects absorbing systematic variation across different dimensions exploiting the unique advantage of our data explained below. Our main interest is the sign of  $\beta$ , which tells how the haircut and the interest rate are jointly determined in repo contract terms after controlling for both observed and unobserved confounding factors. If  $\beta < 0$ , then the haircut decreases in response to an increase in the repo spread.

In the baseline model, these fixed effects include the maturity type of a repo contract, borrower type, lender type, and unique identifier for collateral assets, which all interact with time-fixed effects. Thus, they allow us to control for a variety of individual repo contract characteristics that are not only constant but time-varying. They also capture time-variant components common to all repo contracts within a day, such as macroeconomic and financial market conditions and various policy actions. Note that we only have information on a type of borrower or lender, not exact entities. However, we do not view this as a major constraint given the focus of our paper on the average relationship between

the haircut and the interest rate prevalent in equilibrium. Standard errors are double clustered at the collateral level and date level but we also explore alternative clustering.

### 2.2.2 Determinants of repo trading terms

Before presenting the main findings regarding the relationship between haircuts and repo rates, we analyze the determinants of each of repo trading terms in the Korean tri-party repo market and check whether they are consistent with the previous findings obtained from other markets (e.g., [Hu et al. \(2021\)](#); [Auh and Landoni \(2022\)](#); [Julliard et al. \(2022\)](#); [Macchiavelli and Zhou \(2022\)](#); [Suzuki and Sasamoto \(2022\)](#)). To be specific, we estimate the following regression for both haircuts and spreads, in turn:

$$Term_{B,L,M,c,t} = \gamma Size_{B,L,M,c,t} + I_B \Upsilon_B + I_L \Upsilon_L + I_M \Upsilon_M + I_c \Upsilon_c + \tau_t + \epsilon_{B,L,M,c,t}, \quad (2)$$

where  $Term_{B,L,M,c,t}$  denotes either a haircut or a spread in a given contract.

The main difference from Equation 1 is that we present the coefficients on each categorical variable capturing the characteristics of a given repo contract instead of saturating them via fixed effects. Time-fixed effects are still absorbed to control for time-varying aggregate confounding factors, such as market risk. Moreover, since we have full information on the ISIN of the collateral posted in every repo contract, we can match the security-level information obtained from FnGuide (a provider of financial information services in Korea) to our transaction-level data.

Although security-level data from FnGuide only covers bonds, they account for about 95% of our transaction-level data. Exploiting this security-level information containing the issuance and maturity dates of each ISIN, we can not only classify each ISIN into precise asset types but track the remaining maturity of the underlying collateral at any point in time. This security-level information is used to capture the role of collateral risk (i.e., the type and remaining maturity of collateral) in repo trading terms studied in the

related literature (e.g., [Copeland et al. \(2014\)](#); [Nyborg \(2019\)](#); [Hu et al. \(2021\)](#); [Auh and Landoni \(2022\)](#)).

Table 4 summarizes the findings. The omitted baseline for each category is security companies and their trusts (for borrower type), domestic banks and their trusts (for lender type), overnight (for maturity), government bonds (for collateral class), remaining maturity less than or equal to 3 months (for remaining maturity of collateral).<sup>12</sup> Columns (1) and (2) report the results using haircuts as a dependent variable, whereas the dependent variable in Columns (3) and (4) is repo spreads. While Columns (1) and (3) employ the full sample, Columns (2) and (4) use the subsample that is matched with the security-level data from FnGuide. Since the matched sample with information on collateral assets is our preferred specification, we only discuss the results in Columns (2) and (4).

In this table, types of collateral assets as well as those of borrowers and lenders are presented in the decreasing order in their shares in total observations. As shown in Column (2), there appears no clear pattern in the size of the haircut across the types of the borrower and the lender. For example, the average haircut applied to the repo with government and domestic banks as a cash borrower is higher than that with securities companies, although the default risk of the former is arguably smaller than the latter. This finding implies two possibilities. First, what matters for a haircut is the individual borrower's default risk and the type of borrower is a poor proxy for this risk. For example, using the transaction-level data from the U.K. bilateral repo market, [Julliard et al. \(2022\)](#) found a significant role of borrower-level default risk measured by credit rating, leverage, or asset size in determining the size of haircut, whereas the average haircut is not systematically different across non-hedge fund borrower types. Second, counterparty risk is indeed not a major factor in determining a haircut in the tri-party repo market given its stability, especially during our sample period.

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<sup>12</sup>Here, we aggregate some of similar entities (e.g., banks and bank trusts) in each dimension of repo contract terms to streamline the analysis of haircut and interest rate determinants. In our main analysis in which the constellation of fixed effects is employed, we use the most disaggregated entities in each dimension to control for unobserved heterogeneity as cleanly as possible.

Table 4: Preliminary analysis: determinants of haircut and spread

	Haircut		Repo rate	
	(1)	(2)	(3)	(4)
Size	-0.00631 (-1.54)	0.00293*** (3.03)	-0.000704*** (-3.08)	-0.000340* (-1.94)
Borrower type (baseline: Securities company and its trust)				
Collective investment scheme	0.0855*** (7.12)	0.0579*** (16.70)	0.0177*** (25.78)	0.0172*** (26.47)
Government	0.0759*** (8.11)	0.0527*** (13.86)	0.0170*** (18.35)	0.0161*** (16.57)
Specialized credit finance company	0.0865*** (4.04)	0.124*** (10.94)	-0.0360*** (-8.79)	-0.0339*** (-7.90)
Domestic bank and domestic bank trust	0.220*** (13.68)	0.218*** (14.44)	-0.0703*** (-24.81)	-0.0689*** (-24.71)
Foreign bank	-0.0628 (-0.95)	0.0760*** (5.45)	-0.00199 (-0.61)	0.00273 (1.21)
Pension funds and insurance company	0.0833*** (6.98)	0.0574*** (6.92)	0.0116*** (3.38)	0.00899*** (2.92)
Lender type (baseline: Domestic bank and its trust)				
Collective investment scheme	-0.0309*** (-2.77)	-0.00368 (-1.62)	-0.0184*** (-20.15)	-0.0175*** (-20.60)
Security company and security company trust	-0.388*** (-14.29)	-0.380*** (-14.84)	0.0175*** (10.08)	0.0151*** (9.09)
Pension funds and insurance company	-0.00185 (-0.41)	0.00608* (1.81)	-0.0186*** (-21.87)	-0.0180*** (-21.28)
Specialized credit finance company	0.0550** (2.27)	-0.00857*** (-3.39)	-0.0174*** (-13.70)	-0.0194*** (-19.01)
Government	-0.0288*** (-3.84)	-0.0112* (-1.94)	-0.0163*** (-10.38)	-0.0155*** (-10.07)
Foreign bank	0.0240* (1.69)	0.0237* (1.78)	-0.0304*** (-10.28)	-0.0304*** (-10.19)
Maturity type (baseline: overnight)				
2-day to 1-week	0.0932** (2.45)	0.00671 (0.92)	0.00995*** (3.56)	0.00462** (2.28)
1-week to 1-month	0.540** (2.18)	0.0271*** (4.40)	0.0595*** (7.46)	0.0487*** (13.87)
1-month to 3-month	2.092** (2.23)	0.154*** (4.41)	0.0332 (0.68)	0.0490*** (6.49)
Over 3 months	0.246*** (7.19)	0.164*** (9.20)	0.0219*** (4.48)	0.0189*** (3.77)
Collateral type (baseline: Government bond)				
Bank bond	-0.00865 (-1.53)	0.000641 (0.09)	0.00486*** (7.79)	0.00933*** (9.11)
Monetary stabilization bond	0.0149** (2.08)	0.0213** (2.49)	0.00146* (1.95)	0.00621*** (5.27)
Special bond	-0.00380 (-0.29)	0.0201** (2.13)	0.00683*** (8.77)	0.00974*** (9.33)
Financial bond	0.0178 (1.45)	0.0251** (2.15)	0.0393*** (34.71)	0.0439*** (35.49)
Municipal bond	0.0558*** (9.12)	0.0367*** (8.63)	0.00440*** (5.65)	0.00489*** (6.19)
Corporate bond	0.0442** (2.19)	0.0511*** (2.68)	0.106*** (13.08)	0.0991*** (12.89)
Equity	38.40*** (61.04)		1.066*** (17.92)	
Remaining maturity type (base: ≤ 3-month)				
3-month to 6-month		0.0135*** (2.67)		0.00193*** (3.93)
6-month to 1-year		0.0172*** (2.64)		0.00171*** (3.10)
1-year to 2-year		0.0325*** (4.43)		0.00374*** (5.04)
2-year to 5-year		0.0406*** (4.83)		0.00961*** (8.05)
Over 5 years		0.0203* (1.68)		0.00840*** (5.06)
Date FE	Yes	Yes	Yes	Yes
Cluster Var.	Collateral + Date (17,249)	Collateral + Date (15,216)	Collateral + Date (17,249)	Collateral + Date (15,216)
N.Obs	7,393,437	7,005,499	7,393,437	7,005,499
Adj. R <sup>2</sup>	0.728	0.090	0.965	0.971
Sample	Full	FnGuide	Full	FnGuide

The sample period is 2015m1-2020m6. Contracts with an interest rate higher than 10% or with a haircut lower than 0% or higher than 200% are excluded. *t*-Statistics are reported in parentheses. \*, \*\*, and \*\*\* indicate statistical significance at the 10%, 5%, and 1% level, respectively. Columns (1) and (2) have *haircut* as the dependent variable in contrast to Columns (3) and (4) whose dependent variable is *spread*. Different from Columns (1) and (3) employing the full sample, Columns (2) and (4) use the subsample that is matched with the security information data from FnGuide. The numbers in parentheses in the cluster variable row are the number of clusters.

Regarding the role of contractual terms in determining a repo haircut, the haircut increases in the size of loans, which is consistent with [Suzuki and Sasamoto \(2022\)](#). It is apparent that a repo contract with longer maturity tends to have a higher haircut, which is consistent with the finding in [Julliard et al. \(2022\)](#) and [Suzuki and Sasamoto \(2022\)](#). Importantly, unlike borrower or lender types, the results related to collateral risk show a clear pattern. Haircuts increase with the perceived riskiness of collateral assets, which is consistent with the previous findings ([Gorton and Metrick \(2012\)](#); [Auh and Landoni \(2022\)](#); [Julliard et al. \(2022\)](#); [Suzuki and Sasamoto \(2022\)](#)). For example, compared to government bonds (presumably the safest asset in our sample), a repo contract using riskier asset classes is associated with a higher haircut. Similarly, the longer the remaining maturity of a collateral asset is, the higher haircut is observed as in [Suzuki and Sasamoto \(2022\)](#).

As in the case of haircuts, we do not find clear systematic variation in the repo spreads across the types of borrowers and lenders (see Column (4)). If anything, domestic banks and specialized credit finance companies borrow at a lower interest rate than others in the Korean repo market. Unlike the positive relationship between the haircut and loan size, the spread decreases in loan size, highlighting the distinctive nature between the two in terms of associated risk. Consistent with the findings in [Hu et al. \(2021\)](#) and [Suzuki and Sasamoto \(2022\)](#), the spread tends to increase in the maturity of repo contracts.

Regarding collateral risk, the analysis of the repo spread paints a similar picture. While a repo contract with government bonds is associated with the lowest spread, the spread systematically increases with the perceived risk of collateral assets. For example, a repo contract using other kinds of safe assets, such as bank bonds, special bonds, and municipal bonds has a higher spread but the gap is only marginal. The difference in the spread is much larger for risky assets. Consistent with the findings on haircuts, a repo backed by collateral with longer remaining maturities has a higher spread. A similar finding was also obtained in [Auh and Landoni \(2022\)](#).

### 2.2.3 Relationship between haircut and interest rate

Figure 2 shows the unconditional correlation between haircuts and spreads, revealing that a repo contract with a higher haircut is associated with a higher interest rate. This finding can also be anticipated from the findings in Table 4 in the previous section because both the haircut and the spread increase with the perceived risk of the collateral. This positive unconditional relationship is the first main empirical finding and it aligns with the intuition that increased collateral risk should be compensated by a higher haircut or a higher interest rate or both. This finding is formally substantiated in a regression framework by estimating Equation 1.

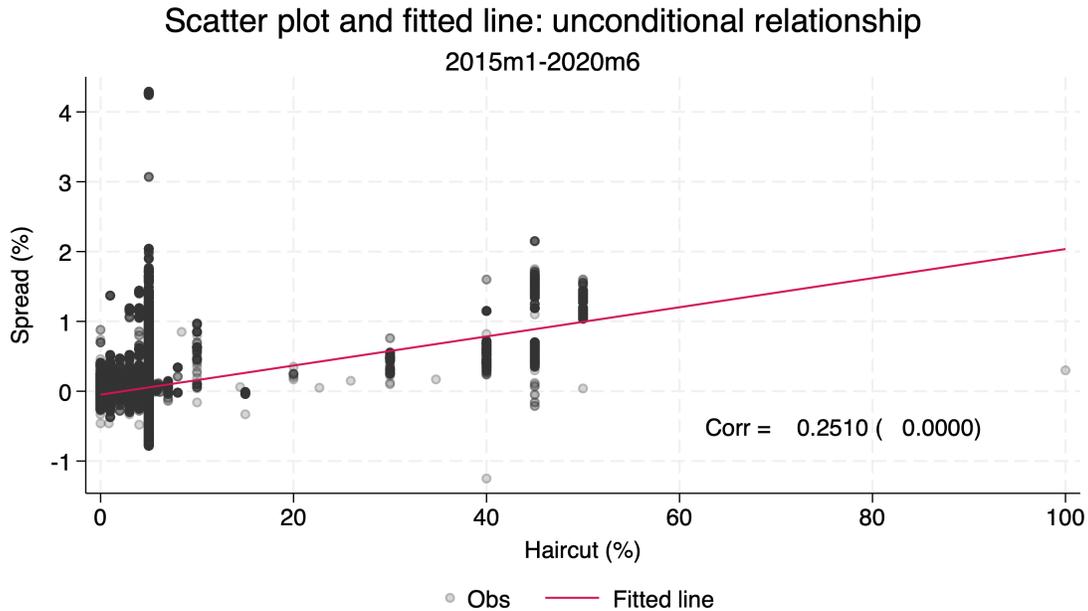


Figure 2: Unconditional relationship between haircut and spread

Table 5 presents the regression results where various fixed effects are sequentially incorporated to account for confounding factors.<sup>13</sup> As expected from Figure 2, the positive correlation is evident when none of the confounding factors is controlled (see Column

<sup>13</sup>To ensure the robustness of our analysis, we exclude contracts with extreme repo rates (higher than 10%) or extreme haircuts (lower than 0% or higher than 200%), amounting to a total of only 29 contracts. This step aims to mitigate the impact of outliers on our results. The mean and standard deviation of haircuts are 4.99% and 1.17%, respectively, while for repo spreads, they are 0.06% and 0.96%, respectively.

(1)). As each fixed effect capturing maturity types, borrower types, lender types, a pair of borrower and lender types (accounting for relationship lending), and types of collateral that interacted with date-fixed effects is successively absorbed from Columns (2) to (7), the positive relationship between the haircut and the spread and the negative relationship between the haircut and loan size continue to hold. Importantly, even with the inclusion of the *collateral type*-fixed effect (Column (7)), these relationships persist, although the size of the coefficient is reduced substantially.

Table 5: Preliminary analysis: haircut, spread, and loan size

Dependent variable:								Baseline
Haircut	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Spread	2.989*** (5.72)	3.863*** (5.72)	2.637*** (4.28)	2.654*** (4.25)	3.276*** (4.83)	2.899*** (4.88)	0.349 (0.83)	-1.361*** (-9.67)
Size	-0.0187*** (-5.48)	-0.0172*** (-4.90)	-0.0241*** (-8.02)	-0.0196*** (-6.22)	-0.0175*** (-5.64)	-0.0134*** (-4.97)	-0.00286 (-1.33)	0.000429 (0.47)
Date FE	Yes							
Maturity type × Date FE			Yes	Yes	Yes	Yes	Yes	Yes
Borrower type × Date FE				Yes	Yes			
Lender type × Date FE					Yes			
Borrower type × Lender type × Date FE							Yes	Yes
Collateral type × Date FE							Yes	Yes
Collateral × Date FE								Yes
Cluster Var.	Collateral + Date (17,249)	Collateral + Date (17,249)	Collateral + Date (17,248)	Collateral + Date (17,248)	Collateral + Date (17,248)	Collateral + Date (17,245)	Collateral + Date (17,241)	Collateral + Date (13,547)
N.Obs	7,393,438	7,393,438	7,393,146	7,392,903	7,392,008	7,388,320	7,387,971	6,162,332
Adj. R <sup>2</sup>	0.064	0.087	0.490	0.500	0.533	0.606	0.828	0.647

The sample period is 2015m1-2020m6. Contracts with an interest rate higher than 10% or with a haircut lower than 0% or higher than 200% are excluded. *t*-Statistics are reported in parentheses. \*, \*\*, and \*\*\* indicate statistical significance at the 10%, 5%, and 1% level, respectively. The numbers in parentheses in the cluster variable row are the number of clusters.

To understand the positive relationship between the haircut and the spread, it is crucial to understand the connection between the quality of posted collateral, borrower default risk, and the lender's expected payoff. Higher collateral risk intensifies the borrower's incentive to default and default makes collateral the property of the lender. Con-

sequently, the diminished expected value of the collateral, owing to poor quality, adversely affects the expected payoff of the lender. Lenders, therefore, are inclined to accept the contract terms only if they are favorable enough to compensate for the associated risk. This rationale is also evidenced by the results in the bottom panels of Table 4.

Intuitively, consider a scenario with only two assets available for a repo contract, in which one asset has higher quality (lower risk) than the other. If a borrower posts the lower-quality asset as collateral, a lender would accept it if the interest rate and/or haircut are higher than those in a contract involving the higher-quality asset for a given loan size. Conversely, for a given haircut, the interest rate should be higher, or the loan size should be smaller, or both, for the lender to participate in the contract.

This poses an empirical challenge for identification. Failing to account for the quality of underlying collateral can introduce an omitted variable bias in estimating the relationship among contract terms. As shown in Column (7), even controlling for asset classes is insufficient, indicating that distinct collateral within the same asset class may possess different properties, such as the issuing entity and time-to-maturity. However, this detailed information is often not available in most existing studies in a consistent manner. This emphasizes the merit of our data with precise information on the unique identifier for every collateral, thereby addressing practical limitations in the literature.

Consistent with our interpretation, a noteworthy reversal in these relationships occurs when collateral-specific fixed effects are included (Column (8)). Here, the coefficient for the haircut becomes negative, and that for loan size becomes statistically insignificant. In other words, the lack of consensus on the relationship between the haircut and the spread is likely driven by the lack of exact information on the collateral used in a repo contract. Once the collateral quality is fully controlled, there is a clear substitution effect between the interest rate and the haircut, which is our second main empirical finding.<sup>14</sup>

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<sup>14</sup>Similarly, we conduct a test of dropping each group of fixed effects from the baseline model (i.e., Column (8)) in Table A-4 in Appendix A. It is clear that the key to identifying the substitution effect in a repo contract is the collateral-fixed effect available from the unique feature of our data.

This reversal also suggests that factors other than collateral characteristics seem to have limited influence on the determinants of repo terms, which is also consistent with the evidence in Table 4, as well as the findings from the U.S. tri-party repo market (Copeland et al. (2014); Krishnamurthy et al. (2014)).

Intuitively, this scenario (i.e., conditioning on the same collateral risk) corresponds to a theoretical setup where only one asset can be used as collateral for a repo contract. In such an environment, a lender cannot simultaneously demand both a higher interest rate and a higher haircut at the expense of risk, as in the case of two assets. Consequently, the lender accepts a lower interest rate for a higher haircut, balancing the expected payoff for a given loan size.

Table 6 presents the baseline results again but separately for the full sample and the sample with one-day maturity only given that 95% of contracts in the institutional repo market are overnight. These results are presented in the first two columns and are quantitatively similar to the baseline results. In terms of economic magnitudes, a 10bp increase in the spread (corresponding to about one standard deviation of the repo spread in our sample) is related to a 0.13% point decline in the haircut. This is not only a statistically significant but economically meaningful relationship. As expected from the inclusion of date-fixed effects, the results using the interest rate instead (Columns (3) and (4)) are nearly identical.

#### **2.2.4 Robustness checks**

To ensure the robustness of our main findings, we perform several sensitivity tests. First, we introduce additional multi-way fixed effects to account for potential confounding factors not absorbed in the baseline model. The nuanced nature of negotiations in the repo market implies that various facets of a contract may interact to determine its terms. For example, there might be systematic variations in the type of collateral used in repo contracts for a specific type of borrower or lender. The specific pairing of borrowers and

Table 6: Main analysis: haircut, spread, and loan size

Dependent variable:	Baseline		Repo rate	
	(1)	(2)	(3)	(4)
Haircut	All maturities	1-day	All maturities	1-day
Spread	-1.361*** (-9.67)	-1.369*** (-9.95)		
Repo rate			-1.361*** (-9.67)	-1.369*** (-9.95)
Size	0.000429 (0.47)	0.000595 (0.65)	0.000429 (0.47)	0.000595 (0.65)
Maturity type × Date FE	Yes	Yes	Yes	Yes
Borrower type × Lender type × Date FE	Yes	Yes	Yes	Yes
Collateral × Date FE	Yes	Yes	Yes	Yes
Cluster Var.	Collateral + Date (13,547)	Collateral + Date (13,347)	Collateral + Date (13,547)	Collateral + Date (13,347)
N.Obs	6,162,332	5,917,734	6,162,332	5,917,734
Adj. R <sup>2</sup>	0.647	0.279	0.647	0.279

The sample period is 2015m1-2020m6. Contracts with an interest rate higher than 10% or with haircut lower than 0% or higher than 200% are excluded. *t*-Statistics are reported in parentheses. \*, \*\*, and \*\*\* indicate statistical significance at the 10%, 5%, and 1% level, respectively. The numbers in parentheses in the cluster variable row are the number of clusters.

lenders could also be influential, possibly due to longstanding relationships. Such practices may vary over time as well. In this regard, the most demanding set of fixed effects is Collateral×Borrower×Lender×Date, which is designed to absorb such variations flexibly. In addition to the baseline fixed effects included in Column (7) of Table 5, we include a more demanding set of fixed effects in turn, starting from Borrowers×Lenders-fixed effects to Collateral×Borrower×Lender×Date-fixed effects. As displayed in Table A-5 in Appendix A, the statistically significant negative relationship between the haircut and the spread conditional on collateral risk survives in every case.

Second, to address the possibility that the negative relationship is driven by a specific

type of collateral, limiting the generalization of our findings, we systematically exclude each collateral type from the estimation in turn. Table A-6 presents the results (Columns (2) to (10)), confirming the resilience of our conclusions. Regardless of the type of excluded assets, the statistical significance and size of the main coefficients are remarkably stable.

Third, we present the estimation results using alternative standard error clustering methods. First, we single cluster standard errors at collateral $\times$ date level. Second, standard errors are double clustered at collateral and year $\times$ month and day $\times$ dow (where *dow* denotes the day of the week) levels. In the last two columns, standard errors are double clustered at collateral $\times$ (year $\times$ month) and day $\times$ dow levels. Irrespective of the level of standard error clustering, the statistically significant substitution effect between the haircut and the spread remains robust (see Table A-7 in Appendix A.)

### 2.2.5 Uncertainty and the substitution between haircut and interest rate

Now we assess the stability of the negative relationship between the haircut and the spread over time. To the extent that market conditions change over time, they may also influence the way haircuts and interest rates interact each other in contract terms. To obtain a visual summary, we estimate Equation 1 in a rolling window basis, employing a one-month window. The coefficients on the spread variable for each month, along with their corresponding 95% confidence intervals, are presented in Figure 3. The red horizontal line indicates the average of the rolling-window coefficients.

The consistently negative and statistically significant coefficients confirm the temporal stability of the negative relationship between the haircut and the spread. However, it appears that its slope changes over time in a systematic way. As shown in Figure 3, the (absolute) size of the slope tends to decrease during a period of heightened market uncertainty—proxied by the VKOSPI (implied volatility of the Korean stock market, corresponding to the VIX in the U.S. stock market). The correlation between the monthly

coefficient  $\beta$  and the VKOSPI is 0.40 and highly statistically significant.

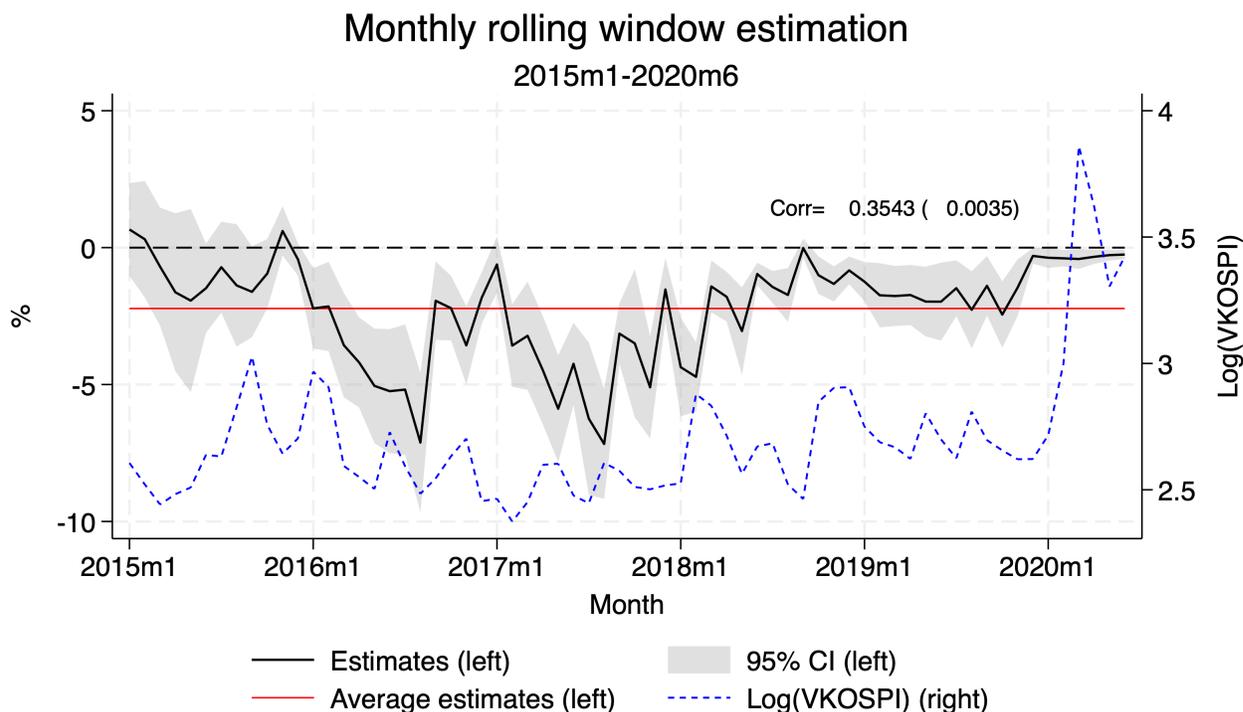


Figure 3: Time-varying relationship: rolling-window estimation

To draw a more robust conclusion about the haircut-spread relationship and market uncertainty, we estimate the following regression:

$$\begin{aligned}
 Haircut_{B,L,M,c,t} = & \beta_1 Spread_{B,L,M,c,t} + \beta_2 Spread_{B,L,M,c,t} \times Unc_t \\
 & + \gamma_1 Size_{B,L,M,c,t} + \gamma_2 Size_{B,L,M,c,t} \times Unc_t + \Upsilon_{B,L,M,c,t} + \epsilon_{B,L,M,c,t}, \quad (3)
 \end{aligned}$$

where the spread and loan size variables are further interacted with a time-varying measure of market uncertainty. If the increased uncertainty indeed lowered the sensitivity of the haircut to the spread in a given repo contract, we would observe a positive sign of the coefficient  $\beta_2$ .

Table 7 summarizes the results of estimating Equation 3. As shown in the first column, the coefficient on the spread variable ( $\beta_1$ ) is still negative, whereas that on the interaction

term ( $\beta_2$ ) is positive and highly statistically significant, lending further support to the suggestive evidence in Figure 3. To confirm the robustness of this novel finding, (i) we lag the VKOSPI variable (to mitigate reverse causality); (ii) use the monthly-averaged VKOSPI variable (to smooth excessive volatility at a daily frequency); (iii) employ the pre-COVID19 sample only (to minimize the influence of outliers); (iv) use the U.S. implied stock market volatility (VIX) instead (to address an endogeneity concern that the repo market development drives stock market volatility); (v) use alternative measures from the real economy (the unemployment rate and the growth of industrial production, respectively) given the countercyclical nature of time-varying uncertainty (Bloom (2014)).

As shown in Columns (2) to (7) in Table 7, the significant role of market uncertainty in dampening the elasticity of the haircut with respect to the spread is highly robust. This systematic variation in the negative relationship between the haircut and the spread constitutes the third main finding, warranting further theoretical exploration.

### 3 Theoretical Model

Our empirical analysis has yielded several intriguing findings. First, we observe an unconditional positive relationship between the repo haircut and the interest rate (or spread). To the extent that this positive relationship continues to hold when further controlling for types of maturities, borrowers, lenders, and collateral, one may conclude that the haircut and the interest rate are complements not substitutes in repo contract terms. Second, however, we uncover a negative relationship between the haircut and the interest rate after fully controlling for collateral risk owing to the advantage of our dataset. This finding confirms and extends the preliminary evidence found in Baklanova et al. (2019) and Auh and Landoni (2022) and provides a rationale for why other studies without precise security-level information (e.g., Suzuki and Sasamoto (2022); Barbiero et al. (2024)) often found different results.

Table 7: Main analysis: uncertainty and haircut-spread relationship

Dependent variable:	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Haircut	Log(KOSPI)	1-day lag	Monthly average	Before COVID	Log(VIX)	Unemployment	IP
Spread	-6.469*** (-8.37)	-6.362*** (-8.33)	-6.582*** (-8.33)	-7.875*** (-4.11)	-5.883*** (-8.80)	-4.334*** (-5.82)	-1.250*** (-10.21)
Spread $\times$ Uncertainty	1.760*** (7.43)	1.722*** (7.37)	1.796*** (7.42)	2.240*** (3.20)	1.536*** (7.73)	0.763*** (4.23)	-0.171*** (-7.16)
Size	0.000676 (0.21)	0.000460 (0.14)	0.000646 (0.18)	-0.00528 (-0.67)	-0.000180 (-0.07)	0.0118*** (3.78)	0.000502 (0.61)
Size $\times$ Uncertainty	0.0000137 (0.01)	0.0000923 (0.09)	0.0000255 (0.02)	0.00235 (0.82)	0.000316 (0.39)	-0.00298*** (-4.07)	0.0000679 (0.53)
Maturity type $\times$ Date FE	Yes						
Borrower type $\times$ Lender type $\times$ Date FE	Yes						
Collateral $\times$ Date FE	Yes						
Cluster Var.	Collateral + Date 13,547	Collateral + Date 13,547	Collateral + Date 13,547	Collateral + Date 12,039	Collateral + Date 13,547	Collateral + Date 13,547	Collateral + Date 13,547
N.Obs	6,162,332	6,162,332	6,162,332	5,211,428	6,162,332	6,162,332	6,162,332
Adj. R <sup>2</sup>	0.649	0.649	0.649	0.651	0.649	0.648	0.649

The sample period is 2015m1-2020m6. Contracts with an interest rate higher than 10% or a haircut lower than 0% or higher than 200% are excluded. *t*-Statistics are reported in parentheses. \*, \*\*, and \*\*\* indicate statistical significance at the 10%, 5%, and 1% level, respectively. The columns *log(VKOSPI)*, *1-day lag*, and *monthly average* use the logarithm of VKOSPI of the current value, 1-day lagged value, and monthly averaged value for *uncertainty*, respectively. The column *before COVID* uses the logarithm of VKOSPI of the current value for *uncertainty* focusing on a subsample from 2015m1 to 2019m12. The column *log(VIX)* uses the logarithm of VKOSPI of the current value for *uncertainty*. The column *unemployment* and *IP* use unemployment rate and year-over-year growth rate of industrial production for *uncertainty*, respectively.

Third, we further document that the elasticity of the haircut with respect to the interest rate decreases in the degree of market uncertainty proxied by various measures. In other words, for a given increase in the repo rate, the reduction in the haircut in an equilibrium is smaller in bad times. This finding has not been documented in the existing literature. Subsequently, we develop a theoretical model of collateralized debt to comprehensively explain these key findings within a unified framework.

### 3.1 Environment

We consider an exchange economy that consists of two risk-neutral agents, namely a borrower ( $b$ ) and a lender ( $l$ ), and two periods,  $t \in \{0, 1\}$ . The utility of each agent is

$$U^b = mc_{b0} + c_{b1}$$
$$U^l = c_{l0} + c_{l1},$$

where  $c_{it}$  is the consumption of goods by the agent  $i \in \{b, l\}$  in period  $t$  and  $m$  is the marginal utility of the borrower from consuming goods in period 0. We assume that  $m > 1$ , which can be interpreted as a borrower having liquidity needs in period 0.

There is a single nondurable consumption good in each period and its endowment process is as follows. In period 0, a lender is endowed with  $e$  units of consumption goods and a borrower does not receive consumption goods. In period 1, the lender receives nothing, while the borrower is endowed with  $e$  units of consumption goods with a probability of  $1 - \alpha$  and receives nothing with the complementary probability  $\alpha$ .

In addition to consumption goods, the borrower is endowed with  $a$  units of the divisible Lucas tree at  $t = 0$ . Trees are subject to a shock: with probability  $1 - \sigma \in (0, 1]$ , a tree yields  $y$  units of consumption goods at the end of period 1. Otherwise, it pays nothing. Letting  $\bar{y} = (1 - \sigma)y$ , we keep  $\bar{y}$  constant throughout the paper, which implies that an increase in  $\sigma$  means an increase in the spread of the distribution of the tree's terminal value while keeping its expected payoff constant.

In period 0, there are gains from trading due to the borrower's liquidity needs, and we assume that the borrower makes a take-it-or-leave-it offer to the lender in period 0. However, unsecured credit is not feasible because of a limited commitment problem. Therefore, trees are necessary as a medium of exchange for trade to occur in period 0. The borrower can finance their liquidity needs in period 0 using trees in one of two ways. On the one hand, they can sell a certain quantity of trees to the lender in exchange for

consumption goods (an asset sale). On the other hand, they can pledge trees as collateral to borrow consumption goods from the lender (collateralized debt).

**Costly information acquisition** In reality, an economic agent may desire to acquire more information about the future value of an asset before purchasing or selling it. For example, an agent may obtain analytical reports regarding a company's financial statements and future prospects, which could offer more precise insights into the value of its equity share before making a transaction, despite a common perception about the expected value of the company's equity.

It is certainly costly to purchase such information or exert her own effort and time to obtain detailed information. To capture this practice, we assume that prior to trading with the borrower in period 0, the lender can obtain private information regarding the quality of the tree similar to [Dang et al. \(2013\)](#) and [Gorton and Ordoñez \(2014\)](#). Specifically, the lender must incur a fixed cost of  $\gamma > 0$  in terms of the consumption goods in period 0 to obtain this information. In principle, the borrower may also desire to acquire information about the quality of her assets at a cost. However, in Appendix C, we demonstrate that the borrower never acquires such information at a cost. Thus, we simply assume that the borrower cannot obtain information about the tree's quality at a cost without loss of generality.

**Defaults on collateralized debt** In practice, a borrower may default on a collateralized debt for two reasons. First, she is unable to repay the loan because of insufficient resources. In the model, the borrower does not receive any consumption goods in period 1 with probability  $\alpha$ , and thus she cannot make a repayment on the loan. Thus,  $\alpha$  represents the probability of exogenous default on collateralized debt.

Second, borrowers may default on collateralized debt even though they have sufficient resources because it is profitable to do so. Specifically, borrowers will compare the current value of collateral assets with the repayment value and will default opportunistically.

tically when the latter is higher than the former. For example, in the real world, on the payment due date of collateralized debt, a borrower can observe the current market price of the collateral asset in spot markets and make a repayment decision based on this price information. To introduce this type of default into the model in a simple way, we assume that the information about the dividend state is revealed at the beginning of period 1 before the settlement of any debts. Thus, the borrower can decide whether to repay the loan in period 1 based on the revealed information.<sup>15</sup>

One may argue that this opportunistic default rarely occurs because trades among the market participants are not a one-shot game in real repo markets: Participants in these markets repeatedly enter into repo contracts over time. Specifically, if we construct our model such that the bargaining game between borrowers and lenders is infinitely repeated, and agents exhibit sufficient patience, then an equilibrium without opportunistic defaults can be achieved, as shown by the folk theorem.

Even in such a repeated game setup, an equilibrium still exists wherein the borrower defaults opportunistically whenever feasible. Moreover, defaults have indeed occurred in repo markets under extreme market conditions, as exemplified by the collapse of Lehman Brothers in 2008. To streamline the analysis and focus on the economic mechanisms driving the main empirical findings outlined in Section 2, we employ a one-shot game model.

**Parameter restriction** The trade volume in period 0 can be bounded by  $e$  if  $e$  is too small. Because our main interest is the impact of the properties of trees on the terms of trade, the size of the lender's endowment  $e$  should not restrict the trade volume. The following assumption implies that the lender's endowment  $e$  is large enough so that the borrower has no restriction on trading all the trees she has if wishes.

**Assumption 1**  $e > ya + \gamma$

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<sup>15</sup>Barro (1976) also studied the implications of opportunistic default on the loan size.

### 3.2 Bargaining problem in period 0

As discussed in the previous section, the borrower can obtain consumption goods from the lender either by selling trees directly (asset sale) or by collateralizing trees (collateralized debt). Here, we focus on collateralized debt in the following analysis and later show that it is optimal, strictly dominating asset sales. Collateralized debt consists of three terms,  $(q, p, a')$ . In period 0, the borrower receives  $q$  units of the consumption goods from the lender and promises to repay  $p$  units of the consumption goods in period 1. At the same time, the borrower posts  $a'$  units of the trees as collateral in period 0. Incomplete payment entitles the lender to possess the collateral trees that belonged to the borrower.

The lender's payoff from accepting a contract  $(q, p, a')$  depends on whether the lender acquires costly information about the quality of the trees before trading with the borrower. When the lender does not acquire the information, her expected payoff is

$$\pi_{IIS} = -q + (1 - \alpha)(1 - \sigma)p + \alpha\bar{y}a'. \quad (4)$$

Once the lender accepts the borrower's offer, the lender transfers  $q$  units of the consumption goods to the borrower in period 0. With probability  $1 - \alpha$ , the borrower receives the consumption goods and decides whether to make the repayment. In particular, she makes repayment  $p$  only if the tree is good which occurs with a probability of  $1 - \sigma$ . Next, with probability  $\alpha$ , the borrower receives nothing so defaults, and the lender receives  $\bar{y}a'$  units of the dividend in expectation from collateral trees.

On the other hand, if the lender checks the tree's dividend state in advance, then she will accept the borrower's offer only if the trees are good. Then, the lender's expected payoff is

$$\pi_{IS} = (1 - \sigma) [-q + (1 - \alpha)p + \alpha ya'] - \gamma. \quad (5)$$

The borrower cannot default opportunistically here because the lender accepts the

borrower's offer only if the dividend state is good. However, the lender must incur the information acquisition cost  $\gamma > 0$ , in terms of the consumption goods. Assuming that the lender's participation constraint holds, if  $\pi_{IIS} \geq \pi_{IS}$ , the lender accepts the borrower's offer without information acquisition, and if  $\pi_{IIS} < \pi_{IS}$ , the lender acquires the costly information and decides whether to accept the borrower's offer based on that information. As in [Gorton and Ordoñez \(2014\)](#), we say that collateralized debt is information sensitive (IS) if it triggers information acquisition by the lender and information insensitive (IIS) otherwise.

In what follows, we examine the terms of the optimal IIS contract and IS contract. Then, by comparing the borrower's value under these two types of contracts, we find the optimal debt contract.

**Information insensitive (IIS) collateralized debt** The borrower's maximized surplus from IIS collateralized debt,  $V_{IIS}$ , is given by

$$V_{IIS} = \max_{q,p,a'} \{mq - (1 - \alpha)(1 - \sigma)p - \alpha(1 - \sigma)ya'\} \quad (6)$$

subject to

$$-q + (1 - \alpha)(1 - \sigma)p + \alpha\bar{y}a' \geq 0 \quad (7)$$

$$-\sigma q + \gamma \geq 0 \quad (8)$$

$$ya' - p \geq 0 \quad (9)$$

$$a - a' \geq 0 \quad (10)$$

$$q, p, a' \geq 0 \quad (11)$$

Here, the inequality (7) is the lender's participation constraint without information acquisition. (8) is the constraint to avoid information acquisition, which means that the lender's payoff with information acquisition (5) should not be higher than the payoff

without information (4), i.e.,  $\pi_{IIS} \geq \pi_{IS}$ . (9) is the borrower's incentive constraint to make repayment when she has the resources to make repayment and knows that tree yields dividends. Finally, (10) and (11) are the feasibility constraints.

The next lemma solves the above maximization problem, describing the terms of IIS collateralized debt.

**Lemma 1** *Define the cutoff level of the information acquisition cost as*

$$\gamma^* \equiv \sigma \bar{y} a. \quad (12)$$

*Then, the terms of IIS collateralized debt are as follows:*

1. [Fully-IIS] *If  $\gamma^* \leq \gamma$ , then  $q = \bar{y} a$ ,  $p = y a$ ,  $a' = a$ , and  $V_{IIS} = (m - 1) \bar{y} a$ .*
2. [Partially-IIS] *If  $\gamma < \gamma^*$ , then  $q = \frac{\gamma}{\sigma}$ ,  $p \in [0, y a']$ , and*

$$a' \in \left[ \frac{\gamma a}{\gamma^*}, \min \left\{ a, \frac{\gamma a}{\alpha \gamma^*} \right\} \right]$$

*are determined simultaneously by*

$$\frac{\gamma}{\sigma} = (1 - \sigma)[(1 - \alpha)p + \alpha y a'], \quad (13)$$

$$\text{and } V_{IIS} = \frac{(m-1)\gamma}{\sigma}.$$

**Proof.** See Appendix B. ■

The terms of IIS collateralized debt depend on whether the constraint to avoid information acquisition (8) binds or not. First, if the information acquisition cost is sufficiently high as  $\gamma \geq \gamma^*$ , the constraint (8) does not bind (Fully-IIS). In this case, the borrower pledges all trees as collateral, namely  $a' = a$ , and sets  $p = y a'$  to maximize the loan size  $q$ , satisfying the lender's participation constraint (7). Second, if  $\gamma < \gamma^*$ , the constraint to avoid information acquisition (8) binds, which limits the trade volume. It implies that if

the borrower increases the size of the loan by an additional unit, the borrower will likely acquire information. Consequently, collateralized debt is partially IIS. Because an increase in  $\sigma$  intensifies the lender's information acquisition incentive, the loan size decreases in  $\sigma$ .

An important property of partially IIS collateralized debt is that there are infinite numbers of partially IIS collateralized debt with different contract terms. The repayment value  $p$  and quantity of collateral trees  $a'$  are simultaneously determined by equation (13), so there are infinite pairs of  $(p, a')$ . This result enables the model to generate the substitution effect between repo rates and haircuts, as will be discussed in detail later.

The intuitive explanation is as follows. The borrower's expected payment on the loan is  $(1 - \alpha)(1 - \sigma)p + \alpha\bar{y}a'$ . In general,  $p$  and  $a'$  affect the loan size  $q$  through their effects on the lender's payoff; however, under partially-IIS contracts,  $q$  is pinned down by the binding constraint to avoid information acquisition (8). Thus, the borrower is indifferent about the choice of  $(p, a')$  as long as  $(p, a')$  gives a zero surplus in expectation to the lender. Then, substituting  $q = \frac{\gamma}{\sigma}$  into the binding (7), we obtain (13).

**Information sensitive collateralized debt (IS)** Under IS loan contracts, the borrower can trade with the lender only if the dividend state is good. Thus, the borrower's maximized value from IS collateralized debt,  $V_{IS}$ , is given by

$$V_{IS} = \max_{q,p,a'} \{(1 - \sigma)[mq - (1 - \alpha)p - \alpha ya']\} \quad (14)$$

subject to

$$(1 - \sigma) [-q + (1 - \alpha)p + \alpha ya'] - \gamma \geq 0 \quad (15)$$

$$\sigma q - \gamma \geq 0 \quad (16)$$

$$ya' - p \geq 0 \quad (17)$$

$$a - a' \geq 0 \quad (18)$$

$$q, p, a' \geq 0. \quad (19)$$

The inequality (15) is the lender's participation constraint with information acquisition. (16) is the constraint that induces the lender to acquire information about the collateral quality. The inequality (17) is the incentive constraint for the borrower to make repayments on the loan, and (18) and (19) are the feasibility constraints.

Triggering information acquisition is costly because it raises the cost of borrowing to compensate for the information acquisition cost  $\gamma$ . Thus,  $\gamma$  must be sufficiently low for IS collateralized debt to exist. In particular, if  $\gamma > \gamma^* \equiv \sigma \bar{y} a$ , then no  $\{q, p, a'\}$  satisfies (15) - (18) at the same time. Furthermore, because the borrower can always choose not to trade with the lender in period 0,  $V_{IS}$  must be higher than 0. Given these arguments, the next lemma describes the terms of IS collateralized debt.

**Lemma 2 [IS]** *If  $\gamma \leq \min \{ \gamma^*, \frac{m-1}{m\sigma} \gamma^* \}$ , then the terms of IS collateralized debt are such that  $q = ya - \frac{\gamma}{1-\sigma}$ ,  $p = ya$ ,  $a' = a$ , and  $V_{IS} = (m-1)(1-\sigma)ya - m\gamma$ . Otherwise, IS collateralized debt is either infeasible or worse than the no-trading option.*

**Proof.** See Appendix B. ■

Because a trade occurs only if the dividend state is good, there is no incentive for the borrower to restrict the quantity of collateral trees and repayment value as partially IIS collateralized debt does. Therefore, the borrower pledges all trees as collateral, namely  $a' = a$ , and sets  $p = ya$  to maximize the loan size  $q$  without violating the lender's participation constraint (15). However, the borrower must compensate for the information

acquisition cost, which reduces the loan size  $q$ . The loan size also decreases with  $\sigma$  because an increase in  $\sigma$  implies a lower probability of having a trade to cover the lender's information acquisition cost.

**Optimal collateralize debt contract** In period 0, the borrower optimally chooses collateralized debt that triggers information acquisition about the collateral trees (IS collateralized debt) or not (IIS collateralized debt) by comparing  $V_{IIS}$  and  $V_{IS}$  given in equations (6) and (14), respectively. Then, the borrower's maximized surplus with collateralized debt is given as  $V = \max\{V_{IIS}, V_{IS}\}$ . The following proposition describes the type of optimal collateralized debt.

**Proposition 3** *Define the cutoff level of the information acquisition cost as*

$$\gamma^{**} \equiv \frac{(m-1)\gamma^*}{m(1+\sigma)-1}. \quad (20)$$

*Then, the optimal collateralized debt is 1) IS if  $\gamma \in [0, \gamma^{**})$ , 2) partially-IIS if  $\gamma \in [\gamma^{**}, \gamma^*)$ , and 3) fully-IIS if  $\gamma \geq \gamma^*$ . Furthermore,  $\gamma^*$  and  $\gamma^{**}$  increase with  $\sigma$ .*

**Proof.** See Appendix B. ■

Figure 4 illustrates proposition 3 graphically. As one can see from the figure, as  $\gamma$  decreases or  $\sigma$  increases, the type of optimal collateralized debt tends to change from fully-IIS to partially-IIS and to IS contracts.<sup>16</sup> First, the effects of  $\gamma$  on the contract type are straightforward from the results of lemmas 1 and 2:  $V_{IIS}$  weakly increases in  $\gamma$  while  $V_{IS}$  decreases in  $\gamma$ . Next, as  $\sigma$  increases, the likelihood of the lender holding bad trees

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<sup>16</sup>Jang and Kang (2021) consider an asset exchange model in which the seller knows the quality of her asset and the borrower can verify the quality of the asset at a cost. In Jang and Kang (2021), the properties of IIS contracts and IS contracts are similar to those of our model. The key difference from our model is that partially IIS contracts cannot be the optimal contract in Jang and Kang (2021). The intuitive explanation is as follows. In Jang and Kang (2021), the seller of the good asset does not face the risk of no-trade under the IS contract in contrast to our model. Thus, the IS contract is more attractive to the seller with a good asset in Jang and Kang (2021) than in our model, so whenever the IIS contract is partially IIS, it is dominated by the IS contract.

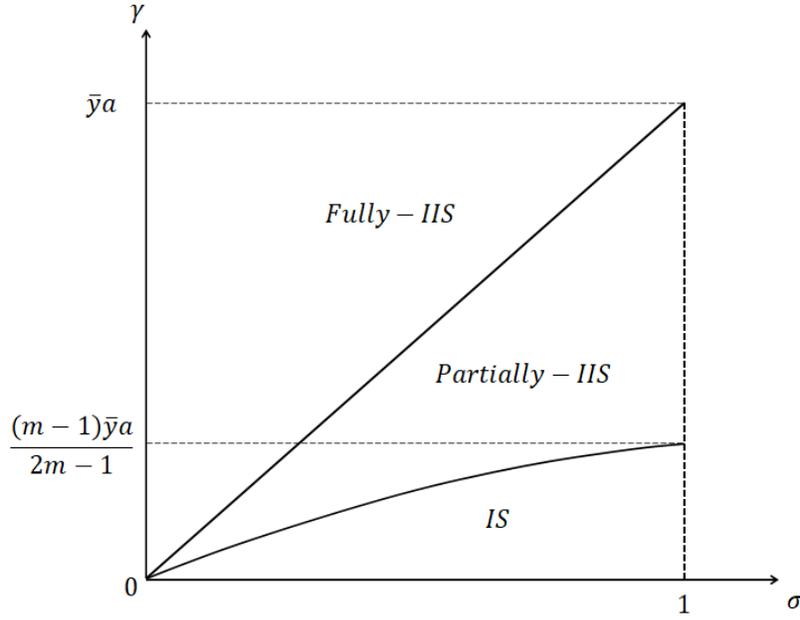


Figure 4: Optimal secured loan contract for different  $\gamma$  and  $\sigma$

upon entering into the collateralized contract rises. Thus, given  $\gamma$ , the lender has a higher incentive to check the tree's quality when  $\sigma$  is high.

**Optimality of collateralized debt** Thus far, our focus has been on finding the optimal collateralized debt contract. However, the borrower can also raise funds by selling trees on the spot: sell  $a'_s$  units of the trees to the lender in exchange for  $q_s$  units of the consumption goods in period 0. The terms of optimal tree sales  $(q_s, a'_s)$  can be obtained by imposing the condition that  $\alpha = 1$  into the terms of the optimal collateralized debt, because if  $\alpha = 1$ , then the lender always seizes the collateral trees, the same as asset sales. More importantly, as shown from lemmas 1 and 2 and (20),  $V_{IIS}$ ,  $V_{IS}$ ,  $\gamma^*$ , and  $\gamma^{**}$  are independent of  $\alpha$ . This implies that collateralized debt and tree sales are equivalent to the borrower in terms of trade surplus.<sup>17</sup>

<sup>17</sup>The equivalence result between collateralized debt and asset sales was also shown by Lagos (2011) under symmetric information and by Rocheteau (2011) under asymmetric information. However, this equivalence result is broken if the borrower incurs any costs from failing to hold trees in period 1. In reality, a financial institution often needs to hold specific assets for hedging purposes or must maintain a designated quantity of liquid assets to comply with regulations, such as reserve requirements. If the financial institution fails to hold those assets on its balance sheet, it may face fees for non-compliance with regulations or incur financial losses from failing to hedge against risks. In such cases, collateralized debt contracts domi-

### 3.3 Haircuts and interest rates

We now explore how our model provides theoretical explanations for the set of empirical findings on the properties of haircuts and interest rates in repo markets that we studied in section 2. For this purpose, we extend the model by introducing heterogeneity across agents (i.e., borrowers and lenders) and collateral trees.

Specifically, let  $\Omega \subset [0, 1]$ ,  $\Sigma \subset [0, 1]$ , and  $\Gamma \subset \mathbb{R}_+$  be the finite sets of  $\alpha$ ,  $\sigma$ , and  $\gamma$ , respectively. For each  $\alpha$  and  $\gamma$ , there is a unit mass of borrowers with the same  $\alpha$  and lenders with the same  $\gamma$ . Thus, there are  $|\Omega|$  number of groups of borrowers, each indexed by  $\alpha$ , where  $|\Omega|$  is the cardinality of set  $\Omega$ , and  $|\Gamma|$  number of groups of lenders, indexed by  $\gamma$ . Each borrower has  $a$  units of trees whose  $\sigma$  is randomly drawn from  $\Sigma$ . Finally, we assume that there are bilateral meetings between a borrower and a lender in period 0 wherein the borrower offers a contract as in the baseline model.<sup>18</sup>

Each meeting is characterized by  $(\alpha, \sigma, \gamma)$ , where  $\alpha$ ,  $\sigma$ , and  $\gamma$  capture the characteristics of the borrower, the lender, and collateral trees, respectively. Note that given  $(\alpha, \sigma, \gamma)$  for each meeting, the results of Proposition 3 delineate the optimal contract between the matched borrower and lender. We assume that if there are multiple optimal contracts for a trade, the borrower randomly chooses one of them.

Given a collateralized debt contract  $(q, p, a')$ , an interest rate is defined as  $r = \frac{p-q}{q}$ . A haircut is the percentage difference between the value of the collateral tree and loan size, defined as  $\theta = \frac{v-q}{v}$ , where  $v$  is the expected value of collateral in period 0. In what follows, we use  $r(\alpha, \sigma, \gamma)$  and  $\theta(\alpha, \sigma, \gamma)$  to represent the equilibrium interest rate and the haircut, respectively, in a meeting with  $(\alpha, \sigma, \gamma)$ . Additionally,  $\hat{r}(\alpha, \sigma, \gamma)$  and  $\hat{\theta}(\alpha, \sigma, \gamma)$  denote the average interest rate and haircut, respectively. The next lemma describes how the interest

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nate asset sales because the borrower relinquishes ownership of pledged trees only in the event of default under the collateralized debt contracts, contrasting with asset sales.

<sup>18</sup>Random matching is often assumed in models of over-the-counter markets. Examples include Berentsen et al. (2014), Geromichalos and Herrenbrueck (2016), Mattesini and Nosal (2016), and Herrenbrueck and Geromichalos (2017). These studies focus on matching frictions in decentralized markets and assets' liquidity, while we focus on the (potential) informational frictions in decentralized markets.

rate and the haircut are determined for each type of collateralized debt.

**Proposition 4** *Given  $(\alpha, \sigma, \gamma)$ , the interest rate  $r$  and the haircut  $\theta$  of each type of collateralized debt are given as follows:*

- 1)  $r = \frac{\sigma}{1-\sigma}$  and  $\theta = 0$  if collateralized debt is fully IIS.
- 2)  $r$  and  $\theta$  are simultaneously determined by

$$1 = (1 - \alpha)(1 - \sigma)(1 + r) + \frac{\alpha}{(1 - \theta)}, \quad (21)$$

*in the range of  $r \in \left[ \max \left\{ 0, \frac{1 - \alpha\gamma^*}{(1 - \alpha)(1 - \sigma)} \right\} - 1, \frac{1}{1 - \sigma} - 1 \right]$  and  $\theta \in \left[ 0, 1 - \max \left\{ \frac{\gamma}{\gamma^*}, \alpha \right\} \right]$  if collateralized debt is partially-IIS.*

- 3)  $r = \frac{\gamma}{y\alpha - \gamma}$  and  $\theta = \frac{\gamma}{y\alpha}$  if collateralized debt is IS.

**Proof.** See Appendix B. ■

Given the loan size  $q$ , a triple  $(q, r, \theta)$  represents an alternative expression of the contract terms  $(q, p, a')$ . Therefore, we refrain from extensively examining the interest rate  $r$  and haircut  $\theta$  for each contract type here, as we have already thoroughly analyzed the terms of contract  $(q, p, a')$  in subsection 3.2. Nevertheless, it is worth exploring the factors that influence the presence of haircuts, which we will delve into.

Under fully IIS debt contracts, the lender has no incentives to acquire information about the quality of the collateral asset. The only problem stemming from informational frictions is opportunistic default. However, the interest rate fully compensates for the risk of opportunistic default, in line with conventional finance theory, which posits that default risk is integrated into interest rates. Thus, haircuts do not come into play, suggesting that the risk of opportunistic default alone cannot account for the presence of haircuts.

In contrast, under the partially IIS and IS contracts, the lender's incentive for information acquisition matters. Specifically, in the partially IIS debt contract, the lender does not obtain information about the collateral quality, but the incentive constraint preventing

the lender from acquiring information is binding. This implies that the lender will check the collateral quality once the borrower increases the loan size, so the binding incentive constraint limits the loan size. Next, under an IS debt contract, the lender indeed acquires the information, and the loan size accounts for the information acquisition cost. Thus, the lender's incentive to acquire information about the collateral quality shapes partially-IIS and IS debt contracts, and it is only when this incentive problem exists that a haircut emerges as an equilibrium outcome.

The result that haircuts are present only when there is a threat of or actual acquisition of information about collateral quality is consistent with the findings in previous studies that haircuts account for the risk associated with the future value of the collateral asset.<sup>19</sup> For example, [Dang et al. \(2013\)](#), [Kang \(2021\)](#), and [Madison \(2024\)](#) have demonstrated that haircuts can exist in the presence of (potential) asymmetric information regarding asset quality. Additionally, [Simsek \(2013\)](#), [Fostel and Geanakoplos \(2015\)](#), and [Barsky et al. \(2016\)](#) have illustrated the presence of haircuts when there are disagreements regarding the belief in the future value of collateral assets. However, in those models, the set of optimal debt contracts is a singleton. This makes it challenging to explain the negative relationship between the haircut and the interest rate after fully controlling for collateral risk, a topic we will explore further below.

In the rest of the paper, we elaborate on how our model can offer a narrative explanation for the main three empirical findings in the following order: (i) the negative relationship between the haircut and the repo rate conditional on collateral quality, (ii) the unconditional positive relationship between the two, and (iii) the decrease in the rate at

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<sup>19</sup>There are papers, such as [Kiyotaki and Moore \(1997\)](#), [Venkateswaran and Wright \(2013\)](#), and [Williamson \(2016\)](#), that study the macroeconomic implications of haircuts by assuming limited pledgeability of collateral assets, rather than explicitly incorporating informational friction about collateral quality into the model. However, in such models, the haircut is solely determined by a parameter capturing this limited pledgeability, failing to elucidate the underlying economic mechanism for its existence. We can also incorporate limited pledgeability into our model. In this case, a fully IIS debt contract also includes positive haircuts. However, the size of haircuts simply depends on parameters capturing the extent of limited pledgeability and does not provide additional insights. Therefore, we opt not to include limited pledgeability, as it would only complicate the analysis without providing additional insight into the determinants of haircuts.

which the repo rate substitutes the haircut during periods of heightened market uncertainty or economic downturn.

In the following analysis of the model's predictions, we exclude the IS debt contracts based on the findings presented in section 2, which indicate that the predominant form of repo transactions involves overnight arrangements. In such scenarios, lenders would be less inclined to bear the added costs and efforts of ascertaining the true value of collateral assets. Rather, they opt for repo contract terms relying on the market value of these assets, especially during normal periods. Since there was no significant financial shock during our sample period, IS-debt contracts would be less prevalent in our dataset. Given this rationale, we impose the following parametric assumption on  $\gamma$  in the analysis below to exclude the existence of IS debt contracts in the extended model with a continuum of agents.

**Assumption 2**  $\gamma > \frac{(m-1)\sigma_{\max}\bar{y}a}{m(1+\sigma_{\max})-1}$ , where  $\sigma_{\max} = \max \Sigma$ .

**Conditional negative relationship** Column (7) of Table 5 shows that the haircut and the interest rate exhibit a negative correlation when all confounding factors, including borrower traits, lender attributes, and collateral asset characteristics, are fully controlled. This finding suggests that they are a substitute in repo contract terms. From our model's perspective, the above result entails analyzing the relationship between  $r(\alpha, \sigma, \gamma)$  and  $\theta(\alpha, \sigma, \gamma)$  within a group of bilateral meetings sharing the same  $(\alpha, \sigma, \gamma)$ .

Consider a group of bilateral meetings characterized by  $(\alpha, \sigma, \gamma)$ . If the contract type is partially IIS in those meetings, participants in each bilateral meeting randomly select a pair  $(r, \theta)$  that satisfies (21). Because the right-hand side of (21) increases with both  $r$  and  $\theta$ , what we observe in equilibrium is the negative correlation between  $r$  and  $\theta$  when the contract type is partially-IIS. We emphasize that this negative relationship stems from the substitution effects between the interest rate and haircut in the negotiation of repo terms, rather than from any heterogeneity in other factors, as we hold  $(\alpha, \sigma, \gamma)$  constant.

The intuition is in line with our earlier observation. A choice of  $(r, \theta)$  does not affect the loan size  $q$  of partially IIS collateralized debt because it is determined by the binding constraint to avoid information acquisition (8). Thus, the choice of  $(r, \theta)$  only affects the composition of the lender's payoff. An increase in  $r$  means an increase in the repayment  $p$  (i.e., the payoff in the non-default state). To break even, the lender's payoff in the default state must decrease, which is obtained by pledging less collateral equivalent to a decrease in the haircut  $\theta$ . As long as the lender receives a zero surplus in expectation, the borrower is indifferent between partially IIS collateralized debt with high  $r$ /low  $\theta$  and partially-IIS collateralized debt with low  $r$ /high  $\theta$ .

As explained earlier, under both partially IIS and IS contracts, the lender's incentive for information acquisition affects contract terms. Thus, both contracts feature a positive haircut. However, only a partially IIS debt contract involves the substitution effect. The key distinction between these contract types lies in the existence of the risk of opportunistic default: There is an opportunistic default risk in addition to the threat of information acquisition in the partially IIS contract, whereas there is no opportunistic risk in the IS contract because the trade occurs only if the dividend state is good. Furthermore, if the borrower is unaware of the dividend state before making the repayment decision in period 1, and thus cannot default opportunistically, the partially IIS debt contract no longer exhibits the substitution effect in the model. Consequently, the substitution effect arises only when both the incentive for information acquisition and the risk of opportunistic default are present.

In our data, the negative relationship exists beyond the substitution effect discussed above. Indeed, it remains evident even without controlling for the characteristics of borrowers and lenders, as long as the characteristics of the posted collateral are controlled via ISIN-fixed effects (Table A-4 in Appendix A). To ascertain whether the model provides a comparable prediction, we analyze the relationship between  $r(\alpha, \sigma, \gamma)$  and  $\theta(\alpha, \sigma, \gamma)$  within a group of bilateral meetings sharing the same  $\sigma$  but with varying sets of  $(\alpha, \gamma)$ .

The subsequent proposition lays the foundation for equilibrium outcomes that align with empirical observations.

**Proposition 5** 1) When  $\alpha$  changes, only one or neither of  $\hat{r}(\alpha, \sigma, \gamma)$  and  $\hat{\theta}(\alpha, \sigma, \gamma)$  changes. 2) If both  $\hat{r}(\alpha, \sigma, \gamma)$  and  $\hat{\theta}(\alpha, \sigma, \gamma)$  respond to a change in  $\gamma$ , they change in the opposite directions.

**Proof.** See Appendix B. ■

The results of Proposition 5 imply that for a given  $\sigma$ ,  $\hat{r}(\alpha, \sigma, \gamma)$  and  $\hat{\theta}(\alpha, \sigma, \gamma)$  do not move in the same direction simultaneously in response to changes in  $\alpha$  and  $\gamma$ . Combined with the property of partially-IIS debt contracts explained earlier, the results of Proposition 5 provide a basis for the negative relationship between  $r(\alpha, \sigma, \gamma)$  and  $\theta(\alpha, \sigma, \gamma)$  after controlling for collateral risk  $\sigma$  in the model economy.

**Unconditional positive relationship** Another key empirical finding from Table 5 is that there exists a positive relationship between the haircut and the interest rate if the characteristics of collateral are not fully controlled via the ISIN-fixed effect. To generate the unconditional positive relationship in the model, it is crucial that both the average interest rate,  $\hat{r}(\alpha, \sigma, \gamma)$ , and the haircut,  $\hat{\theta}(\alpha, \sigma, \gamma)$ , change in the same direction in response to variations in  $\sigma$ . This correlation should persist not only within a specific type of debt contract but across different types of debt contracts. The next proposition demonstrates that this criterion is met within the model economy under certain conditions stated below.

**Proposition 6** Suppose that  $\alpha < \frac{(m-1)2\sigma}{m-1+(2m-1)\sigma}$ . If both  $\hat{r}(\alpha, \sigma, \gamma)$  and  $\hat{\theta}(\alpha, \sigma, \gamma)$  change in response to alterations in  $\sigma$  for a given  $(\alpha, \gamma)$ , they do so in a consistent manner.<sup>20</sup>

**Proof.** See Appendix B. ■

The result of Proposition 6 is important, as it establishes a fundamental basis for deriving the unconditional positive relationship between the haircut and the interest rate.

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<sup>20</sup>When the contract type is partially-IIS and  $\gamma \approx \gamma^{**}$ , if  $\alpha \geq \frac{(m-1)2\sigma}{m-1+(2m-1)\sigma}$ , then changes in  $\sigma$  can cause  $\hat{r}(\alpha, \sigma, \gamma)$  and  $\hat{\theta}(\alpha, \sigma, \gamma)$  to shift in opposite directions. However, if  $\gamma$  is sufficiently higher than  $\gamma^{**}$  when the contract type is partially-IIS, then  $\hat{r}(\alpha, \sigma, \gamma)$  and  $\hat{\theta}(\alpha, \sigma, \gamma)$  respond in the same way to a change in  $\sigma$ .

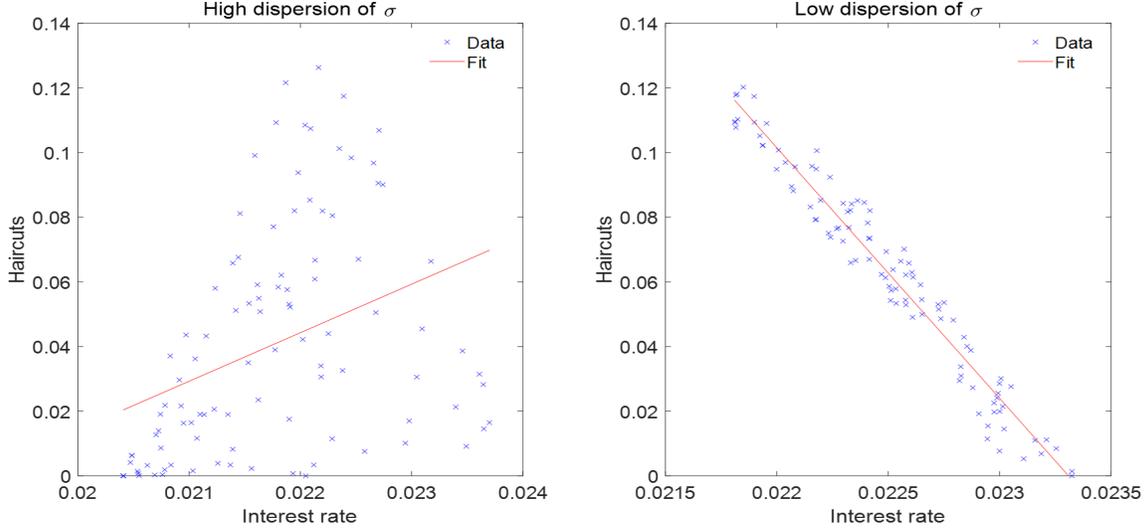


Figure 5: Dispersion of  $\sigma$  and unconditional relation between  $r$  and  $\theta$

However, it is worth noting that the result of Proposition 6 alone is not sufficient to generate an unconditional positive relationship. For example, if  $\hat{r}(\alpha, \sigma, \gamma)$  and  $\hat{\theta}(\alpha, \sigma, \gamma)$  move in the opposite directions when  $\sigma$  increases, the model would still yield a negative correlation even when  $\sigma$  varies for a given  $(\alpha, \gamma)$ , in contrast to the empirical finding in section 2. In other words, the model can also generate an unconditional negative relationship due to the property of the conditional negative relationship of the partially IIS debt contract.

To illustrate this, we conduct a numerical simulation with specific parameter values:  $a = \bar{y} = 1$ ,  $m = 1.1$ ,  $\gamma = 0.02$ ,  $\alpha = 0.01$ . Additionally, we choose two sets of  $\sigma$  as  $\Sigma_1 = [0.02, 0.0234]$  and  $\Sigma_2 = [0.0225, 0.0228]$  such that for all  $\sigma \in \Sigma_i$  and  $i \in \{1, 2\}$ , the optimal contract type is partially-IIS, given our choices for other parameter values. For each  $i \in \{1, 2\}$ , we randomly select 100 samples of  $\sigma$  from  $\Sigma_i$ , and then for each  $\sigma$ , we randomly select a pair of  $(r, \theta)$  that satisfies (21). The left panel of Figure 5 depicts the case when we draw  $\sigma$  from  $\Sigma_1$ , while the right panel illustrates the case when we draw  $\sigma$  from  $\Sigma_2$ . Note that when  $\sigma$  is drawn from  $\Sigma_1$ , it is more widely dispersed than when it is drawn from  $\Sigma_2$  because  $\Sigma_2 \subset \Sigma_1$ .

As shown in (21), an increase in  $\sigma$  shifts up the iso-curves of  $(r, \theta)$  that satisfy (21) given  $(\alpha, \gamma)$ . Thus, when the dispersion of  $\sigma$  is high, a positive unconditional relationship exists

between  $r$  and  $\theta$  as depicted in the left panel of Figure 5. Furthermore, as the dispersion of  $\sigma$  increases so that fully IIS and IS contracts emerge in meetings for a certain set of  $(\alpha, \sigma, \gamma)$ , the positive unconditional relation is intensified given the results of Proposition 4. On the other hand, if the dispersion of  $\sigma$  is low, the iso-curves of  $(r, \theta)$  that satisfy (21) for each  $(\alpha, \sigma, \gamma)$  are close to each other. As a result, a negative unconditional relationship between  $r$  and  $\theta$  can still manifest as illustrated in the right panel of Figure 5. These results imply that the positive unconditional relationship tends to exist when the characteristics of borrowers, lenders, and collateral assets are more diverse.

**Uncertainty and conditional negative relationship** We have demonstrated, as shown in Figure 3 and Table 7, that the rate at which the repo rate substitutes the haircut tends to fall during a period characterized by heightened uncertainty. Through the lens of our model, we can interpret an escalation of uncertainty in financial markets as an exogenous increase in  $\sigma$ , and the coefficient for the spread in column (7) of Table 5 is represented by  $\frac{\partial \theta}{\partial r}$  in each type of debt contract for a given  $(\alpha, \sigma, \gamma)$ .

As previously discussed, the conditional negative relationship exists under partially-IIS debt contracts, where both the interest rate  $r$  and haircut  $\theta$  are simultaneously determined by (21). By taking total derivative of (21), we obtain

$$\frac{\partial \theta}{\partial r} = -\frac{(1-\alpha)(1-\sigma)(1-\theta)^2}{\alpha}, \quad (22)$$

Thus, an increase in  $\sigma$  reduces  $\left| \frac{\partial \theta}{\partial r} \right|$  consistent with empirical finding in Figure 3. The intuitive explanation for this result is as follows.

The interest rate  $r$  and the haircut  $\theta$  serve as compensation to a lender in the non-default and default states, respectively. As  $\sigma$  or  $\alpha$  increases, the default probability increases, leading to a higher chance that the lender is compensated with the collateral trees. This results in a decrease in the relative contribution of the interest rate to the lender's payoff compared to the haircut, as illustrated in (4) in which the repayment  $p$  and the

quantity of collateral trees  $a'$  represent alternative expressions of the interest rate and the haircut, respectively, for a given loan size  $q$ . Thus, the borrower can offset a decrease in the interest rate with a lesser increase in the haircut.

## 4 Conclusion

In this paper, we undertake an empirical and theoretical exploration of the joint determination of interest rates and haircuts in a collateralized debt contract. Leveraging transaction-level data from the Korean repo market, we present three key empirical findings: (i) a positive correlation between the haircut and the interest rate when the risk of the underlying collateral is not controlled, (ii) a negative relationship (i.e., substitution effect) between the two once collateral risk is fully controlled by utilizing data on unique collateral identifiers, and (iii) heightened market uncertainty reduces the interest rate elasticity of the haircut.

To elucidate the economic mechanisms driving these empirical observations, we develop a model of collateralized debt incorporating costly information acquisition and opportunistic default. The model's predictions align with our empirical findings. Specifically, it demonstrates that when lenders refrain from acquiring information about the quality of collateral assets, while the potential threat of information production restricts the loan size, the conditional negative relationship between interest rates and haircuts emerges in equilibrium. In addition, an increase in default risk stemming from a rise in uncertainty about collateral quality makes insurance against the default state particularly more valuable. As a result, changes in the haircut become less sensitive to the same change in the interest rate.

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## Appendix A: Robustness checks and additional results

Table A-1: Collateral class and loan terms: overnight

Collateral class	Haircut (%)	Spread (%)	Principal (bil. of wons)	Maturity (days)	Obs.	Unique ISINs
Government Bond	5	.05	23.3	1	2,951,543 (41.71%)	539
Bank Bond	5	.06	37.6	1	1,735,405 (24.52%)	3,764
Monetary Stabilization Bond	5	.04	28.2	1	676,718 (9.56%)	536
Special Bond	5	.05	32.9	1	640,495 (9.05%)	2,048
Financial Bond	5	.1	17.1	1	551,956 (7.80%)	4,728
Municipal Bond	5	.05	5.9	1	317,845 (4.49%)	1,536
Corporate Bond	5	.11	14	1	199,832 (2.82%)	4,887
ETF Security (else)	5	-.09	50	1	4,045 (0.03%)	16
Equity	30	.33	.2	1	29 (0.00%)	7
Unknown	5	.13	9.3	1	539 (0.01%)	27
All Collateral	5	.05	26	1	7,393,467	15,895

The sample period is 2015m1-2020m6. The table shows the median value of haircut, spread, principal, and maturity, as well as the number of observations and the unique identifier for collateral assets across various collateral classes. The percentages in the *obs* column represent the share of each collateral class in relation to the total number of observations.

Table A-2: Types of cash borrowers and cash lenders (based on the number of contracts)

Type of investors	Borrower type (%)	Lender type (%)
Collective investment scheme	48.70	52.97
Securities company	40.13	3.42
Government	3.79	0.49
Securities company (trust)	3.49	6.97
Specialized credit finance company	2.98	4.00
Domestic bank	0.72	5.63
Foreign bank	0.10	0.25
Insurance company	0.09	3.52
Foreigner	0.00	0.00
Pension funds and guarantee	0.00	2.38
Bank (trust)	.	20.32
ETC (trust)	.	0.04
Credit union and thrift institution	.	0.01
Other financial institution	.	0.00

The sample period is 2015m1-2020m6. The table separately shows the share of each borrower type and lender type in all observations. The share is calculated based on the number of repo contracts.

Table A-3: Top 10 investor pairs and loan terms (based on the number of contracts)

Borrower type	Investor pair Lender type	Most-used collateral type	Spread (%)	Haircut (%)	Principal (bil. of won)	Maturity (days)	Share (%)
Collective investment scheme	Collective investment scheme	Government bond (40.86%)	.06	5	12.5	1	28.28
Securities company	Collective investment scheme	Government bond (47.95%)	.03	5	15	1	20.22
Securities company	Bank (trust)	Bank bond (43.60%)	.04	5	100	1	8.93
Collective investment scheme	Bank (trust)	Bank bond (44.82%)	.07	5	50	1	8.81
Securities company	Securities company (trust)	Government bond (32.56%)	.06	5	34.5	1	3.19
Collective investment scheme	Domestic bank	Government bond (57.76%)	.09	5	36.7	1	3.08
Collective investment scheme	Specialized credit finance company	Bank bond (34.14%)	.08	5	21.7	1	2.51
Government	Collective investment scheme	Government bond (51.09%)	.07	5	15.1	1	2.19
Securities company	Domestic bank	Government bond (74.53%)	.07	5	60	1	2.12
Collective investment scheme	Insurance company	Government bond (37.40%)	.06	5	23.3	1	1.80

The sample period is 2015m1-2020m6. The table shows the median value of haircut, spread, principal, and maturity by investor pairs. The percentages in the *most-used collateral type* column are the share of repo contracts that the corresponding pair of investors agree to set a collateral of the corresponding most-used collateral type. All share in the table is calculated based on the number of contracts.

Table A-4: Robustness check: elimination of fixed effects

Dependent variable: Haircut	Baseline	Elimination of fixed effects		
	(1)	(2)	(3)	(4)
Spread	-1.361*** (-9.67)	-1.291*** (-9.47)	-1.459*** (-9.37)	2.899*** (4.88)
Size	0.000429 (0.47)	0.000260 (0.28)	-0.00312*** (-3.06)	-0.0134*** (-4.97)
Maturity type × Date FE	Yes		Yes	Yes
Borrower type × Lender type × Date FE	Yes	Yes		Yes
Collateral × Date FE	Yes	Yes	Yes	
Cluster Var.	Collateral + Date (13,547)	Collateral + Date (13,549)	Collateral + Date (13,552)	Collateral + Date (17,245)
N.Obs	6,162,332	6,162,662	6,167,994	7,388,320
Adj. R <sup>2</sup>	0.647	0.655	0.585	0.606

The sample period is 2015m1-2020m6. Contracts with an interest rate higher than 10% or with a haircut lower than 0% or higher than 200% are excluded. *t*-Statistics are reported in parentheses. \*, \*\*, and \*\*\* indicate statistical significance at the 10%, 5%, and 1% level, respectively. The numbers in parentheses in the cluster variable row are the number of clusters.

Table A-5: Robustness checks: controlling for additional fixed effects

Dependent variable: Haircut	(1) All maturities	(2) All maturities	(3) All maturities	(4) All maturities
Spread	-1.361*** (-9.67)	-1.380*** (-9.20)	-0.742*** (-6.38)	-0.676*** (-5.93)
Size	0.000429 (0.47)	0.000842 (0.91)	0.000540 (0.74)	0.000445 (0.60)
N.Obs	6,162,332	5,651,717	4,764,059	4,606,518
Adj. R <sup>2</sup>	0.647	0.669	0.706	0.765
Dependent variable: Haircut	(5) 1-day	(6) 1-day	(7) 1-day	(8) 1-day
Spread	-1.369*** (-9.95)	-1.396*** (-9.53)	-0.762*** (-6.72)	-0.701*** (-6.31)
Size	0.000595 (0.65)	0.000954 (1.04)	0.000636 (0.87)	0.000555 (0.75)
N.Obs	5,917,734	5,501,590	4,647,036	4,497,358
Adj. R <sup>2</sup>	0.279	0.296	0.319	0.441
Borrower type × Collateral × Date FE		Yes	Yes	
Lender type × Collateral × Date FE			Yes	
Borrower type × Lender type × Collateral × Date FE				Yes

The sample period is 2015m1-2020m6. Contracts with an interest rate higher than 10% or with a haircut lower than 0% or higher than 200% are excluded. *t*-Statistics are reported in parentheses. \*, \*\*, and \*\*\* indicate statistical significance at the 10%, 5%, and 1% level, respectively. *Maturity type* × *Date*, *Borrower type* × *Lender type* × *Date*, and *Collateral* × *Date* fixed effects are all included in each regression unless they are included in the multi-way fixed effect. The standard error is clustered at the collateral level.

Table A-6: Robustness checks: excluding each of collateral types

Dependent variable: Haircut	Baseline		Elimination of collateral types							
	(1) All maturities	(2) All maturities	(3) All maturities	(4) All maturities	(5) All maturities	(6) All maturities	(7) All maturities	(8) All maturities	(9) All maturities	(10) All maturities
Spread	-1.361*** (-9.67)	-1.234*** (-7.25)	-1.138*** (-7.31)	-1.242*** (-8.87)	-1.394*** (-9.56)	-1.425*** (-9.67)	-1.359*** (-9.65)	-1.561*** (-10.18)	-1.385*** (-9.87)	-1.395*** (-9.95)
Size	0.000429 (0.47)	-0.00216** (-2.37)	0.000783 (0.74)	0.000767 (0.75)	0.000631 (0.68)	0.000447 (0.48)	0.000546 (0.59)	0.000432 (0.47)	0.000439 (0.48)	0.000505 (0.55)
N.Obs	6,162,332	3,203,216	4,640,994	5,477,794	5,676,212	5,875,472	6,044,211	6,052,153	6,160,387	6,161,884
Adj. R <sup>2</sup>	0.647	0.797	0.686	0.675	0.653	0.652	0.650	0.650	0.569	0.467
Dependent variable: Haircut	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
	1-day	1-day	1-day	1-day	1-day	1-day	1-day	1-day	1-day	1-day
Spread	-1.369*** (-9.95)	-1.275*** (-7.72)	-1.167*** (-7.69)	-1.256*** (-9.23)	-1.405*** (-9.85)	-1.434*** (-9.95)	-1.367*** (-9.94)	-1.573*** (-10.52)	-1.369*** (-9.95)	-1.372*** (-9.97)
Size	0.000595 (0.65)	-0.00183** (-2.04)	0.000909 (0.85)	0.000952 (0.92)	0.000792 (0.85)	0.000620 (0.66)	0.000705 (0.76)	0.000582 (0.63)	0.000587 (0.64)	0.000604 (0.66)
N.Obs	5,917,734	3,042,630	4,483,044	5,250,957	5,465,235	5,643,626	5,802,944	5,813,706	5,916,183	5,917,726
Adj. R <sup>2</sup>	0.279	0.411	0.268	0.294	0.248	0.272	0.284	0.280	0.271	0.274
Excluded collateral type		Government bond	Bank bond	Monetary stabilization bond	Special bond	Financial bond	Municipal bond	Corporate bond	ETF security (else)	Equity

The sample period is 2015m1-2020m6. Contracts with an interest rate higher than 10% or with a haircut lower than 0% or higher than 200% are excluded. *t*-Statistics are reported in parentheses. \*, \*\*, and \*\*\* indicate statistical significance at the 10%, 5%, and 1% level, respectively. *Maturity type* × *Date*, *Borrower type* × *Lender type* × *Date*, and *Collateral* × *Date* fixed effects are all included. The standard error is clustered at the collateral level.

Table A-7: Robustness check: alternative standard error clustering

Dependent variable: Haircut	(1)	(2)	(3)	(4)	(5)	(6)
	All maturities	1-day	All maturities	1-day	All maturities	1-day
Spread	-1.361*** (-72.53)	-1.369*** (-77.41)	-1.361*** (-5.67)	-1.369*** (-5.61)	-1.361*** (-24.24)	-1.369*** (-24.53)
Size	0.000429*** (3.70)	0.000595*** (5.17)	0.000429 (0.43)	0.000595 (0.59)	0.000429 (1.30)	0.000595* (1.80)
Maturity type × Date FE	Yes	Yes	Yes	Yes	Yes	Yes
Borrower type × Lender type × Date FE	Yes	Yes	Yes	Yes	Yes	Yes
Collateral × Date FE	Yes	Yes	Yes	Yes	Yes	Yes
Cluster Var.	Collateral × Date	Collateral × Date	Collateral + (Year × Month) + (Day × DoW)	Collateral + (Year × Month) + (Day × DoW)	Collateral × ((Year × Month) + (Day × DoW))	Collateral × ((Year × Month) + (Day × DoW))
N.Obs	(970,175)	(927,149)	(12,259)	(12,059)	(642,901)	(622,147)
Adj. R <sup>2</sup>	0.648	0.279	0.647	0.279	0.648	0.279

The sample period is 2015m1-2020m6. Contracts with an interest rate higher than 10% or with a haircut lower than 0% or higher than 200% are excluded. *t*-Statistics are reported in parentheses. \*, \*\*, and \*\*\* indicate statistical significance at the 10%, 5%, and 1% level, respectively. DoW is an abbreviation of "the day of week". The numbers in parentheses in the cluster variable row are the number of clusters.

## Appendix B: Omitted proofs

**Proof of Lemma 1.** We define the Lagrangian function for the maximization problem (6) as

$$L = mq - (1 - \alpha)(1 - \sigma)p - \alpha\bar{y}a' + \lambda_1 [-q + (1 - \alpha)(1 - \sigma)p + \alpha\bar{y}a'] \\ + \lambda_2 [-\sigma q + \gamma] + \lambda_3 [ya' - p] + \lambda_4 [a - a'] + \lambda_5 q + \lambda_6 p + \lambda_7 a'$$

where  $\lambda_i$  for  $i \in \{1, \dots, 7\}$  are the Lagrange multipliers with  $\lambda_i \geq 0$  for all  $i$ . The first order conditions are

$$\{q\} : m + \lambda_5 = \lambda_1 + \lambda_2\sigma \quad (23)$$

$$\{p\} : \lambda_3 - \lambda_6 = (1 - \alpha)(1 - \sigma)(\lambda_1 - 1) \quad (24)$$

$$\{a'\} : \lambda_4 - \lambda_7 = (\lambda_1 - 1)\alpha\bar{y} + \lambda_3 y. \quad (25)$$

**Case 7 (Case 1 (Fully-IIS))**  $\lambda_2 = 0$

Given  $\lambda_2 = 0$ , we obtain  $\lambda_1 = m + \lambda_5 > 0$ ,  $\lambda_3 - \lambda_6 > 0$ , and  $\lambda_4 - \lambda_7 > 0$  from (23) - (25). Thus, (7), (9), and (10) must bind, which implies  $q = \bar{y}a$ ,  $p = ya$ , and  $a' = a$ . Thus,  $V_{IIS} = (m - 1)\bar{y}a$ . Finally, to have  $\lambda_2 = 0$ , it must  $\gamma \geq \sigma\bar{y}a$ .

**Case 8 (Case 2 (Partially-IIS))**  $\lambda_2 > 0$

Given  $\lambda_2 > 0$ , (8) must bind, which gives  $q = \frac{\gamma}{\sigma}$  and  $\lambda_5 = 0$ . Therefore, it must be  $a' > 0$  and thus  $\lambda_7 = 0$ , because  $q \leq 0$  from (7) otherwise. Suppose  $\lambda_1 = 0$ . Then, from (24) and (25), we obtain  $a' = 0$ , a contradiction. Thus,  $\lambda_1 > 0$  must hold whenever  $\lambda_2 > 0$ . Then, from the binding (7) and (8), we obtain equation (13). Given  $q = \frac{\gamma}{\sigma}$  and the binding (7), the borrower's surplus is given as  $V_{IIS} = \frac{(m-1)\gamma}{\sigma}$ .

For a moment, we assume that  $\lambda_3 = 0$ . Given  $\lambda_3 = 0$ , we obtain, from (24) and (25),  $\lambda_6 = (1 - \alpha)(1 - \sigma)(1 - \lambda_1) \geq 0$  and  $\lambda_4 = \alpha\bar{y}(\lambda_1 - 1) \geq 0$ . This requires  $\lambda_1 = 1$  and  $\lambda_4 = \lambda_6 = 0$ , and  $\lambda_2$  is given as  $\lambda_2 = \frac{m-1}{\sigma}$  from (23). Therefore, it must be  $0 \leq p \leq ya'$  and  $0 < a' \leq a$ , and  $p$

and  $a'$  must satisfy condition (13) but are not determined uniquely. Next, from (9), (10), and (13), we obtain

$$\frac{\gamma}{\sigma} = (1 - \alpha)(1 - \sigma)p + \alpha\bar{y}a' \leq \bar{y}a, \quad (26)$$

and thus the necessary condition for this case is  $\gamma \leq \sigma\bar{y}a$ . Next, if  $p = ya'$ , (13) implies  $\frac{\gamma}{\sigma} = \bar{y}a'$ . Thus,  $a' \geq \frac{\gamma}{\sigma\bar{y}}$  must hold to have  $p \leq ya'$ . On the other hand, if  $p = 0$ , then (13) gives  $\frac{\gamma}{\sigma} = \alpha\bar{y}a'$ . Thus,  $a' \leq \frac{\gamma}{\alpha\sigma\bar{y}}$  must hold to have  $0 \leq p$ . Combined together, we obtain  $p \in [0, ya']$  and  $a' \in \left[ \frac{\gamma}{\sigma\bar{y}}, \min \left\{ a, \frac{\gamma}{\alpha\sigma\bar{y}} \right\} \right]$ .

Now suppose  $\lambda_3 > 0$ , which implies  $p = ya' > 0$  from (9) so  $\lambda_6 = 0$ . Then, from (13), we obtain  $a' = \frac{\gamma}{\sigma\bar{y}}$ , and the necessary condition for this case is again  $\gamma \leq \sigma\bar{y}a$ . This is the knife edge case of the case with  $\lambda_3 = 0$  above.

Finally, by defining  $\gamma^* \equiv \sigma\bar{y}a$  and reorganizing cases 1 and 2 above, we obtain the results of lemma 1. ■

**Proof of Lemma 2.** In the loan contract problem (14), it must be  $q > 0$  and  $a' > 0$  to satisfy (16). It is also obvious that (15) must bind; otherwise, the borrower could increase the surplus without violating any constraints. Then, from (15) and (16), we obtain

$$q = (1 - \alpha)p + \alpha ya' - \frac{\gamma}{1 - \sigma} \geq \frac{\gamma}{\sigma}. \quad (27)$$

Thus,  $\gamma \leq \sigma\bar{y}a = \gamma^*$  must hold, otherwise, constraints (15) - (18) cannot be satisfied simultaneously. Substituting  $q = (1 - \alpha)p + \alpha ya' - \frac{\gamma}{1 - \sigma}$  into the objective function (14), we obtain

$$V_{IS} = \max_{p, a'} \{ (1 - \sigma)(m - 1) [(1 - \alpha)p + \alpha ya'] - m\gamma \}$$

subject to (17) - (19) and (27). Now, it becomes obvious that it must be  $a' = a$  and  $p = ya$  to maximize the objective function. Then,  $q = ya - \frac{\gamma}{1 - \sigma}$  and  $V_{IS} = (m - 1)\bar{y}a - m\gamma$ . Finally, the borrower can always choose not to trade. Thus, it must be  $V_{IS} = (m - 1)\bar{y}a - m\gamma \geq 0$ , which requires  $\gamma \leq \frac{(m-1)\bar{y}a}{m}$ . Thus, the necessary condition for the existence of an IS loan

contract that is not dominated by the no trading option is  $\gamma \leq \min \left\{ \gamma^*, \frac{(m-1)\bar{y}a}{m} \right\}$ . ■

**Proof of Proposition 3.** As one can see from lemmas 1 and 2,  $V_{IIS}$  weakly increases with  $\gamma$  while  $V_{IS}$  decreases with  $\gamma$ . In particular,  $\lim_{\gamma \searrow 0} V_{IIS} < \lim_{\gamma \searrow 0} V_{IS}$  and  $\lim_{\gamma \nearrow \gamma^*} V_{IIS} > \lim_{\gamma \nearrow \gamma^*} V_{IS}$ . Thus, there exists a unique cutoff level of the information acquisition cost  $\gamma^{**} \in (0, \gamma^*)$  such that when  $\gamma = \gamma^{**}$ ,  $V_{IIS} = V_{IS}$ . More specifically, the type of IIS collateralized debt must be partially IIS when  $\gamma = \gamma^{**}$  because  $\gamma^{**} < \gamma^*$ . Therefore,  $V_{IIS} = \frac{(m-1)\gamma^{**}}{\sigma}$  and  $V_{IS} = (m-1)\frac{\gamma^*}{\sigma} - m\gamma^{**}$  when  $\gamma = \gamma^{**}$ , so it must be  $\frac{(m-1)\gamma^{**}}{\sigma} = (m-1)\frac{\gamma^*}{\sigma} - m\gamma^{**}$ . Hence, the cutoff level  $\gamma^{**}$  is given by (20).

Then, the borrower induces the lender to acquire information (i.e., IS collateralized debt) only if  $\gamma < \gamma^{**}$ , and does not trigger information acquisition otherwise. Furthermore, when  $\gamma \geq \gamma^{**}$ , if  $\gamma \in [\gamma^{**}, \gamma^*)$ , the borrower offers partially-IIS collateralized debt to the lender and offers fully-IIS collateralized debt if  $\gamma \geq \gamma^*$ . ■

**Proof of Proposition 4.** The terms of direct sales  $(q_s, a'_s)$  can be obtained by imposing  $\alpha = 1$  into the terms of collateralized debt.

First, when collateralized debt is fully-IIS, we obtain  $(q_s, a'_s) = (\bar{y}a, a)$ , that is, the borrower can sell  $a$  units of the trees in exchange for  $\bar{y}a$  units of the goods. Therefore, the value of  $a$  units of the collateral trees is given as  $v = \bar{y}a$ , and we also have  $q = \bar{y}a$ . Thus, the haircut is  $\theta = \frac{v-q}{v} = 0$ . Substituting  $p = ya$  and  $q = \bar{y}a$  into  $r = \frac{p-q}{q}$ , we obtain the interest rate as

$$r = \frac{ya - (1 - \sigma)ya}{(1 - \sigma)ya} = \frac{\sigma}{1 - \sigma}.$$

Second, when collateralized debt is partially-IIS, we obtain  $(q_s, a'_s) = \left( \frac{\gamma}{\sigma}, \frac{\gamma}{\sigma\bar{y}} \right)$ , which implies that the borrower sells each tree at the price of  $\bar{y}$ , in terms of the consumption goods in period 0. Therefore, the value of  $a'$  units of the collateral trees is given as  $v = \bar{y}a'$  in this case. Then, we obtain the haircut as  $\theta = \frac{v-q}{v} = 1 - \frac{\gamma}{\sigma\bar{y}a'}$ . According to lemma 1, we obtain  $\sigma\bar{y}a' \in [\gamma, \min\{\gamma^*, \frac{\gamma}{\alpha}\}]$ , thus,  $\theta \in \left[ 0, 1 - \max \left\{ \frac{\gamma}{\gamma^*}, \alpha \right\} \right]$ . Given  $q = \frac{\gamma}{\sigma}$ , the interest rate is  $r = \frac{\sigma p}{\gamma} - 1$ . Also, from lemma 1, we obtain  $\sigma p \in \left[ \max \left\{ 0, \frac{\gamma - \alpha\sigma\bar{y}a}{(1-\alpha)(1-\sigma)} \right\}, \frac{\gamma}{1-\sigma} \right]$ , thus,

$r \in \left[ \max \left\{ 0, \frac{1-\alpha \cdot \frac{\gamma^*}{\gamma}}{(1-\alpha)(1-\sigma)} \right\} - 1, \frac{1}{1-\sigma} - 1 \right]$ . Moreover, because  $p$  and  $a'$  are not determined uniquely in this case, the interest rate  $r$  and haircut  $\theta$  also are not determined uniquely. More precisely, substituting  $r = \frac{\sigma p}{\gamma} - 1$  and  $\theta = 1 - \frac{\gamma}{\sigma \bar{y} a'}$  into (13), we obtain (21), which determines the pair of  $(r, \theta)$  for the partially-IIS loan contract.

Finally, when collateralized debt is IS, we obtain  $(q_s, a'_s) = (ya - \frac{\gamma}{1-\sigma}, a)$ . Therefore, the value of  $a$  units of the collateral trees of IS collateralized debt is given as  $v = ya - \frac{\gamma}{1-\sigma}$ , and  $q = ya - \frac{\gamma}{1-\sigma}$ . Thus, we obtain the haircut as  $\theta = 0$ . The interest rate is

$$r = \frac{p - q}{q} = \frac{ya}{ya - \frac{\gamma}{1-\sigma}} - 1 = \frac{1}{1 - \frac{\gamma}{\bar{y}a}} - 1 = \frac{\gamma}{\bar{y}a - \gamma}.$$

■

**Proof of Proposition 5.** Let  $\hat{r}_{FIIS}$  ( $\hat{\theta}_{FIIS}$ ),  $\hat{r}_{PIIS}$  ( $\hat{\theta}_{PIIS}$ ),  $\hat{r}_{IS}$  ( $\hat{\theta}_{IS}$ ) denote the interest rate (haircut) when the type of collateralized debt is fully IIS, partially IIS, and IS, respectively. According to Proposition 4, we have

$$\hat{r}_{PIIS} = \frac{1}{2} \left[ \max \left\{ 0, \frac{1 - \alpha \cdot \frac{\gamma^*}{\gamma}}{(1-\alpha)(1-\sigma)} \right\} + \frac{1}{1-\sigma} \right] - 1, \quad \hat{\theta}_{PIIS} = \frac{1}{2} \left[ 1 - \max \left\{ \frac{\gamma}{\gamma^*}, \alpha \right\} \right].$$

Firstly, we examine the effect of  $\alpha$  on  $\hat{r}$  and  $\hat{\theta}$ . The value of  $M$  is not affected by  $\alpha$ , and the type of collateralized debt is IS if  $\gamma < M\sigma\bar{y}a$ , partially-IIS if  $\gamma \in [M\sigma\bar{y}a, \sigma\bar{y}a)$ , and fully-IIS otherwise. Thus, the type of collateralized debt remains unchanged by variations in  $\alpha$ . According to proposition 4,  $\hat{r}$  and  $\hat{\theta}$  do not change with  $\alpha$  when the type of collateralized debt is either fully IIS or IS. However,  $\frac{\partial \hat{r}_{PIIS}}{\partial \alpha} = 0$  and  $\frac{\partial \hat{\theta}_{PIIS}}{\partial \alpha} < 0$  when  $\alpha\gamma^* \geq \gamma$ , and otherwise,  $\frac{\partial \hat{r}_{PIIS}}{\partial \alpha} = 0$  and

$$\frac{\partial \hat{r}_{PIIS}}{\partial \alpha} = \frac{\partial}{\partial \alpha} \left( \frac{2 - \alpha - \alpha \frac{\gamma^*}{\gamma}}{1 - \alpha} \right) = \frac{1 - \frac{\gamma^*}{\gamma}}{(1 - \alpha)^2} < 0,$$

because  $\gamma < \gamma^*$  when the type of collateralized debt is partially-IIS. Thus,  $\hat{r}$  and  $\hat{\theta}$  never

change concurrently in response to variations in  $\alpha$ .

Next, the impact of  $\gamma$  on  $\hat{r}$  and  $\hat{\theta}$  is assessed. Because  $\hat{\theta}_{FIIS} = \hat{\theta}_{IS} = 0$ , responses in  $\hat{\theta}$  occur only when the type of collateralized debt shifts due to changes in  $\gamma$ , or remains partially-IIS, that is,  $\gamma \in [\gamma^{**}, \gamma^*]$ . First, we check the direction of the responses of  $\hat{r}$  and  $\hat{\theta}$  with  $\gamma \in [\gamma^{**}, \gamma^*]$ . It is verified that  $\frac{\partial \hat{\theta}_{PIIS}}{\partial \gamma} = \frac{\partial \hat{r}_{PIIS}}{\partial \gamma} = 0$  if  $\alpha\gamma^* \geq \gamma$ . Suppose that  $\alpha\gamma < \gamma^*$ . Then  $\frac{\partial \hat{\theta}_{PIIS}}{\partial \gamma} < 0$  while  $\frac{\partial \hat{r}_{PIIS}}{\partial \gamma} > 0$ . Thus, if the change of  $\gamma$  is within  $[\gamma^{**}, \gamma^*]$ , and if both  $\hat{r}(\alpha, \sigma, \gamma)$  and  $\hat{\theta}(\alpha, \sigma, \gamma)$  respond to the change in  $\gamma$ , then they change in opposite directions. Moreover,  $\lim_{\gamma \rightarrow \gamma^*} \hat{r}_{PIIS}(\alpha, \sigma, \gamma) = \frac{\sigma}{1-\sigma} = \hat{r}(\alpha, \sigma, \gamma^*)$ . Therefore, from the fact that  $\frac{\partial \hat{r}_{PIIS}}{\partial \gamma} > 0$  when  $\alpha\gamma^* < \gamma$  and  $\frac{\partial \hat{r}_{PIIS}}{\partial \gamma} = 0$  when  $\alpha\gamma^* \geq \gamma$ , we obtain  $\hat{r}_{FIIS} > \hat{r}_{PIIS}$ . Moreover, because  $\hat{\theta}_{FIIS} = 0$  while  $\hat{\theta}_{PIIS} > 0$ , we obtain  $\hat{\theta}_{FIIS} < \hat{\theta}_{PIIS}$ . Thus, the statement for  $\gamma$  is also valid for  $\gamma \geq \gamma^*$ , i.e., when the type of collateralized debt is not IS. Finally, we have  $\hat{\theta}_{IS} < \hat{\theta}_{PIIS}$  because  $\hat{\theta}_{IS} = 0$  while  $\hat{\theta}_{PIIS} > 0$ . However,  $\lim_{\gamma \rightarrow \gamma^*} \hat{r}_{PIIS}(\alpha, \sigma, \gamma) = \frac{\sigma}{1-\sigma} > 0$  while  $\lim_{\gamma \rightarrow 0} \hat{r}_{IS}(\alpha, \sigma, \gamma) = 0$ . Therefore, when the type of collateralized debt switches between partially-IIS and IS,  $\hat{r}$  and  $\hat{\theta}$  may change in the same direction.

Especially, suppose that we have  $\alpha < \frac{2m\sigma - 2\sigma}{2m\sigma + m - 1 - \sigma}$ . For ease of discussion, denote  $M \equiv \frac{m-1}{m(1+\sigma)-1} < 1$ , which implies  $\gamma^{**} = M\gamma^* = M\sigma\bar{y}a$  and  $\alpha < \frac{2M\sigma}{1+M\sigma}$ . Then

$$\begin{aligned} & \lim_{\gamma \rightarrow \gamma^{**}} \hat{r}_{IS}(\alpha, \sigma, \gamma) - \hat{r}_{PIIS}(\alpha, \sigma, \gamma^{**}) \\ &= \frac{1}{1 - \frac{\gamma^{**}}{\bar{y}a}} - \frac{1}{2} \left[ \frac{1 - \alpha \cdot \frac{\gamma^*}{\gamma^{**}}}{(1 - \alpha)(1 - \sigma)} + \frac{1}{1 - \sigma} \right] \\ &= \frac{1}{1 - M\sigma} - \frac{1}{2(1 - \sigma)} \left[ \frac{1 - \frac{\alpha}{M}}{1 - \alpha} + 1 \right] \\ &= \frac{1}{2(1 - \sigma)} \cdot \frac{(1 - M)[\alpha(1 + M\sigma) - 2M\sigma]}{(1 - M\sigma)(M - M\alpha)} < 0 \end{aligned}$$

because  $\alpha < \frac{2M\sigma}{1+M\sigma}$ . Thus, from the fact that  $\frac{\partial \hat{r}_{IS}}{\partial \gamma} > 0$  and  $\frac{\partial \hat{r}_{PIIS}}{\partial \gamma} \geq 0$ , we have  $\hat{r}_{IS} < \hat{r}_{PIIS}$ . Therefore, given the restriction on  $\alpha$ , both  $\hat{r}$  and  $\hat{\theta}$  rise as the type of collateralized debt switches between partially-IIS and IS. ■

**Proof of Proposition 6.** In the context of the proof of Proposition 5, define  $\hat{r}_{FIIS}$ ,  $\hat{\theta}_{FIIS}$ ,  $\hat{r}_{PIIS}$ ,  $\hat{\theta}_{PIIS}$ ,  $\hat{r}_{IS}$ ,  $\hat{\theta}_{IS}$ ,

and  $M$  as in the proof of Proposition 5. Let  $\sigma^* \equiv \frac{\gamma}{\bar{y}a}$ , and  $\sigma^{**}$  be the level of  $\sigma$  such that  $\gamma = \frac{m-1}{m(1+\sigma^{**})-1} \cdot \sigma^{**}\bar{y}a$ . It can be confirmed that  $\sigma^{**} > \sigma^*$ . Depending on  $\sigma$ , the type of collateralized debt is IS if  $\sigma > \sigma^{**}$ , partially-IIS if  $\sigma \in (\sigma^*, \sigma^{**}]$ , and fully-IIS otherwise.

Observe that  $\frac{\partial \hat{r}_{FIIS}}{\partial \sigma} > 0$ , while  $\frac{\partial \hat{r}_{IS}}{\partial \sigma} = 0$ . Given that  $\hat{\theta}_{FIIS} = \hat{\theta}_{IS} = 0$ ,  $\hat{\theta}$  responds to a change of  $\sigma$  only if either the type of collateralized debt changes or if the type remains partially-IIS with the change of  $\sigma$ . It is noted that

$$\hat{r}_{IS}(\alpha, \sigma, \gamma) = \frac{1}{1 - \frac{\gamma}{\bar{y}a}} - 1 = \frac{\sigma^*}{1 - \sigma^*} = \hat{r}_{FIIS}(\alpha, \sigma^*, \gamma) = \lim_{\sigma \rightarrow \sigma^*} \hat{r}_{PIIS}(\alpha, \sigma, \gamma).$$

This equation shows that  $\hat{r}_{FIIS} = \hat{r}_{IS} < \hat{r}_{PIIS}$ , which is also true for  $\hat{\theta}$  because  $\hat{\theta}_{FIIS} = \hat{\theta}_{IS} = 0$  and  $\hat{\theta}_{PIIS} > 0$ . Now we show that the direction of the responses of  $\hat{r}$  and  $\hat{\theta}$  to a change in  $\sigma$  are the same within partial-IIS. Note that  $\frac{\partial \hat{\theta}_{PIIS}}{\partial \sigma} = 0$  and  $\frac{\partial \hat{r}_{PIIS}}{\partial \sigma} > 0$  if  $\alpha\gamma^* \geq \gamma$ . If  $\alpha\gamma < \gamma^*$ , with  $\gamma^* = \sigma\bar{y}a$ , we obtain  $\frac{\partial \hat{\theta}_{PIIS}}{\partial \sigma} > 0$  and

$$\begin{aligned} \frac{\partial \hat{r}_{PIIS}}{\partial \sigma} &= \frac{\partial}{\partial \sigma} \left( \frac{(2-\alpha)\gamma - \alpha\sigma\bar{y}a}{\gamma(1-\alpha)(1-\sigma)} \right) = \frac{(2-\alpha)\gamma - \alpha\bar{y}a}{\gamma(1-\alpha)(1-\sigma)^2} \\ &\geq \frac{((2-\alpha)M\sigma - \alpha)\bar{y}a}{\gamma(1-\alpha)(1-\sigma)^2} = \frac{(2M\sigma - (M\sigma + 1)\alpha)\bar{y}a}{\gamma(1-\alpha)(1-\sigma)^2} > 0, \end{aligned}$$

given that  $\gamma \geq \gamma^{**} = M\sigma\bar{y}a$  and  $\alpha < \frac{2M\sigma}{1+M\sigma}$ . Therefore, if both  $\hat{r}(\alpha, \sigma, \gamma)$  and  $\hat{\theta}(\alpha, \sigma, \gamma)$  respond to a change in  $\sigma$ , they do so in the same direction. ■

## Appendix C: Robustness to borrower's information acquisition

In this appendix, we clarify that the borrower's accessibility to information does not impact our theoretical results. Specifically, we argue that a borrower, even when allowed to acquire information, would choose not to. For this analysis, we assume the borrower has the option to acquire information at the beginning of the first period. This scenario introduces the possibility of information asymmetry; hence, we utilize the perfect Bayesian equilibrium as our solution concept.

In an equilibrium where the borrower acquires information before making an offer to the lender, the borrower learns the quality of the collateral asset at the beginning of the first period. This information, known only to the borrower, categorizes the borrower into either a good-type borrower (with good-quality collateral) or a bad-type borrower (with bad-quality collateral). It is important to note that no separating equilibrium occurs when the borrower acquires information. The rationale is that a bad-type borrower has a strong incentive to mimic a good-type borrower. If a lender identifies the collateral as bad, she will reject the collateralized debt offered by the bad-type borrower. Given this mimicry by the bad-type borrower in the first period, the lender has a pooling belief about the borrower's type in equilibrium. The choice of collateralized debt by the good-type borrower is influenced by the lender.

The good-type borrower's choice of collateralized debt depends on the lender's belief. Let  $\bar{V}_{IIS}^g$  be the largest possible surplus for the good-type borrower from IIS collateralized debt in equilibrium, and let  $\bar{V}_{IS}^g$  be that from IS collateralized debt. Let  $\bar{V}_{IIS}^b$  and  $\bar{V}_{IS}^b$  be the bad-type borrower's surplus who mimics the good-type borrower who obtains the largest possible equilibrium surplus from IIS collateralized debt and IS collateralized debt, respectively. We show that the terms of the optimal equilibrium collateralized debt for each  $\gamma$  are not different from when the borrower does not acquire the information.

We first solve for the optimal equilibrium IIS collateralized debt contracts for the good-type borrower who already acquired information. The borrower's maximized equilibrium surplus from IIS collateralized debt,  $\bar{V}_{IIS}^g$ , is given by

$$\bar{V}_{IIS}^g = \max_{q,p,a'} \{mq - (1 - \alpha)p - \alpha ya' - \gamma_b\} \quad (28)$$

subject to

$$-q + (1 - \alpha)(1 - \sigma)p + \alpha \bar{y}a' \geq 0 \quad (29)$$

$$-\sigma q + \gamma \geq 0 \quad (30)$$

$$ya' - p \geq 0 \quad (31)$$

$$a - a' \geq 0 \quad (32)$$

$$q, p, a' \geq 0 \quad (33)$$

Given that the bad-type borrower mimics the good-type borrower's IIS collateralized debt  $(q, p, a')$  in the first period and does not repay in the second period,

$$\bar{V}_{IIS}^b = mq - \gamma_b.$$

**Lemma 9** *If  $m(1 - \sigma) < 1$  then no-trade. Given  $m(1 - \sigma) \geq 1$ , the terms of the optimal equilibrium IIS collateralized debt are as follows:*

1. *[Fully-IIS] Consider that  $\gamma^* \leq \gamma$ .  $q = \bar{y}a$ ,  $p = ya$ ,  $a' = a$ , and*

$$\bar{V}_{IIS}^g = \left(m - \frac{1}{1 - \sigma}\right)\bar{y}a - \gamma_b, \quad \bar{V}_{IIS}^b = m\bar{y}a - \gamma_b.$$

2. *[Partially-IIS] If  $\gamma < \gamma^*$ , then  $q = \frac{\gamma}{\sigma}$ ,  $p \in [0, ya']$  and  $a' \in \left[\frac{\gamma a}{\gamma^*}, \min\left\{a, \frac{\gamma a}{\alpha \gamma^*}\right\}\right]$  are determined simultaneously by*

$$\frac{\gamma}{\sigma} = (1 - \alpha)(1 - \sigma)p + \alpha \bar{y}a', \quad (34)$$

and

$$\bar{V}_{IIS}^g = \frac{m\gamma}{\sigma} - \frac{\gamma}{\sigma(1-\sigma)} - \gamma_b, \quad \bar{V}_{IIS}^b = \frac{m\gamma}{\sigma} - \gamma_b.$$

**Proof of Lemma 9.** We define the Lagrangian function for the maximization problem (28)

as

$$\begin{aligned} L = & mq - (1-\alpha)p - \alpha ya' - \gamma_b + \lambda_1 [-q + (1-\alpha)(1-\sigma)p + \alpha \bar{y}a'] \\ & + \lambda_2 [-\sigma q + \gamma] + \lambda_3 [ya' - p] + \lambda_4 [a - a'] + \lambda_5 q + \lambda_6 p + \lambda_7 a' \end{aligned}$$

where  $\lambda_i$  for  $i \in \{1, \dots, 7\}$  are the Lagrange multipliers with  $\lambda_i \geq 0$  for all  $i$ . The first order conditions are

$$\{q\} : m + \lambda_5 = \lambda_1 + \lambda_2\sigma \quad (35)$$

$$\{p\} : \lambda_3 - \lambda_6 = (1-\alpha)((1-\sigma)\lambda_1 - 1) \quad (36)$$

$$\{a'\} : \lambda_4 - \lambda_7 = ((1-\sigma)\lambda_1 - 1)\alpha y + \lambda_3 y. \quad (37)$$

First suppose that  $m(1-\sigma) > 1$ .

**Case 10 (Case 1 (Fully-IIS))**  $\lambda_2 = 0$  and  $m(1-\sigma) > 1$ .

Given  $\lambda_2 = 0$ , we obtain  $\lambda_1 = m + \lambda_5 > 0$ . Also, from  $\lambda_1 > m$ , we have  $(1-\sigma)\lambda_1 > 1$ . Thus,  $\lambda_3 - \lambda_6 > 0$  and  $\lambda_4 - \lambda_7 > 0$  from (35) - (37). Thus, (29), (31), and (32) must bind, which implies  $p = ya$ ,  $q = \bar{y}a$ , and  $a' = a$ . Thus,

$$\bar{V}_{IIS}^g = \left(m - \frac{1}{1-\sigma}\right)\bar{y}a - \gamma_b$$

and  $\bar{V}_{IIS}^b = mq - \gamma_b = m\bar{y}a - \gamma_b$ . To have  $\lambda_2 = 0$ ,  $\gamma \geq \sigma\bar{y}a$  must hold.

Next, suppose that  $m(1-\sigma) < 1$ .

**Case 11 (Case 2 (Fully-IIS))**  $\lambda_2 = 0$  and  $(1-\sigma)m < 1$ .

Given  $\lambda_2 = 0$ , we obtain  $\lambda_1 = m + \lambda_5 > 0$ , thus  $q = (1 - \alpha)(1 - \sigma)p + \alpha\bar{y}a'$ . However,  $\bar{V}_{IIS}^g = (m - (1 - \sigma))q - \gamma_b < 0$ . Thus, the borrower would not make a fully information-insensitive collateralized debt. First, suppose that  $\lambda_5 > 0$ . Then  $q = 0$ , which means no-trade. Now suppose that  $\lambda_5 = 0$ . Then  $\lambda_1 = m$ , which implies  $(1 - \sigma)\lambda_1 < 1$ . Thus,  $\lambda_3 - \lambda_6 < 0$  and  $\lambda_4 - \lambda_7 < 0$  from (35) - (37). Thus, (29) must bind and  $p = a' = 0$ , which in turn implies  $q = 0$ , i.e., no-trade.

Now, suppose that  $m(1 - \sigma) = 1$ .

**Case 12 (Case 3 (Fully-IIS))**  $\lambda_2 = 0$  and  $m(1 - \sigma) = 1$ .

Given  $\lambda_2 = 0$ , we obtain  $\lambda_1 = m + \lambda_5$ . If  $\lambda_5 > 0$  then  $q = 0$ , which means no-trade. Now suppose that  $\lambda_5 = 0$ . Then  $\lambda_1 = m$ , which implies  $(1 - \sigma)\lambda_1 = 1$ . Thus,  $\lambda_3 - \lambda_6 = \lambda_4 - \lambda_7 = 0$  from (35) - (37). If  $\lambda_3 > 0$  and  $\lambda_6 > 0$ , then  $p = a' = 0$ , no-trade. Moreover, both  $\lambda_4 > 0$  and  $\lambda_7 > 0$  cannot hold together, because both  $a' = a$  and  $a' = 0$  cannot hold together. Thus  $\lambda_4 = \lambda_7 = 0$ . Now consider that  $\lambda_3 = \lambda_6 = 0$ . Then  $p \leq ya'$  and  $a' \leq a$ . This is the knife edge case of either case 1, 2, or case 4. We consider this case as the knife edge case of case 1.

**Case 13 (Case 4 (Partially-IIS))**  $\lambda_2 > 0$

Given  $\lambda_2 > 0$ , (30) must bind, which gives  $q = \frac{\gamma}{\sigma}$  and  $\lambda_5 = 0$ . Therefore, it must be  $a' > 0$  and thus  $\lambda_7 = 0$ , because  $q \leq 0$  from (29) otherwise. Suppose  $\lambda_1 = 0$ . Then, from (36) and (37), we obtain  $a' = 0$ , a contradiction. Thus,  $\lambda_1 > 0$  must hold whenever  $\lambda_2 > 0$ . Then, from the binding (29) and (30), we obtain (34). Given  $q = \frac{\gamma}{\sigma}$  and the binding (29), the borrower's surplus is given as  $\bar{V}_{IIS}^g = \frac{m\gamma}{\sigma} - \frac{\gamma}{\sigma(1-\sigma)} - \gamma_b$ .

For a moment, we assume that  $\lambda_3 = 0$ . Given  $\lambda_3 = 0$ , we obtain, from (36) and (37),  $\lambda_6 = (1 - \alpha)(1 - \sigma)(1 - (1 - \sigma)\lambda_1) \geq 0$  and  $\lambda_4 = ((1 - \sigma)\lambda_1 - 1)\alpha y \geq 0$ . This requires  $(1 - \sigma)\lambda_1 = 1$  and  $\lambda_4 = \lambda_6 = 0$ , and  $\lambda_2$  is given as  $\lambda_2 = \frac{m-1}{\sigma}$  from (35). Therefore, it must be  $0 \leq p \leq ya'$  and  $0 < a' \leq a$ , and  $p$  and  $a'$  must satisfy condition (34) but are not determined uniquely. Next, from (31), (32), and (34), we obtain

$$\frac{\gamma}{\sigma} = (1 - \alpha)(1 - \sigma)p + \alpha\bar{y}a' \leq \bar{y}a, \quad (38)$$

and thus the necessary condition for this case is  $\gamma \leq \sigma \bar{y} a$ . Next, if  $p = ya'$ , (34) implies  $\frac{\gamma}{\sigma} = \bar{y} a'$ . Thus,  $a' \geq \frac{\gamma}{\sigma \bar{y}}$  must hold to have  $p \leq ya'$ . On the other hand, if  $p = 0$ , then (34) gives  $\frac{\gamma}{\sigma} = \alpha \bar{y} a'$ . Thus,  $a' \leq \frac{\gamma}{\alpha \sigma \bar{y}}$  must hold to have  $0 \leq p$ . Combined, we obtain  $p \in [0, ya']$  and  $a' \in \left[ \frac{\gamma}{\sigma \bar{y}}, \min \left\{ a, \frac{\gamma}{\alpha \sigma \bar{y}} \right\} \right]$ . Thus,  $\bar{V}_{IIS}^g = mq - (1 - \alpha)p - \alpha ya' - \gamma_b = \frac{m\gamma}{\sigma} - \frac{\gamma}{\sigma(1-\sigma)} - \gamma_b$  and  $\bar{V}_{IIS}^g = mq - \gamma_b = \frac{m\gamma}{\sigma} - \gamma_b$ .

Now suppose  $\lambda_3 > 0$ , which implies  $p = ya' > 0$  from (31) so  $\lambda_6 = 0$ . Then, from (34), we obtain  $a' = \frac{\gamma}{\sigma \bar{y}}$ , and the necessary condition for this case is again  $\gamma \leq \sigma \bar{y} a$ . This is the knife edge case of the case with  $\lambda_3 = 0$  above.

Finally, for the borrower to offer an IIS collateralized debt,  $\bar{V}_{IIS}^g \geq -\gamma_b$  is required, that is,  $m(1 - \sigma) \geq 1$ . Otherwise no-trade.

By applying  $\gamma^* \equiv \sigma \bar{y} a$  and reorganizing cases 1 and 2 above, we obtain the results of lemma 9. ■

We now solve for the optimal equilibrium IS collateralized debt contracts for the good-type borrower who already acquired information that triggers information acquisition by the lender about the collateral quality. In IS loan contracts, transactions between the borrower and the lender are guaranteed. This certainty stems from the borrower's type being good, which is known to the lender. Thus, the borrower's maximized equilibrium surplus from IS collateralized debt,  $\bar{V}_{IS}^g$ , is given by

$$\bar{V}_{IS}^g = \max_{q,p,a'} \{[mq - (1 - \alpha)p - \alpha ya' - \gamma_b]\} \quad (39)$$

subject to

$$(1 - \sigma) [-q + (1 - \alpha)p + \alpha ya'] - \gamma \geq 0 \quad (40)$$

$$\sigma q - \gamma \geq 0 \quad (41)$$

$$ya' - p \geq 0 \quad (42)$$

$$a - a' \geq 0 \quad (43)$$

$$q, p, a' \geq 0. \quad (44)$$

Because the bad-type borrower cannot trade with the lender when the collateralized debt is information sensitive,  $\bar{V}_{IS}^b = -\gamma_b$ .

**Lemma 14** [IS] *If  $\gamma \leq \min \left\{ \gamma^*, \frac{(m-1)\gamma^*}{\sigma m} \right\}$ , then the terms of the optimal equilibrium IS collateralized debt are such that  $q = ya - \frac{\gamma}{1-\sigma}$ ,  $p = ya$ ,  $a' = a$ ,  $\bar{V}_{IS}^g = (m-1)ya - \frac{m\gamma}{1-\sigma} - \gamma_b$ , and  $\bar{V}_{IS}^b = -\gamma_b$ . Otherwise, IS collateralized debt is either infeasible or worse than the no-trading option.*

**Proof of Lemma 14.** In the loan contract problem (39), it must be  $q > 0$  and  $a' > 0$  to satisfy (41) and (42). It is also obvious that (40) must bind; otherwise, the borrower could increase the surplus without violating any constraints. Then, from (40) and (41), we obtain

$$q = (1 - \alpha)p + \alpha ya' - \frac{\gamma}{1 - \sigma} \geq \frac{\gamma}{\sigma}. \quad (45)$$

Substituting  $q = (1 - \alpha)p + \alpha ya' - \frac{\gamma}{1-\sigma}$  into the objective function (39), we obtain

$$\bar{V}_{IS}^g = \max_{p, a'} \left\{ (m - 1) [(1 - \alpha)p + \alpha ya'] - \frac{m\gamma}{1 - \sigma} - \gamma_b \right\}$$

subject to (42) - (44) and (45). Now, it becomes obvious that it must be  $a' = a$  and  $p = ya$  to maximize the objective function. Then,  $q = ya - \frac{\gamma}{1-\sigma}$  and  $\bar{V}_{IS}^g = (m-1)ya - \frac{m\gamma}{1-\sigma} - \gamma_b$ . From (45),  $\gamma \leq \sigma(1 - \sigma)ya = \gamma^*$  must hold. Finally, the borrower can always choose not to trade.

Thus, it must be  $\bar{V}_{IS}^g = (m-1)ya - \frac{m\gamma}{1-\sigma} - \gamma_b \geq -\gamma$ , which requires  $\gamma \leq \frac{(m-1)\gamma^*}{\sigma m}$ . Thus, the necessary condition for the existence of an IS loan contract that is not dominated by the no trading option is  $\gamma \leq \min \left\{ \gamma^*, \frac{(m-1)\gamma^*}{\sigma m} \right\}$ . Finally, the lender would not trade with the bad-type borrower, thus,  $\bar{V}_{IS}^b = m \cdot 0 - \gamma_b$ . ■

Based on the results from lemmas 9 and 14, we argue in the following proposition that even if the borrower is allowed to acquire information, she would not.

**Proposition 15** *The borrower does not acquire information in equilibrium.*

**Proof of Proposition 15.** The borrower's maximized surplus when the borrower does not acquire the information is  $\max \{V_{IIS}, V_{IS}\}$ . If the borrower decides to acquire the information, then she offers a collateralized debt to maximize her surplus with probability  $1 - \sigma$  and mimics the good-type borrower with probability  $\sigma$ , thus the borrower's maximized surplus who acquires the information is at most  $\max \left\{ (1 - \sigma)\bar{V}_{IIS}^g + \sigma\bar{V}_{IIS}^b, (1 - \sigma)\bar{V}_{IS}^g + \sigma\bar{V}_{IS}^b \right\}$ . Thus, we are done if we prove that  $\max \{V_{IIS}, V_{IS}\} > \max \left\{ (1 - \sigma)\bar{V}_{IIS}^g + \sigma\bar{V}_{IIS}^b, (1 - \sigma)\bar{V}_{IS}^g + \sigma\bar{V}_{IS}^b \right\}$ , which will be proven by showing that  $V_{IIS} > (1 - \sigma)\bar{V}_{IIS}^g + \sigma\bar{V}_{IIS}^b$  for all  $\gamma^* > 0$  and  $V_{IS} > (1 - \sigma)\bar{V}_{IS}^g + \sigma\bar{V}_{IS}^b$  for  $\gamma \leq \min \left\{ \gamma^*, \frac{(m-1)\gamma^*}{\sigma m} \right\}$ . Note that  $V_{IIS} = (m-1)\bar{y}a$  if  $\gamma \geq \gamma^*$ ,  $V_{IIS} = \frac{(m-1)\gamma}{\sigma}$  if  $\gamma < \gamma^*$ ,  $V_{IS} = (m-1)\bar{y}a - m\gamma$  if  $\gamma \leq \min \left\{ \gamma^*, \frac{(m-1)\gamma^*}{\sigma m} \right\}$ .

We first show that  $V_{IIS} > (1 - \sigma)\bar{V}_{IIS}^g + \sigma\bar{V}_{IIS}^b$ . It is trivial if  $m(1 - \sigma) < 1$ . Consider that  $m(1 - \sigma) \geq 1$ . First, suppose that  $\gamma \geq \gamma^*$ . Then, by lemma 9,

$$(1 - \sigma)\bar{V}_{IIS}^g + \sigma\bar{V}_{IIS}^b = m\bar{y}a - \gamma_b - \bar{y}a = V_{IIS} - \gamma_b.$$

Now suppose that  $\gamma < \gamma^*$ . Then, by lemma 9,

$$(1 - \sigma)\bar{V}_{IIS}^g + \sigma\bar{V}_{IIS}^b = \frac{m\gamma}{\sigma} - \gamma_b - \frac{\gamma}{\sigma} = V_{IIS} - \gamma_b.$$

Finally, we show that  $V_{IS} > (1 - \sigma)\bar{V}_{IS}^g + \sigma\bar{V}_{IS}^b$ . By lemma 14,

$$(1 - \sigma)\bar{V}_{IS}^g + \sigma\bar{V}_{IS}^b = (m-1)\bar{y}a - m\gamma - \gamma_b = V_{IS} - \gamma_b.$$

■

**Proposition 16** Let  $\gamma^* \equiv \sigma \bar{y} a$  and  $\gamma^{**} \equiv \frac{(m-1)\gamma^*}{m(1+\sigma)-1}$ .

- 1) If  $\gamma \geq \gamma^*$ , then fully-IIS and  $r = \frac{\sigma}{1-\sigma}$  and  $\theta = 0$  if collateralized debt is fully-IIS.
- 2) If  $\gamma \in [\gamma^{**}, \gamma^*)$ , then partially-IIS and  $r$  and  $\theta$  are simultaneously determined by

$$1 = (1 - \alpha)(1 - \sigma)(1 + r) + \frac{\alpha}{(1 - \theta)}, \quad (46)$$

in the range of  $r \in \left[ \max \left\{ 0, \frac{1 - \alpha \gamma^*}{(1 - \alpha)(1 - \sigma)} \right\} - 1, \frac{1}{1 - \sigma} - 1 \right]$  and  $\theta \in \left[ 0, 1 - \max \left\{ \frac{\gamma}{\gamma^*}, \alpha \right\} \right]$  if collateralized debt is partially-IIS.

- 3) If  $\gamma \in [0, \gamma^{**})$ , then IS and  $r = \frac{\gamma}{\bar{y} a - \gamma}$  and  $\theta = 0$  if collateralized debt is IS.