Inflation Indexation and Zero Lower Bound

Daeha Cho* Eunseong Ma†
Hanyang University Yonsei University

June 2024

Abstract

This study quantitatively assesses the effects of inflation indexed loan contracts on business cycle fluctuations using a heterogeneous agent New Keynesian (HANK) model with an occasionally binding zero lower bound (ZLB). Substituting real for nominal government bonds reduces the volatility of output and inflation and decreases the frequency of ZLB events. Real loans sever the link between real interest rates and inflation, preventing a rise in real interest rates at the ZLB. Accordingly, ZLB events become less costly, weakening precautionary savings against aggregate risk. Furthermore, inflation indexing is a more effective policy than raising the inflation target in terms of reducing output and inflation volatility.

Key Words: Inflation indexation, Zero lower bound, Business cycles, Inequality
JEL Classification: D31, E31, E32,

*College of Economics and Finance, Hanyang University, Republic of Korea. e-mail: daehac@hanyang.ac.kr.

†School of Economics, Yonsei University, Republic of Korea. e-mail: masilver@yonsei.ac.kr.

1 Introduction

Most loan agreements specify a nominal interest rate. For example, in the U.S., more than 90% of new loans to households specify fixed nominal interest rates (Badarinza, Campbell and Ramadorai, 2018). One cost associated with nominal loan contracts, as documented in macroeconomics textbooks, is related to unexpected inflation. If inflation turns out to be higher than expected, the debtor benefits while the creditor loses because the ex-post real return that the debtor pays to the creditor is lower than what both parties expected, and vice versa. Under sticky prices, business cycles are linked to changes in both unexpected and expected inflation, influencing both ex-post and ex-ante real returns and, consequently, impacting households' optimal decisions. In the context, inflation-indexed loan contracts play a crucial role in stabilizing aggregate fluctuations, as real interest rates remain unaffected by inflation variability. This underscores the importance of thoroughly examining the role of inflation indexation in comprehending a central bank objective—ensuring price stability. In light of this, our study investigates how inflation indexation contributes to shaping the cyclical variation of economic outcomes and welfare, accounting for heterogeneity across different households.

Our findings demonstrate that inflation indexation substantially contributes to output and inflation volatility. The strength of this aggregate effect arises from the interaction among idiosyncratic income risk, aggregate risk, and the zero lower bound (ZLB) on nominal interest rates. We draw this conclusion by building on Fernández-Villaverde et al. (2023), who constructed and solved a heterogeneous agent New Keynesian (HANK) model with an occasionally binding ZLB constraint. We calibrate the model to match the realistic income and wealth distribution and historical frequency of hitting the ZLB in the U.S. and solve the model using the Krusell and Smith (1998) method.

In the model, households hold nominal government bonds to self-insure not only against idiosyncratic income risk but also against aggregate risk. The precautionary savings against idiosyncratic risk, a component absent in representative agent models, lower the steady-state nominal interest rates, thereby increasing the frequency of hitting the ZLB. Unlike typical HANK models that abstract from the ZLB, the ZLB gives rise to a deeper aggregate output drop, as central banks can no longer accommodate contractionary shocks. The aggregate risk arising from the presence of the ZLB generates much stronger precautionary savings than what is observed in HANK models with unconstrained monetary policy.

These precautionary savings exert further downward pressure on average nominal rates, causing ZLB episodes to occur more often.

To quantify the effect of inflation indexation on business cycles, we compare the variance of output and inflation, as well as the frequency of hitting the ZLB, computed from the ZLB-HANK model with nominal contracts with those implied by the ZLB-HANK model with real contracts. The latter model assumes that government bonds are fully indexed to inflation, which implies that real interest rates are unaffected by inflation variability. If nominal contracts are replaced with their real counterparts, the volatility of output and inflation falls by 26% and 32%, respectively. Additionally, the frequency of the ZLB decreases from 11.5% to 2.8%, indicating a substantial contribution of nominal contracts to aggregate fluctuations.

The mechanism through which real contracts stabilize aggregate fluctuations works as follows. In a world with nominal contracts, contractionary demand shocks induce excess savings that cannot be cleared by a fall in real interest rates due to the ZLB. The only route to clear the excess supply of savings is through a sufficient contraction in output, which leads to a fall in expected inflation as the decline in goods prices is expected to persist because of stick prices. The resulting rise in ex-ante real interest rates aggravates the contraction. However, when government bonds are indexed to inflation, a fall in expected inflation does not affect ex-ante real rates. As households no longer face an increase in ex-ante real rates, the ZLB episodes become much less costly. Accordingly, the demand for precautionary savings against aggregate risk weakens substantially, leading to higher average nominal rates than under nominal contracts and contributing to the reduced ZLB frequency.

At the disaggregate level, nominal contracts hurt the wealth-poor but benefit the wealth-rich during recessions. As the ex-post real interest payment is greater than expected, it partially offsets the decline in labor income of the wealth-rich. However, since the wealth-poor do not benefit from the unexpected increase in real interest income, they prefer real contracts to nominal contracts.

Our main result, which indicates that the real contract reduces the ZLB frequency by lifting up the average nominal interest rates, suggests that issuing inflation-linked bonds can be one way to create more leeway for central banks to adjust nominal interest rates. A commonly discussed policy to expand the scope for conventional monetary policy is to raise the inflation target (Blanchard, Dell'Ariccia and Mauro, 2010). We show that, under the same ZLB frequency, real contracts result in smaller macroeconomic volatility and higher welfare compared to the high inflation target policy. This is because, during ZLB episodes, the high inflation target policy fails to sever the link between expected inflation and real interest rates, whereas real contracts do.

The present paper is connected to the literature that studies propagation of aggregate shocks or policy evaluation in HANK models with the ZLB. McKay, Nakamura and Steinsson (2016) studies the role of the automatic stabilizers at the ZLB, using a perfect foresight solution method. However, in contrast to the present paper, their model does not include households' expectations regarding the risk of hitting the ZLB. Studies that do incorporate the risk of hitting the ZLB include Schaab (2020) and Fernández-Villaverde et al. (2023). Schaab (2020) examines the propagation of macro uncertainty near the ZLB through the interaction with countercyclical unemployment risk, whereas our model does not include unemployment risk. Fernández-Villaverde et al. (2023) investigate how the monetary policy stance influences households' precautionary savings by altering the frequency of ZLB events. In contrast, our focus is on the extent to which indexing loans to inflation matters for the ZLB frequency.

The current work also aligns with the literature that evaluates the redistributive or aggregate effects under nominal contracts. Doepke and Schneider (2006*b*), Adam and Zhu (2016) study the redistributive effects of unanticipated inflation shocks, while Doepke and Schneider (2006*a*) and Meh, Ríos-Rull and Terajima (2010) focus on the aggregate effect in a partial equilibrium model with heterogeneity in household net nominal position. Iacoviello (2005) and Kuncl and Ueberfeldt (2023) study the aggregate effects of monetary policy shocks in a general equilibrium New Keynesian model that operates through the debt deflation channel. Carrillo and Poilly (2013) analyze the fiscal multiplier under nominal contracts when monetary policy is constrained at the ZLB, using a perfect foresight solution method. However, none of these papers investigates how nominal contracts affect household precautionary savings and the ZLB frequency. Moreover, all of these papers assume loan contracts between households or between households and firms, whereas we assume contracts between households and government.

The rest of the paper is structured as follows. Section 2 presents the ZLB-HANK model, in which the degree of inflation indexation can be parameterized. In Section 3, we calibrate the model and describe our solution method and its accuracy. Section 4 compares business cycle moments under nominal and real contracts to quantify the cost of business cycles stemming from nominal contracts. Section 5 compares welfare between the high inflation target policy and real contracts. Section 6 concludes.

2 Model

In this section, we present the ZLB-HANK model, which incorporates both nominal and real contracts. It comprises four main components: a continuum (measure one) of households with identical preferences but different productivity levels, firms, a central bank, and a government. In this model economy, individual households are subject to uninsurable exogenous variations in labor productivity due to the incompleteness of asset markets, as in Huggett (1993) and Aiyagari (1994). Asset market incompleteness, combined with borrowing constraints, results in substantial heterogeneity in households' asset holdings, income, and consumption. Consequently, households respond differently to aggregate risks due to MPC heterogeneity. The government supplies public bonds and collects taxes from households to finance interest payments on these bonds. Notably, government bonds can be indexed to inflation or not. These two extreme forms of indexation enable us to examine the extent to which nominal contracts amplify aggregate risk as opposed to their real counterparts. The remaining model elements are standard in the New Keynesian literature: sticky nominal prices, monopolistically competitive goods markets, and a conventional Taylor rule that is bounded by zero nominal interest rates.

2.1 Households

Households maximize expected lifetime utility by selecting a sequence of consumption, c_t, labor supply, h_t and real bonds, b_{t+1}:

$$\max_{\{c_t,h_t,b_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \zeta_t \left(\frac{c_t^{1-\sigma}-1}{1-\sigma} - \Xi \frac{h_t^{1+1/\gamma}}{1+1/\gamma} \right) \right]$$

subject to the sequence of budget constraints,

$$c_t + b_{t+1} = (1+r_t)b_t + w_t z_t h_t - T_t + d_t, \tag{1}$$

and the borrowing constraint,

$$b_{t+1} \geq \underline{b},$$

where $0 < \beta < 1$ is a discount factor, σ is the degree of relative risk aversion, $\Xi > 0$ is a parameter for disutility from labor, and γ is the Frisch elasticity of labor supply. Each household is endowed with one unit of time in each period, which can be allocated between hours devoted to work and leisure. Additionally, households trade one-period non-contingent bonds, denoted as b, which offer a real rate of return, r_t. The bond position of households is subject to an exogenous limit, denoted as $\underline{b}$. When a household contributes h_t units of labor, it receives $w_t z_t h_t$ as income for its labor. Here, w_t represents the wage rate per unit of effective labor, and z_t is the labor productivity of the household. In addition to labor income, households receive profit income, d_t, from monopolistic firms, and they are also obligated to pay taxes, T_t, to the government.

Households face both individual and aggregate risks. Firstly, they are subject to an uninsurable idiosyncratic labor productivity shock. Labor productivity, z, follows a log-AR(1) process: $\log z_{t+1} = \rho_z \log z_t + \varepsilon_{z,t+1}, \quad \varepsilon_{z,t+1} \sim N(0, {\sigma_z}^2)$. Secondly, concerning aggregate risk, households are affected by a common exogenous preference shock, ζ_t. As discussed by Christiano, Eichenbaum and Rebelo (2011) and others, shifts in households' preferences have a significant impact on aggregate demand, potentially leading to the ZLB if the shock is sizable. ζ_t is assumed to follow an AR(1) process in logs: $\log \zeta_{t+1} = \rho_\zeta \log \zeta_t + \varepsilon_{\zeta,t+1}, \quad \varepsilon_{\zeta,t+1} \sim N(0, {\sigma_\zeta}^2)$.

2.2 Firms

A competitive firm combines a continuum of intermediate inputs, $y_t(j)$, indexed by $j \in [0, 1]$ to produce a homogeneous final good, Y_t, according to a CES production function:

$$Y_t = \left(\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{2}$$

where $\epsilon > 1$ is the input demand elasticity. The profit maximization problem of the final good firm implies the demand for intermediate good j:

$$y_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\epsilon} Y_t, \tag{3}$$

where $P_t = \left(\int_0^1 p_t(j)^{1-\epsilon}\, dj\right)^{\frac{1}{1-\epsilon}}$.

Each intermediate good firm j produces a different type of intermediate good $y_t(j)$ using $n_t(j)$ units of effective labor, by means of a production function:

$$y_t(j) = n_t(j) - f,$$

where and $f \geq 0$ is the fixed cost of production. Each intermediate goods firm j pays quadratic nominal price adjustment costs à la Rotemberg (1982). The problem for intermediate goods firms is to choose a sequence of prices that maximizes their expected discounted profits net of pricing costs:

$$\max_{p_{t+s}(j)} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \left(\prod_{i=0}^{s} \frac{1}{1+r_{t+i}}\right) \left\{ \left(\frac{p_{t+s}(j)}{P_{t+s}} - mc_{t+s}\right) y_{t+s}(j) - \frac{\theta}{2}\left(\frac{p_{t+s}(j)}{p_{t+s-1}(j)} - \overline{\Pi}\right)^2 Y_{t+s} \right\}\right], \tag{4}$$

subject to (3), where θ measures the degree of price stickiness, mc_{t+s} is the firm's real marginal cost, and $\overline{\Pi}$ is the steady-state gross inflation. The first-order condition associated with the optimal price gives rise to a New Keynesian Phillips curve:

$$\theta\left(\Pi_t - \overline{\Pi}\right)\Pi_t + \epsilon\left(\frac{\epsilon - 1}{\epsilon} - mc_t\right) = \theta\mathbb{E}_t\left[\frac{1}{1+r_t}\left\{\Pi_{t+1} - \overline{\Pi}\right\}\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\right], \tag{5}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$.

2.3 Central Bank and Government

The central bank operates under a ZLB constraint. It determines the policy rate based on a Taylor rule when the shadow rates are greater than zero, but sets the policy rate at zero if the shadow rates are 0 or below. Specifically, the gross nominal interest rate, R_t, is set according to:

$$R_t = \max \left\{ 1, \widetilde{R_t} \right\}. \tag{6}$$

$\widetilde{R_t}$ is the desired (or shadow) interest rate, which is set according to the Taylor rule:

$$\log \widetilde{R_t} = \log \overline{R} + \phi_\pi \left(\log \Pi_t - \log \overline{\Pi} \right) + \phi_y \left(\log Y_t - \log \overline{Y} \right), \tag{7}$$

where $\overline{X}$ is the deterministic steady-state value of variable X, and ϕ_π and ϕ_y are coefficients on inflation and the output gap, respectively.

The government plays three roles in the economy: i) collecting taxes from households, ii) issuing public bonds, and iii) redistributing profits from intermediate good firms to households. Following McKay, Nakamura and Steinsson (2016), we assume that taxes increase with households' labor productivity, z_t:

$$T(z_t) = \tau_t z_t, \tag{8}$$

where τ_t is a tax rate.[1] The government supplies bonds with a real face value of B_t, and adjusts taxes to cover interest payments on public debt. Specifically, in line with McKay, Nakamura and Steinsson (2016), we assume a constant level of public debt, i.e., $B_t = \overline{B}$, and assume that the government maintains a balanced budget in each period:

$$r_t \overline{B} = \int T_t(z_t) d\mu_t, \tag{9}$$

where $\overline{B}$ is the deterministic steady-state value of public debt. The government also has the responsibility of distributing monopoly profits to households. We assume that ag-

[1]While McKay, Nakamura and Steinsson (2016) assume a non-linear tax system with a positive tax rate applicable only to the highest productivity levels, we employ a linear tax system in which the tax rate is proportional to individual productivity. Given that individual labor productivity follows an exogenous process, this assumption does not influence or distort households' optimal choices.

gregate profits, D_t, are proportionally distributed according to productivity:[2]

$$d_t(z_t) = \frac{z_t}{\int z_t d\mu_t} D_t. \tag{10}$$

Importantly, government bonds can be indexed to inflation. Following Carrillo and Poilly (2013), we assume that nominal public debt is indexed to inflation at a rate of $\chi \in [0, 1]$. Formally, the real interest rate, r_t, follows a simple indexation rule:

$$r_t = \log\left(\frac{R_{t-1}}{\Pi_t}\left(\frac{\Pi_t}{\overline{\overline{\Pi}}}\right)^{\chi}\right), \tag{11}$$

where $\left(\frac{\Pi_t}{\overline{\overline{\Pi}}}\right)^{\chi}$ is a term that captures the adjustment of nominal rates, R_t, to inflation. When $\chi = 0$, it implies that government bonds are purely denominated in nominal terms. Consequently, the return on the bond becomes lower (or greater) than expected in the presence of unexpectedly high (or low) inflation. On the other hand, with $\chi = 1$, government debt fully adjusts to inflation, and the return paid to households is unaffected by surprise inflation.

3 Calibration and Numerical Solution

3.1 Calibration

We calibrate the parameters of the model following the existing literature. Table 1 provides a summary of the parameter values used in the model.

The time discount factor, β, is chosen in a way that yields a steady-state real interest rate of 1 percent per year. For the risk aversion parameter, σ, we assigned a value of one. The Frish elasticity of labor supply, γ, is set to one. We determine the disutility parameter of working, Ξ, to match the steady-state hours, which are equal to 0.233.[3]

Regarding individual labor productivity shocks, we adopt the values used in Debortoli and Gali (2018), with ρ_z set to 0.978 and σ_z set to 0.193. These parameter values imply that individual wages exhibit an autoregressive coefficient of 0.914 and an innovation

[2]It should also be noted that $\int z_t d\mu_t = Z_t = \overline{Z}$ by construction.

[3]This value is obtained by multiplying the average hours conditional on working (1/3) by the long-run employment rate (70 percent).

Table 1. Model Parameters

Parameter	Value	Description	Source/Target Moments
		Households	
β	0.974477	Time discount factor	1% real return to bond
σ	1.0	Risk-aversion	Standard
Ξ	15.2	Disutility parameter	See text
γ	1.0	Labor supply elasticity	Standard
ρ_z	0.978	Persistence of z shocks	Debortoli and Gali (2018)
σ_z	0.193	Standard deviation of z shocks	Debortoli and Gali (2018)
$\underline{b}$	0	Borrowing limit	McKay, Nakamura and Steinsson (2016)
ρ_ζ	0.6	Persistence of ζ shocks	Fernández-Villaverde et al. (2023)
σ_ζ	0.0048	Standard deviation of ζ shocks	11% ZLB frequency
		Firms	
f	0.0412	Production fixed cost	Zero profits
ϵ	10	Elasticity of substitution	Standard
θ	100	Price adjustment cost	See text
		Government and Monetary Authority	
ϕ_π	2.0	Coefficient on inflation dev.	Standard
ϕ_y	0.1	Coefficient on output dev.	Standard
$\overline{B}/\overline{Y}$	1.4	Public debt to annual GDP	McKay, Nakamura and Steinsson (2016)
$\overline{\Pi}$	1.0025	Steady state gross inflation	2% inflation target

standard deviation of 0.258 at an annual frequency, similar to the estimates provided by Floden and Linde (2001). The AR(1) process is converted into a 17-state Markov chain, using the Tauchen (1986)'s method. The borrowing limit, $\underline{b}$, is set to zero following McKay, Nakamura and Steinsson (2016) and Hagedorn et al. (2019). Regarding the demand shock process, we adopt $\rho_\zeta = 0.6$ as suggested by Fernández-Villaverde et al. (2023) and set $\sigma_\zeta = 0.0048$, leading to an 11% frequency of hitting the ZLB.[4]

We choose the fixed cost of production, f, in order to ensure zero profits for intermediate goods firms in the steady state. The elasticities of substitution across intermediate goods, ϵ is set equal to 10. The Rotemberg adjustment cost parameter, θ is set to be consistent with the Calvo stickiness parameter of 0.75.[5]

In accordance with McKay, Nakamura and Steinsson (2016), we set the debt-to-annual

[4]This frequency is consistent with US interest rates at the ZLB during 2009Q1-2015Q4 and 2020Q2-2020Q4, where the total sample period is from 1952Q1 until late 2020Q4.

[5]Given a Calvo parameter ϱ, the Rotemberg adjustment cost parameter can be recovered from: $\theta = \frac{\varrho(\epsilon-1)}{(1-\varrho)(1-\beta\varrho)}$.

Table 2. Accuracy of Forecasting Rule for Inflation

	R^2	Den Haan Error (pp)	
		Mean	Max.
Non-ZLB Sample	0.9998	0.0346	0.2072
ZLB Sample	0.9998	0.0497	0.2311

Note: The accuracy of forecasting rules is evaluated based on the statistics proposed by Den Haan (2010). The unit is expressed in percentage points on an annualized basis. 'Non-ZLB Sample' refers to simulated periods excluding the ZLB, while 'ZLB Sample' denotes periods when the ZLB is binding. The errors are computed using the parameters estimated from all simulated periods.

GDP ratio at 1.4, while the tax rate τ_t is chosen to ensure a balanced budget for the government each period. The Taylor rule coefficients of inflation and output, ϕ_π and ϕ_y, are chosen to be 2.0 and 0.1, respectively. These choices are conventional values in the New Keynesian DSGE literature and are consistent with the estimates in the empirical literature. The steady-state gross inflation, $\overline{\Pi}$, is set so that the annual inflation is 2% in the steady state, in line with the target inflation rate set by the US. In the baseline model, we assume that public bonds are purely denominated in nominal terms, i.e., $\chi = 0$.

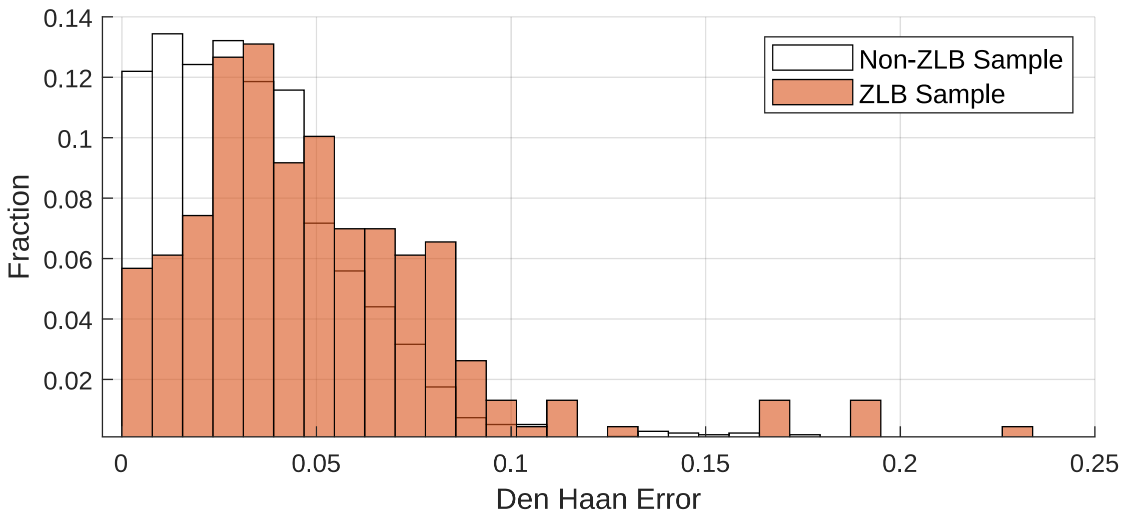
3.2 Numerical Solution

To solve the model with wealth distribution, we adopt the widely recognized approach proposed by Krusell and Smith (1998). Given the inherent nonlinearity of our model due to the presence of the ZLB, we modify the Krusell and Smith approach as follows. We distribute grid points for $\widetilde{R_t}$ in a non-uniform manner, allocating more points near the zero shadow rates. Furthermore, for the sake of a more accurate fit, we enhance the forecasting functions by incorporating interaction and quadratic terms. Remarkably, our modified technique yields a highly accurate fit, even when the ZLB constraint is active.[6]

Table 2 presents a summary of the goodness of fit and precision of the inflation forecasting rule. This assessment covers two scenarios: i) simulated periods excluding the ZLB period and ii) periods when the ZLB is binding. Evidently, the R^2 values for the forecasting function are notably high in both sets of time periods. In evaluating the accuracy of the forecasting rule, we utilize the metrics introduced by Den Haan (2010). In the non-ZLB sample, the mean errors prove to be sufficiently small, staying below 0.04 percentage points on an annualized basis. Furthermore, the maximum errors remain reasonably modest, at around 0.21 percentage points. Regarding the periods in which the ZLB constraint

[6]Further details can be found in Appendix XX, which outlines the computational procedures.

Figure 1. Distribution of Den Haan Errors



Note: On the x-axis, errors are expressed in percentage points on an annualized basis, while the y-axis represents the fraction of errors (normalized to sum to one). 'Non-ZLB Sample' refers to simulated periods excluding the ZLB period, while 'ZLB Sample' denotes periods when the ZLB is binding. The errors are computed using the parameters estimated from all simulated periods.

binds, our methodology effectively captures inflation dynamics. In this case, the mean and maximum errors are around 0.05 and 0.23 percentage points, respectively—slightly larger than those in the non-ZLB sample but still showcasing strong accuracy.[7] Confirming the accuracy, Figure 1 depicts the distribution of the Den Haan errors for the inflation forecasting rule in both ZLB and non-ZLB periods. The x-axis represents errors in percentage points on an annualized basis, while the y-axis illustrates the frequencies of errors, normalized to sum to one. Remarkably, the non-ZLB samples exhibit a relatively high number of zero or very small errors. However, the forecasting rule maintains its accuracy in the ZLB sample. Although there are some relatively large outliers in this case, they do not significantly undermine the overall accuracy of the forecasting rule, as these outliers are rare. This observation underscores the robustness of our methodology, showcasing its ability to sustain accurate predictions even in ZLB scenarios. We argue that this level of performance is comparable to that achieved by neural networks in the work of Fernández-Villaverde et al. (2023).

[7]The Den Haan errors in both non-ZLB and ZLB periods are computed using the parameters estimated from all simulated periods.

Table 3. Income and Wealth Distributions

	Quintile					Gini
	1st	2nd	3rd	4th	5th	
U.S. Data						
Share of Income	2.8	6.7	11.3	18.3	60.9	0.58
Share of Wealth	-0.2	1.1	4.5	11.2	83.4	0.82
Model Economy						
Share of Income	3.1	8.0	8.9	19.7	60.4	0.56
Share of Wealth	0.0	0.1	1.7	11.6	86.6	0.82

Note: Income and wealth data are from the Survey of Consumer Finances (SCF) 2007 (source: Diaz-Gimenez, Glover and Rios-Rull (2011)).

4 Results

4.1 Cross-sectional Distributions

We examine whether the model economy effectively reproduces income and wealth distribution among households observed in the data. Table 3 reveals the comparison between the income and net asset holdings in the model and their data counterparts in the U.S.[8] The model economy demonstrates a reasonable reproduction of the income distribution found in the data, resulting in an income Gini coefficient of 0.56, which closely aligns with its empirical counterpart (0.58). Similarly, wealth inequality, characterized by a Gini coefficient of 0.82, is accurately replicated by the model economy. Overall, the model economy successfully achieves a realistic representation of heterogeneity across households.

4.2 Aggregate Effects of Inflation Indexation

In this subsection, we discuss the extent to which the inflation indexation contributes to business cycle fluctuations. We assess this contribution by examining how much output and inflation volatility is reduced when nominal contracts are replaced with contracts that are indexed to inflation. We demonstrate that the stabilizing effect of inflation indexation is particularly powerful in the presence of the ZLB. To do so, we compare the volatility of

[8] Information for income and wealth in the data is from the Survey of Consumer Finances (SCF) 2007 in Diaz-Gimenez, Glover and Rios-Rull (2011).

Table 4. Business Cycle Statistics

	Std(Y)	Std(Π)	Pr$(\tilde{R} < 1)$	Mean$(\tilde{R})$	Mean(r)	Mean(Π)
Steady state	-	-	-	3.00	1.00	2.00
ZLB-HANK	0.57	0.22	11.50%	2.75	0.88	1.87
HANK w/o ZLB	0.53	0.20	6.55%	2.90	0.94	1.95
ZLB-HANK+Index	0.42	0.15	2.80%	3.02	1.01	2.01
HANK w/o ZLB +Index	0.43	0.16	2.85%	2.96	0.98	1.98

Note: Y, Π, $\tilde{R}$, and r denote output, inflation, the shadow nominal interest rate, and the real interest rate, respectively. When computing the standard deviations, all variables are logged and then detrended using the HP filter.

output, inflation, and the frequency of the shadow nominal interest rates being less than zero, computed from the time series of the HANK models with and without indexation when the ZLB is not present. We then compare the volatility of output, inflation, and the frequency of the ZLB in the ZLB-HANK models with and without indexation.

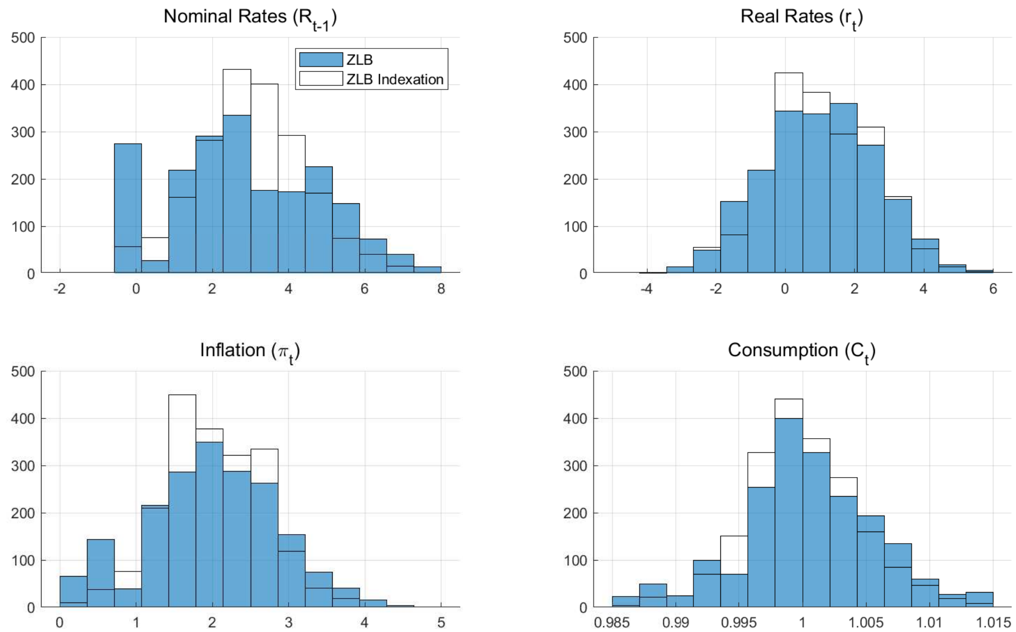
Rows 2 to 5 of Table 4 illustrate the standard deviations and the mean of variables of interest across different model specifications. Row 1 reports nominal interest rate, real interest rate, and inflation in the deterministic steady state. Compare these values with those shown in Row 3, which represents the HANK model without the ZLB constraint. Unlike linearly solved heterogeneous agent models such as McKay and Reis (2016) and Auclert, Rognlie and Straub (2020), the long-run mean implied from the model is not equal to the steady state. The precautionary savings motive with respect to aggregate uncertainty drives the average nominal and real interest rates below their steady-state counterparts. With sticky prices, the contraction in aggregate demand associated with precautionary motives makes the mean output and inflation lower than their steady-state levels. This nonlinear effect of aggregate uncertainty is also present in nonlinear representative agent models, but the effect is stronger in HANK models in which aggregate income volatility is larger due to the well-known interaction between MPCs and nominal rigidities.

Compare the moments presented in Row 2, which corresponds to the ZLB-HANK economy without inflation indexation, and those in Row 3. Row 3 shows that the volatility of inflation and output is larger when the HANK economy is faced with occasionally binding ZLB constraints. In addition, the presence of ZLB further lowers the long-run mean of nominal interest rates, real interest rates, and inflation. This outcome confirms the findings of Fernández-Villaverde et al. (2023). Since income is expected to drop more in a recession when monetary policy is constrained than when it is unconstrained, wealth-poor households attempt to accumulate more buffer stock of savings to hedge against their consumption drop during ZLB episodes. This stronger precautionary motive, caused by the risk of hitting the ZLB, further reduces output, inflation, and interest rates during recessions, generating greater macroeconomic volatility. Because the utility loss associated with an income change is greater in recessions than the utility gain in expansions, a disproportionately large precautionary savings motive in recessions, inherent in the ZLB-HANK economy, puts further downward pressure on mean output, inflation, and interest rates.

To illustrate the extent to which full inflation indexing stabilizes macroeconomic volatility when the ZLB is present, compare Rows 2 and 4. Expressing the stabilization effect in percentage terms, the volatility of output and inflation is reduced by 26% ($= (0.57 - 0.42)/0.57 \times 100$) and 32% ($= (0.22 - 0.15)/0.22 \times 100$), respectively. A substantial decrease in macroeconomic volatility due to the inflation indexation weakens the precautionary savings motives, and thus the average real and nominal interest rates become higher than in the world without the indexation, as observed in Columns 4 and 5. A higher mean nominal interest makes the future realization of the ZLB events less likely. As observed in the table, the frequency of hitting the ZLB changes from 11.5% to 2.8%, indicating that indexing loan contracts to inflation greatly expands the room for maneuver for central banks. Now, compare Rows 2 and 4 to evaluate the power of inflation indexing when the ZLB is absent. Output and inflation volatility is reduced by 19% ($= (0.53 - 0.43)/0.53 \times 100$) and 20% ($= (0.20 - 0.16)/0.20 \times 100$), respectively, displaying a smaller stabilization effect. The ZLB dependence of the stabilization effect of inflation indexation suggests that the business cycle effect of nominal contracts depends heavily on the presence of the ZLB.

The decreased variations in output and inflation under inflation indexation are not an artifact of the less asymmetric monetary policy that results from reduced likelihood of the ZLB per se. That is, macroeconomic variables in the presence of indexation not only fall by less but also increase by less. One way to appreciate this point is by comparing the ergodic distribution of the aggregate variables in ZLB-HANK models with and without inflation indexation. Figure 2 shows that inflation indexation makes the ZLB less frequent. Reduced ZLB frequency is associated with less frequent drops in inflation and consumption, evident in the thinner left tail of the distribution of these variables, and less frequent rises in real interest rates, as seen in the thinner right tail of the distribution of real rates. The figure also shows that an increase in inflation and consumption occurs less often with indexation. The thinner left and right tails of the distribution of consumption and inflation indicate that

Figure 2. Ergodic Distributions: Indexation vs. No-Indexation



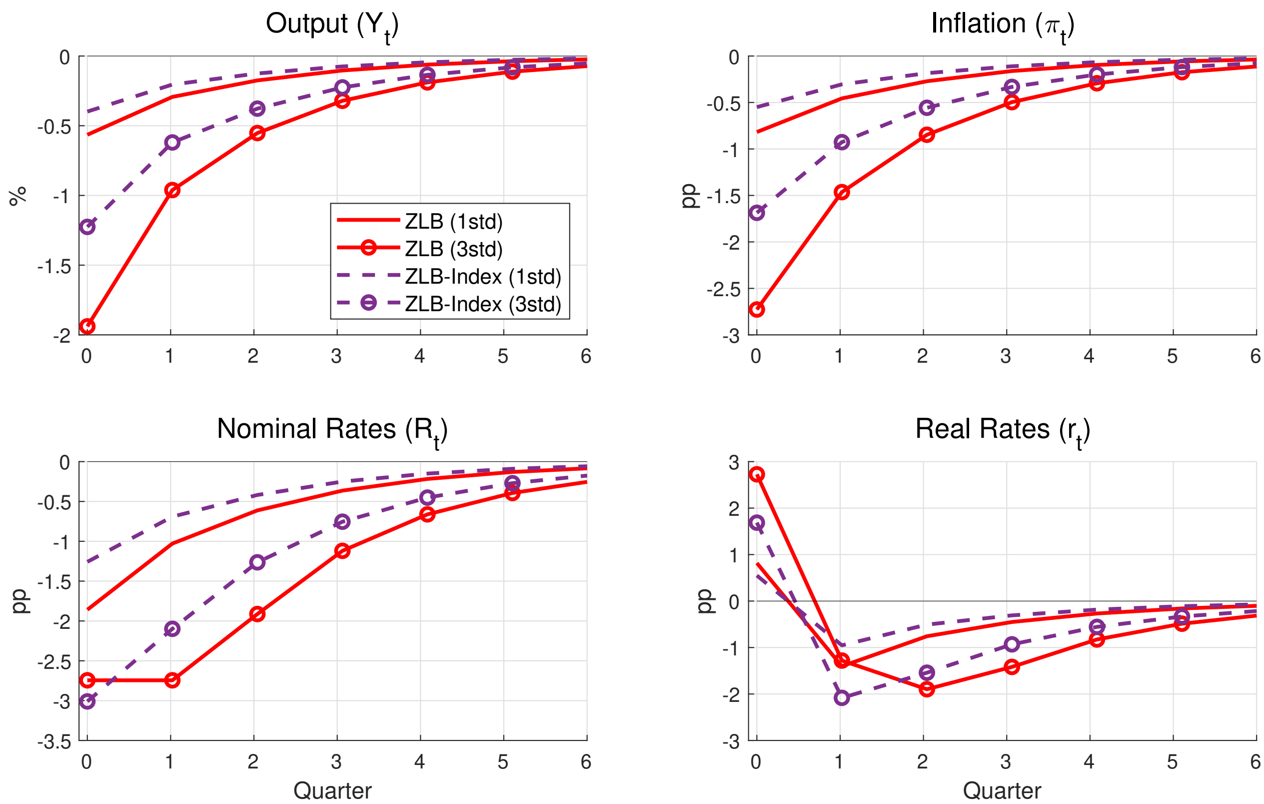
Note: The ergodic distributions of aggregate variables in the ZLB-HANK models with and without inflation indexation.

inflation indexation not only limits the severity of contractions but also the magnitude of expansions.

Why does the stabilizing effect of inflation indexation arise, and why is it more pronounced in the presence of the ZLB? Consider a world without the ZLB first. A fall in inflation during contractions leads to a decline in nominal interest rates via a Taylor rule. When loans are fully indexed to inflation, as seen in Equation (11), a decrease in nominal rates is entirely translated into a decrease in ex-ante real rates, boosting aggregate demand through Euler equation. This offsetting force works to mitigate the drop in inflation and output during contractions. However, in a world without inflation indexation, ex-ante real rates are affected not only by nominal rates but also by inflation expectations, which decrease during contractions. A fall in expected inflation weakens the drop in ex-ante real rates, thereby stimulating aggregate demand less than in the world with inflation indexation. The disconnection between real rates and inflation expectations constitutes a key mechanism in the stabilizing channel of inflation indexation.

In a world with ZLB, inflation indexation is even more stabilizing because it ameliorates the feedback loop between aggregate demand and deflation, which is a popular transmission mechanism of demand shocks documented in the New Keynesian literature (Chris-

Figure 3. IRFs: Indexation vs. No-Indexation



Note: The impulse responses to demand shocks in the ZLB-HANK models with and without inflation indexation. The responses represent the deviations from the long-run mean, which differs across model economies.

tiano, Eichenbaum and Rebelo, 2011). Excess savings induced by adverse demand shocks cannot be cleared by a fall in real interest rates due to the constraint on nominal interest rates. In this setting, output must decline significantly to eliminate the excess supply of savings. In the absence of inflation indexation, the fall in output leads to expected deflation, which, in conjunction with zero nominal rates, drives up ex-ante real interest rates. The increase in real rates depresses aggregate demand further. However, when loans are indexed to inflation, expected deflation does not influence real rates. Thus, households no longer face the increase in ex-ante real rates, making ZLB episodes less contractionary. Accordingly, the precautionary savings motive against the risk of hitting the ZLB diminishes substantially, exerting upward pressure on nominal rates. The resulting decrease in the frequency of hitting the ZLB further reduces the macroeconomic volatility.

Figure 3 illustrates the stabilizing power of inflation indexation via impulse responses to demand shocks of different magnitudes. The responses of variables represent deviations from their long-run mean, which differs across the models with different degrees of indexation. In our ZLB-HANK models, with and without indexation, the ZLB does not bind in

Table 5. Welfare Gains of Inflation Indexation

Wealth Percentile				Total
1-40	40- 80	80-99	99-100 (Top 1%)	
0.2392	0.1438	-0.1689	-0.4887	0.1173

Note: The welfare difference between the ZLB-HANK without indexation and the ZLB-HANK with indexation, expressed as a fraction of steady-state consumption in the ZLB-HANK without indexation. The welfare in each economy is computed as the welfare conditional on the highest preference level and the lowest shadow policy rate. The positive number indicates that the ZLB-HANK with indexation is more desirable.

response to a 1-standard deviation shock, as the shock does not lead to a sufficiently large reduction in nominal interest rates. In response to a 3-standard deviation shock, the ZLB binds only when debt is denominated in nominal terms. This is because, without inflation indexation, the long-run mean of nominal interest rates is lower, as shown in Table 4. Additionally, output and inflation drop more in response to a given size of demand shocks in a model without indexation due to an insufficient fall in ex-ante real rates faced by households. The further reduced inflation decreases the nominal rates by more. The figure illustrates that, irrespective of the magnitude of the demand shock, both output and inflation exhibit greater contraction when loans are denominated in nominal terms, making policy rates more likely to be constrained.

4.3 Disaggregate Effects of Inflation Indexation

Having demonstrated that the contribution of real debt contracts to aggregate fluctuations is particularly pronounced in a model with ZLB, in this subsection, we study the distributional consequences. Specifically, we investigate whether inflation-indexed public bonds are beneficial for all households. To answer this question, for each wealth level group, we first compute the welfare of business cycles in the ZLB-model without indexation and then compare it with the welfare in the model with indexation. Welfare is conditioned on the highest preference level and the lowest shadow policy rate. We then use impulse responses for each group to explain the intuition behind the welfare gains or losses associated with nominal contracts.

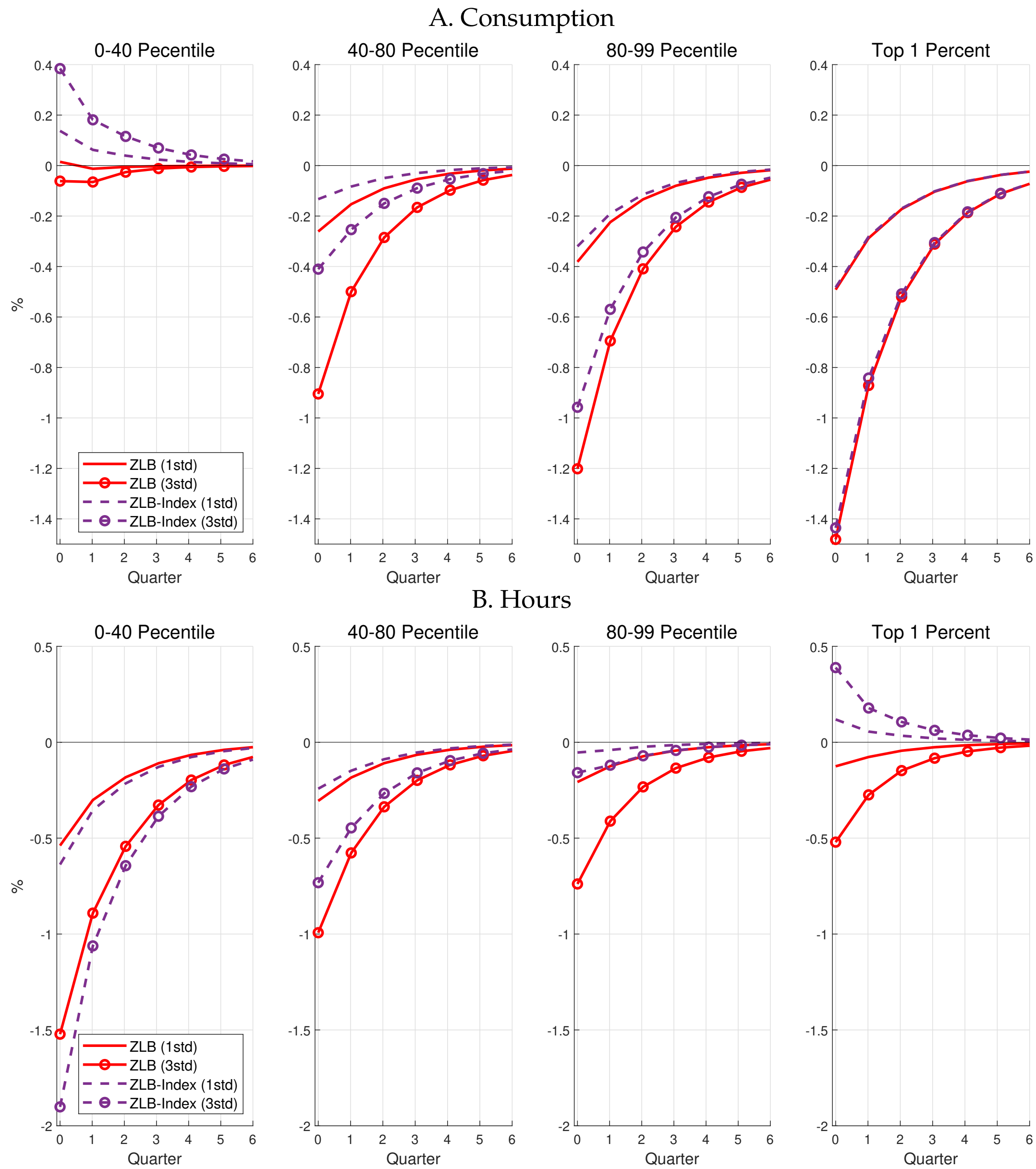
Table 5 describes the welfare difference between the economy without indexation and the economy with indexation, where a positive number indicates that the latter is more welfare-enhancing. Clearly, when it comes to aggregate welfare, the economy with indexation is better off than the one without indexation. This is consistent with the lower macro-volatility in the former, as shown in Table 4. However, the table illustrates that not all households benefit from inflation indexation. While the welfare gain from indexation becomes greater for households with lesser amounts of wealth, those in the 80th percentile of the wealth distribution and above are worse off due to indexation.

To interpret these different welfare outcomes across households, it is useful to pay attention to Figure 4, which displays the impulse responses of consumption and hours to negative demand shocks for each wealth group. For a given magnitude of demand shock, households in the bottom 80th percentile of the distribution experience a large stabilization in consumption compared to those in the top 20%, explaining a substantial welfare gain of indexation for households in the bottom 80%. Because the bottom 80% are the ones relatively poorly insured against aggregate income fluctuations, their consumption is more sensitive to changes in aggregate conditions. Accordingly, indexing loan contracts to inflation, which mitigates the drop in aggregate income, stabilizes the consumption of wealth-poor households more than that of wealth-rich households. Notice that the consumption of households in the 0-40th percentile increases when indexation is in place. This is because in sticky price models, profits are countercyclical, and so households receive positive profit income during recessions.[9] Since the precautionary savings motive, which drags down their consumption, is largely absent under inflation indexation, increased profits income works to boost the consumption of wealth-poor households.

Observe that top 20% of the wealth distribution are worse-off when nominal contracts are replaced with real contracts, despite a less drop is consumption. The adverse effect of real contracts can be explained by the absence of the realized real interest rate channel. In the absence of inflation indexation, realized real rates increase during recessions as the nominal rates are predetermined, but inflation falls. The increased realized real interest rate in response to adverse demand shocks is confirmed in Figure 3, and this is beneficial for wealthy households, for whom a large portion of income is derived from interest income. However, under inflation indexation, the absence of such a beneficial effect partly offsets the stabilization effect of indexation at the ZLB discussed in Section 4.2. To make up for the loss of real interest income, wealthy households increase hours worked more than they would under nominal contracts, which is a force that decreases their welfare.

[9] To address the positive consumption response among individuals with lower wealth, we adjust the distribution of countercyclical profits, as outlined in Section C of the Appendix. The modifications result in individuals within the 0-40th percentile range showing a negative consumption response, regardless of inflation indexation. Importantly, households at the bottom of the wealth distribution experience a substantial stabilization in consumption when compared to their wealthier counterparts.

Figure 4. IRFs: Indexation vs. No-Indexation



Note: The impulse responses to demand shocks in the ZLB-HANK models with and without inflation indexation. The responses represent the deviations from the long-run mean, which differs across model economies.

Figure 4 shows that households in the 80th percentile of the wealth distribution and above increase hours more when nominal loan contracts are replaced with real contracts for a given size of demand shock. Observe that the increase in hours worked is particularly pronounced for the top 1% of households, who are hurt the most by the loss of real interest income. In contrast, households in the 0-40th percentile of the distribution do not increase hours worked when nominal contracts are replaced with real contracts, as interest income

Table 6. Inequality and Business Cycle Statistics

	Std(Y)	Std(Π)	Pr($\tilde{R} < 1$)
ZLB-HANK (high inequality)	0.73	0.28	26.00%
ZLB-HANK	0.57	0.22	11.50%
ZLB-HANK+Index (high inequality)	0.42	0.16	6.50%
ZLB-HANK+Index	0.42	0.15	2.80%

Note: Y, Π, and $\tilde{R}$ denote output, inflation, and the shadow nominal interest rate, respectively. When computing the standard deviations, all variables are logged and then detrended using the HP filter.

is not a large component of their total income. The large increase in hours worked, in conjunction with a mild stabilization in consumption, explains why inflation indexation is welfare-detrimental as households become wealthy.

4.4 The Role of Inequality

In this subsection, we discuss the extent to which household heterogeneity matters when assessing the business cycle effect. We consider different level of labor income risk, which determines the dispersion of labor income and wealth across households, and show that the nominal contracts become more destabilizing as the level of labor income risk increases. Table 6 reports the standard deviations of output, inflation, and the ZLB-frequency for ZLB-HANK models, and those for ZLB-HANK models with inflation indexation, by varying degrees of labor income risk. To construct the highly unequal economy, we increase the standard deviation of the idiosyncratic productivity shock from 0.193 to 0.198, implying a 2% increase in the income Gini coefficient.

As found in Fernández-Villaverde et al. (2023), aggregate volatility depends substantially on income inequality. Even a small increase in idiosyncratic income risk results in increased aggregate volatility, with output and inflation rising by 28% ($= (0.73 - 0.57)/0.57 \times 100$) and 27% ($= (0.28 - 0.22)/0.22$), respectively. The reason for the increased volatility is as follows. Higher idiosyncratic risk leads to more households precautionary savings, implying a lower steady-state real and nominal interest rate. The lower steady-state nominal rates make the economy more prone to encountering the ZLB when aggregate demand shocks kick in. As households anticipate more frequent deep recessions associated with the ZLB, the precautionary savings motive against aggregate uncertainty becomes more potent. Consequently, the average nominal interest rates decrease further, leading to an increase in both the frequency of encountering the ZLB and aggregate volatility. The ZLB

Table 7. Household Debt and Business Cycle Statistics

	Std(Y)	Std(Π)	Pr($\tilde{R} < 1$)
ZLB-HANK (borrowing)	0.59	0.22	11.85%
ZLB-HANK (no borrowing)	0.57	0.22	11.50%
ZLB-HANK+Index (borrowing)	0.42	0.15	2.80%
ZLB-HANK+Index (no borrowing)	0.42	0.15	2.80%

Note: Y, Π, and $\tilde{R}$ denote output, inflation, and the shadow nominal interest rate, respectively. When computing the standard deviations, all variables are logged and then detrended using the HP filter.

hitting frequency is 26% in our high-inequality benchmark, whereas it is 11.5% under the baseline calibration.

Notice that the effect of inflation indexation in reducing aggregate volatility increases with the level of inequality. When loans are indexed to inflation, output and inflation volatility are reduced by 42% $(= (0.73 - 0.42)/0.73 \times 100)$ and 43% $(= (0.28 - 0.16)/0.28 \times 100)$, respectively, under high productivity risk, compared to 26% $(= (0.57 - 0.42)/0.57 \times 100)$ and 32% $(= (0.22 - 0.15)/0.22 \times 100)$ under baseline productivity risk. This striking difference implies that nominal contracts are more costly in a more unequal economy.

One interesting observation is that, despite the higher frequency of hitting the ZLB in an economy with higher idiosyncratic risk, the volatility of output and inflation is very similar across the models with different degrees of idiosyncratic risk when loans are indexed to inflation, as observed in Rows 3 and 4. This indicates that in a setting where the deflationary spiral does not affect ex-ante real interest rates, the cost of the ZLB is very low. If the costs were high, macroeconomic volatility would have increased with the frequency of the ZLB.

4.5 The Role of Borrowing

In our previous discussions, we quantified the effect of inflation indexation on business cycle fluctuations in a setting where households are all lenders. Originally, the perils of nominal contracts were discussed in a setting where deflation aggravates recessions through the Fisherian debt-deflation channel (Fisher, 1933; Eggertsson and Krugman, 2012). That is, deflation increases the real value of liabilities or the real interest rate on liabilities, thereby causing debtors to cut spending more than savers in recessions. This reduction in spending contributes to a decrease in aggregate demand, leading to lower production. A natural question is whether the debt-deflation channel is quantitatively important in amplifying demand shocks. To answer this question, we extend our model by introducing household borrowing and set the borrowing limit, $\underline{b} < 0$, equal to the model-implied average labor income, as in Kaplan, Moll and Violante (2018). We then recalibrate the model with household borrowing to match the steady-state targets described in Section 3.1.

Table 7 reports the volatility of aggregate variables and the frequency of hitting the ZLB for models with and without borrowing, along with the changes in these moments when loans are indexed to inflation. Two observations stand out. First, allowing for household debt does increase the output volatility, as seen in rows 1 and 2 of the table, but only by a small degree. The small increase in output variations due to the inclusion of household debt is associated with a slight increase in the ZLB-frequency, from 11.5% to 11.9%, indicating the insignificance of the Fisherian debt-deflation channel. Secondly, as shown in rows 3 and 4 of the table, the stabilizing effect of inflation indexation remains robust even with the inclusion of household debt. Macroeconomic volatility and the ZLB frequency in models with and without borrowing are virtually identical under inflation indexation.

5 High Inflation Target vs. Inflation Indexation

In the previous section, we demonstrated that the stabilizing effect of inflation indexation is powerful. Indexing loan contracts to inflation decreases macroeconomic volatility by severing the link between real interest rates and inflation, thus reducing households' demand for precautionary savings. Accordingly, it raises the average level of real and nominal interest rates, decreasing the likelihood of encountering the ZLB. In this regard, loan contracts that index to inflation can be an effective way to provide more room for monetary policy to ease during a crisis. One policy that implements such contracts is the issuance of inflation-indexed government bonds.[10] Like nominal government bonds, inflation-indexed bonds pay interest and principal, but the payments are fixed in real terms.

An existing proposed policy in academic and policy circles to reduce the risk of hitting the ZLB is to raise the inflation target (Blanchard, Dell'Ariccia and Mauro, 2010). Coibion, Gorodnichenko and Wieland (2012) show that raising the inflation target in a representative agent model lifts up the nominal interest rates in the deterministic steady state, reducing the frequency of hitting the ZLB. Fernández-Villaverde et al. (2023) show that raising

[10] As of September 2021, the share of inflation-linked government bonds in total government debt outstanding is 7.5%, while in the U.K., it is 24%.

Table 8. Business Cycle Statistics: High Inflation Target vs. Inflation Indexation

	Std(Y)	Std(Π)	Pr($\tilde{R} < 1$)
ZLB-HANK	0.57	0.22	11.50%
ZLB-HANK+Infl. target	0.51	0.19	2.80%
ZLB-HANK+Index	0.42	0.15	2.80%

Note: Y, Π, and $\tilde{R}$ denote output, inflation, and the shadow nominal interest rate, respectively. When computing the standard deviations, all variables are logged and then detrended using the HP filter.

the inflation target in a heterogeneous agent model reduces the ZLB frequency to a larger extent than that in a representative agent model because of a greater reduction in households' savings demand. A natural question is to ask, between raising the inflation target and indexing loans to inflation, which policy is more desirable in terms of welfare. The goal of this section is to compare the stabilization power and welfare of these two policies both at the aggregate and disaggregate levels.

For a fair comparison, we raise the inflation target in the ZLB-HANK model without indexation so that the implied frequency of hitting the ZLB is equal to that in the ZLB-HANK model with indexation. When raising the inflation target, we also adjust the steady-state inflation that appears in the price adjustment cost term in Equation (4) so that the welfare between the two policies is not affected by the difference in steady-state price adjustment costs. The two model economies that we compare exhibit the same quantities and cross-sectional distribution in deterministic steady state. The inflation target in the model in which the target is modified is 2.8%, whereas the model with inflation indexation is 2%. This difference implies that the steady-state nominal rates is 3.8% in the former, while 3% in the latter.

Rows 1 and 2 in Table 8 report that raising the inflation target from 2% to 2.8% reduces the volatility of output and inflation, consistent with the findings by Coibion, Gorodnichenko and Wieland (2012) and Fernández-Villaverde et al. (2023). Rows 2 and 3 show that the model with a high inflation target is less stabilizing than the model with indexation, indicating that raising the inflation target is less stimulative than inflation indexing, despite the equal frequency of the ZLB. Why does the model with a higher inflation target show larger aggregate volatility than the model with indexation? The reason is that while raising the inflation target lifts up the average nominal rates, it does not eliminate the link between expected inflation and real interest rates. Unlike in the model with inflation indexation, households still face a rise in the ex-ante real rate caused by the expected

Table 9. Welfare Gains of High Inflation Target and Inflation Indexation

	Wealth Percentile				Total
	1-40	40- 80	80-99	99-100 (Top 1%)	
ZLB-HANK+Infl. target	0.1405	0.0084	-0.0974	-0.2803	0.0693
ZLB-HANK+Index	0.2392	0.1438	-0.1689	-0.4887	0.1173

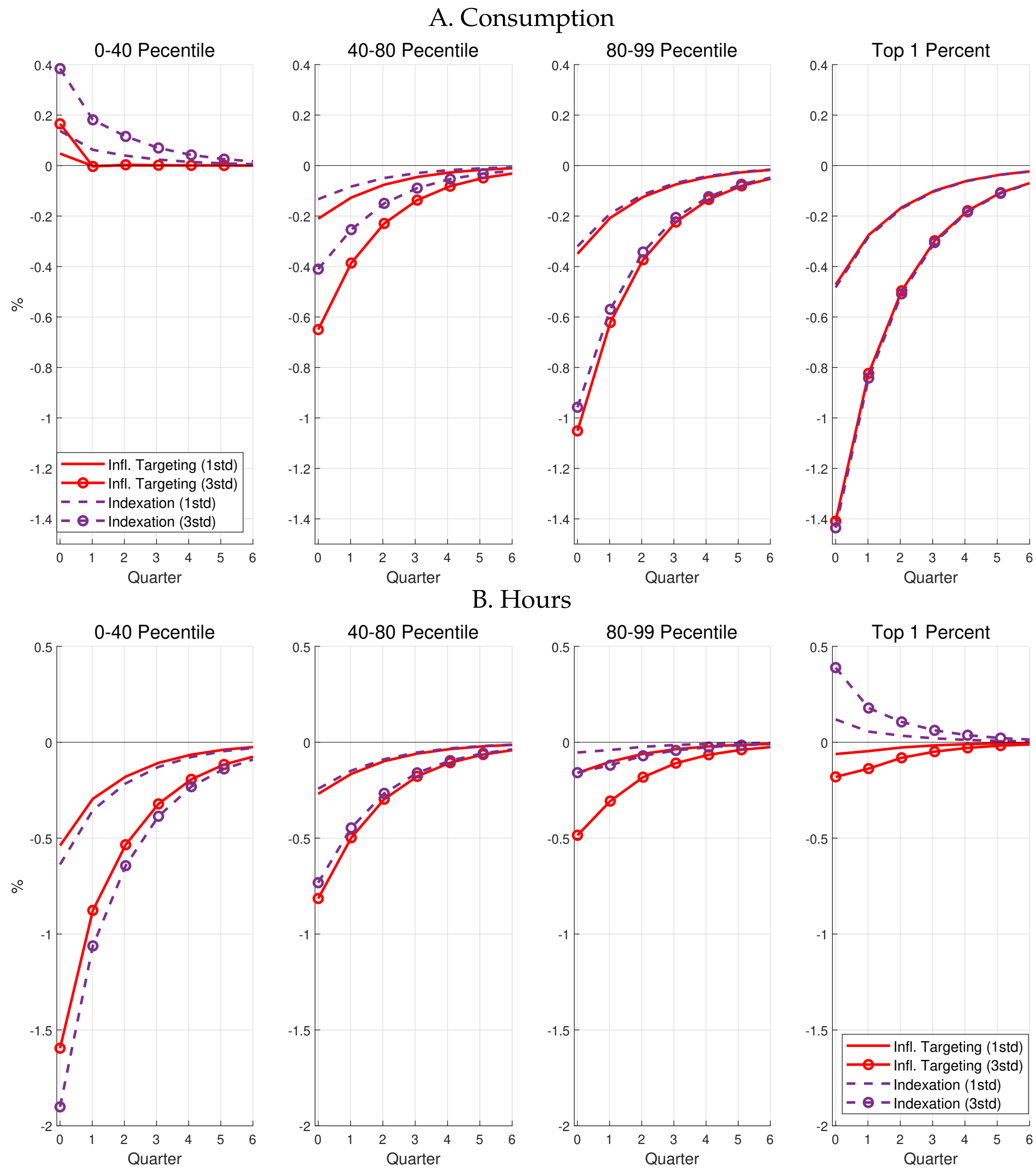
Note: The first row shows the welfare difference between the ZLB-HANK with 2.8% inflation target and the ZLB-HANK, while the second row shows the welfare difference between the ZLB-HANK without indexation and the ZLB-HANK, expressed as a fraction of steady-state consumption in the ZLB-HANK. The welfare in each economy is computed as the welfare conditional on the highest preference level and the lowest shadow policy rate. The positive number indicates that the ZLB-HANK with indexation or ZLB-HANK with high inflation target is more desirable.

deflation when the economy is at the ZLB, which makes adverse demand shocks more contractionary. Moreover, during expansions, the rise in expected inflation ameliorates the increase in real rates encountered by households, leading to higher increases in output and inflation compared to the model with indexation.

We now compare the welfare effect of business cycles in the model with high inflation target and that in the model with inflation indexation at the disaggregate level. The first row in Table 9 reports the welfare change when the target inflation rate is raised from 2.0% to 2.8% in the ZLB-HANK model across different wealth levels, where positive numbers indicate that, all else being equal, households prefer a higher inflation target. As indicated by the total welfare, a positive trend in inflation enhances overall welfare thanks to the reduction in aggregate volatility resulting from the decreased ZLB frequency. The second row in the table reproduces the result reported in Table 5, that is, the welfare change when nominal contracts are replaced with real contracts, holding the inflation target at 2%. The comparison between rows 1 and 2 indicates that inflation indexation is more welfare-enhancing than raising the inflation target in aggregate, reflecting the lower aggregate volatility in the former.

However, the welfare gain of indexing loans to inflation over raising the inflation target is not uniform across households. While households below the 80th percentile of the wealth distribution prefer indexation over a high inflation target, the top 20% are worse off. The reason for this disproportionate gain can be understood from Figure 5, which compares the impulse responses under indexation and high trend inflation to 1 and 3 standard deviation demand shocks. As noted in the figure, the bottom 80% of households enjoy a higher level of consumption under indexation than under a high inflation target for a given magnitude of demand shock. As the consumption of these households is closely tied to individual labor and dividend income, which moves proportionately with aggre-

Figure 5. IRFs: Indexation vs. High Inflation Target



Note: The impulse responses to demand shocks in the ZLB-HANK with inflation indexation and the ZLB-HANK with a 2.8% inflation target. The responses represent the deviations from the long-run mean, which differs across model economies.

gate income, a less severe recession under indexation makes these households consume more than under a high inflation target.

Notice that the consumption stabilization effect of inflation indexation relative to high inflation target is smaller for households in the 80-99th percentile of the distribution than those below the 80th percentile. In fact, inflation indexation even destabilizes consumption relative to high inflation target for top 1% in wealth. Again, by severing the link between realized real interest rates and inflation, indexation makes the real interest income of wealth-rich households decrease more than under a high inflation target during deflationary episodes. In response, wealthy households work more to compensate for the reduction in interest income under indexation, which explains why the top 20% are worse off under indexation than under a high inflation target.

The desirability of real contracts relative to raising the inflation target to enlarge the room for conventional monetary policy may appear stronger if the two are compared in models in which positive steady-state inflation is costly. As discussed in Coibion, Gorodnichenko and Wieland (2012), the most prominent welfare cost associated with a higher trend inflation under staggered price setting is greater price dispersion, which leads to an inefficient allocation of factor inputs, thereby lowering aggregate output. In this context, inflation indexation may be even more beneficial as it does not involve such a significant welfare cost.

6 Conclusion

In this paper, we employed a ZLB-HANK model with nominal government bonds to assess the extent to which inflation-indexed loan contracts affect macroeconomic volatility. We did so by computing the reduction in volatility when real contracts replace nominal contracts. We found that if loans were fully indexed to inflation, output and inflation would have greatly stabilized, with a significant reduction in ZLB frequency.

When government bonds are indexed to inflation, a fall in expected inflation does not affect ex-ante real rates, indicating that households do not encounter an increase in ex-ante real rates at the ZLB, which would have occurred under nominal bonds. Accordingly, the reduced aggregate risk under real contracts weakens the demand for precautionary savings substantially, leading to higher average nominal rates and contributing to a reduction in ZLB frequency.

The aggregate effect of nominal contracts is amplified when idiosyncratic income risk is high. With higher income risk, steady-state nominal interest rates fall, increasing the likelihood of hitting the ZLB. In this case, the stabilizing effect of real contracts becomes more pronounced, implying a greater cost of nominal contracts.

We demonstrated that a policy that indexes loans to inflation is more effective in stabilizing aggregate volatility than setting a high inflation target. This result suggests that the issuance of inflation-linked bonds, or at least the combination of these bonds with raising the inflation target, may be a more desirable policy than raising the inflation target alone in the face of a declining natural rate of interest, a phenomenon observed in many advanced countries.

References

Adam, Klaus, and Junyi Zhu. 2016. "Price-Level Changes and the Redistribution of Nominal Wealth across the Euro Area." *Journal of the European Economic Association*, 14(4): 871–906. 1

Aiyagari, S. Rao. 1994. "Uninsured Idiosyncratic Risk and Aggregate Saving." *The Quarterly Journal of Economics. Vol. 109, No. 3, 659-684*. 2

Auclert, Adrien, Matthew Rognlie, and Ludwig Straub. 2020. "Micro Jumps, Macro Humps: Monetary Policy and Business Cycles in an Estimated HANK Model." NBER Working Paper No. 26647. 4.2

Badarinza, Cristian, John Y. Campbell, and Tarun Ramadorai. 2018. "What Calls to ARMs? International Evidence on Interest Rates and the Choice of Adjustable-Rate Mortgages." *Management Science*, 64(5): 2275–2288. 1

Blanchard, Olivier, Giovanni Dell'Ariccia, and Paolo Mauro. 2010. "Rethinking Macroeconomic Policy." *Journal of Money, Credit and Banking*, 42(s1): 199–215. 1, 5

Carrillo, Julio A., and Céline Poilly. 2013. "How do financial frictions affect the spending multiplier during a liquidity trap?" *Review of Economic Dynamics*, 16(2): 296–311. 1, 2.3

Christiano, Lawrence, Martin Eichenbaum, and Sergio Rebelo. 2011. "When Is the Government Spending Multiplier Large?" *Journal of Political Economy*, 119(1): 78–121. 2.1, 4.2

Coibion, Olivier, Yuriy Gorodnichenko, and Johannes Wieland. 2012. "The Optimal Inflation Rate in New Keynesian Models: Should Central Banks Raise Their Inflation Targets in Light of the Zero Lower Bound?" *The Review of Economic Studies*, 79(4): 1371–1406. 5, 5, 5

Debortoli, Davide, and Jordi Gali. 2018. "Monetary policy with heterogeneous agents: Insights from TANK models." Department of Economics and Business, Universitat Pompeu Fabra Economics Working Papers 1686. 3.1, 1

Den Haan, Wouter J. 2010. "Assessing the accuracy of the aggregate law of motion in models with heterogeneous agents." *Journal of Economic Dynamics and Control*, 34(1): 79 – 99.

Computational Suite of Models with Heterogeneous Agents: Incomplete Markets and Aggregate Uncertainty. 2, 3.2, 1, 7

Diaz-Gimenez, Javier, Andrew Glover, and Jose-Victor Rios-Rull. 2011. "Facts on the distributions of earnings, income, and wealth in the United States: 2007 update." *Quarterly Review*. 3, 8

Doepke, Matthias, and Martin Schneider. 2006*a*. "Aggregate Implications of Wealth Redistribution: The Case of Inflation." *Journal of the European Economic Association*, 4(2-3): 493–502. 1

Doepke, Matthias, and Martin Schneider. 2006*b*. "Inflation and the Redistribution of Nominal Wealth." *Journal of Political Economy*, 114(6): 1069–1097. 1

Eggertsson, Gauti B., and Paul Krugman. 2012. "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach*." *The Quarterly Journal of Economics*, 127(3): 1469–1513. 4.5

Fernández-Villaverde, Jesús, Joël Marbet, Galo Nuño, and Omar Rachedi. 2023. "Inequality and the Zero Lower Bound." Working Paper 31282. 1, 1, 3.1, 3.2, 4.2, 4.4, 5, 5

Fisher, Irving. 1933. "The Debt-Deflation Theory of Great Depressions." *Econometrica*, 1(4): 337–357. 4.5

Floden, Martin, and Jesper Linde. 2001. "Idiosyncratic Risk in the United States and Sweden:Is There a Role for Government Insurance?" *Review of Economic Dynamics 4, 406-437*. 3.1

Hagedorn, Marcus, Jinfeng Luo, Iourii Manovskii, and Kurt Mitman. 2019. "Forward guidance." *Journal of Monetary Economics*, 102: 1 – 23. 3.1

Huggett, Mark. 1993. "The risk-free rate in heterogeneous-agent incomplete-insurance economies." *Journal of Economic Dynamics and Control, Elsevier, vol. 17(5-6), 953-969*. 2

Iacoviello, Matteo. 2005. "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle." *American Economic Review*, 95(3): 739–764. 1

Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante. 2018. "Monetary Policy According to HANK." American Economic Review 108(3), 697-743. Working Papers 1602. 4.5

Krusell, Per, and Anthony A. Smith. 1998. "Income and Wealth Heterogeneity in the Macroeconomy." *Journal of Political Economy, 106(5): 867-96*. 1, 3.2, B.2

Kuncl, Martin, and Alexander Ueberfeldt. 2023. "Monetary policy and the persistent aggregate effects of wealth redistribution." *Journal of Monetary Economics*. 1

McKay, Alisdair, and Ricardo Reis. 2016. "The Role of Automatic Stabilizers in the U.S. Business Cycle." *Econometrica*, 84(1): 141–194. 4.2

McKay, Alisdair, Emi Nakamura, and Jon Steinsson. 2016. "The Power of Forward Guidance Revisited." *American Economic Review*, 106(10): 3133–58. 1, 2.3, 2.3, 1, 1, 3.1

Meh, Césaire A., José-Víctor Ríos-Rull, and Yaz Terajima. 2010. "Aggregate and welfare effects of redistribution of wealth under inflation and price-level targeting." *Journal of Monetary Economics*, 57(6): 637–652. 1

Rotemberg, Julio J. 1982. "Sticky Prices in the United States." *Journal of Political Economy*, 90(6): 1187–1211. 2.2

Schaab, Andreas. 2020. "Micro and Macro Uncertainty." Working Paper. 1

Tauchen, George. 1986. "Finite state markov-chain approximations to univariate and vector autoregressions." *Economics Letters*, 20(2): 177 – 181. 3.1, B.1, 13

Appendix

A Definition of Recursive Competitive Equilibrium

A.1 The Recursive Problem of the Household

The household's problem can be recursively written as follows. Define x and X as the vectors of individual and aggregate state variables, respectively: $x \equiv (b, z)$ and $X \equiv (\mu, \zeta, \widetilde{R}_{-1})$, where $\mu(x)$ is the type distribution of households, and $\widetilde{R}_{-1}$ is the shadow interest rate set in the previous period.[11] The value function for an individual household, denoted by $V(x, X)$, is defined as:

$$V(x, X) = \max_{c,b',h} \left\{ \frac{c^{1-\sigma} - 1}{1 - \sigma} - \Xi \frac{h^{1+1/\gamma}}{1 + 1/\gamma} + \zeta\beta\mathbb{E}\left[V(x', X')|z, \zeta\right] \right\} \tag{A.1}$$

subject to

$$c + b' = w(X)zh + (1 + r(X))b - T + d, \; b' \geq \underline{b},$$

and

$$\mu' = \mathbb{T}(X),$$

where $\mathbb{T}$ denotes the law of motion for μ, time subindices are suppressed to simplify notation, and primes denote variables in the next period.

A.2 Definition of Equilibrium

A recursive competitive equilibrium is a value function $V(x, X)$, a transition operator $\mathbb{T}(X)$, a set of policy functions $\{c(x, X)$,$b'(x, X)$,$h(x, X), n_j(X), p_j(X), y_j(X)\}$, and a set of prices $\{w(X), r(X), R(X)$,$P(X)\}$ such that:

1. Individual households' optimization: given $w(X)$ and $r(X)$, optimal decision rules $c(x, X)$, $b'(x, X)$, and $h(x, X)$ solve the Bellman equation, $V(x, X)$.

2. Intermediate goods firms' optimization: given $w(X)$, $r(X)$, and $P(X)$, the associated optimal decision rules are $n_j(X)$ and $p_j(X)$.

3. Final good firm's optimization: given a set of prices $P(X)$ and $p_j(X)$, the associated optimal decision rules are $y_j(X)$ and $Y(X)$.

4. The gross nominal interest rate, $R(X)$, satisfies the Taylor rule (Equation 6).

5. Balanced budget of the government: $r(X)\overline{B} = \int T(x, X)d\mu$.

[11]Denote $\mathcal{B}$ and $\mathcal{Z}$ for sets of all possible realizations of b and z, respectively. Then, the measure $\mu(b, z)$ is defined over a σ-algebra of $\mathcal{B} \times \mathcal{Z}$.

6. For all Ω,

   - (Labor market) $N(X) = \int zh(x,X)d\mu = \int n_j(X)dj$
   - (Bond market) $\overline{B} = \int b'(x,X)d\mu$
   - (Goods market) $Y(X) = C(X)$ where $Y(X) = N(X) - f$ and $C(X) = \int c(x,X)d\mu$.

7. Consistency of individual and aggregate behaviors: for all $A^0 \subset \mathcal{A}$ and $Z^0 \subset \mathcal{Z}$,

$$\mu'(A^0, Z^0) = \int_{A^0,Z^0} \left\{ \int_{\mathcal{A},\mathcal{Z}} \mathbf{1}_{b'=b'(x,X)} d\Gamma_z(z'|z) d\mu \right\} db'dz',$$

   where $\Gamma_z(z'|z)$ is a transition probability distribution function for z.

B Computational Procedures

B.1 Steady-State Equilibrium

We summarize the computational algorithm used for the steady-state economy. In this step, we find the stationary measure, $\overline{\mu}$. The procedures are as follows.

Step 1. Have guesses for endogenous values such as β, Ξ, τ, and w.

Step 2. Construct grids for individual-state variables, such as bond holdings, b and logged individual labor productivity, $\hat{z} = \ln z$. N_b and N_z, denote the number of grids for b and z, respectively. Choose $N_b = 101$ and $N_z = 17$. The range of b is $[0, 40]$. More bond grid points are assigned on the lower range with a convex function. $\widehat{z}$ is equally spaced in the range of $[-3\sigma_{\hat{z}}, 3\sigma_{\hat{z}}]$, where $\sigma_{\hat{z}} = \sigma_z / \sqrt{1-\rho_z^2}$.

Step 3. Approximate the transition probability matrices for individual labor productivity, Γ_z, using Tauchen (1986).

Step 4. Solve the individual value functions at each grid point. In this step, we obtain the optimal decision rules for saving, $b'(b,z)$ and hours worked, $h(b,z)$, the value functions, $V(b,z)$. The detailed steps are as follows:

   (a) Make an initial guess for the value function, $V_0(b,z)$ for every grid point.

   (b) Solve the individual household's problem, and obtain $V_1(b,z)$:

$$V_1(b,z) = \max_{\{b',h\}}$$
$$\left\{\log\left(wzh + (1+r)b - T + d - b'\right) - \Xi\frac{h^{1+1/\nu}}{1+1/\nu} + \beta\sum_{z'=1}^{N_z}\Gamma_z(z'|z)V_0(b',z')\right\}$$

(c) If V_0 and V_1 are close enough for each grid point, go to the next step. Otherwise, update the value functions ($V_0 = V_1$), and go back to (b).

Step 5. Obtain the time-invariant measure, $\overline{\mu}$, with finer grid points for b. Using cubic spline interpolation, compute the optimal decision rules for bond holdings with the new grid points. We compute $\overline{\mu}$ using the optimal decision rules with the finer grid points and transition probabilities for z, Γ_z.

Step 6. Compute aggregate variables using $\overline{\mu}$. If the aggregate values become sufficiently close to the targeted values, then the steady-state equilibrium of the economy is found. Otherwise, update the endogenous parameters, and go back to Step 4.

B.2 Equilibrium with Aggregate Fluctuations

We summarize the computational algorithm used for the model economy with aggregate preference shocks. To address the intricacies of our model, we employ the well-established methodology introduced by Krusell and Smith (1998).

Step 1. Construct grids for aggregate-state variables, and the individual-state variables such as the individual labor productivity and bond holdings. For preference shocks, ζ, construct 15 grid points in the range of $[-3\widetilde{\sigma}_\zeta, 3\widetilde{\sigma}_\zeta]$, where $\widetilde{\sigma}_\zeta = \sigma_\zeta/\sqrt{1-\rho_\zeta^2}$. Distribute grid points for $\widetilde{R}_{-1}$ in a non-uniform manner, allocating more points for the ZLB range. The grids for individual-state variables are the same as those in the steady-state economy.

Step 3. Parameterize the forecasting functions for Y,Π, w, $\widetilde{R}$, and d.[12]

Step 4. Given the forecasting functions, solve the optimization problems for individual households. Obtain the policy functions for asset holdings, $b'(b, z, R_{-1}, \zeta)$, and the hours decision rule, $h(a, z, R_{-1}, \zeta)$.[13]

[12]In pursuit of better fit, we include interaction terms and quadratic terms in the forecasting functions.
[13]As discussed earlier, the transition probabilities for z and ζ are approximated using Tauchen (1986).

Step 5. Generate simulated data for 2,500 periods using the value functions obtained in Step 4. The details are as follows.

(a) Set the initial conditions for $\widetilde{R}_{-1}, \zeta$, and $\mu(a, z)$.

(b) Given the forecasting functions, the evaluated value function obtained in Step 4, and the obtained new prices. Solve the optimization problems for individual households to get the policy functions for asset holdings, $b'(b, z)$, and the hours decision rule, $h(b, z)$.

(c) Compute aggregate variables using μ: $C = \int c(b, z) d\mu$, $N = \int z h(b, z) d\mu$, $H = \int h(b, z) d\mu$, and $Y = N - f$.

(d) Obtain the next period measure $\mu'(b, z)$ using $b'(b, z)$ and transition probabilities for z.

(e) Save the time series of Y,Π, w, $\widetilde{R}$, and d.

Step 6. Obtain the new coefficients for the forecasting functions by the OLS estimation using the simulated time series.[14] If the new coefficients are close enough to the previous ones, the simulation is done. Otherwise, update the coefficients, and go to Step 4.

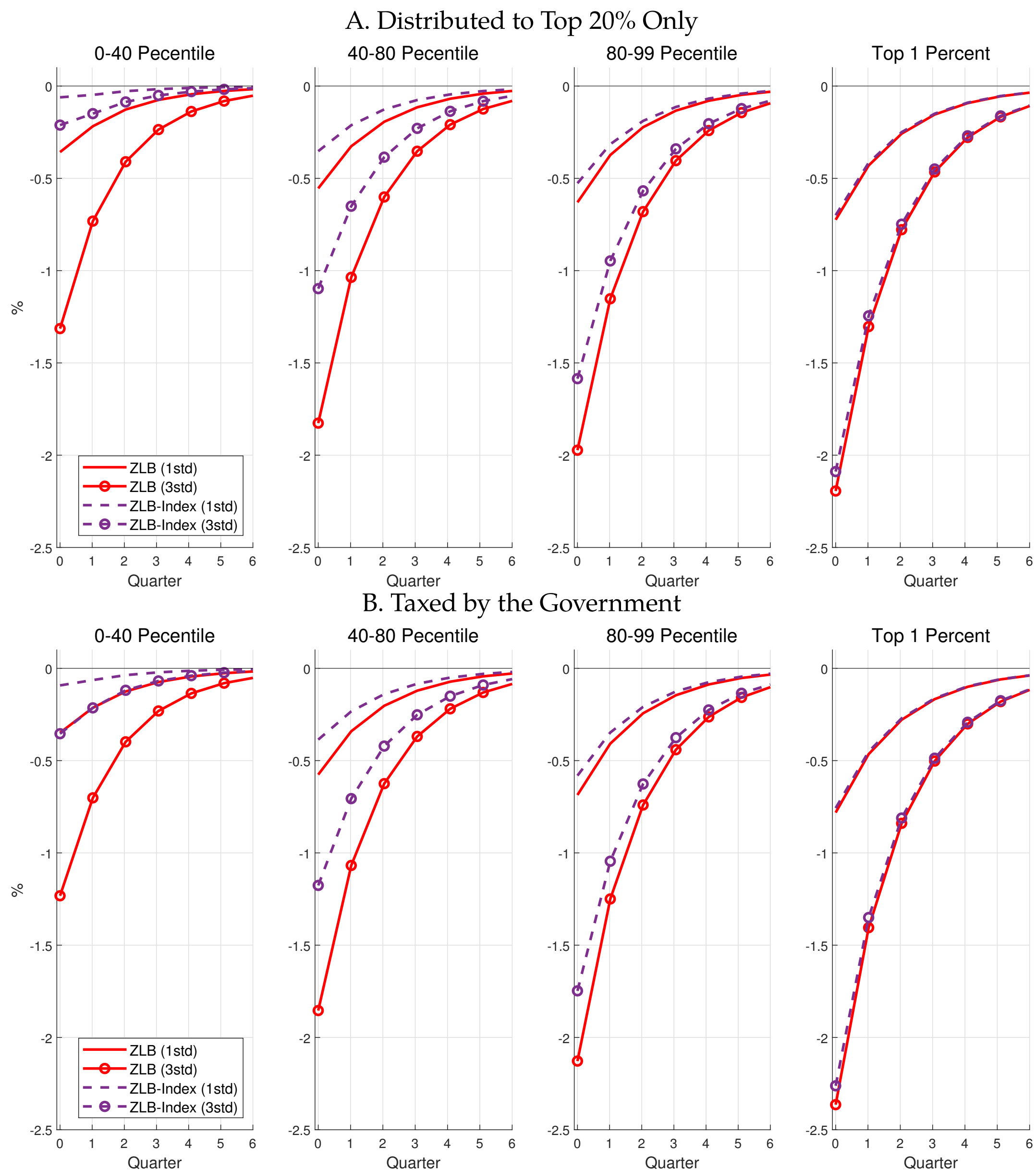
C Distribution of Countercyclical Profits

In model economies characterized by monopolistic competition and nominal price rigidities, markups exhibit countercyclical behavior, leading to positive profits in response to negative demand shocks. This phenomenon significantly influences consumption dynamics within the model economy. In this section, we conduct a comparative analysis of two distinct economic scenarios: i) an economy wherein profits are exclusively distributed to the top 20% of households in the productivity distribution, and ii) an economy where profit incomes are subjected to a 100 percent tax rate, with the government utilizing the proceeds for wasteful government consumption.

Figure A.1 illustrates the consumption responses across the wealth distribution in these two economies. Two noteworthy findings emerge. Firstly, the manner in which countercyclical profits are distributed has a profound impact on consumption dynamics, particularly for households with lower wealth. Specifically, irrespective of inflation indexation,

[14] I drop the first 500 periods to eliminate the impact of the arbitrary choice of initial aggregate state variables.

Figure A.1. Consumption IRFs across Wealth Distribution



Note: The impulse responses to demand shocks in the ZLB-HANK models with and without inflation indexation. The responses represent the deviations from the long-run mean, which differs across model economies.

households in the 0-40th percentile exhbit a decline in consumption. Secondly, the heterogeneous consumption responses across wealth distribution persist in both cases: households at the bottom of the wealth distribution exhibit a significant stabilization in consumption compared to their counterparts at the top.