# Optimal Information Disclosure in Credit Market: Default History & Transaction History*

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**Abstract**

We develop a dynamic model of unsecured debt contracts with adverse selection. An entrepreneur borrows investment goods from lenders to run a project whose return depends on both entrepreneurial and aggregate productivity. Entrepreneurial productivity is the entrepreneur's private information. Lenders evaluate the entrepreneur's productivity and the credit risk based on historical aggregate productivity, as well as the entrepreneur's operation history and credit history. We investigate how equilibrium outcomes depend on the type of information available to lenders regarding the borrower's credit history and explore the optimal information disclosure policy in credit markets.

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# 1 Introduction

The adverse selection problem is pervasive in credit markets and lenders employ various sources of information to alleviate informational disparities when evaluating borrowers' credit risks. One essential resource on which lenders rely is a credit report. Specifically, in the U.S., credit reports issued by credit bureaus like Equifax and Experian show an individual's default history, although they do not include an individual's transaction history (referring to the terms of debt contracts in which an individual has previously engaged).

However, the majority of previous studies exploring the economic implications of the adverse selection problem in credit markets have relied on static models, which constrain our comprehensive understanding of credit risk assessment. The following questions still need to be addressed: How do lenders utilize a borrower's credit history when assessing credit risk? How does the fact that future lenders can observe a borrower's credit history affect the terms of debt contracts in the current period? How does the type of credit history information available to lenders impact the terms of debt contracts and economic activities? What constitutes the optimal level of information disclosure in credit markets?

We address these questions by developing a dynamic model of unsecured debt contracts with adverse selection. We examine how borrowers' operational history, credit history, and information on past aggregate economic conditions are utilized to assess a borrower's credit risk. The borrower's credit history includes default history and/or transaction history, depending on the information regime. We investigate how the accessibility of different types of information to lenders, regarding the borrower's credit history, influences equilibrium outcomes. Furthermore, we explore the dynamics of borrowing costs under different information regimes and examine the optimal level of information disclosure regarding the borrower's credit history.

***Model preview***. The model involves an entrepreneur borrowing an investment good from lenders to run the project, where the returns depend on both entrepreneurial and aggregate productivity. The aggregate productivity is a random variable and is independent across periods. There are two types of entrepreneurs: those with high productivity (H-type) and those with low productivity (L-type), and entrepreneurial productivity is the entrepreneur's private information.

In the model, the realized aggregate productivity in the past is assumed to be public information, similar to GDP data which is publicly available in most countries. We also assume that operation history — whether the entrepreneur ran the project in the past — is public information. Furthermore, we assume that lenders can observe the entrepreneur's credit history, which includes debt contract terms and/or default decisions, depending on the information regime. Thus, lenders could use information about the history of aggregate productivity and the entrepreneur's economic decisions in the past to evaluate the entrepreneur's productivity and, hence, credit risk.

We consider four information regimes: 1) no-information regime, where lenders cannot observe any information about the entrepreneur's credit history, 2) default history regime, where lenders can only observe the default history, 3) transaction history regime, where lenders can observe the terms of contracts that the entrepreneur made in the past, and 4) full-information regime, where lenders can observe both the transaction history and the default history.

***Results preview***. If lenders can correctly verify the entrepreneur's type, i.e., the entrepreneurial productivity, by observing the available history information, the terms of the contract are customized to each type. In particular, the repayment or interest payment is lower for the H-type compared to the L-type because the L-type has a higher default risk. Consequently, the H-type has an incentive to reveal his/her type.

In particular, if lenders have access to the entrepreneur's transaction history, the H-type entrepreneur may be willing to pay higher repayments that disincentivizes the L-type from mimicking in the current period, which allows the H-type to have better terms of contract in the next period. As a result, a separating equilibrium can emerge, in which the H-type and the L-type choose different debt contracts, subject to certain parametric conditions.

The cost that the H-type incurs to be separated from the L-type in the current period is lower under the full-information regime than under the transaction history regime, because the default history also provides useful information about verifying the entrepreneur's type, disincentivizing the L-type from mimicking.

On the other hand, under the no-information regime or the default history regime, where lenders cannot observe the entrepreneur's transaction history, separating equilibrium is not feasible, and only pooling equilibrium exists. Nonetheless, even in a pooling equilibrium, lenders can verify the entrepreneur's true type by observing the default history, contingent upon the realized aggregate productivity in the past. This is because the H-type exhibits a lower default risk than the L-type.

Having characterized the equilibrium debt contract under each regime, we proceed to conduct a welfare analysis. Specifically, we show that the no-information regime is the optimal information regime, which resonates with the findings regarding the optimality of withholding information in asset markets as presented by Andolfatto et al. (2014) and Andolfatto and Martin (2013). However, if the productivity of the L-type is sufficiently low that lenders would refrain from lending their investment goods to the L-type upon discovering the true type, disclosing the default history can enhance welfare. This is because the default history can reveal the entrepreneur's type, allowing a lender to avoid lending to the socially unproductive L-type. In particular, the model shows that disclosing the default history tends to be optimal when the L-type's productivity and the average common productivity are sufficiently low.

When the productivity of the L-type is sufficiently low, the transaction history is uninformative because the L-type always has an incentive to mimic the H-type. Conversely, when the L-type's productivity is sufficiently high, the transaction history enables a separating equilibrium, resulting in lower welfare compared to a pooling equilibrium. Thus, in the model, the disclosure of transaction history is either suboptimal or irrelevant. This finding suggests that the government should refrain from such disclosure, providing a theoretical justification for the current legal system's practices.

***Literature review***. This paper adds to the tradition of studying adverse selection problems in credit markets. While most existing literature focuses on one-time transactions in a single-period model, we construct a dynamic model that allows us to investigate reputation formation in credit markets.[1] Hennessy et al. (2010), Morellec and Schürhoff (2011), and Strebulaev et al. (2016) construct dynamic models of credit market with adverse selection, but these papers essentially reduce the signaling problem to a static one by assuming that private information is short-lived or that debt is a one-time choice in a real options framework. In contrast, our model enables a more comprehensive range of signaling strategies and delves into the process of reputation formation, as well as the dynamic evolution of lenders' beliefs over time.[2] Boot and Thakor (1994) studies on the dynamics of loan interest rates in credit markets with a moral hazard problem and shows that loan interest rates decline over time, but our focus remains on adverse selection problems in credit markets.

More relatedly, Diamond (1989) studies reputation formation through the default history in credit markets with adverse selection problems in a dynamic setting, and explores the dynamics of an incentive problem between borrowers and lenders. Specifically, Diamond (1989) shows that the incentive problems are mitigated once a borrower manages to acquire a good reputation. However, in Diamond (1989), ex ante separation by choice of contract, i.e., separating contracts, is not feasible, and he focuses on ex-post separation through default history. Ordoñez et al. (2019) goes a step further and shows that good borrowers can effectively signal their credit quality through repayment history when uncertainty in collateral value is low. However, the role and importance of borrower's credit history in collateralized

[1]See Bester (1985), Besanko and Thakor (1987), Figueroa and Leukhina (2015), Jaffee and Russell (1976), and Milde and Riley (1988) for single period models of credit market with adverse selection for instance.

[2]While models of debt contracts with dynamic adverse selection are limited, several papers have studied the multi-period adverse selection problems in other areas. For example, Kreps and Wilson (1982) consider a finite-period reputation formation model in the context of industrial organization to show a high type's precommitment to its action. Noldeke and van Damme (1990) and Swinkels (1999) extend the Spence (1973) job market signaling model into a multi-period environment. Toxvaerd (2017) studies a multi-period model of limit pricing with one-sided incomplete information in which a simple entry game is repeated until entry occurs. However, these papers still do not study how other aggregate variables can be used for constructing the beliefs of less informed agents.

credit markets would differ from that in an unsecured credit market given that collateral limits the lender's loss from defaults and collateral itself can work as a signaling device (see Bester (1985)).

Furthermore, our paper differs from Diamond (1989) and Ordoñez et al. (2019) in the following perspective. First, we introduce aggregate shocks to study how lenders incorporate the history of aggregate economic conditions alongside borrowers' individual history to evaluate credit risks. Second, we examine the impact of future economic perspectives on the effectiveness of credit history as a signaling device in credit markets. Third, we analyze the optimal level of information disclosure about the borrower's credit history in credit markets.

***Layout***. The rest of this paper is organized as follows. Section 2 presents the economic environment of the model. Section 3 describes the game structure between a borrower and a lender, and section 4 characterizes equilibrium. In section 5, we conduct welfare analysis to find the optimal information regime, and in section 6 we examine a model economy when the productivity of the L-type entrepreneur is sufficiently low, leading a lender to avoid lending to the L-type. Section 7 concludes.

# 2 Model

In this section, we set up the model.

**Physical environment** The economy consists of two dates, $t = 1, 2$, and each period $t$ is divided into two subperiods: morning and afternoon. Morning is the investment period and consumption occurs in the afternoon. The actors in the model are an entrepreneur and two lenders 1 and 2. The entrepreneur lives for both periods with the discount factor $\beta \in (0, 1)$ across periods, lender 1 lives in period 1, and lender 2 lives in period 2. Thus, the entrepreneur faces a different lender in each period. The instantaneous utility of all agents in each period equals the quantity of consumption in the afternoon, i.e., agents have a constant marginal utility of 1. In what follows, we use lender $t$ to denote the lender who lives in period $t \in \{1,2\}$.

Lender $t$ receives an indivisible endowment of one unit of an investment good in the morning in period $t$. The investment good can be either lent to the entrepreneur or invested in a saving technology that yields a certain return of $\gamma > 0$ units of the consumption good in the afternoon. The entrepreneur does not receive any endowments in the morning. Instead, the entrepreneur has access to an investment project that produces $w_t$ units of consumption goods in the afternoon in each period $t \in \{1,2\}$ if the project is funded with one unit of the investment good in the morning, and produces zero units otherwise. The outcome of the project depends on the aggregate productivity, $A_t$, and the entrepreneurial productivity $\theta$, as $w_t = A_t\theta$.

Aggregate productivity $A_t$ is uniformly distributed with the support of $\left[0, \overline{A}_t\right]$ in period $t \in \{1,2\}$ and it is independent across periods. The entrepreneur has productivity $\theta_H$ with the probability $\sigma \in (0,1)$ and has $\theta_L$, where $0 < \theta_L < \theta_H$, with the complement probability. The entrepreneurial productivity is realized at the beginning of the morning in period 1 and remains fixed until the end of period 2. We refer the entrepreneur with $\theta_H$ and $\theta_L$ the H-type and the L-type, respectively.

We assume that the distributions for $A_t$ and probability $\sigma$ are public information. However, entrepreneurial productivity $\theta$ is the entrepreneur's private information. Furthermore, the aggregate productivity $A_t$ is not observable in the afternoon in period $t$. Thus, only the entrepreneur can observe the exact realized return of his/her project. Finally, we assume that $\min\{\overline{A}_1, \overline{A}_2\}\theta_L \geq 4\gamma$, which supports loan contract between the entrepreneur and lenders feasible in both periods. On the other hand, if $4\gamma > \overline{A}_t\theta_L$, lender $t$ will never lend the investment good to the L-type once the entrepreneur's type is revealed. We will investigate the model economy with $4\gamma > \overline{A}_t\theta_L$ in section 6.

**Borrowing with a debt contract** In the model, the entrepreneur must borrow the investment good from lender $t$ to run his/her project in the morning in period $t = 1,2$. We assume that the entrepreneur offers a contract to lender $t$ in each period and lender $t$ either accepts or rejects the offer.

The contract between the entrepreneur and lender $t$ is assumed to be a debt contract. Although we focus on a debt contract, a debt contract often emerges as the optimal contract when project returns are unobservable with private information because equity contracts are infeasible. For example, Gale and Hellwig (1985) and Townsend (1979) show that unobservability implies that contracts are optimal for the debt form. In our setup, it can be shown that a debt contract is optimal if the amount of payment that the entrepreneur made at $t = 1$ is unobservable in period $t = 2$.[3]

A debt contract is described by the repayment $r_t \in \mathbb{R}_+$ at time $t$ given the fixed loan size. We assume that a debt contract involves a commitment to liquidation for all payments less than $r_t$ and liquidation implies the destruction of all output from the project the same as in Diamond (1989). Thus, if the project's return is higher than $r_t$, the entrepreneur repays $r_t$ and consumes the remainder, and otherwise, the entrepreneur defaults. In the following analysis, we say that contract $r'$ is lower than $r''$ if $r' < r''$.

A model of unsecured credit often assumes that borrowers face limited access to the credit market following defaults, and there have been extensive studies on how the severity of penalties for defaulters affects equilibrium allocations (e.g., Azariadis and Kass (2013) and Kehoe and Levine (1993)). In contrast, our focus lies in the channel through which a borrower's history of economic decisions in the past influences the terms of debt contracts and real allocations. To focus on the main issue, we assume that there is no penalty for defaulters. Thus, the entrepreneur can access to the credit market in period 2 following

[3] To elaborate this argument, note that the repayment on any equity contract in the model economy must depend on the information provided by the entrepreneur because only the entrepreneur can observe the exact realized return from the project. Specifically, a contract defines a repayment function $R_1(w_1')$ such that the entrepreneur pays $R_1(w_1')$ units of consumption goods in period 1 after reporting a signal $w_1'$ about the output from the project to lender 1. Now suppose that lender 2 cannot observe the exact signal $w_1'$ and repayment that the entrepreneur made in period 1 although lender 2 can observe the terms of contract $R_1(\cdot)$. Then, the entrepreneur will always choose $w_1'$ so as to minimize the payment to the lender in period 1 whenever he/she decides to honor the contract. Thus, the payment is constant and, hence, the contract has the form of the debt contract similar to results in Jang and Kang (2023) and Williamson (1986).

defaults in period 1. However, the main implications do not change with the assumption that the entrepreneur can meet lender 2 to borrow the investment good in period 2 with the probability $\rho \in (0, 1)$, following a default in period 1.[4]

**History information** We now describe the three types of histories that lender 2 can use to evaluate the entrepreneur's credit risk in the model economy.

First, many countries in the world release time-series data on gross domestic production (GDP) and total factor productivity (TFP) to the public. We incorporate this reality into the model in the following way: In the morning in period 2, all agents, including lender 2, can observe the realized aggregate productivity $A_1$ (common prouctivity history). Note that the realized $A_t$ is not observable in the afternoon in period $t \in \{1, 2\}$, which is also consistent with the real-world observation that GDP and TFP data being published with a lag.

The second type of history information is the entrepreneur's operation history. Specifically, let $o = 1$ if the entrepreneur runs the project in period 1 and $o = 0$ otherwise. Then, $o \in \{0, 1\}$ summarizes the entrepreneur's operation history and we assume that $o \in \{0, 1\}$ is the public information in the morning in period 2, so lender 2 can observe whether the entrepreneur ran his/her project in period 1.

Finally, the last type of history information is the entrepreneur's credit history. To make the definition of credit history concrete, suppose that the entrepreneur entered into a debt contract $r_1$ with lender 1 in period 1, and let $d = 1$ if the entrepreneur defaults on contract $r_1$ and $d = 0$ otherwise. Then, $r_1$ and $d$ capture the entrepreneur's transaction history and default history, respectively, that the entrepreneur made in period 1, and we call the set $\{r_1, d\}$ the entrepreneur's *intrinsic credit history.* Note that if the entrepreneur does not run the project in period 1, then the entrepreneur has no intrinsic credit history, and, in this case, we let $r_1 = d = 0$.

In the model, lender 2 can observe some or none of the entrepreneur's intrinsic credit

[4]Specifically, if there is a penalty on defaulters, there are additional requirements for pooling and separating equilibrium without a break to exist in section 4. Otherwise, the main implications do not change.

history depending on the information regime. Let $\omega$ be a set of available information to lender 2 about $\{r_1, d\}$ and call $\omega$ the credit history of the entrepreneur. Given that intrinsic credit history consists of transaction history and default history, we have four cases about the information regime:

1. (No-information regime) Lender 2 can observe no information about the entrepreneur's credit history: $\omega = \varnothing$.
2. (Full-information regime) Lender 2 can observe both the transaction history and default history: $\omega = \{r_1, d\}$.
3. (Default history regime) Lender 2 can observe the default history: $\omega = \{d\}$.
4. (Transaction history regime) Lender 2 can observe the transaction history: $\omega = \{r_1\}$.

# 3 Game structure

In this section, we describe the game between the entrepreneur and lender $t$ in each period $t \in \{1, 2\}$, agents' strategy, and lender $t$'s belief system.

**Game structure in each period** In each period, there is a game between the long-lived entrepreneur and short-lived lenders. A sequence of moves in each period is as follows. In the morning in period $t$, the entrepreneur offers a contract $r_t$ to lender $t$. Then, lender $t$ decides whether to accept the offered contract or not. If lender $t$ rejects the offer, the game ends. On the other hand, if lender $t$ accepts the offer, lender $t$ provides the investment good to the entrepreneur and the entrepreneur runs the project in the morning. Then, after observing the return from the project in the afternoon, the entrepreneur decides whether to repay $r_t$ units of consumption goods to lender $t$ or to default.

**Agents' strategies** To analyze the entrepreneur's strategy, we define $I_t$ for each $t \in \{1, 2\}$ such that $I_1 = \varnothing$ and $I_2 = \{A_1, o, \omega\}$. Thus, $I_t$ represents the set of histories available to lender $t$ in period $t$. Let $\mathbb{I}_t$ denote the set of all feasible $I_t$. Then, a period $t \in \{1, 2\}$ strategy of the entrepreneur specifies a contract $r_t \in \mathbb{R}_+$ as a function of $(\theta, I_t)$ and a set $D_t \subset [0, 1]$ of $A_t$ as a correspondence of $(\theta, I_t, r_t)$ such that the entrepreneur defaults on the debt contract $r_t$ if and only if $A_t \in D_t$. Next, the strategy of lender $t$ is an acceptance rule that specifies a set $\mathcal{B}_t \subset \mathbb{R}_+$ of acceptable contracts $r_t$ as a correspondence of $I_t$.

If there is no risk of confusion, we drop arguments for each decision rule: We use $r_t$, $D_t$, and $\mathcal{B}_t$ instead of $r_t(\theta, I_t)$, $D_t(\theta, I_t, r_t)$, and $\mathcal{B}_t(I_t)$, respectively. Further, we use $r_{i,t}$ to denote the equilibrium contract that the type $i \in \{H, L\}$ entrepreneur offers in period $t \in \{1, 2\}$ in what follows.

**Belief system** Because the entrepreneur's productivity $\theta$ is the entrepreneur's private information, lender $t$ must form beliefs about $\theta$ before making an acceptance decision for the proposed contract $r_t$. Specifically, lender $t$ constructs the belief using all available information which includes the terms of the offered contract $r_t$ and the public information $I_t$. We write $\mu_t : \mathbb{R}_+ \times \mathbb{I}_t \to [0, 1]$ for the lender's belief function in period $t \in \{1, 2\}$, assigning the probability that the entrepreneur is the H-type.

We impose the following restrictions on the belief $\mu_t$, which we believe reasonable assumptions. First, we assume that if lender 2 observes that the entrepreneur defaulted on contract $r_1$ when the realized $A_1$ was sufficiently high as $A_1 \geq \frac{r_1}{\theta_H}$ so that the H-type entrepreneur could make repayment $r_1$, then lender 2 believes that the entrepreneur is the L-type. Second, suppose that in period 1, the H-type chooses $r'$, while the L-type chooses $r'' \neq r'$, so that separating equilibrium exists, and assume that lender 2 can observe the entrepreneur's transaction history in the morning in period 2, i.e., $r_1 \in \omega$. In this case, we assume that lender 2 believes that the entrepreneur is the H-type (and L-type) if lender 2 observes that the entrepreneur offered $r'$ (and $r''$) in period 1: $\mu_2(\cdot, I_2) = 1$ if $r' \in \omega$ and $\mu_2(\cdot, I_2) = 0$ if $r'' \in \omega$. Similarly, we assume that if the H-type has operation history $o$ while the L-type has $o' \neq o$, then $\mu_2(\cdot, \{A_1, o, \omega\}) = 1$ and $\mu_2(\cdot, \{A_1, o', \omega\}) = 0$.

**Optimal strategies** In the model, defaults destroy all output from the project and do not increase the probability that lender 2 believes the entrepreneur is the H-type. Consequently, the entrepreneur will always make repayment whenever it is feasible and the optimal default strategy for the entrepreneur is $D_t \in \left[0, \frac{r_t}{\theta}\right)$ for any $r_t > 0$. Then, the entrepreneur's expected payoff from making contract $r$ with lender $t$ in period $t \in \{1, 2\}$ is given as

$$u_t(r|\theta) = \frac{1}{\overline{A}_t} \int_{\min\left\{r/\theta, \overline{A}_t\right\}}^{\overline{A}_t} (A\theta - r) dA. \tag{1}$$

Note that the entrepreneur always has incentives to run the project in period 2 because the economy ends in period 2. Thus, the entrepreneur will always offer a contract that is accepted by the lender 2. Specifically, given the lender's acceptance rule $\mathcal{B}_2$ and public information set $I_2 = \{A_1, o, \omega\}$, the entrepreneur solves

$$V_2(\theta, I_2) = \max_{r \in \mathcal{B}_2(I_2)} \left\{u_2(r|\theta)\right\}, \tag{2}$$

in period 2.

In period 1, on the other hand, the entrepreneur may or may not run the project. If the entrepreneur offers $r_1 \in \mathcal{B}_1(I_1)$, then the entrepreneur runs the project and updates the history set as $I_2 = \{A_1, 1, \omega\}$, where specific elements of $\omega$ depend on the information regime. On the other hand, if the entrepreneur offers $r_1 \notin \mathcal{B}_1(I_1)$, then the entrepreneur does not run the project in period 1 and he/she has $I_2 = \{A_1, 0, \varnothing\}$ as the history in period 2. Based on these observations, the entrepreneur's problem in period 1 is given as

$$\max_{r \in \mathbb{R}_+} \left\{ \begin{array}{c} \mathbf{1}_{\mathcal{B}_1(I_1)}(r) \dfrac{1}{\overline{A}_1} \left\{ \int_{\frac{r}{\theta}}^{\overline{A}_1} (A_1\theta - r + \beta V(\theta, I_2^1)) dA_1 + \int_0^{\frac{r}{\theta}} \beta V_2(\theta, I_2^1) dA_1 \right\} \\ + \left(1 - \mathbf{1}_{\mathcal{B}_1}(r)\right) \beta V_2(\theta, I_2^0) \end{array} \right\} \tag{3}$$

where $\mathbf{1}_{\mathcal{B}_1(I_1)}(r)$ is an indicator function that is equal to one if $r \in \mathcal{B}_1(I_1)$, $I_2^1 = \{A_1, 1, \omega\}$, and $I_2^0 = \{A_1, 0, \varnothing\}$. In what follows, we call equilibrium in which the entrepreneur of any type does not run the project in period 1 *equilibrium with a break* and call equilibrium where both types run the project in period 1 *equilibrium without a break*.

Next, given a belief system $\mu_t$ and the public information set $I_t$ in period $t \in \{1,2\}$, the optimal strategy for lender $t$ is the set of acceptable contracts which is given as

$$\mathcal{B}_t^*(\mu_t, I_t) = \left\{ r_t \in \mathbb{R}_+ : \mu_t\left(r_t, I_t\right) r_t \left(1 - \frac{r_t}{\overline{A}_t \theta_H}\right) + (1 - \mu_t\left(r_t, I_t\right)) r_t \left(1 - \frac{r_t}{\overline{A}_t \theta_L}\right) \geq \gamma \right\}. \tag{4}$$

For a contract to be acceptable, the expected revenue from the entrepreneur's repayment should not be lower than the payoff from investing the investment good in the saving technology that yields $\gamma$ units of consumption goods in the afternoon with certainty.

# 4 Equilibrium

In this section, we characterize the equilibrium of the model economy. We adopt Perfect Bayesian Equilibrium (PBE) as our equilibrium concept for the signaling game, which is formally stated in the following definition.

**Definition 1** *An equilibrium is a profile of strategies and a belief system,* $\langle \{r_t, D_t\}, \{\mathcal{B}_t\}, \mu_t \rangle_{t \in \{1,2\}}$ *such that for all* $t \in \{1,2\}$*, 1)* $\{r_1, D_1\}$ *solves (2) and* $\{r_2, D_2\}$ *solves (3) for all* $(\theta, I_t) \in \Theta \times \mathbb{I}$*, 2)* $\mathcal{B}_t = \mathcal{B}_t^*(\mu_t, I_t)$*,*[5] *and 3)* $\mu_t(r_t, I_t)$ *is consistent with Bayes' law whenever it is applicable for all* $(r_t, I_t) \in \mathbb{R} \times \mathbb{I}_t$*.*

In the model, the entrepreneur with a different observable history $I_2 = \{A_1, o, \omega\}$ could offer different debt contract in period 2. In this case, we refrain from stating that contracts are separating because lender 2 views the entrepreneur with a distinct $I_2$ as an entirely different borrower. We say that equilibrium is separating only if the entrepreneur of a different type but possessing identical $I_t$ offers different contracts. Note that the entrepreneur with

[5] We assume that a lender accepts a contract that makes the lender indifferent between accepting or rejecting the contract, so that the set of acceptable offers is closed.

different intrinsic credit history $\{r_1, d\}$ could have the same credit history $\omega$ depending on the information regime and what matters is the credit history $\omega$.

As is standard in PBE models, we have multiple equilibria depending on how we construct the lender's belief for off the equilibrium path. In particular, we can have multiple pooling or multiple separating equilibria. When multiple pooling equilibria, with or without a break, exist, we choose the pooling equilibrium with the lowest $r_t$ for each period $t \in \{1, 2\}$, which we refer to as the least pooling equilibrium, with or without a break. When multiple separating equilibria exist, we pick separating equilibrium with the lowest $r_{H,t}$ for each period $t \in \{1, 2\}$, which we call the least separating equilibrium. Here, we focus on $r_{H,t}$ in separating equilibrium because $r_{L,t}$ is the same in any separating equilibrium, which will be manifested later. We call a contract in the least equilibrium of any type the least contract in what follows.

Before characterizing equilibrium, suppose that lender $t$ knows the entrepreneur's type in period $t$. For instance, the entrepreneur's type could be revealed in period 2 through different observable histories. However, we still keep the assumption that lender $t$ cannot observe the realized $A_t$ in the afternoon in period $t$, which precludes the viability of the equity contract.

Given that it is optimal for the entrepreneur to default only if he/she has no choice but to default, we obtain the lender $t$'s participation condition as

$$r_{i,t}\left(1 - \frac{r_{i,t}}{\overline{A}_t \theta_i}\right) \geq \gamma \tag{5}$$

for each $t \in \{1, 2\}$ and $i \in \{H, L\}$. Define a function $\hat{r} : [4\gamma, \infty) \to \mathbb{R}$ and a threshold value $\bar{r}_{i,t}$ as

$$\hat{r}(x) = \frac{x - \sqrt{x^2 - 4x\gamma}}{2} \tag{6}$$

$$\bar{r}_{i,t} = \frac{\overline{A}_t \theta_i + \sqrt{\overline{A}_t^2 \theta_i^2 - 4\overline{A}_t \theta_i \gamma}}{2}, \tag{7}$$

for each $t \in \{1, 2\}$ and $i \in \{H, L\}$. Then, the participation constraint (5) holds if and only if $r_{i,t} \in \left[\hat{r}(\overline{A}_t\theta_i), \bar{r}_{i,t}\right]$.

Since we are focusing on the least equilibrium, if the entrepreneur chooses to run the project in period $t$ when his/her type is revealed to lender $t$, the entrepreneur will offer $\hat{r}(\overline{A}_t\theta_i)$ in equilibrium. Note, from (6), that $\hat{r}'(x) < 0$. Thus, we have $\hat{r}(\overline{A}_t\theta_H) < \hat{r}(\overline{A}_t\theta_L)$, indicating that the H-type makes lower repayments than the L-type. This is because the L-type has a higher default risk which must be compensated by the higher repayment.

We now investigate the equilibrium outcomes when the entrepreneur's type is not revealed starting from period 2. In the model, the economy ends in period 2, so an increase in the repayment $r$ only decreases the entrepreneur's expected payoff as shown (1) and (2). Thus, the terms of contract $r$ cannot work as a signaling device in period 2, and the entrepreneur will always opt for the minimum contract from the pool of acceptable contracts for lender 2. This implies that the entrepreneur with the same observable history $I_2$ will offer the same contract because the set of acceptable contracts $\mathcal{B}_2^*(\mu_2, I_2)$ only depends on $I_2$ given the lender 2's belief system $\mu_2$. Thus, the equilibrium contract in period 2 is pooling, which is formally stated in the next lemma whose proof is omitted.

**Lemma 1** *The entrepreneur of the both types with the same observable history $I_2$ offers the same contract in period 2: $r_2(\theta_H, I_2) = r_2(\theta_L, I_2)$ for all $I_2 = (A_1, o, \omega) \in \mathbb{I}$.*

The entrepreneur's economic decisions in period 1, on the other hand, have a dual impact: They not only affect the entrepreneur's payoff in period 1 but also influence the entrepreneur's value in period 2 by shaping the observable history $I_2$. In particular, the H-type has the incentive to disclose his/her type to lender 2 by building a different history $I_2$ from that of the L-type. Consequently, the H-type entrepreneur might choose different economic decisions from those of the L-type, generating a separating equilibrium in period 1.

Specifically, a separating equilibrium can occur in two different ways. First, a separating equilibrium can exist when the H-type and the L-type offer different contracts to lender 1 (separating equilibrium without a break). Second, the H-type may make a different decision about running the project in period 1 compared to the L-type (separating equilibrium with a break). For instance, the H-type might choose not to run the project in period 1 while the L-type decides to run the project, and vice versa. Nevertheless, the following proposition shows that there is no equilibrium in which the entrepreneur's decision to run the project in period 1 varies based on the type.

**Proposition 1** *There is no equilibrium in which one type of entrepreneur runs the project in period 1 while the other type does not.*

**Proof.** See Appendix. ■

The intuitive explanation for the result of proposition 1 is as follows. First, suppose that the L-type does not run the project in period 1 while the H-type runs the project. Then, by mimicking the H-type, the L-type not only enjoys returns from the project in period 1 but also can achieve better terms of debt contract in period 2 unless his/her true type is disclosed to lender 2 through the credit history. Second, consider the equilibrium in which the H-type does not run the project in period 1. In this case, if the L-type mimics the H-type by taking a break in period 1, lender 2 will perceive the L-type as the H-type, allowing the deviating L-type to secure more favorable contract terms in period 2. Thus, it may be profitable for the L-type to take a break in period 1 as the H-type does. In the model, in particular, whenever it is optimal for the H-type to take a break in period 1, it is also optimal for the L-type to do so (see the proof of proposition 1 for the detail). Consequently, equilibrium in which only one type takes a break cannot exist.

Meanwhile, pooling equilibrium can exist either with or without a break, as illustrated below. Thus, there can exist three different types of equilibrium under each regime: 1) pooling equilibrium without a break, 2) pooling equilibrium with a break, and 3) separating equilibrium without a break. For the sake of clarity, henceforth, we will refer to the separating equilibrium without a break simply as separating equilibrium.

**Pooling equilibrium without a break** We first analyze pooling equilibrium without a break. In this case, both types of the entrepreneur offers the same contract to lender 1 and runs the project in period 1. Thus, for any equilibrium contract $r_1$, we have $\mu_1(r_1, I_1) = \sigma$ by the consistency of the lender 1's belief system. Substituting this result into (4), we obtain

$$\sigma r_1 \left(1 - \frac{r_1}{\overline{A}_1 \theta_H}\right) + (1-\sigma) r_1 \left(1 - \frac{r_1}{\overline{A}_1 \theta_L}\right) \geq \gamma, \tag{8}$$

which determines the terms of the least contract in period 1.

Although both types enter into the same contract in period 1, they can make different default decisions depending on the realized common productivity $A_1$. As a result, the H-type and L-type can have distinct credit histories depending on the information regime, which reveals the entrepreneur's true type. Building upon these arguments, the next proposition analyzes terms of contracts in pooling equilibrium without a break.

**Proposition 2** *In pooling equilibrium without a break, the terms of contract in period 1 are given as* $r_{H,1} = r_{L,1} = \hat{r}(\overline{A}_1 \theta_\sigma)$, *where* $\theta_\sigma \equiv \dfrac{\theta_H \theta_L}{(1-\sigma)\theta_H + \sigma \theta_L}$, *for all information regimes, and the terms of the contract in period 2 are given as follows:*

*1) Suppose that* $d \notin \omega$. *Then,*

$$r_{H,2} = r_{L,2} = \hat{r}(\overline{A}_2 \theta_\sigma).$$

*2) Suppose that* $d \in \omega$. *Then,*

$$r_{i,2} = \begin{cases} \hat{r}(\overline{A}_2 \theta_i) \text{ for each } i \in \{H, L\} \text{ if } A_1 \in \left[\dfrac{\hat{r}(\overline{A}_1 \theta_\sigma)}{\theta_H}, \dfrac{\hat{r}(\overline{A}_1 \theta_\sigma)}{\theta_L}\right) \\ \hat{r}(\overline{A}_2 \theta_\sigma) \text{ otherwise.} \end{cases}$$

**Proof.** See Appendix. ■

The results of proposition 2 show that the terms of the contract in period 2 in pooling equilibrium without a break depends on whether lender 2 can observe the entrepreneur's default history. Specifically, suppose that $A_1 \in \left[\dfrac{\hat{r}(\overline{A}_1 \theta_\sigma)}{\theta_H}, \dfrac{\hat{r}(\overline{A}_1 \theta_\sigma)}{\theta_L}\right)$. Then, the H-type makes repayment while the L-type defaults in period 1 given that $r_{H,1} = r_{L,1} = \hat{r}(\overline{A}_1\theta_\sigma)$. This implies that if the lender 2 can observe the entrepreneur's default history $d$ in period 2, the entrepreneur's type is revealed whenever $A_1 \in \left[\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_H}, \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_L}\right)$. Thus, the entrepreneur of type $i \in \{H, L\}$ can offer $r_{i,2} = \hat{r}(\overline{A}_2\theta_i)$ to lender 2. On the other hand, if lender 2 cannot observe the entrepreneur's default history or both types have the same default history, the entrepreneur's type is not revealed in period 2. Thus, $\mu_2(r_2, I_2) = \sigma$ for any equilibrium contract $r_2$ by consistency in equilibrium, and hence, the entrepreneur offers $\hat{r}(\overline{A}_2\theta_\sigma)$.

For pooling equilibrium without a break to exist, both types should not have an incentive to deviate. Note that the L-type always prefers not to disclose his/her true type, so does not have incentives to deviate. What matters is the H-type's incentive. In a single-period model of PBE, pooling equilibrium always exists with an appropriate belief system. However, our model with a dynamic setting requires more delicate analysis because the H-type might have incentives to deviate from the equilibrium path to reveal his/her true type to lender 2. In particular, the next proposition shows that pooling equilibrium without a break may not exist under the full information regime.

**Proposition 3** *Pooling equilibrium without a break always exists under the no-information regime, transaction history regime, and default history regime, while it exists under the full-information regime if and only if*

$$
\begin{aligned}
&u_1(\hat{r}(\overline{A}_1\theta_\sigma)|\theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_\sigma)|\theta_H) + \beta\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)\left\{\begin{array}{c} u_2(\hat{r}(\overline{A}_2\theta_H)|\theta_H) \\ -u_2(\hat{r}(\overline{A}_2\theta_\sigma)|\theta_H) \end{array}\right\} \\
\geq\, &u_1(\bar{r}_{L,1}|\theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_L)|\theta_H) + \beta\frac{\bar{r}_{L,1}}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)\left\{\begin{array}{c} u_2(\hat{r}(\overline{A}_2\theta_H)|\theta_H) \\ -u_2(\hat{r}(\overline{A}_2\theta_L)|\theta_H) \end{array}\right\}. \qquad (9)
\end{aligned}
$$

**Proof.** See Appendix. ■

In pooling equilibrium without a break, lender 2 cannot verify the true type of entrepreneur who opted for a deviating offer in period 1 through the sole observation of transaction history or default history. In this case, offering a deviating contract can only hurt the entrepreneur's payoff in period 1 if lender 1 believes that the entrepreneur with an out-of-equilibrium offer is the L-type. On the other hand, under the full information regime, lender 2 can observe the specific contract in which the entrepreneur defaulted in period 1 and the corresponding aggregate productivity. Thus, lender 2 can correctly infer the true type of the entrepreneur, and the H-type might find a profitable deviation from offering an out-of-equilibrium contract.

In particular, contract $\bar{r}_{1,L}$ in proposition 3 is the highest contract that lender 1 can accept when lender 1 believes that the entrepreneur offering $\bar{r}_{1,L}$ is of the L-type. This, in turn, gives the highest payoff to the H-type along the off-the-equilibrium path, which is given by the right-hand side of (9). Thus, as long as the constraint (9) holds, the H-type has no incentives to deviate from pooling equilibrium without a break under the full information regime.

**Pooling equilibrium with a break** Next, in pooling equilibrium with a break, the entrepreneur of both types does not run the project in period 1 and moves to period 2 with operation history $o = 0$. Thus, both types have the same observable history $I_2$, so we have $r_2(\theta_H, I_2) = r_2(\theta_L, I_2)$ by the result of lemma 1. Furthermore, for any equilibrium contract $r_2$, it must be that $\mu_2(r_2, I_2) = \sigma$ for the lender 2's belief system to be consistent. Substituting this result into (4), we obtain

$$\sigma r_2 \left(1 - \frac{r_2}{\overline{A}_2 \theta_H}\right) + (1 - \sigma) r_2 \left(1 - \frac{r_2}{\overline{A}_2 \theta_L}\right) \geq \gamma \tag{10}$$

for any equilibrium contract $r_2$ to be acceptable. Then, the terms of the contract in the least pooling equilibrium are obtained from the binding (10) by the break-even condition.

**Proposition 4** *For any information regime, the terms of the contract in period 2 in pooling* *equilibrium with a break are given as*

$$r_{H,2} = r_{L,2} = \hat{r}(\overline{A}_2\theta_\sigma).$$

**Proof.** See Appendix. ■

For pooling equilibrium with a break to exist, the entrepreneur of both types should not have an incentive to run the project in period 1. These deviating incentives can be deterred if the lenders believe that the entrepreneur is the L-type whenever he/she runs the project in period 1. Specifically, pooling equilibrium with a break exists only if both types have no incentive to run the project by offering $\hat{r}(\overline{A}_1\theta_L)$ in period 1, as stated in the next proposition.

**Proposition 5** *Pooling equilibrium with a break exists if and only if*

$$\begin{aligned}\beta u_2(\hat{r}(\overline{A}_2\theta_\sigma)|\theta_H) \geq\ & u_1(\hat{r}(\overline{A}_1\theta_L)|\theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_L)|\theta_H) \\ & + \mathbf{1}_{\{d\in\omega\}}\beta\left(\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_L} - \frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_H}\right)\left[\begin{array}{c} u_2(\hat{r}(\overline{A}_2\theta_H)|\theta_H) \\ -u_2(\hat{r}(\overline{A}_2\theta_L)|\theta_H)\end{array}\right], \end{aligned} \tag{11}$$

*where $\mathbf{1}_{\{d\in\omega\}}$ is an indicator function that takes the value of 1 if $d \in \omega$ and zero otherwise.*

**Proof.** See Appendix. ■

In proposition 5, the right hand side of (11) is the payoff of the H-type with a deviating offer $\hat{r}(\overline{A}_1\theta_L)$ in period 1, while the left-hand side is the payoff along the equilibrium path. Thus, (11) is the incentive compatibility constraint for the H-type not to run the project in period 1. In principle, the L-type also should not have a deviating incentive but the constraint (11) implies the incentive compatibility constraint for the L-type (see the proof of proposition 5 for detail). Here, we point out two features in the incentive compatibility constraint (11).

First, when $A_1 \in \left[\frac{\hat{r}(\overline{A}_1\theta_L)}{\theta_H}, \frac{\hat{r}(\overline{A}_1\theta_L)}{\theta_L}\right)$, only the H-type can make a payment of $\hat{r}(\overline{A}_1\theta_L)$ in period 1 while the L-type defaults. Thus, lender 2 can correctly deduce the entrepreneur's true type by observing the default history. Because the H-type can have better terms of contract in period 2 by disclosing his/her type, the H-type has a higher incentive to deviate in pooling equilibrium with a break when the default history is observable. Consequently, pooling equilibrium with a break is more likely to exist when the default history is unobservable.

Second, as $\overline{A}_1$ increases, the economic payoff from running the project in period 1, $u_1(\hat{r}(\overline{A}_1\theta_L)|\theta_i)$ for $i \in \{H, L\}$, increases. This, in turn, raises the entrepreneur's incentive to run the project in period 1. However, when the default history is observable, an increase in $\overline{A}_1$ reduces the probability that the entrepreneur's true type is disclosed to lender 2, which decreases the deviating incentive of the H-type. As a result, the effects of an increase in $\overline{A}_1$ on the H-type's incentive for running the project in period 1 is not clear when the default history is observable.

**Separating equilibrium** We now characterize separating equilibrium. In the model, the entrepreneur can reveal his/her type to lender 2 through the credit history unless $\omega \neq \varnothing$. In particular, in separating equilibrium, the H-type and the L-type offer distinct contracts to lender 1. Thus, lender 2 can ascertain the entrepreneur's type by observing the entrepreneur's transaction history. In contrast, the entrepreneur's default history can disclose the entrepreneur's type to lender 2 only if one type defaults while the other type does not in period 1. Consequently, the transaction history is more informative to lender 2 and is, therefore, a more effective signaling device than the default history. In particular, the next proposition shows that separating equilibrium exists only if lender 2 can observe transaction history.

**Proposition 6** *Separating equilibrium does not exist if* $r_1 \notin \omega$.

**Proof.** See Appendix. ■

The result of proposition 6 allows us to restrict our attention to the full-information regime, $\omega = \{r_1, d\}$, and transaction history regime, $\omega = \{r_1\}$, when characterizing separating equilibrium. Given that $r_1 \in \omega$, the entrepreneur's type is always revealed in period 2 because the H-type and the L-type make different contracts in period 1. Thus, it must be that $r_{i,2} = \hat{r}(\overline{A}_2\theta_i)$ for each $i \in \{H, L\}$ in period 2. Similarly, the L-type offers $r_{L,1} = \hat{r}(\overline{A}_1\theta_L)$ in period 1 because that is the least contract that the L-type can offer when his/her type is revealed to lender 1.

Next, the terms of the contract for the H-type in period 1 can be obtained by analyzing the incentive compatibility constraints of each type — the entrepreneur of each type must have no incentives to mimic the other type in period 1. In separating equilibrium, the type $i \in \{H, L\}$ entrepreneur can deceive lender 1 about his/her type by offering $r_{-i,1}$, where $-i \in \{H, L\}\backslash\{i\}$, in period 1. This deviation, in turn, also leads lender 2 to believe the entrepreneur is of type $-i$ unless the credit history discloses the true type. Hence, when deriving the incentive compatibility constraints for each type, one must also take into account how deviating offers affect the lender 2's belief.

Specifically, the incentive compatibility constraint for the L-type is given as

$$\begin{aligned} & u_1\left(\hat{r}(\overline{A}_1\theta_L)|\theta_L\right) + \beta u_2\left(\hat{r}(\overline{A}_2\theta_L)|\theta_L\right) \\ & \geq u_1\left(r_{H,1}|\theta_L\right) + \beta\left(1 - \frac{r_{H,1}}{\overline{A}_1\theta_L} + \frac{r_{H,1}}{\overline{A}_1\theta_H}\right) u_2\left(\hat{r}(\overline{A}_2\theta_H)|\theta_L\right) \\ & + \beta\left(\frac{r_{H,1}}{\overline{A}_1\theta_L} - \frac{r_{H,1}}{\overline{A}_1\theta_H}\right)\left\{\begin{array}{c} (1 - \mathbf{1}_{\{\omega=\{r_1\}\}})u_2\left(\hat{r}(\overline{A}_2\theta_L)|\theta_L\right) \\ +\mathbf{1}_{\{\omega=\{r_1\}\}}u_2\left(\hat{r}(\overline{A}_2\theta_H)|\theta_L\right) \end{array}\right\} \end{aligned} \tag{12}$$

where $\mathbf{1}_{\{\omega=\{r_1\}\}}$ is an indicator function that takes the value of 1 if $\omega = \{r_1\}$ and zero if $\omega = \{r_1, d\}$. Here, when $\overline{A}_1 \in \left(\frac{r_{H,1}}{\theta_H}, \frac{r_{H,1}}{\theta_L}\right)$, only the L-type defaults on $r_{H,1}$. Thus, under the full information regime, lender 2 can correctly infer that the entrepreneur is the L-type although he/she chose $r_{H,1}$ in period 1 if lender 2 observes that the entrepreneur defaulted on $r_{H,1}$ when $\overline{A}_1 \in \left(\frac{r_{H,1}}{\theta_H}, \frac{r_{H,1}}{\theta_L}\right)$. Consequently, the L-type offers $\hat{r}(\overline{A}_2\theta_L)$ in period 2.

**Proposition 7** *Let $r_H^*$ and $r_H^{**}$ be the values of $r_{H,1}$ that make (12) bind when $\omega = \{r_1, d\}$ and $\omega = \{r_1\}$, respectively. Then, in separating equilibrium, $(r_{H,1}, r_{L,1}) = (r_H^*, \hat{r}(\overline{A}_1\theta_L))$ if $\omega = \{r_1, d\}$ and $(r_{H,1}, r_{L,1}) = (r_H^{**}, \hat{r}(\overline{A}_1\theta_L))$ if $\omega = \{r_1\}$ . Furthermore, $r_H^{**} > r_H^* > \hat{r}(\overline{A}_1\theta_L)$.*

**Proof.** See Appendix. ■

In the model, the L-type always has an incentive to mimic the H-type whenever it is feasible to reduce the repayment. To be separated from the L-type in period 1, the H-type pays higher repayment than the L-type in period 1, i.e., $r_{H,1} > r_{L,1}$, which disincentivizes the L-type from mimicking the H-type. The H-type bears this higher repayment in period 1 because he/she can enjoy the better terms of the contract in period 2 by revealing his/her type through the terms of the contract in period 1. Note that in separating equilibrium, $r_{H,1} > \hat{r}(\overline{A}_1\theta_L) > \hat{r}(\overline{A}_1\theta_H)$. Thus, lender 1 makes positive profits from trading with the H-type although the entrepreneur has the whole bargaining power.

Proposition 7 shows that the cost that the H-type incurs to be separated from the L-type is lower under the full-information regime than under the transaction history regime, i.e., $r_H^{**} > r_H^*$. The intuition for this finding is in line with our earlier observation. Suppose that the L-type offers a deviating offer $r_{H,1}$ in period 1 to mimic the H-type. In this case, lender 2 cannot discern the entrepreneur's true type by observing the transaction history. However, if lender 2 can observe default history, then lender 2 can verify the true type when $\overline{A}_1 \in \left(\frac{r_{H,1}}{\theta_H}, \frac{r_{H,1}}{\theta_L}\right)$ because only the L-type defaults on $r_{H,1}$ in this case. Thus, the L-type has a lower incentive to mimic the H-type when $\omega = \{r_1, d\}$ than when $\omega = \{r_1\}$. Consequently, the observability of default history reduces the cost that the H-type incurs to be separated in period 1.

For this separating equilibrium to exist, the H-type also should not have incentives to mimic the L-type, i.e., offering $\hat{r}(\overline{A}_1\theta_L)$ in period 1. Specifically, the incentive compatibility constraint for the H-type is given as

$$\begin{aligned}
&u_1\left(r_{H,1}|\theta_H\right)+\beta u_2\left(\hat{r}(\overline{A}_2\theta_H)|\theta_H\right)\\
&\geq u_1\left(\hat{r}(\overline{A}_1\theta_L)|\theta_H\right)+\beta\left(1-\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_L}+\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_H}\right)u_2\left(\hat{r}(\overline{A}_2\theta_L)|\theta_H\right)\\
&+\beta\left(\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_L}-\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_H}\right)\left\{\begin{array}{c}(1-\mathbf{1}_{\{\omega=\{r_1\}\}})u_2\left(\hat{r}(\overline{A}_2\theta_H)|\theta_H\right)\\ +\mathbf{1}_{\{\omega=\{r_1\}\}}u_2\left(\hat{r}(\overline{A}_2\theta_L)|\theta_H\right)\end{array}\right\}. \qquad (13)
\end{aligned}$$

Here, when $\overline{A}_1\in\left(\frac{\hat{r}(\overline{A}_1\theta_L)}{\theta_H},\frac{\hat{r}(\overline{A}_1\theta_L)}{\theta_L}\right)$, only the H-type can make repayment on $\hat{r}(\overline{A}_1\theta_L)$. Thus, if lender 2 observes that the entrepreneur did not default on $\hat{r}(\overline{A}_1\theta_L)$ when $\overline{A}_1\in\left(\frac{r_{L,1}}{\theta_H},\frac{r_{L,1}}{\theta_L}\right)$, lender 2 can correctly infer the entrepreneur's true type although he/she offered the deviating contract $\hat{r}(\overline{A}_1\theta_L)$. Thus, the H-type can offer $\hat{r}(\overline{A}_2\theta_H)$ in period 2 in this case.

Obviously, any equilibrium offer $r_{H,1}$ must satisfy both incentive compatibility constraints (12) and (13) for separating equilibrium to exist. Furthermore, it must be that $r_{H,1}\leq\bar{r}_{1,H}$ for a contract $r_{H,1}$ to be accepted by lender 1. Finally, for separating equilibrium to exist, the entrepreneur of both types should not have incentives to take a break in period 1. However, as long as lender 2 holds the belief that the entrepreneur taking a break in period 1 is the L-type off the equilibrium path, opting for a break only results in forfeiting the expected return from running the project in period 1 without any improvement in the contract terms in period 2. Based on the above analysis, the next proposition characterizes the existence of a separating equilibrium.

**Proposition 8** *Under the full information regime or transaction history regime, separating equilibrium exists if and only if there exists $r_{H,1}\leq\bar{r}_{1,H}$ that simultaneously satisfies both (12) and (13).*

**Proof.** See Appendix. ■

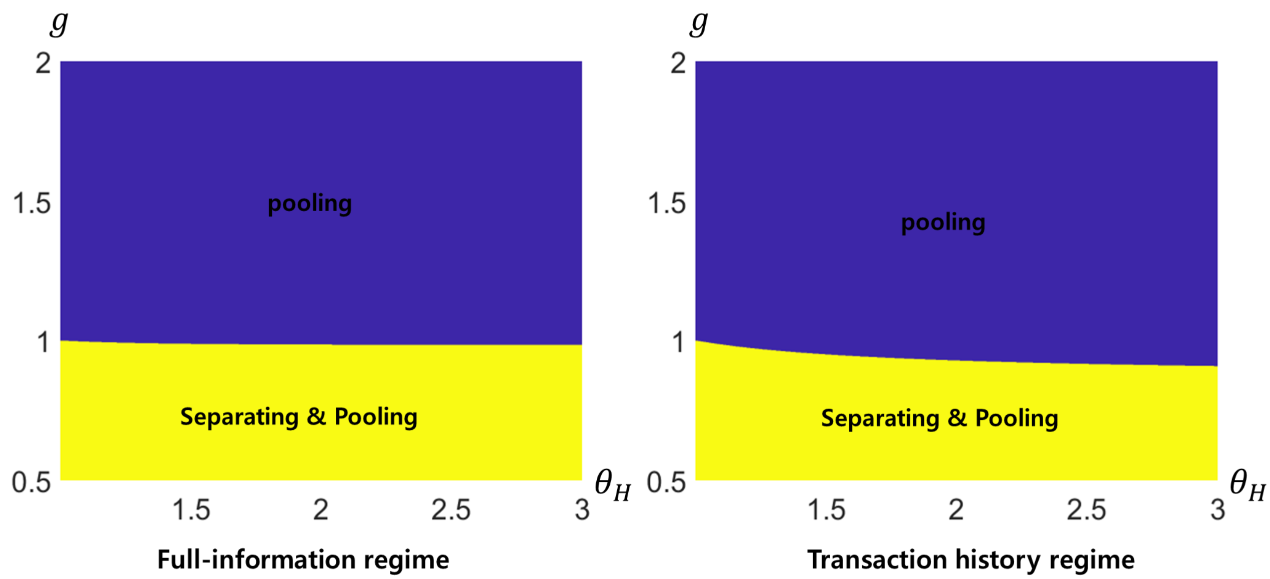


Figure 1: Typology of equilibria in $(\theta_H, g)$-space

**Numerical analysis for the existence of each equilibrium** We have examined the necessary conditions for the existence of pooling equilibrium without a break, pooling equilibrium with a break, and separating equilibrium under each information regime. To achieve a more profound understanding of how the existence of each type of equilibrium is influenced by the average growth rate of productivity, represented as $\frac{\overline{A}_2}{\overline{A}_1}$, and the disparity in entrepreneurial productivity between the H-type and the L-type, we undertake a numerical analysis. For this purpose, we set $\beta = 0.9$, $\overline{A}_1 = 8, \theta_L = 1, \gamma = 1$, and $\sigma = 0.5$. We set $\overline{A}_2 = g\overline{A}_1$, where $g \in [0.5, 2]$, and we investigate the existence of each type equilibrium under each regime for different values of $g$ and $\theta_H \in [1, 3]$.

In our numerical analysis, pooling equilibrium with a break can exist only if $g$ is sufficiently high such that $g > 10^{17}$. Given our choices for parameter values, pooling equilibrium with a break does not exist. This implies that under the no-information regime and default history regime, only pooling equilibrium without a break exists for all $g \in [0.5, 2]$ and $\theta_H \in [1, 3]$ because separating equilibrium is not feasible under those two information regimes. Thus, we focus on the full-information regime and transaction history regime, and Figure 1 illustrates the existence of each type of equilibrium under these two regimes.

Figure 1 shows that the full information regime supports separating equilibrium better than the transaction history. This is because the cost that the H-type incurs in period 1 to be separated is lower under the full information regime than under the transaction history regime as the default history disincentivizes the L-type from mimicking.

Another finding in Figure 1 is that separating equilibrium is more likely to exist when $\theta_H$ and $g$ are sufficiently low. In the model, as $\theta_H$ increases, the L-type entrepreneur's incentives to mimic the H-type rise because the payoff from mimicking increases. This, in turn, raises the cost that the H-type must incur to be separated in period 1, which makes it harder for separating equilibrium to exist. Next, an increase in $\overline{A}_2$ relaxes the incentive constraint (12) given our parameter choices. This, in turn, reduces $r_H^*$ or $r_H^{**}$ depending on the information regime. However, at the same time, an increase in $\overline{A}_2$ tightens the incentive compatibility constraint (13). In particular, an increase in $\overline{A}_2$ has a greater impact on tightening incentive compatibility constraint (13) compared to relaxing constraint (12), making it harder for separating equilibrium to exist as $\overline{A}_2$ increases.

# 5 Welfare Analysis

In this section, we explore welfare under each information regime and investigate the optimal information regime. We use the sum of utility across agents and periods in equilibrium as our welfare measure. Note that in any equilibrium under any information regime, lender 2 obtains $\gamma$ units of the expected payoff in period 2, while lender 1 can earn a payoff higher than $\gamma$ in seperating equilibrium. Thus, we use the following as our welfare measure:

$$W = \sum_{t=\{1,2\}} \beta^{t-1} \left[\sigma u_t(r_{t,H}|\theta_H) + (1-\sigma) u_t(r_{t,L}|\theta_L)\right] + \sigma \left(1 - \frac{r_{1,H}}{\overline{A}_1 \theta_H}\right) r_{1,H} + (1-\sigma) \left(1 - \frac{r_{1,L}}{\overline{A}_1 \theta_L}\right) r_{1,L}. \tag{14}$$

In what follows, we use $W_\omega$ to denote welfare under the information regime $\omega$: Specifically, $W_\varnothing$, $W_{\{r_1,d\}}$, $W_{\{d\}}$, and $W_{\{r_1\}}$ represent welfare under the no-information, full-information, default history, and transaction history regimes, respectively.

In principle, pooling equilibrium without a break and with a break can coexist under each information regime. However, we rule out pooling equilibrium with a break when we analyze welfare for the following three reasons. First, pooling equilibrium with a break rarely exists in our model as shown in the previous section: Pooling equilibrium with a break does not emerge in the numerical analysis unless $\frac{\overline{A}_2}{\overline{A}_1}$ is unreasonable high as $\frac{\overline{A}_2}{\overline{A}_1} > 10^{17}$. Second, real allocations in pooling equilibrium with a break are the same across all regimes. Thus, all regimes achieve the same level of welfare and there is no optimal regime. Third, when default history is not observable, i.e., $d \notin \omega$, welfare in pooling equilibrium without a break is higher than in pooling equilibrium with a break. This is because when $d \notin \omega$, pooling equilibrium with and without a break achieves the same real allocations in period 2, while there is an additional return in period 1 in pooling equilibrium without a break.

**Lemma 2** *In pooling equilibrium without a break, welfare is higher when $d \notin \omega$ than when $d \in \omega$.*

**Proof.** See Appendix. ■

The intuitive explanation for the result of lemma 2 is as follows. In pooling equilibrium without a break, real allocations are different only in period 2 depending on whether lender 2 can verify the entrepreneur's type. Specifically, if the entrepreneur's type is not revealed to lender 2, both types of the entrepreneur offer the pooling contract $\hat{r}(\overline{A}_2\theta_\sigma)$ in period 2, while the entrepreneur of type $i \in \{H, L\}$ offers $\hat{r}(\overline{A}_2\theta_i)$ if the true type is disclosed to lender 2. Then, due to the convexity of $\hat{r}(\cdot)$, defined in 6, liquidation cost from the pooling contract $\hat{r}(\overline{A}_t\theta_\sigma)$ is lower than that from $\hat{r}(\overline{A}_t\theta_i)$ for each $i \in$\{H,L\} on average. Consequently, in pooling equilibrium without a break, welfare is higher when $d \notin \omega$ than when $d \in \omega$ because whenever $A_1 \in \left[\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_H}, \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_L}\right)$, lender 2 can correctly infer the entrepreneur's type by observing the entrepreneur's default history.

**Lemma 3** *Welfare is higher in separating equilibrium under the full information regime* *than in separating equilibrium under the transaction history regime.*

**Proof.** See Appendix. ■

In separating equilibrium, the only difference between the full information regime and the transaction history regime in terms of real allocation is the debt contract $r_{H,1}$ for the H-type in period 1. In particular, the contract for the H-type in period 1 is higher under the transaction history regime than under the full information regime as shown in proposition 7. An increase in $r_{H,1}$ implies an increase in the default probability and hence higher expected social cost from defaults without affecting real allocations in period 2. As a result, welfare is higher under the full information regime than under the transaction history regime in separating equilibrium.

However, the result of lemma 3 does not necessarily imply that the full information regime dominates the transaction history regime. For instance, consider a scenario where welfare is higher in separating equilibrium than in pooling equilibrium without a break, and separating equilibrium can exist only under the transaction history regime. In such a situation, the transaction history regime emerges as the optimal regime. However, the next lemma shows that this specific scenario does not occur in the model.

**Lemma 4** *Welfare is higher in pooling equilibrium without a break than in separating equilibrium.*

**Proof.** See Appendix. ■

The economic mechanism behind the result of lemma 4 is similar to that of lemmas 2 and 3. Again, because of the convexity of $\hat{r}(\cdot)$, the average liquidation cost is lower when both types of the entrepreneur offer $\hat{r}(\overline{A}_t\theta_\sigma)$ than when the entrepreneur of type $i \in \{H, L\}$ offers $\hat{r}(\overline{A}_t\theta_i)$ in each period $t \in \{1, 2\}$. Moreover, in separating equilibrium, $r_{H,1}$ for the H-type in period 1 is greater than $\hat{r}(\overline{A}_1\theta_H)$, implying a higher default probability and, consequently, a greater social cost in expectation resulting from defaults.

We now explore the optimal information regime by comparing welfare across different regimes. First, the default history regime is dominated by the no-information regime by the result of lemma 2 given that only pooling equilibrium without a break can exist under these two regimes.[6] Second, the transaction history regime achieves the same level of welfare as the no-information regime in a pooling equilibrium without a break. However, under the transaction history regime, separating equilibrium that results in lower welfare compared to pooling equilibrium without a break is also feasible, while separating equilibrium cannot exist under the no-information regime. Hence, from this perspective, the no-information regime is superior to the transaction history regime. Finally, under the full-information regime, pooling equilibrium without a break and separating equilibrium can exist. In either case, welfare is lower than welfare in pooling equilibrium without a break under the no-information regime by the results of lemmas 2 and 4.

From the above analysis, we can conclude that the no-information regime is the optimal information regime. This result is re-emphasized in the next proposition whose proof is omitted.

**Proposition 9** *The no-information regime is the optimal information regime.*

# 6 Extension: Entrepreneur of the L-type to avoid

In the baseline model, we have assumed that $\min\{\overline{A}_1, \overline{A}_2\}\theta_L \geq 4\gamma$. This implies that the L-type entrepreneur has sufficiently high productivity to engage in contracts with lenders even after disclosure of his/her true type. In this section, to comprehend the impact of this assumption on the main findings and obtain further policy implications, we examine a model economy wherein the productivity of the L-type is so low that lenders would refrain from lending their investment goods to the L-type upon discovering the true type.

Specifically, we impose the following two assumptions. First, we assume that $\overline{A}_t\theta_L < 4\gamma$,

[6] Note that we do not consider pooling equilibrium with a break because it rarely exists in the model economy and welfare in this equilibrium is the same across all regimes.

which implies that $r\left(1-\dfrac{r}{\overline{A}_t\theta_L}\right)<\gamma$ for all $r>0$. Thus, lender $t$ would refrain from entering into a debt contract with the entrepreneur if lender $t$ was aware that the entrepreneur is of the L-type. However, we assume that $4\gamma \le \overline{A}_t\theta_H$ so that the H-type entrepreneur can borrow the investment good from lender $t$ once his/her true type is disclosed. Second, we assume that the entrepreneur of type $i \in \{H, L\}$ does not offer a contract $r \ge \overline{A}_t\theta_i$ as he/she would inevitably default on such a contract, resulting in zero gains, unless offering that contract $r$ provides any other benefits to the entrepreneur.[7]

Under this environment, if the entrepreneur's type is revealed to lender $t$, then only the H-type can borrow the investment good from lender $t$ and offers $\hat{r}(\overline{A}_t\theta_H)$. On the other hand, when the type is not disclosed yet, the L-type may be able to borrow the investment good by mimicking the H-type. In what follows, we focus on analyzing the equilibrium when the type has not yet been disclosed, starting from period 2.

**Proposition 10** *Assume that $\overline{A}_2\theta_L < 4\gamma \le \overline{A}_2\theta_H$ and the entrepreneur's type has not been revealed to lender 2. Then, equilibrium outcomes in period 2 are given as:*

1) *[Market collapse] If $4\gamma > \overline{A}_2\theta_\sigma$ and $\bar{r}_{2,H} < \overline{A}_2\theta_L$, the entrepreneur of both types cannot borrow the investment good.*
2) *[Pooling] If $4\gamma \le \overline{A}_2\theta_\sigma$ and $\bar{r}_{2,H} < \overline{A}_2\theta_L$, the entrepreneur of the both types offers $\hat{r}(\overline{A}_2\theta_\sigma)$.*
3) *[Pooling & Separating] If $4\gamma \le \overline{A}_2\theta_\sigma$ and $\hat{r}(\overline{A}_2\theta_\sigma) < \overline{A}_2\theta_L \le \bar{r}_{2,H}$, pooling equilibrium where the entrepreneur of both types offers $\hat{r}(\overline{A}_2\theta_\sigma)$ and separating equilibrium where only the H-type offers $\overline{A}_2\theta_L$ co-exist.*
4) *[Separating] Otherwise, only the H-type offers* $\max\left\{\hat{r}(\overline{A}_2\theta_H), \overline{A}_2\theta_L\right\}$.

**Proof.** See Appendix. ■

[7] For example, if the L-type cannot borrow the investment good from lender 2 even though his/her true type is not disclosed, then the L-type will never offer a contract $r' \ge \overline{A}_1\theta_L$ in period 1. On the other hand, if the L-type can increase the probability of borrowing the investment good in period 2 by offering $r' \ge \overline{A}_1\theta_L$ in period 1, then the L-type might have incentives to offer $r'$ in period 1.

To understand the intuitions for the results of proposition 10, it is worthwhile to note that if the H-type offers a contract $r_2' \geq \overline{A}_2\theta_L$ in period 2, the L-type has no incentive to mimic the H-type because the L-type will inevitably default on $r_2'$ and the economy ends in period 2. Thus, the H-type can effectively separate himself/herself from the L-type by offering a contract $r_2' \geq \overline{A}_2\theta_L$. However, the H-type can offer such a contract only if $\overline{A}_2\theta_L \leq \bar{r}_{2,H}$, because $\bar{r}_{2,H}$ is the highest contract that the H-type can offer once his/her type is disclosed. Thus, the equilibrium outcome critically depends on whether $\overline{A}_2\theta_L \leq \bar{r}_{2,H}$ or not.

First, suppose that $\overline{A}_2\theta_L > \bar{r}_{2,H}$, so that the H-type cannot offer a separating contract. In this case, if $4\gamma > \overline{A}\theta_\sigma$, there is no pooling contract $r_2'$ that can be accepted by lender 2 with consistent belief $\mu_2\left(r_2', I_2\right) = \sigma$. Thus, pooling equilibrium is not feasible and the market collapses. Note that if lender 2 knows the entrepreneur's type, then the H-type can borrow the investment good and run the project. The credit market collapses because of an adverse selection problem. On the other hand, if $4\gamma \leq \overline{A}_2\theta_\sigma$, lender 2 will accept the pooling contract $\hat{r}(\overline{A}_2\theta_\sigma)$ if $\mu_2\left(\hat{r}(\overline{A}_2\theta_\sigma), I_2\right) = \sigma$, which must hold in pooling equilibrium by consistency. Thus, pooling equilibrium exists. Note, from proposition 2, that $\theta_\sigma$ increases in $\sigma$. Thus, when $\overline{A}_2\theta_L > \bar{r}_{2,H}$, it is more likely that the market collapses (pooling equilibrium exists) when $\sigma$ is sufficiently low (high), which is intuitive.

Second, if $\overline{A}_2\theta_L \leq \bar{r}_{2,H}$, the H-type can offer a contract $r_2' \geq \overline{A}_2\theta_L$ that can deter the L-type from mimicking so separating equilibrium can exist. Because $\overline{A}_2\theta_L - \bar{r}_{2,H}$ decreases in $\overline{A}_2$ and $\theta_H$, separating equilibrium is more likely to exist when $\overline{A}_2$ and $\theta_H$ are sufficiently high. Note that for any contract $r_2'$ offered by the H-type to be accepted by lender 2, it must be that $r \in \left[\hat{r}(\overline{A}_2\theta_H), \bar{r}_{2,H}\right]$. Thus, the H-type will offer $r_{H,2} = \max\left\{\hat{r}(\overline{A}_2\theta_H), \overline{A}_2\theta_L\right\}$ in this equilibrium. Notice that when $4\gamma \leq \overline{A}_2\theta_\sigma$ and $\hat{r}(\overline{A}_2\theta_\sigma) < \overline{A}_2\theta_L \leq \bar{r}_{2,H}$, pooling equilibrium, where the both types offer $\hat{r}(\overline{A}_2\theta_\sigma)$, is also feasible. Thus, pooling and separating equilibria co-exist.

Equilibrium characterization in period 1 requires a more detailed analysis because the L-type may have incentives to mimic the H-type in order to obtain the investment good in period 2 even though the H-type offers $r \geq \overline{A}_1\theta_L$ in period 1. In particular, we divide the analysis depending on whether pooling equilibrium exists in period 2 when the entrepreneur's type is not disclosed.

Suppose that the market collapses or separating equilibrium exists in period 2 when the entrepreneur's type has not been disclosed to lender 2. In this case, the L-type cannot secure the investment good in period 2 by mimicking the H-type in period 1. Thus, the L-type has no incentive to mimic the H-type whenever the H-type offers a contract $r_1' \geq \overline{A}_1\theta_L$ in period 1. This implies that the equilibrium outcomes in period 1 are equivalent to the results in proposition 10 when we replace $\overline{A}_2$ and $\bar{r}_{2,H}$ with $\overline{A}_1$ and $\bar{r}_{1,H}$, respectively. This leads to the next proposition whose proof is omitted.

**Proposition 11** *Suppose that $\overline{A}_t\theta_L < 4\gamma \leq \overline{A}_t\theta_H$ for each $t \in \{1,2\}$ and the market collapses or only the H-type offers in period 2 when the type is not disclosed. Then, the equilibrium outcomes in period 1 are given as:*

1) *If $4\gamma > \overline{A}_1\theta_\sigma$ and $\overline{A}_1\theta_L > \bar{r}_{1,H}$, the market collapses.*
2) *If $4\gamma \leq \overline{A}_1\theta_\sigma$ and $\bar{r}_{1,H} < \overline{A}_1\theta_L$, the entrepreneur of the both types offers $\hat{r}(\overline{A}_2\theta_\sigma)$.*
3) *If $4\gamma \leq \overline{A}_1\theta_\sigma$ and $\hat{r}(\overline{A}_1\theta_\sigma) < \overline{A}_1\theta_L \leq \bar{r}_{1,H}$, pooling equilibrium where the entrepreneur of both types offers $\hat{r}(\overline{A}_1\theta_\sigma)$ and separating equilibrium where only the H-type offers $\overline{A}_1\theta_L$ co-exist.*
4) *Otherwise, only the H-type offers* $\max\left\{\hat{r}(\overline{A}_1\theta_H), \overline{A}_1\theta_L\right\}$.

On the other hand, if economic environments support pooling equilibrium in period 2 when the true type is not disclosed, the L-type can borrow the investment good in period 2 by concealing his/her type from lender 2. This implies that the L-type has an incentive to mimic the H-type, even if the H-type offers a contract $r' \geq \overline{A}_1\theta_L$ in period 1 to hide his/her true type from lender 2. This incentive makes separating equilibrium infeasible in period 1. Thus, either the market collapses or pooling equilibrium exists as stated in the next proposition.

**Proposition 12** *Suppose that* $\overline{A}_t\theta_L < 4\gamma \leq \overline{A}_t\theta_H$ *for each* $t \in \{1, 2\}$ *and both types offer a pooling contract in period 2 when the type is not disclosed. Then, the equilibrium outcomes in period 1 are given as:*

*1) If* $4\gamma > \overline{A}_1\theta_\sigma$*, the market collapses.*

*2) If* $4\gamma \leq \overline{A}_1\theta_\sigma$*, the entrepreneur of the both types offers* $\hat{r}(\overline{A}_1\theta_\sigma)$*.*

**Proof.** See Appendix. ■

We now investigate how the information regime affects the equilibrium outcomes. First, suppose that the market collapses in period 1. Then, the credit history contains no information, i.e., $\omega = \varnothing$, independent of the information regime. Second, if separating equilibrium exists in period 1, then, only the H-type can borrow the investment good in period 2 because the operation history reveals the entrepreneur's type to lender 2. Consequently, when the market collapses or separating equilibrium exists in period 1, the information regime is irrelevant. Finally, suppose that pooling equilibrium exists in period 1. In this case, for all $A_1 \in \left[\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_H}, \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_L}\right)$, the H-type can make repayment while the L-type defaults in period 1, so lender 2 can verify the entrepreneur's type by observing the default history. Consequently, the real allocations in period 2 depend on the information regime, specifically whether lender 2 can observe the default history.

Then, when pooling equilibrium exists in period 1, between the no-information regime and the default history regime, what is the optimal information regime?[8] When the type is not disclosed in period 2, if separating equilibrium exists, it is optimal to disclose the default history because the H-type can borrow the investment good at a weakly lower cost when the type is disclosed.[9] Next, if the market collapses, the default history regime is optimal because it allows the H-type to run the project in period 2 by disclosing the true type when $A_1 \in \left[\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_H}, \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_L}\right)$. Finally, suppose that pooling equilibrium exists in period 2 when

[8]To find the optimal information regime, we compare welfare under the no-information regime and the default history regime because the observability of transaction history does not affect equilibrium outcomes.

[9]In separating equilibrium, the H-type offers $\max\left\{\hat{r}(\overline{A}_2\theta_H), \overline{A}_2\theta_L\right\}$, while when the type is disclosed the H-type offers $\hat{r}(\overline{A}_2\theta_H)$. Thus, the average liquidation cost is weakly lower when the type is disclosed.

the type is undisclosed. In this case, when $A_1 \in \left[\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_H}, \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_L}\right)$, only the H-type can run the project in period 2 with contract $\hat{r}(\overline{A}_2\theta_H)$ under the default history, while both types run the project with a contract $\hat{r}(\overline{A}_2\theta_\sigma)$ under the no-information regime. Thus, the welfare difference between these two regimes is given as

$$\frac{W_{\{d\}} - W_\varnothing}{\beta} = \sigma\left\{u_2\left(\hat{r}(\overline{A}\theta_H)|\theta_H\right) - u_2\left(\hat{r}(\overline{A}\theta_\sigma)|\theta_H\right)\right\} - (1-\sigma)u_2\left(\hat{r}(\overline{A}\theta_\sigma)|\theta_L\right), \tag{15}$$

and it is optimal to disclose the default history if the term in (15) is positive, and vice versa.

To investigate the optimal information regime when pooling equilibrium exists in period 2, it is useful to express the necessary conditions for the existence of pooling equilibrium in terms of $\sigma$. Specifically, note that when $\overline{A}_2\theta_L < 4\gamma \leq \overline{A}_2\theta_H$, there exists unique $\sigma_1 \in (0,1]$ such that $4\gamma = \overline{A}_2\theta_\sigma$ when $\sigma = \sigma_1$. Next, it can be verified that if $\overline{A}_2\theta_L > 2\gamma$, $\hat{r}(\overline{A}_2\theta_\sigma) < \overline{A}_2\theta_L$ for all $\sigma \in [0,1]$, while if $\overline{A}_2\theta_L \leq 2\gamma$, there exists unique $\sigma_2 \in [\sigma_1, 1]$ such that $\hat{r}(\overline{A}_2\theta_\sigma) = \overline{A}_2\theta_L$ when $\sigma = \sigma_2$.[10] Now define the threshold value of $\sigma$ as

$$\sigma^* = \begin{cases} \sigma_1 & \text{if } 2\gamma < \overline{A}_2\theta_L < 4\gamma \\ \sigma_2 & \text{if } \overline{A}_2\theta_L \leq 2\gamma. \end{cases} \tag{16}$$

Then, for all $\sigma > \sigma^*$, pooling equilibrium exists in period 2.[11] Note that if $\sigma > \sigma^*$ and $\overline{A}_2\theta_L \leq \bar{r}_{2,H}$, separating equilibrium is also feasible as shown in proposition 10. However, when separating equilibrium exists in period 2, the information regime becomes irrelevant. Therefore, we assume that pooling equilibrium exists when analyzing the optimal information regime in cases where multiple equilibria exist.

Using the definition of $\sigma^*$, the following proposition analyzes the optimal information

[10]Note, from (6), that $\hat{r}(\cdot)$ is a decreasing function and $\hat{r}(4\gamma) = 2\gamma$. Hence, $4\gamma \leq \overline{A}_2\theta_\sigma$ implies $\hat{r}(\overline{A}_2\theta_\sigma) \leq 2\gamma$. This, in turn, implies $\hat{r}(\overline{A}_2\theta_\sigma) < \overline{A}_2\theta_L$ when $\overline{A}_2\theta_L > 2\gamma$. On the other hand, when $\overline{A}_2\theta_L \leq 2\gamma$, $\sigma_2 \in [0,1]$ is well defined by the condition that $\hat{r}(\overline{A}_2\theta_\sigma) = \overline{A}_2\theta_L$ with $\sigma = \sigma_2$. Furthermore, $4\gamma \leq \overline{A}_2\theta_\sigma$ holds when $\sigma = \sigma_2$ in this case. Thus, it must be that $\sigma_2 \geq \sigma_1$.

[11]Note that by construction of $\sigma^*$, for all $\sigma > \sigma^*$, $4\gamma < \overline{A}\theta_\sigma$ and $\hat{r}(\overline{A}\theta_\sigma) < \overline{A}\theta_L$. In the knife-edge case, in which $4\gamma \leq \overline{A}\theta_\sigma$ and $\hat{r}(\overline{A}\theta_\sigma) < \overline{A}\theta_L$ when $\sigma = \sigma^*$, pooling equilibrium can also exist with $\sigma = \sigma^*$.

regime when pooling equilibrium exists in both periods.

**Proposition 13** *Suppose that $\overline{A}_t\theta_L < 4\gamma \leq \overline{A}_t\theta_H$ and pooling equilibrium exists for each $t \in \{1,2\}$ when the entrepreneur's type is not disclosed. Then, there is $\theta_L^* \in \left(\frac{2\gamma}{\overline{A}_2}, \frac{4\gamma}{\overline{A}_2}\right)$ such that if $\theta_L \leq \theta_L^*$, there exist $\sigma^{**} \in (\sigma^*, 1]$ such that for all $\sigma \in (\sigma^*, \sigma^{**})$, $W_{\{d\}} > W_\varnothing$ and for all $\sigma \in [\sigma^{**}, 1]$, $W_\varnothing \geq W_{\{d\}}$.*

**Proof.** See Appendix. ■

In the extended model, the L-type's productivity $\theta_L$ is sufficiently low such that the lender's expected payoff from trading with the L-type is lower than $\gamma$. Thus, lenders do not lend the investment good to the L-type once the true type is disclosed. However, it does not necessarily imply social inefficiency in having the L-type run the project, as the L-type can still enjoy positive return from making a pooling contract with lenders. The sum of the expected payoffs for the lender and the L-type from entering into a debt contract between them can exceed $\gamma$.

However, if $\theta_L$ is sufficiently low, the L-type defaults on the pooling contract $\hat{r}(\overline{A}\theta_\sigma)$ with high probability. Hence, the L-type's expected payoff $u_2\left(\hat{r}(\overline{A}_2\theta_\sigma)|\theta_L\right)$ from making a debt contract with lender 2 becomes sufficiently low. Specifically, as $\overline{A}_2\theta_L$ converges to $\hat{r}(\overline{A}_2\theta_\sigma)$, $u_2\left(\hat{r}(\overline{A}_2\theta_\sigma)|\theta_L\right)$ goes to zero, and, hence $W_{\{d\}} > W_\varnothing$ as shown in (15). However, as $\sigma$ increases, $\hat{r}(\overline{A}_2\theta_\sigma)$ decreases, which raises $u_2\left(\hat{r}(\overline{A}_2\theta_\sigma)|\theta_L\right)$, while the gap $u_2\left(\hat{r}(\overline{A}_2\theta_H)|\theta_H\right) - u_2\left(\hat{r}(\overline{A}_2\theta_\sigma)|\theta_H\right)$ in (15) falls. Thus, if $\sigma$ is sufficiently high, then it becomes socially efficient to have the L-type run the project, and hence, the no-information regime is optimal.

We close this section with numerical analysis for equilibrium outcomes in period 2 to obtain more policy implications in the modified model. We set $\beta = 0.9$ and $\theta_L = 1$ the same as in section 5. Next, we set $\gamma = 2.8$, $\theta_H = 1.5$, $\sigma \in [0,1]$, and $\overline{A}_2 \in [7.5, 11]$, so $\overline{A}_2\theta_L < 4\gamma \leq \overline{A}_2\theta_H$ holds. Based on our parameter choices, Figure 2 illustrates the existence of each type of equilibrium in period 2 when the entrepreneur's type is not disclosed to lender 2. In the figure, pooling-1 is pooling equilibrium in which the default history regime is the

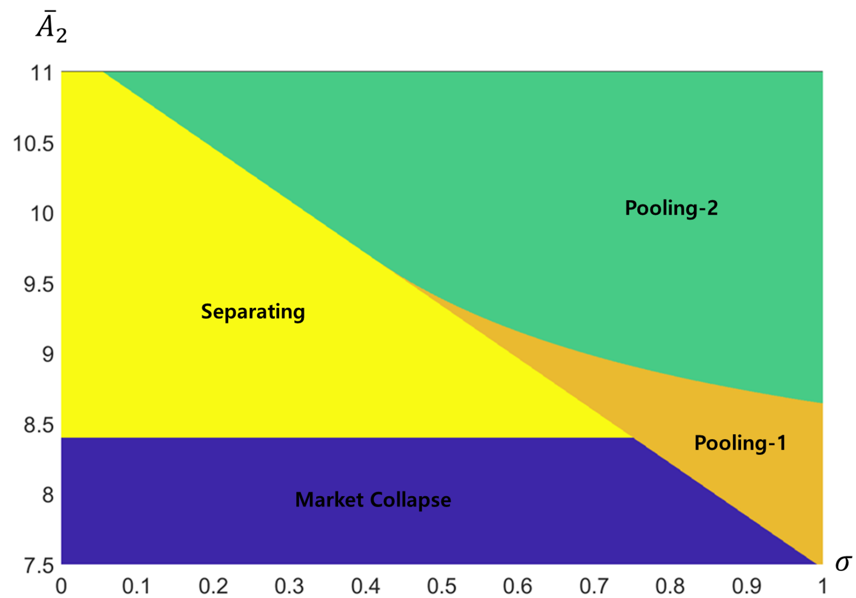


Figure 2: Typology of equilibria in period 2 in $(\sigma, \overline{A}_2)$-space when $\overline{A}_t\theta_L < 4\gamma \leq \overline{A}_t\theta_H$

optimal regime and pooling-2 is pooling equilibrium in which the no-information regime is optimal.

Figure 2 shows that the effects of $\overline{A}_2$ and $\sigma$ on the existence of each type of equilibrium are consistent with our earlier analysis.[12] Furthermore, as $\sigma$ increases, the equilibrium type switches from a separating equilibrium (or market collapse) to a pooling-1 equilibrium, and then it changes to a pooling-2 equilibrium as $\sigma$ increases further. This result is consistent with our theoretical findings in proposition 13. Another finding is that as $\overline{A}_2$ increases, the equilibrium type changes from the market collapse to pooling-1 equilibrium and then to pooling-2 equilibrium. Note that the default history regime is optimal if the market collapses or pooling-1 equilibrium exists in period 2 when the entrepreneur's type has not been disclosed. Thus, Figure 2 shows that disclosing the default history tends to be optimal when $\overline{A}_2$ is sufficiently low.

[12]When investigating the optimal information regime, we assume that pooling equilibrium exists in cases where pooling and separating equilibria co-exist. Thus, in Figure 2, pooling equilibrium exists if $4\gamma \leq \overline{A}_2\theta_\sigma$ and $\hat{r}(\overline{A}_2\theta_\sigma) < \overline{A}_2\theta_L$ as shown in proposition 10. Consequently, as $\overline{A}_2$ increases, it is more likely that pooling equilibrium exists because $\hat{r}(\overline{A}_2\theta_\sigma)$ decreases in $\overline{A}_2$.

# 7 Conclusion

In this paper, we have developed a dynamic model of debt contracts with adverse selection. An entrepreneur borrows the investment goods from lenders to finance a project. The project's return depends on both aggregate productivity and entrepreneurial productivity, the latter being the entrepreneur's private information. Lenders estimate the entrepreneur's productivity by investigating the entrepreneur's operational and credit history, which may include transaction history and/or default history, in conjunction with past aggregate productivity. We have demonstrated that if an entrepreneur with low productivity is sufficiently productive to still secure the investment good even after revealing their true productivity, it is optimal not to disclose any credit history. Conversely, if the less productive entrepreneur's productivity is significantly low, disclosing the default history can be optimal.

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# Appendix

**Proof of proposition 1.** We first show that there does not exist a separating equilibrium where only the L-type runs the project in period 1. Suppose that there exists such equilibrium, and let $r_\ell$ be the contract that the L-type offers in period 1. To bolster the credibility of this equilibrium, we introduce the assumption that any deviation from the established equilibrium path results in the worst belief regarding the entrepreneur's type unless the type is revealed. Notice that both types have different histories at the beginning of period 2.

Thus, by the restriction on the belief system and the following claim, $r_{i,2} = \hat{r}(\overline{A}_2\theta_i)$ for $i \in \{H, L\}$ as we focus on the least-contract equilibrium.

**Claim 1** *In each period $t$ and $q \in \{0, \sigma, 1\}$, letting $\theta_0 = \theta_L$ and $\theta_1 = \theta_H$, $r = \hat{r}(\overline{A}_t\theta_q)$ is the lowest $r$ that satisfies*

$$qr\left(1 - \frac{r}{\overline{A}_t\theta_H}\right) + (1-q)r\left(1 - \frac{r}{\overline{A}_t\theta_L}\right) \geq \gamma.$$

**Proof of claim 1.** By rearranging the inequality, we get

$$r\left(1 - \frac{r}{\overline{A}_t\theta_q}\right) \geq \gamma,$$

which, according to (6), finishes the proof. ■

We prove the non-existence of such separating equilibrium by showing that the incentive compatibility conditions for both types in period 1 cannot hold together, i.e., at least one type of entrepreneur has the incentive to mimic the other type. Suppose that, to strongly support this equilibrium in this regime, lender 2 who observes the entrepreneur's history indicating that he/she ran the project in period 1 believes that he/she is the L-type for sure unless the type is revealed. If the L-type mimics the H-type, i.e., does not run the project in period 1, his/her history at the beginning of period 2 becomes the same as that of the H-type in both regimes, because this entrepreneur mimics both the operation history and the intrinsic credit history of the H-type. Thus, this entrepreneur offers $\hat{r}(\overline{A}_2\theta_H)$ in period 2. The incentive compatibility condition for the L-type not to mimic the H-type is:

$$IC_L : u_1(r_\ell \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L) \geq 0 + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L).$$

In this equilibrium, the H-type does not run the project in period 1 and offers $\hat{r}(\overline{A}_2\theta_H)$ in period 2. Consider that the H-type decides to mimic the L-type to offer $r_\ell$ in period 1. If the default history is not observable, then the credit history of this H-type at the beginning of period 2 becomes the same as that of the L-type. Now consider that the default history is observable. In the default history regime, the type is revealed if the credit history indicates that the entrepreneur operated the business and $A_1 \in \left[\frac{r_\ell}{\theta_H}, \frac{r_\ell}{\theta_L}\right)$. In the full-information regime, the type is revealed if $r_1 = r_\ell$ and $A_1 \in \left[\frac{r_\ell}{\theta_H}, \frac{r_\ell}{\theta_L}\right)$. That is, given that $d \in \omega$ and the H-type mimics the L-type, if $A_1 \in \left[\frac{r_\ell}{\theta_H}, \frac{r_\ell}{\theta_L}\right)$, then the default history for this entrepreneur becomes different from that of the L-type. According to the restriction on the belief system in period 2, this entrepreneur's type is revealed in period 2 as $\theta_H$ by the realization of $A_1$, thus offers $\hat{r}(\overline{A}_2\theta_H)$, according to claim 1. If $A_1 \notin \left[\frac{r_\ell}{\theta_H}, \frac{r_\ell}{\theta_L}\right)$, then the default history for this entrepreneur coincides with that of the L-type, and both the transaction history and the operation history also coincide with that of the L-type, thus this H-type offers $\hat{r}(\overline{A}_2\theta_L)$ in period 2. The incentive compatibility condition for the H-type not to mimic the L-type is:

$$
\begin{aligned}
IC_H : 0 + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) \geq{}& u_1(r_\ell \mid \theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H) \\
&+ \mathbf{1}_{d\in\omega} \cdot \beta \left(\frac{r_\ell}{\overline{A}_1\theta_L} - \frac{r_\ell}{\overline{A}_1\theta_H}\right) \cdot \left(u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H)\right),
\end{aligned}
$$

where $\mathbf{1}_{d\in\omega}$ is an indicator function that is equal to one if the default history is observable, and zero otherwise.

Suppose that $IC_H$ holds. To show that $IC_L$ cannot be satisfied, we first introduce the following claim.

**Claim 2** *For any $\hat{\theta} > \theta_L$,*

$$
\left(1 - \frac{\hat{r}(\overline{A}_2\hat{\theta})}{\overline{A}_2\theta_L}\right)^2 - \left(1 - \frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta_L}\right)^2 > \left(1 - \frac{\hat{r}(\overline{A}_2\hat{\theta})}{\overline{A}_2\theta_H}\right)^2 - \left(1 - \frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta_H}\right)^2.
$$

**Proof or claim 2.** First of all, $\hat{r}(\overline{A}_2\theta_L) > \hat{r}(\overline{A}_2\hat{\theta})$ because $\hat{\theta}(q) > \theta_L$. It suffices to show that $\left(1 - \frac{\hat{r}(\overline{A}_2\hat{\theta})}{\overline{A}_2\theta}\right)^2 - \left(1 - \frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta}\right)^2$ decreases in $\theta$. As we take the derivative of this term with respect to $\theta$, noticing that

$$\begin{aligned}
&\left(1-\frac{\hat{r}(\overline{A}_2\hat{\theta})}{\overline{A}_2\theta}\right)^2-\left(1-\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta}\right)^2\\
&=\left(\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2}-\frac{\hat{r}(\overline{A}_2\hat{\theta})}{\overline{A}_2}\right)\cdot\left[\frac{2}{\theta}-\left(\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2}+\frac{\hat{r}(\overline{A}_2\hat{\theta})}{\overline{A}_2}\right)\frac{1}{\theta^2}\right],
\end{aligned}$$

we get

$$\begin{aligned}
&\frac{\partial}{\partial\theta}\left[\left(1-\frac{\hat{r}(\overline{A}_2\hat{\theta})}{\overline{A}_2\theta}\right)^2-\left(1-\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta}\right)^2\right]\\
&=\left(\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2}-\frac{\hat{r}(\overline{A}_2\hat{\theta})}{\overline{A}_2}\right)\cdot\left[-\frac{2}{\theta^2}+2\left(\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2}+\frac{\hat{r}(\overline{A}_2\hat{\theta})}{\overline{A}_2}\right)\frac{1}{\theta^3}\right],
\end{aligned}$$

which is negative if and only if $\theta > \frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2}+\frac{\hat{r}(\overline{A}_2\hat{\theta})}{\overline{A}_2}$ for $\theta\in[\theta_L,\theta_H]$. We are done if we show that $\theta_L > \frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2}+\frac{\hat{r}(\overline{A}_2\hat{\theta})}{\overline{A}_2}$ for $\theta\in[\theta_L,\theta_H]$. Based on the definition of $\hat{r}(\cdot)$, we have $\overline{A}_2\theta_L \geq 2\hat{r}(\overline{A}_2\theta_L)$, and from the fact that $\hat{r}(\overline{A}_2\theta_L) > \hat{r}(\overline{A}_2\hat{\theta})$, we have $\overline{A}_2\theta_L > 2\hat{r}(\overline{A}_2\hat{\theta})$. Thus,

$$\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2}+\frac{\hat{r}(\overline{A}_2\hat{\theta})}{\overline{A}_2} < \frac{\theta_L}{2}+\frac{\theta_L}{2}=\theta_L,$$

which completes the proof of the claim. ■

Now, suppose that $IC_L$ holds along with and $IC_H$ hold and rewrite both inequalities as follows:

$$\begin{aligned}
IC_L &: \beta\overline{A}_2\left[\left(1-\frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_L}\right)^2-\left(1-\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta_L}\right)^2\right]\leq\overline{A}_1\left(1-\frac{r_\ell}{\overline{A}_1\theta_L}\right)^2,\\
IC_H &: \beta\overline{A}_2\left[\left(1-\frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_H}\right)^2-\left(1-\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta_H}\right)^2\right]\geq\overline{A}_1\left(1-\frac{r_\ell}{\overline{A}_1\theta_H}\right)^2\\
&\quad+\mathbf{1}_{d\in\omega}\cdot\beta\left(\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_L}-\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_H}\right)\cdot\overline{A}_2\left[\left(1-\frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_H}\right)^2-\left(1-\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta_H}\right)^2\right],
\end{aligned}$$

Note that

$$\overline{A}_1\left(1-\frac{r_\ell}{\overline{A}_1\theta_H}\right)^2+\mathbf{1}_{d\in\omega}\cdot\beta\left(\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_L}-\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_H}\right)\cdot\overline{A}_2\left[\left(1-\frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_H}\right)^2-\left(1-\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta_H}\right)^2\right]$$
$$>\overline{A}_1\left(1-\frac{r_\ell}{\overline{A}_1\theta_L}\right)^2.$$

However, claim 2 indicates that

$$\beta\overline{A}_2\left[\left(1-\frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_L}\right)^2-\left(1-\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta_L}\right)^2\right]>\beta\overline{A}_2\left[\left(1-\frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_H}\right)^2-\left(1-\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta_H}\right)^2\right],$$

which leads to a contradiction.

Now we show that there does not exist a separating equilibrium where only the H-type runs the project in period 1 by proving that the L-type will mimic the H-type. Suppose that there exists such equilibrium, and let $r_h$ be the contract that the H-type offers in period 1. We introduce the assumption again that any deviation from the established equilibrium path results in the worst belief regarding the entrepreneur's type unless that type is revealed. Also, note that the entrepreneur's behavior in period 2 is deterministic, in line with the findings above.

If the L-type deviates by offering $r_h$, he/she will be treated as the H-type in period 2, unless his/her type is revealed. So the condition for the L-type not to mimic the L-type entrepreneur in period 1, i.e., deviate by offering $r_h$ in period 1, is:

$$\begin{aligned}IC_L:\ &0+\beta u_2(\hat{r}(\overline{A}_2\theta_L)\mid\theta_L)\\ \geq\ &u_1(r_h\mid\theta_L)+\beta u_2(\hat{r}(\overline{A}_2\theta_H)\mid\theta_L)\\ -&\mathbf{1}_{d\in\omega}\cdot\beta\left(\frac{r_h}{\overline{A}_1\theta_L}-\frac{r_h}{\overline{A}_1\theta_H}\right)(u_2(\hat{r}(\overline{A}_2\theta_H)\mid\theta_L)-u_2(\hat{r}(\overline{A}_2\theta_L)\mid\theta_L)).\end{aligned}$$

However, if cannot hold because $u_2(\hat{r}(\overline{A}_2\theta_L)\mid\theta_L)<u_2(\hat{r}(\overline{A}_2\theta_H)\mid\theta_L)$. ■

**Proof of proposition 2.** We first argue that $r_{H,1}=r_{L,1}=\hat{r}(\overline{A}_1\theta_\sigma)$. It is trivial that

$r_{H,1} = r_{L,1}$, by the definition of pooling equilibrium without a break. Since we restrict our focus on the least-contract equilibrium and by claim 1 in the proof of proposition 1, we have $r_{H,1} = r_{L,1} = \hat{r}(\overline{A}_1\theta_\sigma)$. The claim also indicates that the equilibrium contract in period 2 is $\hat{r}(\overline{A}_2\theta_\sigma)$ unless the type is revealed.

To study the equilibrium contract offers in period 2, first suppose that $d \in \omega$. By the same logic in the previous paragraph, given that both types of entrepreneurs offer $\hat{r}(\overline{A}_1\theta_\sigma)$ in period 1, both types do not default if $A_1 \geq \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_L}$ and both types default if $A_1 < \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_H}$. On the other hand, if $A_1 \in \left[\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_H}, \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_L}\right)$, then only the H-type does not default. Thus, even if the transaction history and the operation history are the same in period 1 for both types, the default history differentiates the type when $A_1 \in \left[\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_H}, \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_L}\right)$. Thus, the H-type offers $\hat{r}(\overline{A}_2\theta_H)$ and the L-type offers $\hat{r}(\overline{A}_2\theta_H)$ in period 2 if $A_1 \in \left[\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_H}, \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\theta_L}\right)$, while both types offer $\hat{r}(\overline{A}_2\theta_\sigma)$ otherwise.

Next, suppose that $d \notin \omega$. Then, because we consider a pooling equilibrium without a break, the entrepreneur's type is not revealed by the credit history. Thus, both types of the entrepreneurs offer $\hat{r}(\overline{A}_2\theta_\sigma)$ in period 2. ■

**Proof of proposition 3.** To provide robust support for the existence of this equilibrium, we introduce an assumption: any deviation from the equilibrium trajectory leads to the worst belief about the entrepreneur's type, that the entrepreneur is undoubtedly the L-type unless the type is revealed. Because the period-2 behavior of the entrepreneur is deterministic, according to claim 1 in the proof of proposition 1, it suffices to check whether each type of entrepreneur has no incentive to deviate from the equilibrium strategy in period 1. Notice that any deviation in period 1 by the entrepreneur will result in being regarded as the L-type. Therefore, any contract offer $r \neq \hat{r}(\overline{A}_1\theta_\sigma)$ will be accepted only if $r \in [\hat{r}(\overline{A}_1\theta_L), \bar{r}_L]$.

We first show that the L-type has no incentive to deviate from the equilibrium strategy. If the L-type deviates by either offering a contract $r \notin [\hat{r}(\overline{A}_1\theta_L), \bar{r}_L] \cup \{\hat{r}(\overline{A}_1\theta_\sigma)\}$ or not offering a contract in period 1, then such contract offer will be rejected in period 1, also resulting in offering $\hat{r}(\overline{A}_2\theta_L)$ in period 2. It is not beneficial for the L-type to deviate by either not offering a contract or offering a contract to be rejected, because

$$
\begin{aligned}
&u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_L) + \beta P(\hat{r}(\overline{A}_1\theta_\sigma))u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)) + \beta(1 - P(\hat{r}(\overline{A}_1\theta_\sigma)))u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L)) \\
&> 0 + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)),
\end{aligned}
$$

where $P(r)$ is the probability that the type is revealed in period 2 after the contract $r$ is accepted in period 1. Therefore, any deviation in period 1 must be offering a contract in $[\hat{r}\overline{A}_1\theta_L), \bar{r}_L]$, which will be accepted by lender 1. If $d \notin \omega$, then $P(\cdot) = 0$. If $\omega = \{d\}$, then the contract offer in period 1 is not observed, thus, $P(\cdot) = \dfrac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_L} - \dfrac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_H}$. If $\omega = \{r_1, d\}$, then $P(r) = \dfrac{r}{\overline{A}_1\theta_L} - \dfrac{r}{\overline{A}_1\theta_L}$, which increases in $r$. Therefore $P(r) \geq P(\hat{r}(\overline{A}_1\theta_\sigma))$ holds in every information regime. If $r_1 \in \omega$, then a deviation in period 1 by offering a contract in $[\hat{r}(\overline{A}_1\theta_L), \bar{r}_L]$ will make the entrepreneur be regarded to be the L-type in period 2, and such deviation is not beneficial because

$$
\begin{aligned}
&u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_L) + \beta P(\hat{r}(\overline{A}_1\theta_\sigma))u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)) + \beta(1 - P(\hat{r}(\overline{A}_1\theta_\sigma)))u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L)) \\
&> u_1(r \mid \theta_L) + u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)).
\end{aligned}
$$

Further, when $r_1 \notin \omega$, deviating by offering a contract $r \in [\hat{r}(\overline{A}_1\theta_L), \bar{r}_L]$ is also not beneficial because

$$
\begin{aligned}
&u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_L) + \beta P(\hat{r}(\overline{A}_1\theta_\sigma))u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)) + \beta(1 - P(\hat{r}(\overline{A}_1\theta_\sigma)))u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L)) \\
&> u_1(r \mid \theta_L) + \beta P(r)u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)) + \beta(1 - P(r))u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L)),
\end{aligned}
$$

and because $P(r) \geq P(\hat{r}(\overline{A}_1\theta_\sigma))$.

To study the H-type's incentive compatibility condition, we first argue that the H-type would not deviate not to running a project or make a contract offer not to be accepted in period 1, because

$$u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_H) + \beta P(\hat{r}(\overline{A}_1\theta_\sigma))u_1(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H)) + \beta(1 - P(\hat{r}(\overline{A}_1\theta_\sigma)))u_1(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H))$$
$$> 0 + \beta u_1(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)).$$

Next, we argue that the H-type would not deviate by offering $r \in [\hat{r}(\overline{A}_1\theta_L), \bar{r}_L]$ in period 1 when $\omega \notin \{r_1, d\}$. The H-type's expected equilibrium utility is

$$u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_H) + \beta P(\hat{r}(\overline{A}_1\theta_\sigma))u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H)) + \beta(1 - P(\hat{r}(\overline{A}_1\theta_\sigma)))u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H)).$$

If $d \notin \omega$, then a deviation in period 1 by offering a contract in $[\hat{r}(\overline{A}_1\theta_L), \bar{r}_L]$ will make the probability that the entrepreneur be regarded to be the H-type in period 2 0 if $\omega = \{r_1\}$ and $P(\hat{r}(\overline{A}_1\theta_\sigma))$ if $\omega = \varnothing$, and such deviation is not beneficial because

$$u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_H) + \beta P(\hat{r}(\overline{A}_1\theta_\sigma))u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H)) + \beta(1 - P(\hat{r}(\overline{A}_1\theta_\sigma)))u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H))$$
$$> u_1(r \mid \theta_L) + \beta P(\hat{r}(\overline{A}_1\theta_\sigma))u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H)) + \beta(1 - P(\hat{r}(\overline{A}_1\theta_\sigma)))u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H)).$$

Now consider that $\omega = \{d\}$. Deviating by offering a contract $r \in [\hat{r}(\overline{A}_1\theta_L), \bar{r}_L]$ gives the H-type the expected utility of

$$u_1(r \mid \theta_H) + \beta P(r)u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H)) + \beta(1 - P(r))u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H)).$$

Note that for all $r \in [\hat{r}(\overline{A}_1\theta_L), \bar{r}_L]$, $P(\hat{r}(\overline{A}_1\theta_\sigma)) = P(r)$ given that $r_1 \notin \omega$, and $u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_H) > u_1(r \mid \theta_H)$. Therefore, the H-type would not deviate by offering $r \in [\hat{r}(\overline{A}_1\theta_L), \bar{r}_L]$ in period 1 if $\omega = \{d\}$.

Finally, we provide the existence condition for the pooling equilibrium without a break when $\omega = \{r_1, d\}$. The condition coincides with the condition that the H-type does not have an incentive to deviate from the equilibrium strategy. Because $P(\hat{r}(\overline{A}_1\theta_\sigma)) = \dfrac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1\theta_L} -$

$\dfrac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1\theta_H}$, the H-type's equilibrium utility is

$$u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H) + \beta\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)(u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H)),$$

and the expected utility of the H-type who deviates by offer $r \in \left(\hat{r}(\overline{A}_1\theta_L), \bar{r}_L\right]$ is

$$f(r) \equiv u_1(r \mid \theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H) + \beta\frac{r}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)(u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H)).$$

It is easy to verify that $f''(r) = \dfrac{1}{\overline{A}_1\theta_H} > 0$, that is, $f$ is convex. Thus, it suffices to check whether the H-type has the incentive to deviate to offer $\bar{r}_L$ in period 1. That is, the H-type does not have the incentive to deviate to offering $\overline{A}_1\theta_L$ in period 1 if and only if

$$\begin{aligned} & u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H) \\ +\beta & \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1}\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right)(u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H)) \\ & \geq f(\bar{r}_L). \end{aligned}$$

■

**Proof of proposition 4.** As we consider pooling equilibrium with a break, both types do not run a business in period 1, which means that both types proceed to period 2 with the same history. Because there is no way for lender 2 to differentiate the entrepreneur's type by the history in period 2, $\mu_2\left(r_{H,2}, I_2\right) = \mu_2\left(r_{L,2}, I_2\right) = \sigma$ holds in equilibrium. Moreover, because $u_2(r \mid \theta)$ decreases in $r$ and according to (2), both types offer $\min \mathcal{B}_2^*(\mu_2, I_2)$, i.e., $r_{H,2} = r_{L,2} = \min \mathcal{B}_2^*(\mu_2, I_2) \equiv r_2$. That is, both types offer the lowest $r_2$ such that $\sigma r_2\left(1 - \dfrac{r_2}{\overline{A}_2\theta_H}\right) + (1-\sigma)r_2\left(1 - \dfrac{r_2}{\overline{A}_2\theta_L}\right) \geq \gamma$.

Notice that we restrict our focus to the least-contract equilibrium. Thus, according to claim 1 in the proof of proposition 1, the lowest $r_2$ is $\hat{r}(\overline{A}_2\theta_\sigma)$, which finishes the proof. ■

**Proof of proposition 5.** In an equilibrium in which both types of entrepreneur do not offer in period 1, both types offer $\hat{r}(\overline{A}_2\theta_\sigma)$ in period 2 in any regime, according to proposition 4. We need to check whether each type has an incentive to deviate by running the project in period 1. To strongly support this pooling equilibrium with a break, we consider that lender 1 believes in period 1 that the entrepreneur is the L-type for sure whenever he/she offers a contract in period 1, i.e., observes that $o = 1$. Then a contract offers $r$ will be accepted by lender 1 in period 1 if and only if $r \in [\hat{r}(\overline{A}_1\theta_L), \bar{r}_L]$. Also, lender 2 believes in period 2 that the entrepreneur is the L-type for sure if an entrepreneur runs a business in period 1 and either *i)* the default history is not observable or *ii)* the default history is observable and $A_1 \notin \left[\frac{\hat{r}(\overline{A}_1\theta_L)}{\theta_L}, \frac{\hat{r}(\overline{A}_1\theta_L)}{\theta_H}\right)$[13].

It is trivial that given such a belief system, deviating by offering a contract above $\hat{r}(\overline{A}_1\theta_L)$ in period 1 is dominated by deviating by offering $\hat{r}(\overline{A}_1\theta_L)$ in period 1. Therefore, we only need to check whether each type has an incentive to deviate by offering $\hat{r}(\overline{A}_1\theta_L)$ in period 1. If $d \notin \omega$, the $i$-type for each $i = H, L$ who offers $\hat{r}(\overline{A}_1\theta_L)$ defaults in period 1 if $A_1 < \frac{\hat{r}(\overline{A}_1\theta_L)}{\theta_i}$, i.e., with probability $\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_i}$, and then offers $\hat{r}(\overline{A}_2\theta_L)$ in period 2. Now consider that $d \in \omega$. Then, the L-type who offers $\hat{r}(\overline{A}_1\theta_L)$ in period 1 defaults if $A_1 < \frac{\hat{r}(\overline{A}_1\theta_L)}{\theta_L}$ and then offers $\hat{r}(\overline{A}_2\theta_L)$ in period 2. Whereas, the H-type who offers $\hat{r}(\overline{A}_1\theta_L)$ in period 1 *i)* defaults if $A_1 < \frac{\hat{r}(\overline{A}_1\theta_L)}{\theta_H}$ and then offers $\hat{r}(\overline{A}_2\theta_L)$ in period 2, *ii)* does not default and the type is revealed when $A_1 \in \left[\frac{\hat{r}(\overline{A}_1\theta_L)}{\theta_L}, \frac{\hat{r}(\overline{A}_1\theta_L)}{\theta_H}\right)$ so that offers $\hat{r}(\overline{A}_2\theta_L)$ in period 2, and *iii)* does not default and the type is not revealed otherwise so that offers $\hat{r}(\overline{A}_2\theta_L)$ in period 2. Thus, the incentive compatibility constraints not to deviate from not running the business in period

[13] Under such $A_1$, either both types default or both types do not default in period 1.

1 for each type are as follow:

$$IC_L: 0+\beta u_2(\hat{r}(\overline{A}_2\theta_\sigma)\mid\theta_L)\geq u_1(\hat{r}(\overline{A}_1\theta_L)\mid\theta_L)+\beta u_2(\hat{r}(\overline{A}_2\theta_L)\mid\theta_L),$$
$$IC_H: 0+\beta u_2(\hat{r}(\overline{A}_2\theta_\sigma)\mid\theta_H)\geq u_1(\hat{r}(\overline{A}_1\theta_L)\mid\theta_H)+\beta u_2(\hat{r}(\overline{A}_2\theta_L)\mid\theta_H)$$
$$+\mathbf{1}_{d\in\omega}\cdot\beta\left(\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_L}-\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_H}\right)\cdot\left(u_2(\hat{r}(\overline{A}_2\theta_H)\mid\theta_H)-u_2(\hat{r}(\overline{A}_2\theta_L)\mid\theta_H)\right).$$

For the rest of the proof, we show that if $IC_H$ holds, then $IC_L$ also holds, thus, the only necessary condition for the existence of this equilibrium is $IC_H$. We first rewrite the two inequalities as follows:

$$IC_L: \beta\overline{A}_2\left[\left(1-\frac{\hat{r}(\overline{A}_2\theta_\sigma)}{\overline{A}_2\theta_L}\right)^2-\left(1-\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta_L}\right)^2\right]\geq\overline{A}_1\left(1-\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_L}\right)^2,$$
$$IC_H: \beta\overline{A}_2\left[\left(1-\frac{\hat{r}(\overline{A}_2\theta_\sigma)}{\overline{A}_2\theta_H}\right)^2-\left(1-\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta_H}\right)^2\right]\geq\overline{A}_1\left(1-\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_H}\right)^2$$
$$+\mathbf{1}_{d\in\omega}\cdot\beta\left(\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_L}-\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_H}\right)\cdot\overline{A}_2\left[\left(1-\frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_H}\right)^2-\left(1-\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta_H}\right)^2\right],$$

Because

$$\overline{A}_1\left(1-\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_H}\right)^2+\mathbf{1}_{d\in\omega}\cdot\beta\left(\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_L}-\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_H}\right)\cdot\overline{A}_2\left[\left(1-\frac{\hat{r}(\overline{A}_2\theta_H)}{\overline{A}_2\theta_H}\right)^2-\left(1-\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta_H}\right)^2\right]$$
$$>\overline{A}_1\left(1-\frac{\hat{r}(\overline{A}_1\theta_L)}{\overline{A}_1\theta_L}\right)^2,$$

we are done if we show that $\left(1-\frac{\hat{r}(\overline{A}_2\theta_\sigma)}{\overline{A}_2\theta}\right)^2-\left(1-\frac{\hat{r}(\overline{A}_2\theta_L)}{\overline{A}_2\theta}\right)^2$ decreases in $\theta$, which is indicated by claim 2 in the proof of proposition 1. ■

**Proof of proposition 6.** Assume that $r_1\notin\omega$ and a separating equilibrium without a break exists. Let $r_h$ and $r_\ell$ be the contract offers that the H-type and the L-type make in period 1, respectively. Note that $r_h\neq r_\ell$ must hold, i.e., either $r_\ell>r_h$ or $r_\ell<r_h$.

First, we argue that $r_\ell>r_h$ cannot hold. Specifically, we show that the L-type has an incentive to offer $r_h$ instead of $r_\ell$ if $r_\ell > r_h$. The L-type in equilibrium offers $r_\ell$ in period 1, and offers $\hat{r}(\overline{A}_2\theta_\sigma)$ if the type is not revealed and offers $\hat{r}(\overline{A}_2\theta_L)$ otherwise in period 2. Note that the type is revealed in period 2 only if $d \in \omega$ and $A_1 \in \left[\frac{r_h}{\theta_H}, \frac{r_\ell}{\theta_L}\right)$. Thus, the expected equilibrium utility for the L-type is

$$U_L^*(r_\ell) \equiv u_1(r_\ell \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L) - \mathbf{1}_{d\in\omega} \cdot \left(\frac{r_\ell}{\overline{A}_1\theta_L} - \frac{r_h}{\overline{A}_1\theta_H}\right)(u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)).$$

Consider that the L-type mimics the H-type in period 1, i.e., offers $r_h$ in period 1. Then this entrepreneur defaults when $A_1 < \frac{r_h}{\theta_L}$. Notice that if $A_1 \in \left[\frac{r_h}{\theta_L}, \frac{r_\ell}{\theta_L}\right)$ and $d \in \omega$, then this entrepreneur does not default in period 1, thus lender 2 regards this entrepreneur as the H-type, because $A_1 \in \left[\frac{r_h}{\theta_L}, \frac{r_\ell}{\theta_L}\right)$ implies $A_1 \in \left[\frac{r_h}{\theta_H}, \frac{r_\ell}{\theta_L}\right)$. Also notice that if $A_1 \in \left[\frac{r_h}{\theta_H}, \frac{r_h}{\theta_L}\right)$ and $d \in \omega$, then this entrepreneur does not default in period 1, thus lender 2 regards this entrepreneur as the L-type. Thus, in period 2, the L-type who mimics the H-type in period 1 will offer $\hat{r}(\overline{A}_2\theta_H)$ if $A_1 \in \left[\frac{r_h}{\theta_L}, \frac{r_\ell}{\theta_L}\right)$, offer $\hat{r}(\overline{A}_2\theta_L)$ if $A_1 \in \left[\frac{r_h}{\theta_H}, \frac{r_h}{\theta_L}\right)$, and offer $\hat{r}(\overline{A}_2\theta_\sigma)$ otherwise. Thus, the expected payoff for this entrepreneur, when he/she mimics the H-type, is

$$\begin{aligned} U_L(r_h) \equiv\ & u_1(r_h \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L) \\ & + \mathbf{1}_{d\in\omega} \cdot \left(\frac{r_\ell}{\overline{A}_1\theta_L} - \frac{r_h}{\overline{A}_1\theta_L}\right)(u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L) - u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L)) \\ & - \mathbf{1}_{d\in\omega} \cdot \left(\frac{r_h}{\overline{A}_1\theta_L} - \frac{r_h}{\overline{A}_1\theta_H}\right)(u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)). \end{aligned}$$

We show from the following expression that the L-type has an incentive to mimic the H-type in period 1 if $r_\ell > r_h$:

$$\begin{aligned}
U_L(r_h) - U_L^*(r_\ell) &= u_1(r_h \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L) \\
&\quad - u_1(r_\ell \mid \theta_L) - \beta u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L) \\
&\quad + \mathbf{1}_{d\in\omega} \cdot \left(\frac{r_\ell}{\overline{A}_1\theta_L} - \frac{r_h}{\overline{A}_1\theta_L}\right)(u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L) - u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L)) \\
&\quad + \mathbf{1}_{d\in\omega} \cdot \left(\frac{r_\ell}{\overline{A}_1\theta_L} - \frac{r_h}{\overline{A}_1\theta_L}\right)(u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)) \\
&\geq u_1(r_h \mid \theta_L) - u_1(r_\ell \mid \theta_L) \\
&> 0.
\end{aligned}$$

Finally, suppose that $r_\ell < r_h$. Consider that the H-type deviates by offering $r_\ell$ in period 1 and strategically defaults whenever $A_1 < \dfrac{r_h}{\theta_H}$, i.e., as if he/she offered $r_h$. Then such deviation makes his/her period-1 utility increase, while his/her period-2 utility is unchanged from playing the equilibrium strategy. That is, offering $r_h$ in period 1 is not optimal. ■

**Proof of proposition 7.** We begin by showing that $r_{L,1} = \hat{r}(\overline{A}_1\theta_L)$. First, based on the lender 1's rationality, it is not possible for $r_{L,1}$ to be less than $\hat{r}(\overline{A}_1\theta_L)$. Additionally, considering that $\hat{r}(\overline{A}_1\theta_L)$ would be accepted regardless of lender 1's belief, there is no rationale for the L-type to offer above $\hat{r}(\overline{A}_1\theta_L)$ in period 1. Thus, $r_{L,1} = \hat{r}(\overline{A}_1\theta_L)$ holds.

Next, we show that $r_{H,1} > \hat{r}(\overline{A}_1\theta_L)$ and $r_{H,1} = r_H^*$ if $d \in \omega$ and $r_{H_1} = r_H^{**}$ if $d \notin \omega$. We prove this through the L-type's incentive compatibility constraint. If the L-type mimics the H-type, i.e., deviates by offering $r_{H,1}$ in period 1, then the type will not revealed in period 2 unless $d \in \omega$ and $A_1 \in \left[\dfrac{r_{H,1}}{\theta_H}, \dfrac{r_{H,1}}{\theta_L}\right]$. Further, the L-type who offers $r_{H,1}$ in period 1 will offer $\hat{r}(\overline{A}_2\theta_H)$ in period 2 if the type is not revealed, and will offer $\hat{r}(\overline{A}_2\theta_L)$ in period 2 otherwise. Thus, as we define

$$F_L(r) = u_1(r \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L) - \mathbf{1}_{d\in\omega} \cdot \beta \left(\frac{r}{\overline{A}_1\theta_L} - \frac{r}{\overline{A}_1\theta_H}\right)(u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)),$$

the L-type's incentive compatibility constraint is as follows:

$$u_1(\hat{r}(\overline{A}_1\theta_L) \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L) \geq F_L(r_{H,1}).$$

It is easy to verify that $F_L$ decreases in $r$. Also,

$$F_L(\hat{r}(\overline{A}_1\theta_L)) > u_1(\hat{r}(\overline{A}_1\theta_L) \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L),$$

which implies $F_L(\hat{r}(\overline{A}_1\theta_L)) > F_L(r_{H,1})$. Therefore, $r_{H,1} > \hat{r}(\overline{A}_1\theta_L)$. Because we restrict our attention to the least-contract equilibria, this incentive compatibility constraint for the L-type binds, given that $F_L$ is decreasing. Thus, $r_{H,1} = r_H^*$ if $d \in \omega$ and $r_{H,1} = r_H^{**}$ if $d \notin \omega$. Thus, both $r_H^* > \hat{r}(\overline{A}_1\theta_L)$ and $r_H^{**} > \hat{r}(\overline{A}_1\theta_L)$ hold.

We finish the proof by showing $r_H^{**} > r_H^*$. According to the definition of $r_H^*$,

$$\begin{aligned}&u_1(r_H^* \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L) - \beta\left(\frac{r_H^*}{\overline{A}_1\theta_L} - \frac{r_H^*}{\overline{A}_1\theta_H}\right)(u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L))\\ &= u_1(\hat{r}(\overline{A}_1\theta_L) \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L),\end{aligned}$$

which implies that

$$u_1(r_H^* \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L) > u_1(\hat{r}(\overline{A}_1\theta_L) \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L).$$

Because $u_1(r_H^* \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L)$ decreases in $r_H^*$ and

$$u_1(r_H^{**} \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_L) = u_1(\hat{r}(\overline{A}_1\theta_L) \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)$$

holds by the definition of $r_H^{**}$, $r_H^{**} > r_H^*$ must hold. ■

**Proof of proposition 8.** According to the proof of proposition 7, $(r_{H,1}, r_{L,1}) = (r_H^*, \hat{r}(\overline{A}_1\theta_L))$ holds if $\omega = \{r_1, d\}$ and $(r_{H,1}, r_{L,1}) = (r_H^{**}, \hat{r}(\overline{A}_1\theta_L))$ holds if $\omega = \{r_1\}$. To provide robust support for the existence of this equilibrium, we introduce an assumption that any deviation from the equilibrium trajectory leads to the worst belief about the entrepreneur's type, that the entrepreneur is the L-type unless the type is revealed.

Firstly, it is worth noting that the type of entrepreneur is revealed at the beginning of period 2 in the separating equilibrium. Therefore it is trivial that the H-type offers $\hat{r}(\overline{A}_2\theta_H)$, while the L-type offers $\hat{r}(\overline{A}_2\theta_L)$ in period 2, given that we focus on the least-contract equilibria and according to claim 1 in the proof of proposition 1. It is also trivial that the L-type in such equilibrium offers $\hat{r}(\overline{A}_1\theta_L)$ in period 1. The necessary condition we need to check is that the H-type's contract offer $r_{H,1}$ in period 1 would be accepted by lender 1 who believes that such a contract is offered by the H-type entrepreneur for sure. According to the lender 1's rationality, $r_{H,1}\left(1-\dfrac{r_{H,1}}{\overline{A}_1\theta_H}\right) \geq \gamma$ must hold, which is equivalent to $r_{H,1} \in \left[\hat{r}(\overline{A}_1\theta_H), \bar{r}_H\right]$. We already have $r_{H,1} > \hat{r}(\overline{A}_1\theta_L)$ from proposition 7, which implies $r_{H,1} > \hat{r}(\overline{A}_1\theta_H)$. Therefore, the above necessary condition eventually requires that $r_{H,1} \leq \bar{r}_H$.

To establish the conditions for the existence of a separating equilibrium without a break, it is necessary to examine whether each type of entrepreneur has an incentive to deviate from the equilibrium in period 1. A deviation by either type in period 1 would lead lender 1 to adopt the worst belief - that the entrepreneur is the L-type - unless the deviation is mimicking the other type. Consequently, if the entrepreneur deviates by offering below $\hat{r}(\overline{A}_1\theta_L)$, the contract would be rejected, precluding the running of the project in the first period.

We first focus on the H-type's incentive to deviate from the equilibrium strategy. First, it is trivial that the H-type has no incentive to offer higher than $r_{H,1}$ in period 1. Now consider that the H-type decides to deviate by offering a contract $r \in \left[\hat{r}(\overline{A}_1\theta_L), r_{H,1}\right)$. If $\omega = \{r_1\}$, then this entrepreneur will be treated as the L-type entrepreneur by lender 2 in period 2, thus, offering $\hat{r}(\overline{A}_2\theta_L)$. If $\omega = \{r_1, d\}$, then this entrepreneur's type will be revealed if $A_1 \in \left[\dfrac{r}{\theta_H}, \dfrac{r}{\theta_L}\right)$, thus, offering $\hat{r}(\overline{A}_2\theta_H)$ in period 2, and will be treated as the L-type by the lender 2 in period 2 otherwise, thus, offering $\hat{r}(\overline{A}_2\theta_L)$ in period 2. Considering all the cases in both information regimes, the H-type's expected utility who deviates by offering

$r \in \left[\hat{r}(\overline{A}_1\theta_L), r_{H,1}\right)$ in period 1 is

$$\begin{aligned}\hat{f}(r) &= u_1(r \mid \theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H) \\ &\quad + \mathbf{1}_{d\in\omega}\beta \cdot \left(\frac{r}{\overline{A}_1\theta_L} - \frac{r}{\overline{A}_1\theta_H}\right)(u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H)).\end{aligned}$$

Whereas, the H-type's expected equilibrium utility is

$$u_1(r_{H,1} \mid \theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H).$$

It is trivial that $\hat{f}$ is continuous, $\hat{f}$ is convex because $\frac{\partial^2}{\partial r^2}\hat{f}(r) = \frac{\partial^2}{\partial r^2}u_1(r_{H,1} \mid \theta_H) = \frac{1}{\overline{A}_1\theta_H} > 0$, and

$$\hat{f}(r_{H,1}) < u_1(r_{H,1} \mid \theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H).$$

Therefore, the H-type having no incentive to deviate by offering $\hat{r}(\overline{A}_1\theta_L)$ in period 1, i.e., mimicking the L-type, implies that the H-type has no incentive to deviate by offering a contract within a range $\left[\hat{r}(\overline{A}_1\theta_L), r_{H,1}\right)$ in period 1. Thus, the H-type would not deviate from the equilibrium strategy whenever it is not beneficial for the H-type to mimic the L-type.

The L-type who deviates by offering a contract higher than $\hat{r}(\overline{A}_1\theta_L)$ would be unequivocally identified as the L-type in the second period unless the contract offer is $r_{H,1}$. Thus, the L-type has no incentive to deviate by offering a contract above $\hat{r}(\overline{A}_1\theta_L)$, unless he/she mimics the H-type. In conclusion, to ensure the stability of this separating equilibrium, it suffices to verify two conditions: *i)* each type of entrepreneur does not have an incentive to mimic the other type of entrepreneur in period 1, and *ii)* neither type of entrepreneur has an incentive not to run the project in period 1. From *i)*, we need both (12) and (13) to be satisfied. Finally, the constraint for the H-type and the L-type not to break in period 1, respectively, are

$$u_1(r_{H,1} \mid \theta_H) + \beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) \geq 0 + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_H)$$

and

$$u_1(\hat{r}(\overline{A}_1\theta_L) \mid \theta_L) + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L) \geq 0 + \beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L),$$

both of which hold trivially. ■

**Proof of lemma 2.** Let $W_\omega^P$ denote the welfare under the pooling equilibrium of each regime. We have $W_\varnothing^P = W_{r_1}^P \equiv W_{d\notin\omega}^P$, and $W_d^P = W_{r_1,d}^P \equiv W_{d\in\omega}^P$. Then, from proposition 2, we have

$$W_{d\notin\omega}^P = \sigma u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_H) + (1-\sigma)u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_L) + \sigma\beta u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H) + (1-\sigma)\beta u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L)$$

and

$$\begin{aligned} W_{d\in\omega}^P &= \sigma u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_H) + (1-\sigma)u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_L) \\ &\quad + \sigma\beta u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H) + \sigma\beta\left(\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1\theta_L} - \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1\theta_H}\right)(u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H)) \\ &\quad + (1-\sigma)\beta u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L) - (1-\sigma)\beta\left(\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1\theta_L} - \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1\theta_H}\right)(u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L) - u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)), \end{aligned}$$

and therefore we have

$$\begin{aligned} &W_{d\notin\omega}^P - W_{d\in\omega}^P = \beta\left(\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1\theta_L} - \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1\theta_H}\right) \cdot \\ &\left[\sigma u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H) + (1-\sigma)u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L) - \sigma u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - (1-\sigma)u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)\right]. \end{aligned}$$

Therefore, $W_{d\notin\omega}^P > W_{d\in\omega}^P$ is satisfied if and only if $\sigma u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H) + (1-\sigma)u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L) > \sigma u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) + (1-\sigma)u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)$ holds, which is shown in the following claim.

**Claim 3** *For $i = 1, 2$,*

$$\sigma u_i(\hat{r}(\overline{A}_i\theta_\sigma) \mid \theta_H) + (1-\sigma)u_i(\hat{r}(\overline{A}_i\theta_\sigma) \mid \theta_L) > \sigma u_i(\hat{r}(\overline{A}_i\theta_H) \mid \theta_H) + (1-\sigma)u_i(\hat{r}(\overline{A}_i\theta_L) \mid \theta_L).$$

**Proof of claim 3.** First, notice that

$$\hat{r}(\overline{A}_i\theta_H)\left(1 - \frac{\hat{r}(\overline{A}_i\theta_H)}{\overline{A}_i\theta_H}\right) = \hat{r}(\overline{A}_i\theta_L)\left(1 - \frac{\hat{r}(\overline{A}_i\theta_L)}{\overline{A}_i\theta_L}\right) = \gamma$$

and

$$\sigma \cdot \hat{r}(\overline{A}_i\theta_\sigma)\left(1 - \frac{\hat{r}(\overline{A}_i\theta_\sigma)}{\overline{A}_i\theta_H}\right) + (1-\sigma)\cdot\hat{r}(\overline{A}_i\theta_\sigma)\left(1 - \frac{\hat{r}(\overline{A}_i\theta_\sigma)}{\overline{A}_i\theta_L}\right) = \gamma.$$

Therefore

$$\begin{aligned}
&\sigma u_i(\hat{r}(\overline{A}_i\theta_\sigma) \mid \theta_H) + (1-\sigma)u_i(\hat{r}(\overline{A}_i\theta_\sigma) \mid \theta_L) - \sigma u_i(\hat{r}(\overline{A}_i\theta_H) \mid \theta_H) - (1-\sigma)u_i(\hat{r}(\overline{A}_i\theta_L) \mid \theta_L) \\
&= \sigma\left(u_i(\hat{r}(\overline{A}_i\theta_\sigma) \mid \theta_H) + \hat{r}(\overline{A}_i\theta_\sigma)\left(1 - \frac{\hat{r}(\overline{A}_i\theta_\sigma)}{\overline{A}_i\theta_H}\right)\right) + (1-\sigma)\left(u_i(\hat{r}(\overline{A}_i\theta_\sigma) \mid \theta_L) + \hat{r}(\overline{A}_i\theta_\sigma)\left(1 - \frac{\hat{r}(\overline{A}_i\theta_\sigma)}{\overline{A}_i\theta_L}\right)\right) \\
&- \sigma\left(u_i(\hat{r}(\overline{A}_i\theta_H) \mid \theta_H) + \hat{r}(\overline{A}_i\theta_H)\left(1 - \frac{\hat{r}(\overline{A}_i\theta_H)}{\overline{A}_i\theta_H}\right)\right) - (1-\sigma)\left(u_i(\hat{r}(\overline{A}_i\theta_L) \mid \theta_L) + \hat{r}(\overline{A}_i\theta_L)\left(1 - \frac{\hat{r}(\overline{A}_i\theta_L)}{\overline{A}_i\theta_H}\right)\right) \\
&= \sigma\frac{\overline{A}_i\theta_H}{2}\left(1 - \left(\frac{\hat{r}(\overline{A}_i\theta_\sigma)}{\overline{A}_i\theta_H}\right)^2\right) + (1-\sigma)\frac{\overline{A}_i\theta_L}{2}\left(1 - \left(\frac{\hat{r}(\overline{A}_i\theta_\sigma)}{\overline{A}_i\theta_L}\right)^2\right) \\
&- \sigma\frac{\overline{A}_i\theta_H}{2}\left(1 - \left(\frac{\hat{r}(\overline{A}_i\theta_H)}{\overline{A}_i\theta_H}\right)^2\right) - (1-\sigma)\frac{\overline{A}_i\theta_L}{2}\left(1 - \left(\frac{\hat{r}(\overline{A}_i\theta_L)}{\overline{A}_i\theta_L}\right)^2\right).
\end{aligned}$$

Thus, it suffices to show that

$$\frac{\sigma}{\theta_H}\left(\hat{r}(\overline{A}_i\theta_H)\right)^2 + \frac{(1-\sigma)}{\theta_L}\left(\hat{r}(\overline{A}_i\theta_L)\right)^2 > \frac{\sigma}{\theta_H}\left(\hat{r}(\overline{A}_i\theta_\sigma)\right)^2 + \frac{(1-\sigma)}{\theta_L}\left(\hat{r}(\overline{A}_i\theta_\sigma)\right)^2. \tag{17}$$

Let $P \equiv \dfrac{\sigma}{\theta_H}$, and $Q \equiv \dfrac{1-\sigma}{\theta_L}$. Then (17) can be rewritten as

$$P\left(\hat{r}\left(\frac{\sigma\overline{A}_i}{P}\right)\right)^2 + Q\left(\hat{r}\left(\frac{(1-\sigma)\overline{A}_i}{Q}\right)\right)^2 > P\left(\hat{r}\left(\frac{\overline{A}_i}{P+Q}\right)\right)^2 + Q\left(\hat{r}\left(\frac{\overline{A}_i}{P+Q}\right)\right)^2,$$

which holds if and only if $(\hat{r}(\cdot))^2$ is convex, because

$$P \cdot \frac{\sigma}{P} + Q \cdot \frac{1-\sigma}{Q} = P \cdot \frac{1}{P+Q} + Q \cdot \frac{1}{P+Q} = 1.$$

We first show that $\hat{r}(\cdot)$ is convex. Because

$$\frac{\partial \hat{r}(x)}{\partial x} = \frac{1}{2}\left[1 - (x-2r)(x^2-4xr)^{-\frac{1}{2}}\right] \leq 0,$$

we have

$$\begin{aligned}\frac{\partial^2 \hat{r}(x)}{\partial x^2} &= \frac{1}{4}(x-2r)(x^2-4xr)^{-\frac{3}{2}} \cdot (2x-4r) - \frac{1}{2}(x^2-4xr)^{-\frac{1}{2}} \\ &= \frac{1}{2}(x^2-4xr)^{-\frac{3}{2}}\left[(x-2r)^2 - (x^2-4xr)\right] \\ &= (x^2-4xr)^{-\frac{3}{2}} 2r^2 \geq 0.\end{aligned}$$

We finish the proof by showing that $(\hat{r}(\cdot))^2$ is convex in the following expression:

$$\frac{\partial^2 (\hat{r}(x))^2}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial (\hat{r}(x))^2}{\partial x}\right) = \frac{\partial}{\partial x}\left(2\hat{r}(x) \cdot \frac{\partial \hat{r}(x)}{\partial x}\right) = 2\left(\frac{\partial \hat{r}(x)}{\partial x}\right)^2 + 2\hat{r}(x) \cdot \frac{\partial^2 \hat{r}(x)}{\partial x^2} > 0.$$

■ ■

**Proof of lemma 3.** Let $W_\omega^S$ denote the welfare under the separating equilibrium under each $\omega = \{r_1\}, \{r_1, d\}$. From proposition 7, we have

$$\begin{aligned}W_{\{r_1,d\}}^S &= \sigma u_1(r_H^* \mid \theta_H) + (1-\sigma)u_1(\hat{r}(\overline{A}_1\theta_L) \mid \theta_L) + \sigma\left(r_H^*\left(1 - \frac{r_H^*}{\overline{A}_1\theta_H}\right) - \gamma\right) \\ &\quad + \sigma\beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) + (1-\sigma)\beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L), \\ W_{\{r_1\}}^S &= \sigma u_1(r_H^{**} \mid \theta_H) + (1-\sigma)u_1(\hat{r}(\overline{A}_1\theta_L) \mid \theta_L) + \sigma\left(r_H^{**}\left(1 - \frac{r_H^{**}}{\overline{A}_1\theta_H}\right) - \gamma\right) \\ &\quad + \sigma\beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) + (1-\sigma)\beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L),\end{aligned}$$

and therefore we have

$$W^S_{\{r_1,d\}} - W^S_{\{r_1\}} = \sigma \left[ \left( u_1(r^*_H \mid \theta_H) + r^*_H \left( 1 - \frac{r^*_H}{\overline{A}_1\theta_H} \right) \right) - \left( u_1(r^{**}_H \mid \theta_H) + r^{**}_H \left( 1 - \frac{r^{**}_H}{\overline{A}_1\theta_H} \right) \right) \right].$$

Note that $r^{**}_H > r^*_H$, according to proposition 7. Thus, $W^S_{\{r_1,d\}} > W^S_{\{r_1\}}$ if $\left( u_1(r \mid \theta_H) + r \left( 1 - \frac{r}{\overline{A}_1\theta_H} \right) \right)$ decreases in $r$, which is shown in the following claim.

**Claim 4** $\left( u_1(r \mid \theta_H) + r \left( 1 - \frac{r}{\overline{A}_1\theta_H} \right) \right)$ *decreases in* $r$*.*

**Proof.**

$$\begin{aligned}
\left( u_1(r \mid \theta_H) + r \left( 1 - \frac{r}{\overline{A}_1\theta_H} \right) \right) &= \frac{\overline{A}_1\theta_H}{2} \left( 1 - \frac{r}{\overline{A}_1\theta_H} \right)^2 + r \left( 1 - \frac{r}{\overline{A}_1\theta_H} \right) \\
&= \left( 1 - \frac{r}{\overline{A}_1\theta_H} \right) \cdot \left[ \frac{\overline{A}_1\theta_H}{2} \left( 1 - \frac{r}{\overline{A}_1\theta_H} \right) + r \right] \\
&= \left( 1 - \frac{r}{\overline{A}_1\theta_H} \right) \cdot \left[ \frac{\overline{A}_1\theta_H}{2} \left( 1 + \frac{r}{\overline{A}_1\theta_H} \right) \right] \\
&= \frac{\overline{A}_1\theta_H}{2} \left( 1 - \left( \frac{r}{\overline{A}_1\theta_H} \right)^2 \right),
\end{aligned}$$

thus $\left( u_1(r \mid \theta_H) + r \left( 1 - \frac{r}{\overline{A}_1\theta_H} \right) \right)$ decreases in $r$. ■ ■

**Proof of lemma 4.** Because separating equilibrium exists only if $r_1 \in \omega$, we focus on the two cases only, when $\omega = \{r_1\}$ and $\omega = \{r_1, d\}$. From lemmas 2 and 3, we have $W^P_{\{r_1\}} > W^P_{\{r_1,d\}}$ and $W^S_{\{r_1,d\}} > W^S_{\{r_1\}}$. Therefore, it suffices to show that $W^P_{\{r_1,d\}} > W^S_{\{r_1,d\}}$ holds. First, notice that

$$\hat{r}(\overline{A}_1\theta_H) \left( 1 - \frac{\hat{r}(\overline{A}_1\theta_H)}{\overline{A}_1\theta_H} \right) = \gamma.$$

Also, from the fact that $r^*_H > \hat{r}(\overline{A}_1\theta_H)$ and by claim 4 in the proof of lemma 3, we have

$$\begin{aligned}
&\sigma u_1(r^*_H \mid \theta_H) + \sigma \left( r^*_H \left( 1 - \frac{r^*_H}{\overline{A}_1\theta_H} \right) - \gamma \right) \\
&< \sigma u_1(\hat{r}(\overline{A}_1\theta_H) \mid \theta_H) + \sigma \left( \hat{r}(\overline{A}_1\theta_H) \left( 1 - \frac{\hat{r}(\overline{A}_1\theta_H)}{\overline{A}_1\theta_H} \right) - \gamma \right) = \sigma u_1(\hat{r}(\overline{A}_1\theta_H) \mid \theta_H).
\end{aligned}$$

Therefore

$$
\begin{aligned}
&W^P_{\{r_1,d\}} - W^S_{\{r_1,d\}} \\
&> \sigma u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_H) + (1-\sigma)u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_L) + \sigma\beta u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H) + (1-\sigma)\beta u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L) \\
&- \beta\left(\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1\theta_L} - \frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1\theta_H}\right)\cdot \\
&\left[\sigma u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H) + (1-\sigma)u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L) - \sigma u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - (1-\sigma)u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)\right] \\
&- \left[\sigma u_1(\hat{r}(\overline{A}_1\theta_H) \mid \theta_H) + (1-\sigma)u_1(\hat{r}(\overline{A}_1\theta_L) \mid \theta_L) + \sigma\beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) + (1-\sigma)\beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)\right] \\
&= \sigma u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_H) + (1-\sigma)u_1(\hat{r}(\overline{A}_1\theta_\sigma) \mid \theta_L) - \sigma u_1(\hat{r}(\overline{A}_1\theta_H) \mid \theta_H) - (1-\sigma)u_1(\hat{r}(\overline{A}_1\theta_L) \mid \theta_L) \\
&+ \left(1 - \beta\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1\theta_L} + \beta\frac{\hat{r}(\overline{A}_1\theta_\sigma)}{\overline{A}_1\theta_H}\right)\cdot \\
&[\sigma\beta u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H) + (1-\sigma)\beta u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L) - \sigma\beta u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - (1-\sigma)\beta u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L)].
\end{aligned}
$$

The proof is done if $\sigma u_i(\hat{r}(\overline{A}_i\theta_\sigma) \mid \theta_H) + (1-\sigma)u_i(\hat{r}(\overline{A}_i\theta_\sigma) \mid \theta_L) \geq \sigma u_i(\hat{r}(\overline{A}_i\theta_H) \mid \theta_H) + (1-\sigma)u_i(\hat{r}(\overline{A}_i\theta_L) \mid \theta_L)$ for $i = 1, 2$, which is shown in claim 3 in the proof of lemma 2. ■

**Proof of proposition 10.** First of all, the lender's expected payoff from a contract $r$ offered by the entrepreneur that the lender believes as the $i$-type ($i = H, L$) is $r\left(1 - \frac{r}{\overline{A}_2\theta_i}\right)$, which has the maximum value of $\frac{\overline{A}_2\theta_i}{4}$ at $r = \frac{\overline{A}_2\theta_i}{2}$. Therefore, if $\overline{A}_2\theta_i < 4\gamma$, then $r\left(1 - \frac{r}{\overline{A}_2\theta_i}\right) < \gamma$ for all $r > 0$. In other words, $\overline{A}_2\theta_i < 4\gamma$ indicates that the lender will not accept any contract from the entrepreneur that the lender believes would be $i$-type. By the same logic, if $\overline{A}_2\theta_\sigma < 4\gamma$, then any contract the lender believes that is offered by both types will be rejected. Therefore, $\overline{A}_2\theta_\sigma < 4\gamma$ implies that both types offering $\hat{r}(\overline{A}_2\theta_\sigma)$ and running the project in period 2 cannot hold in equilibrium.

From now on, we analyze the three cases in the statement. To strongly support the existence of the equilibrium, we assume that any deviation from the equilibrium trajectory leads to the worst belief about the entrepreneur's type, that the entrepreneur is undoubtedly the L-type unless the type is revealed. Notice that $4\gamma > \overline{A}_2\theta_L$ is premised which indicates that any contract offered by an entrepreneur who is regarded as the L-type by the lender will be rejected. Therefore, under the belief system introduced, the H-type never deviates from the equilibrium strategy, and the L-type in any separating equilibrium would never deviate to a different strategy other than mimicking the H-type. It also indicates that there does not exist a separating equilibrium in which the L-type runs the project in period 2.

1) First consider that $4\gamma > \overline{A}_2\theta_\sigma$ and $\overline{A}_2\theta_L > \bar{r}_{2,H}$. Then only the H-type can run the project in period 2 in any equilibrium. Notice that the H-type would never deviate from the equilibrium strategy. Thus, to show that the market collapses, we only need to show that the L-type would mimic the H-type. Because the L-type earns zero in the equilibrium, he/she does not have the incentive to mimic the H-type only if $\overline{A}_2\theta_L \leq r_{H,2}$, where $r_{H,2}$ is the H-type's contract offer in period 2, which is the necessary condition that the L-type's expected payoff by mimicking the H-type is also zero. Howver, $r_{H,2} \leq \bar{r}_{2,H}$ must hold for $r_{H,2}$ to be accepted, which requires $\overline{A}_2\theta_L \leq \bar{r}_{2,H}$. Therefore, there is no equilibrium in which any type of entrepreneur runs the project.

2) Second, consider that $\overline{A}_2\theta_L < 4\gamma \leq \overline{A}_2\theta_\sigma$, $\hat{r}(\overline{A}_2\theta_\sigma) < \overline{A}_2\theta_L$, and $\overline{A}_2\theta_L > \bar{r}_{2,H}$. By $\overline{A}_2\theta_L > \bar{r}_{2,H}$ and according to the logic in the previous paragraph, it cannot happen that only the H-type offers a contract. We show that, given $\overline{A}_2\theta_L < 4\gamma \leq \overline{A}_2\theta_\sigma$ and $\hat{r}(\overline{A}_2\theta_\sigma) < \overline{A}_2\theta_L$, it is valid that both types offer $\hat{r}(\overline{A}_2\theta_\sigma)$ and run the project.[14] With the support of the worst belief system, we only need to check whether each type of entrepreneur would not deviate by not offering a contract, i.e., $\hat{r}(\overline{A}_2\theta_\sigma) < \overline{A}_2\theta_i$ for $i = H, L$. It holds because $\hat{r}(\overline{A}_2\theta_\sigma) < \overline{A}_2\theta_L$ is given, which implies $\hat{r}(\overline{A}_2\theta_\sigma) < \overline{A}_2\theta_H$.

3) Third, consider that $\overline{A}_2\theta_L < 4\gamma \leq \overline{A}_2\theta_\sigma$, $\hat{r}(\overline{A}_2\theta_\sigma) < \overline{A}_2\theta_L$, and $\overline{A}_2\theta_L \leq \bar{r}_{2,H}$. From the previous paragraph, given $\overline{A}_2\theta_L < 4\gamma \leq \overline{A}_2\theta_\sigma$ and $\hat{r}(\overline{A}_2\theta_\sigma) < \overline{A}_2\theta_L$, there exists equilibrium in which both types offer $\hat{r}(\overline{A}_2\theta_\sigma)$ and run the project. We now show that

[14]The pooling contract is $\hat{r}(\overline{A}_2\theta_\sigma)$ because we focus on considering the least-contract equilibria.

only H-type offering $\overline{A}_2\theta_L$ in period 2 can also hold in equilibrium. According to the logic in 1), only the H-type runs the project in any equilibrium and it exists if and only if the L-type has no incentive to mimic the H-type while the H-type's equilibrium contract offer is accepted. Both conditions are $\overline{A}_2\theta_L \leq r_{H,2}$ and $r_{H,2} \in \left[\hat{r}(\overline{A}_2\theta_H), \bar{r}_{2,H}\right]$ respectively, which are feasible to be satisfied together given that $\overline{A}_2\theta_L \leq \bar{r}_{2,H}$. Notice that we focus on the least-contract equilibria, thus one of both conditions binds. Also notice that $\overline{A}_2\theta_L > \hat{r}(\overline{A}_2\theta_H)$ because $\hat{r}(\overline{A}_2\theta_\sigma) < \overline{A}_2\theta_L$ is given. Therefore $r_{H,2} = \overline{A}\theta_L$ holds.

4) Finally, suppose that either *i)* $4\gamma > \overline{A}_2\theta_\sigma$ and $\overline{A}_2\theta_L \leq \bar{r}_{2,H}$, or *ii)* $4\gamma \leq \overline{A}_2\theta_\sigma$ and $\hat{r}(\overline{A}_2\theta_\sigma) \geq \overline{A}_2\theta_L$. We first consider the case when $4\gamma > \overline{A}_2\theta_\sigma$ and $\overline{A}_2\theta_L \leq \bar{r}_{2,H}$. Because $4\gamma > \overline{A}_2\theta_\sigma$, it cannot hold in equilibrium that both types offer the same contract in period 2. Also, because $\overline{A}_2\theta_L \leq \bar{r}_{2,H}$ and from the previous paragraph, there exists an equilibrium in which only the H-type runs the project in period 2, and the period-2 contract $r_{H,2}$ satisfies both conditions $\overline{A}_2\theta_L \leq r_{H,2}$ and $r_{H,2} \in \left[\hat{r}(\overline{A}_2\theta_H), \bar{r}_{2,H}\right]$. Because we focus on the least-contract equilibria, one of both conditions binds, which implies that $r_{H,2} = \max\{\hat{r}\overline{A}_2\theta_H), \overline{A}_2\theta_L\}$.

   We finish the proof by considering that $4\gamma \leq \overline{A}_2\theta_\sigma$ and $\hat{r}(\overline{A}_2\theta_\sigma) \geq \overline{A}_2\theta_L$. First, suppose that there exists an equilibrium in which both types offer the same contract in period 2. Then, the pooling contract must be $\hat{r}(\overline{A}_2\theta_\sigma)$ or above, by the lender's rationality. However, from $\hat{r}(\overline{A}_2\theta_\sigma) \geq \overline{A}_2\theta_L$, the pooling contract is weakly above $\overline{A}_2\theta_L$, thus, the L-type would not offer such a contract. Therefore, in any equilibrium, only the H-type runs the project in period 2. With the support of the worst belief system, we only need to check whether the L-type has no incentive to mimic the H-type while $r_{H,1}$ is accepted. As in the previous paragraph, both conditions are $\overline{A}_2\theta_L \leq r_{H,2}$ and $r_{H,2} \in \left[\hat{r}(\overline{A}_2\theta_H), \bar{r}_H\right]$ respectively, which are feasible to be satisfied together given that $\overline{A}_2\theta_L \leq \hat{r}(\overline{A}_2\theta_\sigma)$, because $\hat{r}(\overline{A}_2\theta_\sigma) < \bar{r}_{2,H}$.[15] Finally, because we focus on the least-

[15] It is easy to verify that $\bar{r}_{2,H} > \bar{r}_\sigma$, and $\bar{r}_\sigma > \hat{r}(\overline{A}_2\theta_\sigma)$ also holds by the definitions of $\bar{r}_\sigma$ and $\hat{r}(\overline{A}_2\theta_\sigma)$.

contract equilibria, we have $r_{H,1} = \max\{\hat{r}(\overline{A}_2\theta_H), \overline{A}_2\theta_L\}$.

■

**Proof of proposition 12.** We first argue that separating equilibrium cannot exist. Suppose conversely that there exists separating equilibrium, and let $r_{H,1}$ be the H-type's contract offer in period 1. Because $\overline{A}_1\theta_L < 4\gamma$ and $\overline{A}_2\theta_L < 4\gamma$, L-type earns zero payoff in this equilibrium. We show that separating equilibrium cannot exist by arguing that the L-type always has an incentive to mimic the H-type in period 1. Notice that the parametric conditions only support that both types offer the pooling contract in period 2, which will be accepted by lender 2. If the L-type mimics the H-type in period 1, the type is not revealed if both types default or both types do not default, i.e., $A_1 \in \left[0, \frac{r_{H,1}}{\theta_H}\right) \cup \left[\frac{r_{H,1}}{\theta_L}, 1\right]$. That is, with probability $1 - \frac{r_{H,1}}{\overline{A}_1\theta_L} + \frac{r_{H,1}}{\overline{A}_1\theta_H} > 0$, the L-type who mimics the H-type in period 1 will be regarded as the H-type in period 2 by the lender 2, resulting in earning a positive payoff by offering $\hat{r}(\overline{A}_2\theta_\sigma)$ in period 2.

Consider that $4\gamma > \overline{A}_1\theta_\sigma$. Then pooling equilibrium cannot exist, thus, the market collapses, given that separating equilibrium cannot exist. Finally, $4\gamma \leq \overline{A}_1\theta_\sigma$ implies the existence of the pooling equilibrium in which both types offer $\hat{r}(\overline{A}_1\theta_\sigma)$, which is the only feature of equilibrium because separating equilibrium cannot exist. ■

**Proof of proposition 13.** Let $w(\theta_H, \sigma) \equiv W_{\{d\}} - W_\varnothing$. Then, we obtain

$$w(\theta_H, \sigma) = \sigma u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) - \sigma u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_H) - (1-\sigma)u_2(\hat{r}(\overline{A}_2\theta_\sigma) \mid \theta_L),$$

which is continuous with respect to both $\theta_L$ and $\sigma$.

First, assume $\overline{A}_2\theta_L \leq 2\gamma$. According to proposition 10, the market collapses in period 2 if $4\gamma > \overline{A}_2\theta_\sigma$ and $\overline{A}_2\theta_L > \bar{r}_{2,H}$. However, since $4\gamma \leq \overline{A}_2\theta_H$, it follows that $\bar{r}_{2,H} \geq 2\gamma$. Thus, $\overline{A}_2\theta_L > \bar{r}_{2,H}$ cannot be true given $\overline{A}_2\theta_L \leq 2\gamma$. Therefore, when $\overline{A}_2\theta_L \leq 2\gamma$, the market does not collapse. Furthermore, only the H-type offers a contract in period 2 if either *i)* $4\gamma > \overline{A}_2\theta_\sigma$

or *ii)* $4\gamma \leq \overline{A}_2\theta_\sigma$ and $\hat{r}(\overline{A}_2\theta_\sigma) \geq \overline{A}_2\theta_L$. It is worth noting that $4\gamma > \overline{A}_2\theta_\sigma$ is equivalent to $\sigma < \sigma_1$, and the combination of $4\gamma \leq \overline{A}_2\theta_\sigma$ and $\hat{r}(\overline{A}_2\theta_\sigma) \geq \overline{A}_2\theta_L$ is equivalent to $\sigma_1 \leq \sigma \leq \sigma_2$. In summary, only the separating equilibrium exists if $\sigma \leq \sigma_2$. Conversely, the pooling equilibrium exists if $\sigma > \sigma_2$. Note that by the definition of $\sigma_2$, we have $u_2(\hat{r}(\overline{A}_2\theta_{\sigma_2}) \mid \theta_L) = 0$. Thus, $w(\theta_H, \sigma_2) > 0$. Finally, due to the continuity of $w$ with respect to $\sigma$, either $w(\theta_H, \sigma) > 0$ for all $\sigma > \sigma_2$ or there exists $\sigma^{**} \in (\sigma_2, 1)$ such that $w(\theta_H, \sigma) > 0$ for all $\sigma \in (\sigma_2, \sigma^{**})$. As we define $\sigma^{**} = 1$ if $w(\theta_H, \sigma) > 0$ holds for all $\sigma > \sigma_2$, we can say that $W_{\{d\}} > W_\varnothing$ whenever $\sigma \in (\sigma_2, \sigma^{**})$ for some $\sigma^{**} \in (\sigma_2, 1]$.

Next, suppose that $\overline{A}_2\theta_L \in (2\gamma, 4\gamma)$. Notice that, because $4\gamma \leq \overline{A}_2\theta_\sigma$ implies $\hat{r}(\overline{A}_2\theta_\sigma) \leq 2\gamma$, if $4\gamma \leq \overline{A}_2\theta_\sigma$ then $\hat{r}(\overline{A}_2\theta_\sigma) < \overline{A}_2\theta_L$ also holds, from $\overline{A}_2\theta_L > 2\gamma$. Therefore, pooling equilibrium exists in period 2 if and only if $\sigma \geq \sigma_1$. Suppose that $w(\theta_H, \sigma_1) > 0$ for a while. Then, given the continuity of $w$ with respect to $\sigma$, it can be inferred that $W_{\{d\}} > W_\varnothing$ holds whenever $\sigma \in (\sigma_1, \sigma^{**})$ for some $\sigma^{**} \in (\sigma_1, 1]$. Further, notice that $\sigma^*$ is defined so that $\sigma^* = \sigma_2$ if $\overline{A}_2\theta_L \leq 2\gamma$ and $\sigma^* = \sigma_1$ if $\sigma \in (2\gamma, 4\gamma)$. Then, $W_{\{d\}} > W_\varnothing$ whenever $\sigma \in (\sigma^*, \sigma^{**})$ for some $\sigma^{**} \in (\sigma^*, 1]$.

To wrap up the proof, we need to demonstrate the existence of a $\theta_L^* \in \left(\frac{2\gamma}{\overline{A}_2}, \frac{4\gamma}{\overline{A}_2}\right)$ such that for all $\theta_L < \theta_L^*$, $w(\theta_H, \sigma_1) > 0$ holds. Suppose that $\overline{A}_2\theta_L = 2\gamma$. Then $\hat{r}(\overline{A}_2\theta_{\sigma_2}) = 2\gamma$, which implies $\overline{A}_2\theta_{\sigma_2} = 4\gamma$, thus, $\sigma_1 = \sigma_2$. Additionally, given that $w(\theta_H, \sigma_2) > 0$, we also have $w(\theta_H, \sigma_1) > 0$ when $\theta_L = 2\gamma$. If we now assume that $\overline{A}_2\theta_L = 4\gamma$, then $\hat{r}(\overline{A}_2\theta_L)$ is well-defined, and we have $u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L) = 0$. Thus, we have

$$
\begin{aligned}
w(\theta_H, \sigma_1) = {} & \sigma_1 u_2(\hat{r}(\overline{A}_2\theta_H) \mid \theta_H) + (1 - \sigma_1) u_2(\hat{r}(\overline{A}_2\theta_L) \mid \theta_L) \\
& - \sigma_1 u_2(\hat{r}(\overline{A}_2\theta_{\sigma_1}) \mid \theta_H) - (1 - \sigma_1) u_2(\hat{r}(\overline{A}_2\theta_{\sigma_1}) \mid \theta_L).
\end{aligned}
$$

By claim 3, we have $w(\theta_H, \sigma_1) < 0$ when $\theta_L = 4\gamma$. Finally, because $w(\cdot, \cdot)$ is continuous on $\theta_L$, there exists $\overline{A}_2\theta_L^* \in (2\gamma, 4\gamma)$ such that $w(\theta_H, \sigma_1) > 0$ whenever $\theta_L < \theta_L^*$. ∎