CHONSEI AS LEVERAGED FINANCING: REAL OPTIONS THEORY AND EVIDENCE IN KOREA

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SUMMARY

Chonsei is a popular lease contract in the Korean housing market and similar forms exist in other international housing markets. Previous studies acknowledge that chonsei contracts represent an informal housing finance mechanism, whereby a landlord borrows a chonsei deposit, which is to be returned in full to a tenant at the maturity of a rental lease contract. Nevertheless, few studies have examined the role of chonsei that plays as a financing vehicle for housing investment. Using an optimal-stopping framework, this article shows that an increase in the accessiblity of chonsei contracts promotes housing investments. In particular, uncertainty is less of a deterrent to housing investments under a chonsei contract than with other arrangments. This theory is empirically supported by a cointegration model for national residential building starts in Korea.

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I. INTRODUCTION

Chonsei is a popular rental lease contract in Korea, under which a landlord receives an upfront lump-sum desposit that is to be returned in full to a tenant at the maturity of the contract.[2] The amount of the deposit ranges from 50 to 90 per cent of the value of a house. During the contract period, furthermore, the landlord receives no regular rental payments. Data from Statistics Korea indicate that in 2000 chonsei occupied 61.6 per cent of the rental market in Korea.

Previous studies on chonsei contracts (Gyourko and Han, 1989; Renaud, 1989; Kim, 1990; Son, 1997; Ambrose and Kim, 2003; Cho, 2005; Kim, 2013; and Moon, 2018) acknowledge that such contracts play a role in informal financing for landlords, who invest in real property by borrowing from their tenants. Nonetheless, such a finacing mechanism has received little attention in the literature. From the perspective of tenants, a chonsei contract, as an asset-based lease contract, plays the role of an informal savings vechicle. Kim and Park (2016) argue that renters consider the payment of a chonsei deposit a stepping stone to reaching the top of the housing ladder to homeownership. Kim (2013) provides an empirical analysis showing that chonsei renters tend to save more of their wealth than other renters do. Therefore chonsei contracts can contribute to explaining household portfolio choices and wealth accumulation in Korea (Cho, 2005).

This paper proposes, from the landlord perspective, that chonsei contracts play the role of a financing vehicle for landlords who invest in the rental housing market. During the term of a chonsei contract, a landlord can avoid bad states of nature, using the option to make an *ex-post* strategic decision to default on a chonsei contract.[3] Making an *ex-ante* investment decision, as a consequence such an investor considers whether chonsei contracts are available in the housing

[2] Around the world landlords use rental lease contracts similar to chonsei constracts, including bogey and girvi contracts in cities in India (Gilbert, 2003) and antichresis contracts in Bolivia and several civil law countries (Navarro and Turnbull, 2010), including Louisiana in the United States (Slovenko, 1958) and Australia

[3] Based on the moment at which a chonsei lease contract is signed, this paper distinguishes between e*x-ante* decisions and *ex-post* decisions.

market; we conjecture that an investment decision can be a function of the availability of chonsei deposits.

Ambrose and Kim (2003) document that chonsei contracts are instrumental as a hedging tool for housing investment because, in adverse market conditions, chonsei landlords have the option of giving a property to a tenant. The value of a chonsei deposit embeds the value of landlord default risk in the chonsei contract (Moon, 2018). In the lease literature, pricing a lease contract and evaluating the contract subject to credit risk have been investigated in Grenadier (1995) and Grenadier (1996), respectively. By extending Grenadier's theories to the Korean chonsei system, Ambrose and Kim (2003) derive the value of a chonsei contract as a European-style lease option, the maturity of which is fixed based on the Tenants Protection Act in Korea.

By introducing chonsei, this paper makes three contributions to the literature on real options in investment. First, the paper combines two seemingly distinct fields that cannot be separated when considering housing investments in Korea. Pricing a lease contract has been widely investigated in the literature and the theory has been successfully extended to the chonsei market by both Ambrose and Kim (2003) and Bae (2012). On the other hand, the value of an investment option in real estate development has also rececieved wide attention in the literature. This paper builds a theoretical bridge between the pricing of a lease contract and the valuation of an investment option, as housing investments often accompany lease contracts.

Second, the paper derives the value of a compound option, which differs from a compound option in Geske (1979). [4] By signing a chonsei contract, a landlord earns the right to default, making a chonsei similar to a put option. In the context of housing developments, as in Capozza and Helsley (1990), Williams (1991), and Capozza and Li (1994), a developer as investor has an option to invest (or to delay investment). Therefore an investor who rents a property in which he or she has invested

[4] Our theory requires the assumption that all claims, including an investment option and a default option, are tradable in the absence of any market frictions such as taxes, bankruptcy costs, information asymmetry, and investment lags. The assumption is shared by several studies on real options development, such as Williams (1991), Riddiough (1997), and You (2012).

through a chonsei contract has an (American) investment option on a (European) lease option. This paper investgates the value of an investment option that incorporates a chonsei lease contract.

Third, the paper presents empirical evidence that new housing supply can be determined by rental housing market conditions. Unlike studies of chonsei that highlight a housing consumer's investment strategy, this paper highlights a housing supplier's investment strategy with chonsei. By considering long-run relationships in housing market (Grimes and Aitken, 2010), the paper builds an empirical housing supply model that supports our theory; housing starts can be a function of the accessibility of chonsei contracts proxied by the ratio of value of chosei deposit to house value or chonsei deposit ratio (CDR). [5]

The remainder of the paper is structured as follows. In Section 2 we review the litature on chonsei and in section 3 we build an investment model that includes chonsei contracts. The theoretical model has a closed-form solution for both an option value with a chonsei contract and its trigger value. A numerical example is also provided for comparative statics. In Section 4 we provide empirical evidence for our theory and conclude in section 5.

II. LITERATURE REVIEW ON CHONSEI CONTRACTS

Chonsei contracts have been receiving increasing attention from academia (Gyourko and Han, 1989; Renaud, 1989; Kim, 1990; Son, 1997; Ambrose and Kim, 2003; Cho, 2005; Kim, 2013; and Moon, 2018). Similar forms exist in India (Gilbert, 2003) and antichresis contracts, the Bolivian version of chonsei contracts mentioned in Navarro and Turnbull (2010), are widely accepted in several civil law countries. In addition, rental transactions similar to Chonsei are found Louisiana in the United States (Slovenko, 1958) and Australia.[6]

[5] Based on housing-market conditions, the size of a chonsei deposit can vary in accordance with a mutual agreement between a tenant and a landlord. When the deposit is reduced, the tenant pays more in monthly rent.

[6] In Australia, residents in an aged care home can pay a lump sum payment called Refundable Accomdation Deposit (RAD) or Daily Accommdation Payment (DAP). They can RAD to DAP and vice versa. See information on the website of Department of Health, Australia Government

Renaud (1989) argued that chonsei serves as a vehicle for informal financing between tenants and landlords in the undeveloped housing finance market in Korea. As a result of financial repression during the period of economic development, a chonsei system rapidly expanded in Korea. Nonetheless, Gyourko and Han (1989) predicted that chonsei contracts would disappear; recently Kim (2013) also argued that chonsei contracts would disappear with no arbitrage gain from housing investments. Indeed it is entirely possible that chonsei contracts would have disappeared if they did not benefit both landlords and tenants. And in fact chonsei contracts remain a common form of rental contract, which suggests that they offer something of value to both tenants and landlords.[7]

From the tenant perspective, a chonsei deposit is a rung up the housing ladder to homeownership (Cho, 2005), as such chonsei operates somewhere between renting and homeownership (Kim and You, 2020; and Kim and Park, 2016). Tenants prefer chonsei lease contracts over monthly rental leases (Kim, 2013), as they can save more with an asset-based contract than with an income-based rental contract, which is similar to a Western-style rental contract. Cho (2005) proposes that chonsei contracts can promote homeownership by enabling general households to accumulate wealth and that, without such contracts, accounting for household portfolios would not be feasible in Korea. Chonsei contracts can affect household financial strategies (Kim, 2013; and Cho, 2005) and the strategies that both tenants and landlords pursue are incorporated into the value of a chonsei contract (Moon, 2018).

From the landlord perspective, on the other hand, a chonsei contract includes an option to default. Facing adverse conditons in the housing market, a landlord can transfer a rental property to a chonsei tenant even though in practice this does not occur very often (Ambrose and Kim, 2002). The value of a chonsei contract shifts the risks to the tenant and this theory is reintepretated by Bae

(https://www.health.gov.au/initiatives-and-programs/residential-aged-care/managing-residential-aged-care-services/managing-accommodation-payments-and-contributions-for-residential-aged-care, Mar, 26, 2021)

[7] According to the Korea Housing Survey, in 2019 58.0 per cent of households lived in their own homes, 15.1 per cent lived in chonsei-financed housing, and 23.0 per cent lived in monthly rental housing, either with deposits (19.7 per cent) or without deposits (3.3 per cent).

(2012), who separates continuous rental dividends from total returns on housing investments.[8] Within the framework of optimal stopping timing, our investor in the next section exercises an investment option with the understanding that the invested property could be transferred to the tenant.

It is worth noting that previous studies derive the value of a chonsei contract recursively. To estimate the value of a chonsei contract, this paper adopts a methodoloigy used by both Ambrose and Kim (2002) and Bae (2012). Moreover, the paper posits that the value of an option to invest under a chonsei contract resembles the value of an option to develop, as shown in Williams (1991), Bar-Ilan and Strange (1996), You (2012), and Adkins and Paxson (2017). As a result, the paper contributes to the literature by building a theoretical bridge between real estate investment and real estate leasing.

It seems reasonble to assume that a landlord, as an investor, makes an investment decision by considering a rental contract that is available in the market.[9] This paper shows that a landlord can exercise such a investment strategy when he knows that his property will be rented through a chonsei contract; the investor can change his investment decision if there is a default put option attached to a lease.[10]

III. THEORETICAL MODEL

1. Basic Assumptions

Our model begins with an investor who owns a parcel of land on which he builds a residential building in compliance with local zoning regulations. Such an investor can choose when to start

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[8] The value of a chonsei contract may include the landlord's default cost (Moon, 2018), which can be considered in our theoretical model.

[9] Kim (1990) argues that a chonsei landlord can use a chonsei deposit as a new investment oppourtunity.

[10] Previous studies assert that financial represession engendered the chonsei system and that the development of a well-fuctioning mortgage market should lead to the end of chonsei contracts. Unlike previous studies, this paper is silent as to why the chonsei system emerged or how the chonsei market might be reshaped.

the development project, the construction of which is completed immediately with no development lag, as in Bar-Ilan and Strange (1996). With our model, we assume that after completing the development projet, the investor as a landlord will operate the residential building.[11] For conveninence, the building is a single housing unit, the value of which follows a geometric Brownian motion

$$\mathrm{dH(t)} = (\mu - \delta)\mathrm{H(t)dt} + \sigma_{\mathrm{H}}\mathrm{H(t)dz(t)}, \tag{1}$$

where $\mathrm{t}(> 0)$ is the time, μ is the total expected return on the house, δ is rental flows, σ_{H} is the volatility of housing returns, and $\mathrm{dz(t)}$ is a standard Wiener process. As in Kau and Kim (1994), Kau *et al.(*1993) and Titman and Torous (1989), the future rental flows in equation (1) are a constant proportion of $\mathrm{H(t)}$, beginning instantaneously after the development is completed.

Our model derives the value of an investment option with a rental lease contract. By investing cost k, an investor owns a housing unit to be rented according to a chonsei contract. Upon completing the development project, therefore, the investor receives upfront deposit D, where $\mathrm{D} \leq \mathrm{k}$, which is to be fully returned to the tenant at the termination of the contract;[12] the maturity of a chonsei contract is legally determined by $\mathrm{T} > \mathrm{t}$.[13] As in Ambrose and Kim (2003) and Moon (2018), such an investor may default on the contract if $\mathrm{H(T)} \leq \mathrm{D}$. Such a decision is considered rational in the mortgage literature, as in Kau et al. (1993) and Kau and Kim (1994).

The optimal timing of an investment decision can be analyzed by backward induction, as in Wang and Zhou (2006) and Yao and Pretorius (2014). In the first stage, we derive the value of the

[11] The model produces the same implications as long as a residential building is traded at fair maket value in the housing market with no frictions, even though the developer, the landlord, and the investor are distinct agents.

[12] The size of chonsei deposit D is to be endogenouly determined. In a development project, the value of a housing unit is known after the completion of the project. Taking into account of the value of such a unit, market participants negotitate its rental value. This sequential process seems to be reasionable for new developments.

[13] Before July 2020, the legal maturity of a chonsei lease contract was 2 years. Starting in July 2020, however, such a contract can be extended once for another 2 years if the tenant wants to do so. Nonetheless, we treat T as two years, as our dates were collected when legal maturity was 2 years.

chonsei deposit by assuming that we know the optimal timing of the investment τ, where $0 < \tau < \mathrm{T}$ and where $\mathrm{T} - \tau$ is 2 years based on the Tenants Protection Act. Given the value of the chonsei deposit, in the second stage, we derive the value of an investment option and the optimal trigger $\mathrm{H}(\tau)$ using an optimal timing framework.

Following Buttimer *et al*. (2008) and Grenadier (1995) on the lease market and Ambrose *et al.* (1997), Kau and Keenan (1996), Hilliard *et al.* (1998), and Bardhan *et al*. (2006) on the mortgage market, we work in a risk-neutral world by replacing equation (1) with

$$\mathrm{dH(t)} = (\mathrm{r} - \delta)\mathrm{H(t)dt} + \sigma_{\mathrm{H}}\mathrm{H(t)dz(t)}, \tag{2}$$

where r is the fixed risk-free interest rate.[14] The house value in equation (2) is expected to appreciate at a constant rate.

1. The Value of a Chonsei Contract

The value of a lease contract is the sum of the expected discounted service flows until lease maturity (Grenadier, 1996; 2005). The value of the lease contract at origination with known $H(\tau)$ is

$$\begin{aligned} \mathrm{Y}(H(\tau), \tau, \mathrm{T} - \tau) &= \int_{\tau}^{T} e^{-\mu(t-\tau)} E_{\tau}\ \left(\delta H(t)\right) \mathrm{dt} \\ &= \int_{\tau}^{T} \left(e^{-\mu(t-\tau)} \delta H(\tau) e^{(\mu-\delta)(t-\tau)}\right) \mathrm{dt}\ = \int_{\tau}^{T} \left(\delta H(\tau) e^{-\delta(t-\tau)}\right) \mathrm{dt}, \end{aligned} \tag{3}$$

as in Majd and Pindyck (1987). In equation (3), as $\mathrm{T} \rightarrow \infty$, $\mathrm{Y}(\mathrm{H}(\tau), \tau, \mathrm{T} - \tau) = \mathrm{H}(\tau)$; if a tenant stays for $\mathrm{T} = \infty$, the value of the lease is the same as the value of the house. The value of an asset at τ can be divided into the use of the asset for the contract period and the discounted value of the expected house value at T:

[14] See Dixit (1994) for details on equation (2).

$$H(\tau) = \mathrm{Y}(H(\tau), \tau, \mathrm{T} - \tau) + \mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)} E_\tau\big(H(T-\tau)\big) \\ = \int_\tau^T e^{-\mu(t-\tau)} E_\tau\big(\delta H(\tau)\big)\,\mathrm{dt} + \mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)} E_\tau\big(H(T-\tau)\big), \tag{4}$$

as in Dixit (1994: 124). As in Ambrose and Kim (2009), the value of a chonsei contract is

$$\mathrm{Y}(H(\tau), \tau, \mathrm{T} - \tau) = \mathrm{D}\left(1 - \mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)}\right) + \mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)} E_\tau[\max\{D - H(T-\tau), 0\}], \tag{5}$$

as the landlord has an implicit put option to transfer the property to the tenant, as represented by the second term of the right-hand side of equation (5). We assume that there are no costs related to default, which is a common assumption in Ambrose *et al.* (1997), Riddiough (1997), You (2012), and Adkins and Paxson (2017). [15] Moon (2018) proposes that a chonsei landlord has an implicit option to default and that the foreclosure process requires a court decision. After a court auction, according to the Civil Execution Act amended in 2003, the landlord is not allowed to redeem a property, as the property right was already transferred to the winning bidder.[16]

By equating equation (3) to equation (5), we can derive the value of a chonsei contract

$$\mathrm{D} = \mathrm{H}(\tau) - [\mathrm{H}(\tau)\mathrm{e}^{-\delta(\mathrm{T}-\tau)}\Phi\,(\,\omega) - \mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)}\mathrm{D} * \Phi\left(\,\omega - \sigma\sqrt{T-\tau}\right)] \tag{6}$$

where

$$\omega = [\ln(\mathrm{H}(\tau)/\mathrm{D}) + (\mathrm{r} - \delta + \sigma^2/2)(\mathrm{T} - \tau)]/\sigma\sqrt{T-\tau}\,. \tag{7}$$

The proof is shown in Appendix 1 A).

The second term on the right-hand side of equation (6) is the value of a call option on a dividend-paying asset. The value of the chonsei deposit in equation (6) is the sum of the long position in a

[15] Considering default costs would complicate our model even though its implications would remain invariant.

[16] The Civil Execution Act is to "prescribe the procedures for compulsory execution, an auction to exercise a security right, an auction." As a court decision is final and conclusive, the old landlord is not allowed to revoke the decision; no call option in Ambrose and Kim (2003) is embedded in the default option by the amendment of the Civil Execution Act. A call option embedded in the default opton can be incoporated into our model. Nonetheless, this would require a numerical analysis, as we fail to provide a closed-form solution for both a default strategy and an investment strategy with chonsei. The solution helps us understand the implications of the theory more fully.

housing asset and the short position in a call option, such that the landlord has a right to take the asset back if the value of the asset exceeds the value of the deposit at maturity.

Proposition 1. The D has a unique solution.

The proof of proposition 1 is shown in Appendix 1. Equations (6) and (7) provide the closed-form solution of the value of the chonsei deposit.[17] These equations appear in Bae (2012), even though, to derive D, our model adopts a different approach from Bae's approach. Robichek and Van Horne (1967) propose that capital budgeting needs to consider possible abandonment. Their argument is reflected in the second term of the right-hand side of equation (5). With known $\mathrm{H}(\tau)$, we can derive the value of the chonsei deposit; immediately after starting a project, D is determined by the tenant and the landlord.

Corollary 1.

1) With an increase in δ, the equilibrium D increases; 2) with an increase in σ, the equilibrium D decreases; and 3) with an increase in r, the equilibrium D decreases.

See appendix 1 C) for the proof.

It is stragithforward to derive the effects of δ on D, as the size of the deposit increases with an increase in δ. With an increase in σ, the value of the option increases and, with given $\mathrm{H}(\tau)$, D decreases as the value of the call option increases. An increase in r leads to an increase in the value of the call option, which reduces the value of a chonsei contract.

2. The Value of a Development Option under a Chonsei Contract

[17] Unlike the proofs of equations (6) and (7) that appear in the appendix, the put–call parity relationship in Guo and Su (2006) can be used for the derviation of those equations.

Now suppose an investor determines when to start developing a residential building that is to be rented through a chonsei contract. Such a development option with no rental contract is investigated by Capozza and Helsley (1990), Williams (1991), and Capozza and Li (1994), all of which can be distinguished from our paper.

1) Analytic Results

The value of an investment option $\mathrm{F}(t)$ that an investor holds satisfies the partial differential equation

$$\frac{1}{2}\sigma_{\mathrm{H}}^{2}\mathrm{H}^{2}F''(H)+(\mathrm{r}-\delta)\mathrm{H}\mathrm{F}'(\mathrm{H})-rF(H)=0. \tag{8}$$

The general solution to equation (8) is $\mathrm{F(H)}=\Omega_1\, H^{\beta}+\Omega_2\, H^{\gamma}$, where both constants, Ω_1 and Ω_2, are to be determined. Moreover, $\beta=\frac{1}{2}-\frac{(\mathrm{r}-\delta)}{\sigma^2}+\sqrt{\left[\frac{(\mathrm{r}-\delta)}{\sigma^2}-\frac{1}{2}\right]^2+2\frac{\mathrm{r}}{\sigma^2}]}>1$ and $\gamma=\frac{1}{2}-\frac{(\mathrm{r}-\delta)}{\sigma^2}-\sqrt{\left[\frac{(\mathrm{r}-\delta)}{\sigma^2}-\frac{1}{2}\right]^2+2\frac{\mathrm{r}}{\sigma^2}]}<0$, where $\frac{\partial\beta}{\partial r}<0$, $\frac{\partial\beta}{\partial\delta}>0$, and $\frac{\partial\beta}{\partial\sigma}<0$. See Dixit (1994:144) for the details.

Three boundary conditions—the initial, the value-matching, and the smooth-pasting conditons—are required. The initial condition $\lim_{H\to 0}F(H)=0$ leads to $\mathrm{F(H)}=\Omega_1\, H^{\beta}$. The value-matching condition is

$$\mathrm{F(H(}\tau\mathrm{))}=\int_{\tau}^{\infty}e^{-\mu(t-\tau)}E_{\tau}\ \left(\delta H(t)\right)\mathrm{dt}-\mathrm{k}. \tag{9}$$

And the smooth-pasting condition is

$$\partial\mathrm{F}\big(\mathrm{H}(\tau)\big)/\ \partial\mathrm{H}(\tau)=\partial[\int_{\tau}^{\infty}e^{-\mu(t-\tau)}E_{\tau}\ \left(\delta H(t)\right)\mathrm{dt}-\mathrm{k}]/\ \partial\ \mathrm{H}(\tau). \tag{10}$$

At τ, the developer receives D, which is to be returned to the tenant at $T-\tau$, when the option in equation (5) may be exercised. Consequently the developer has an incentive to take the less marginal benefits of waiting further, as he takes no account of uncertainty transferred through the chonsei. The option protects the developer from bad states of nature at the maturity of the rental contract.

When assuming the abovementioned conditions, the optimal trigger is

$$\mathrm{H}(\tau) = [\beta/(\beta - \mathrm{e}^{-\delta(T-\tau)}\Phi(\omega))]k. \tag{11}$$

See appendix 1 for the details.

Proposition 2. There exists a unique solution that generates the optimal trigger $\mathrm{H}(\tau)$.

The proof of proposition 2 is shown in Appendix 1.

The value of a development option is

$$\mathrm{F(H)} = (\mathrm{H}(\tau) - \mathrm{k}) * [H/\mathrm{H}(\tau)]^{\beta}, \tag{12}$$

where we take $\mathrm{H}(\tau)$ as in equation (11). As a result, we can prove proposition 3.

Proposition 3. A developer exercises a development option earlier under a chonsei contract than he would without a chonsei contract.

The proof of proposition 3 is shown in Appendix 1.

In an uncertain business environment, the optimal trigger with no chonsei contract s $\mathrm{H}(\tau_n) = [\beta/(\beta - 1)]k$. With a chonsei contract, $\mathrm{H}(\tau)$is lower than $\mathrm{H}(\tau_n)$; the difference between the two investment timings can be significantly large, even though the difference between the trigger values is very small.

Our theory is similar to theories proposed in Williams (1991), Bar-Ilan and Strange (1996), You (2012), and Adkins and Paxson (2017). Nontheless, Williams (1991), Bar-Ilan and Strange (1996), and Adkins and Paxson (2017) consider an investor with an abandonment option and You (2012) proposes an investor with a default option on a loan. Those options are distinguished from a default option on a lease contract for our investor.

Corollary 2.

1)With an increase in δ, the equilibrium $H(\tau)$ decreases; 2) with an increase in r, the equilibrium $H(\tau)$ increases; and 3) with an increase in k, the equilibrium $H(\tau)$ increases

The proof of corollary 2 is in appendix 1.

An investor is more likely to invest at the higher level of δ, as future rental flows increase. In addition, with an increase in δ the equilibrium D increases, as shown in Corollary 1, and the amount that our investor invests decreases. With an increase in r, the present value of the rental flows decreases, even though δ is fixed and an investor has an incentive to wait longer for the project with higher r. The investor spends more with an increase in k and is more likely to hesitate to start a development project.

2) Numerical Simulations

Appendix 2 includes a numerical analysis of both propositions 2 and 3. The average quarterly 1-year bond rate between Q1 2001 and Q1 2020 in Korea ranges from 6.03 per cent to 1.18 per cent with an average of 3.31 per cent. We assume that δ and σ are 3.0 per cent and 1.0 per cent, respectively; it is unfortunate that δ and σ cannot be measured with market data. In addition, k is assumed to be 100 and the investment cost is financed through a 2-year chonsei contract; according to the real options literature, an investment trigger value that we expect is higher than the cost.[18] The first graph shows that the optimal trigger under the chonsei contract $H(\tau)$ is determined at the intersection of $f(H)$ and $g(H)$, both of which are defined in appendix1 (B). This trigger is also lower than the trigger with no chonsei contract, as indicated by the red circle in the first graph in appendix 2.

[18] Using a backward induction methodology, we derive $H(\tau)$ with known D. For our numerical analysis, nevertheless, D must be assumed.

In the literature on real options, it widely held that as uncertainty increases an investor delays an investment decision, which can be proved by $\frac{\partial \mathrm{H}(\tau_n)}{\partial \sigma} = \frac{-1}{(\beta-1)^2}\frac{\partial \beta}{\partial \sigma}>0$. Nonetheless, we cannot prove this relationship analytically for the trigger with a chonsei contract.[19] Therefore, the numerical relationship between $\mathrm{H}(\tau)$ and σ is shown in the second graph in appendix 2, which shows that $\mathrm{H}(\tau)$ increases with an increase in σ. As is the case with other models, our model shows that an investor (who depends on a chonsei contract) delays investment with greater uncertainty.

IV. EMPIRICAL EVIDENCE

This paper's key theory is that chonsei contracts promote housing investment. as in proposition 3; an investor has a stronger incentive to invest in a rental property in a chonsei market than in a property that is not in a chonsei market. In Korea, chonsei contracts are readily available for housing investors and we build an empirical model of new housing supply using time-series data on nationwide housing starts. Our empirical analysis shows that greater accessibility of chonsei deposits promotes housing investment, which in this section means housing starts.

1. Empirical model

Our empirical analysis uses quarterly time-series data that cover the period running from Q1 2001 through Q1 2020; a new housing supply may include both housing starts and housing permits. According to the Ministry of Land, Infrastructure and Transport, housing starts are a coincidental indicator whereas housing permits are a leading indicator. Housing starts seem to be more apporopriate for our analysis, as some housing permits can be cancelled even though both variables can proxy for new housing supply (Somerville, 2001); some cancelled permits appear to be

[19] The sign of $\frac{\partial \mathrm{g(H)}}{\partial \sigma} = \underset{(+)}{\frac{e^{-\delta(T-\tau)}\Phi(\omega)}{\beta^2}}\frac{\partial \beta}{\partial \sigma} - \underset{(+)}{\frac{e^{-\delta(T-\tau)}\varphi(\omega)}{\beta}}\frac{\partial \omega}{\partial \sigma}$ is undetermined, as we have no analytic information indicating the actual values of both $\frac{\Phi(\omega)}{\beta}\frac{\partial \beta}{\partial \sigma}$ and $\varphi(\omega)\frac{\partial \omega}{\partial \sigma}$.

included in the number of housing permits. Housing starts represent the total number of residential buildings nationally, where those starts are reported to the local governenment.[20]

We assume the presence of a representative developer in our model, as in Mayer and Somervill (2000). New housing supply is a function of housing prices, the cost of land, construction costs, and interest rates.[21] In an equilibrium urban growth model, as in Capozza and Helsley (1989) and Rosenthal and Helsley (1994), housing prices have a long-run relationship with development costs. Grimes and Aitken (2010) propose, among those variables, the existence of a cointegration relationship and recently Winkler (2016) finds long-term relationships in several countries. To consider such a cointegration relationship, we adopt a Dynamic Ordinary Least Squares (DOLS) approach, as in Stock and Watson (1988).[22]

The logarithm of housing starts is

$$\mathrm{LHS_t} = \alpha_0 + \beta \mathrm{X_t} + \sum_{j=-k_1}^{k_2} \theta_j \Delta \mathrm{X_{t+j}} + e_t, \quad (13)$$

where X includes a vector of regressors that include housing prices, the cost of land, construction costs, and interest rates, as in <Table 1>. β is the the vector of coefficients and α_0 is a constant term, while k_1 and k_2 are the lag and the lead, respectively, Δ is the differential, and e_t is the residual.

To test our theory, the availability of chonsei contracts needs to be measured and we use the CDR as a proxy for the availability of such deposits. The ratio has been released by the KB (Kookmin) Bank since December 1998. Its affiliated real estate agents provide the bank with assessed chonsei deposits and the assessed value of condominiums. With those figures the bank

[20] Residential buildings include condominiums, multi-unit buildings, single-family detached houses, and other types; the number of buildings of each type is unavailable. Starts in any of these categories should be reported to the government.
[21] Our use of housing starts is distinguished from other housing supply proxies such as capital investments (Caldera and Johansson, 2013; Winkler, 2016), the growth of existing stock (Ball et al, 2010), and supply constraints (Glaeser et al., 2006).
[22] To analyze housing supplies, Winkler (2016) also adopted a DOLS model.

calculates the ratio, considering the distribution of condominium dwellings at the national level according to the National Census.

We build a model of housing starts that includes the CDR. Note that real estate agents affiliated with the KB Bank provide information on existing units but do not report predicted CDRs for uncompleted houses. When starting building projects, developers can observe a CDR for existing stock, which varies during the period of development. As a result of construction lags, we can set aside the issue of endogenity. [23] In addition, there are far fewer housing starts than houses in the existing stock. Moreover, the CDR represents (existing) condomimums, even though the dependent variable includes starts for all types of residential buildings.[24] Nevertheless, the inclusion of the lag and lead terms in equation (12) can help to control for the endogeneity of regressors;[25] we test an alternative model in subsection 4.

2. Data and Basic Data Analyses

1) Variables and definitions

LHS in equation (12) is the logarithm of quarterly housing starts (HS), which is the sum of monthly residential building starts released by the Ministry of Land, Infrastructure and Transport and Statistics Korea. We analuze residential building starts, as the number of housing unit starts is officially available from 2011. The regressors are quarterly variables of housing prices (HP), land costs (LP), construction costs (CC), and the 1-year government bond rate (BRY1), each of which is converted to real terms with the consumer price index (CPI). The variables are drawn from KB Bank, the Korea Appraisal Board, the Korea Institute of Civil Engineering and Building Technology, the Bank of Korea, and Statistics Korea, as in <Table 1>. Except for the real bond rate

[23] A CDR for a new development is determined when the development is completed. For condomunium buildings in general in Korea, the lag between start and completion is two or three years.
[24] The CDR index for all types of housing units is available starting in Q2 2011.
[25] The optimal k_1 and k_2 are determined automatically based on Schwarz Information Criteria (SIC) with maximum lags of 3 (i.e. within a year).

(RBR), the real-term variables are coverted to logarithmic variables: LRHP for real housing prices, LRLP for real land costs, LRCC for real construction costs).

<Table 2> shows that the average HS is 18.1 thousand during the sample period, varying from a minimum of 6.5 thousand in Q3, 2005 to a maximum of 33.6 thousand in Q2, 2016. HP, LP, CC, and CPI are an index. As real estate developers often depend on short-term debt, we use the 1-year bond rate, which averages 3.31 per cent, varying from a minimum of 1.18 per cent to a maximum of 6.03 per cent. Over the 77 quarters running from Q1, 2001 to Q1, 2020, the average CDR is 63.7 per cent, varying from a minimum of 52.4 in Q1 2009 to a maximum of 75.7 per cent in Q3 2017.[26]

2) Unit root test and cointegration test

To determine the stationarity of time-series data, we run the Augmented Dickey-Fuller (ADF) test, based on the following equation:

$$\Delta \mathrm{y_t} = \alpha_\mathrm{u} + \rho\, \mathrm{y_t} + \sum_{j=1}^{k} \theta_j \Delta \mathrm{y_{t-j}} + e_t, \tag{14}$$

where $\mathrm{y_t}$ is each variable in the model, α_u is the constant term, ρ is the autoregressive coefficient and k is an optimal lag to be determined by the Schwarz Information Criteria (SIC). The null hypothesis of a unit root is $\mathrm{H_0}: \rho = 1$. <Table 3> shows the results; LHS is I (1), LCDR is I(1), LRHP is I(0), LRLP is I(1), LRCC is I(1), and RBR is I(0). These results imply that the four variables are integrated of order one. Regressions that use non-stationary variables in levels may produce spurious regression results but if non-stationary time-series variables are cointegrated they form an equilibrium in the long run.

House prices can be cointegrated with cost variables such as land prices, construction costs, and others. We perform a cointegration test, as in Engle and Granger (1987), which is commonly used. The ordinary least-squares regression (OLS) is

[26] The CDR of 63.7 per cent implies that, for example, if the value of a condo is KRW 100 million, the chonsei deposit the owner receives by renting the same condo through a 2-year chonsei contract is KRW 637 million.

$$\mathrm{LHS_t} = \mathrm{\alpha_c} + \mathrm{\beta_c X_t} + e_t, \qquad (15)$$

$$\text{where } \widehat{e_t} = \mathrm{LHS_t} - \widehat{\mathrm{\alpha_c}} + \widehat{\mathrm{\beta_c}}\mathrm{X_t}$$

where $\widehat{\alpha_c}$ and $\widehat{\beta_c}$ are the estimated intercept and the estiated coefficient, respectively, by OLS. e_t represent deviations from the long-term relationship and, as shown in < Table 4>, $\widehat{e_t}$ is a stationary process and we can conclude that there is a cointegration relationship between $\mathrm{LHS_t}$ and the vector of covariates $\mathrm{X_t}$, which includes LRHP, LRLP, LRCC, RBR and LCDR; model 1 excludes LCDR and model 2 includes LCDR. The results are the same as expected.[27]

3. Empirical Results

1) Basic DOLS regression

In the previous section we found a cointegration relationship in the Korean housing market, and using the DOLS with equation (12) is appropriate. The regression results are reported in <Table 5>. Model 1 is the base model with four covarates: LRHP, LRLP, LRCC, RBR. Models 2–4 include the key covariate LCDR (logarithm of the CDR), which is salient in our theory. All the models include a time trend and a constant term.

The estimated coefficent of LHRP for model 1 is positive and the other estimated coefficients are negative; the results are what we expected to find. The estimated price elasticity of the new housing supply is larger than one unit; a one per cent increase in real house prices is associated with a 1.5 per cent increase in the number of residential building starts. Residential building starts can be differentiated to some extent from housing unit starts (Mayer and Somerville, 2000); Phang et al. (2010) find that the price elasticities of housing starts tend to be higher than the price elastisticities of changes in the housing stock.[28] The estimated coefficient of LRCC is negative, as

[27] Johansen's (1995) cointegration tests also support the existence of cointegration in the Korean housing market.

[28] Nonetheless, residential building starts as a proxy for new supply is differentiated from other proxies of new housing supply in Caldera and Johansson (2013), Winker (2016), Ball et al. (2010). and Glaeser et al(2006).

predicted in Corollary 2. The estimated coefficient of LRLP is also negative and our model can incorporate land costs, an increase in which negatively impacts new development. Interest rates can be critical for new investment and, as in Corollary 2, the estimated coefficient of RBR is also negative. As a consequence, model 1 produces estimated results that are consistent what our theory predicts.

2) The Effects of chonsei on residential building starts

To see the effects of chonsei contracts on residential building starts, model 2 includes LCDR. A one per cent increase in the CDR rate leads to a 0.89 per cent increase in the number of residential building starts. These results support proposition 3; it is worth noting that we use the ratio instead of the amount of a chonsei contract. In a real options model, uncertainty matters and, using a GARCH (Generalized AutoRegressive Conditional Heteroskedasticity) model, we propose a proxy for the volatility of LRHP. In Appendix 3 we explain the process of estimating $\hat{\sigma}^2_{\mathrm{lrhp}}$. As predicted by real options theory, the estimated coefficient of $\hat{\sigma}^2_{\mathrm{lrhp}}$ is negative, which supports numerical analysis we present in Appendix 2.

During the sample period, moreover, the housing market suffered through the Global Financial Crisis (GFC). To stimulate dampened housing markets during the the crisis period, the Korean government nonetheless proposed several policy measures; during the period running from June through November 2008, for example, it announced seven policy packages (Kim and You, 2020). As a consequence, for the crisis period of 2008 through 2010 the estimated coefficient of GFC in model 4 is positive.

3) Robustness test: Fully Modiflied Least Squares (FMOLS)

To incorporate the cointegration relationship discussed in the previous subsection into our analysis, we implemented a parametric DOLS model, which can manage edogeneity efficiently. Unlike DOLS, Fully Modiflied Least Squares (FMOLS) is a semiparametric approach to correcting this issue that is associated with a long-run relationship. In the presence of a cointegrating relationship, the FMOLS adjusts for the effects of serial correlations and endogeneity (Phillips, 1995; see also Phillips and Hansen, 1990). We run an FMOLS regression as a robustness check, even though the FMOLS is known to best fit I(1) regressors; note that LRHP and RBR are I(0) in <Table 3>.

The regression results are reported in <Table 6> and the results for Model 1 show that, except for the results for RBR, we find similar results for the other variables, as is the case with Model 1 in the previous subsection. The key variable is LCDR, whose estimated coefficient under model 2 is positive and statistically significant at the 1 per cent level.

V. CONCLUSION

This paper investigates the effects of chonsei contracts on investment in the housing market. Previous papers agree that the chonsei system expanded in response to financial repression during the period of economic development. It is known that chonsei contracts impacted financial decisions and portfolio choices at the household level, especially for those seeking to reach the top of the housing ladder to achieve homeownership. This paper finds that real estate developers' investment strategies can be determined by chonsei contracts.

This paper shows that chonsei contracts play the role of leveraged investments for housing investors. Based on a chonsei contract, a housing investor has the option to transfer a property to a tenant at the maturity of the chonsei contract. While making a timing decision, such an investor considers the advantage of such an opton that is embedded in a chonsei contract. We therefore argue that chonsei contracts promote new investments in the housing market. This argument is validated by a real options model and is confirmed by national-level housing starts data in Korea, highlighting the effects of chonsei contracts on the supply of new housing.

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<Table 1> Quarterly Variables and Their Sources

	Description	Source
HS	Residental building starts	Ministry of Land, Infrastructure and Transport
CDR	Ratio of a chonsei deposit to a housing unit's value	KB Kookmin Bank
HP	House price index	KB Kookmin Bank
LP	Land price index	Korea Apprsal Board
CC	Construction cost index	Korea Institute of Civil Engineering and Building Technology
BRY1	Government 1-year bond rate	Bank of Korea
CPI	Consumer price index	Statistics Korea
GFC	GFC dummy (1 if 2018≤year≤ 2010, 0 otherwise)	
LHS	Log(HS)	
LCDR	Log(CDR)	
LRHP	Log(HP/CPI)	
LRLP	Log(LP/CPI)	
LRCC	Log(CC/CPI)	
RBR	BRY1 - annual % change in CPI	

<Table 2> Summary Statistics: from Q1, 2001 through Q1, 2020

	Obs	Mean	Max	Min	Std. Dev.	Unit
HS	77	18,102	33,651	6,500	6,399	No. of residential buildings
CDR	77	63.7	75.7	52.4	7.5	%
HP	77	85.8	107.5	52.2	14.8	100 as of Dec 2015
LP	77	89.3	116.2	65.1	12.5	100 as of Nov 2016
CC	77	87.0	118.5	56.7	18.0	100 as of 2015
BRY1	77	3.31	6.03	1.18	1.39	%
CPI	77	89.8	105.7	69.0	11.7	100 as Dec 2015

<Table 3> Augmented Dickey-Fuller Test Results

		Lag[1)]	t-statistics	Prob[2)]
LHS	Level	9	-1.67	0.44
	1st Difference	10	-4.03	0.00***
LCDR	Level	2	-1.51	0.52
	1st Difference	1	-3.52	0.02***
LRHP	Level	0	-3.58	0.00***
	1st Difference	0	-7.72	0.00***
LRLP	Level	2	-1.10	0.71
	1st Difference	1	-4.37	0.00***
LRCC	Level	0	-0.07	0.94
	1st Difference	0	-6.79	0.00***
RBR	Level	3	-3.07	0.03**
	1st Difference	2	-14.06	0.00***

Note: 1) The number of optimal lags is determined based on SIC
2) *** p<0.01, ** p<0.05, * p<0.1

<Table 4> Cointegration Test

	Lag[1)]	t-statistics	Prob[2)]
Model 1[3)]: $\hat{e_t}$	8	-3.72	0.003***
Model 2[4)]: $\hat{e_t}$	8	-4.19	0.000***

Note: 1) The number of optimal lags is determined based on SIC
2) *** p<0.01, ** p<0.05, * p<0.1
3) The covariates include LRHP, LRLP, LRC, and RBR.
4) The covaraites include LRHP, LRLP, LRC, RBR and LCDR.

<Table 5> Cointegration Regression Results: DOLS

	Model 1	Model 2	Model 3	Model 4
LCDR		0.89**	2.03***	2.27***
		(0.37)	(0.41)	(0.36)
LRHP	1.52*	2.86***	1.90**	2.39***
	(0.82)	(0.91)	(0.95)	(0.83)
LRCC	-5.49***	-4.77***	-2.86**	-3.68***
	(1.49)	(1.45)	(1.22)	(1.11)
LRLP	-3.85***	-3.07***	-1.95**	-2.26***
	(0.68)	(0.79)	(0.77)	(0.67)
RBR	-0.05*	-0.05**	-0.04***	-0.04***
	(0.03)	(0.02)	(0.01)	(0.01)
$\hat{\sigma}^2_{\mathrm{lrhp}}$			-1491.47***	-1229.12***
			(477.98)	(420.04)
GFC				0.19 ***
				(0.07)
TREND	0.03***	0.02***	0.00	0.00
	(0.01)	(0.01)	(0.00)	(0.00)
CONSTANT	8.39***	5.15***	1.48	0.26
	(0.30)	(1.34)	(1.44)	(0.19)
Obs	76	76	76	76
R-Squared	0.852	0.870	0.858	0.872

Note: 1) The number of optimal lags is determined based on SIC criteria with maximum = 3.
2) *** p<0.01, ** p<0.05, * p<0.1

<Table 6> Robustness Test: FMOLS

	Model 1	Model 2
LCDR		1.00***
		(0.26)
LRHP	2.37**	3.89***
	(0.76)	(0.73)
LRCC	-3.67***	-2.81***
	(1.21)	(1.04)
LRLP	-4.58***	-3.73***
	(0.67)	(0.56)
RBR	0.02**	0.02**
	(0.01)	(0.01)
TREND	0.02***	0.01***
	(0.00)	(0.00)
CONSTANT	8.79***	5.14***
	(0.24)	(0.99)
Obs	77	77
R-Squared	0.636	0.681

Note: *** p<0.01, ** p<0.05, * p<0.1

[Appendix 1] The Proofs

A) The proof of equation (6).

Following Buchanan (2012), who derives the value of a European put option on an asset that pays a continuous dividend, we can define the value of the put option in equation (5). As equation (3) is equivalent to equation (5),

$$\mathrm{H}(\tau)-\mathrm{H}(\tau)\mathrm{e}^{-\delta(\mathrm{T}-\tau)}$$
$$=\mathrm{D}\left(1-\mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)}\right)+\mathrm{D}\mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)}*\Phi\left(\sigma\sqrt{T-\tau}-\omega\right)-\mathrm{H}(\tau)\mathrm{e}^{-\delta(\mathrm{T}-\tau)}*\Phi(-\omega)$$
$$=\mathrm{D}-\mathrm{D}\,\mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)}*\Phi\left(\omega-\sigma\sqrt{T-\tau}\right)+\mathrm{H}(\tau)\mathrm{e}^{-\delta(\mathrm{T}-\tau)}(\Phi(\omega)-1)$$

where $\omega=\frac{\ln\left(\frac{\mathrm{H}(\tau)}{\mathrm{D}}\right)+\left(\mathrm{r}-\delta+\frac{\sigma^2}{2}\right)(\mathrm{T}-\tau)}{\sigma\sqrt{T-\tau}}$.

B) The proof of a unique D.

From equation (6), we can define both $\mathrm{f}(\mathrm{D})=\mathrm{H}(\tau)-\mathrm{D}$, where $0<\mathrm{D}<\mathrm{H}(\tau)$ and $\mathrm{g}(\mathrm{D})=\mathrm{H}(\tau)\mathrm{e}^{-\delta(\mathrm{T}-\tau)}\Phi(\omega)-D\mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)}\Phi\left(\omega-\sigma\sqrt{T-\tau}\right)$, which is a call option with the underlying asset-paying dividend.

A unique solution is satisfied by the following four conditions:

1) $\frac{\partial\mathrm{g}(\mathrm{D})}{\partial\mathrm{D}}=\mathrm{H}(\tau)\mathrm{e}^{-\delta(\mathrm{T}-\tau)}\frac{\partial\Phi(\omega)}{\partial\mathrm{D}}-D\mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)}\frac{\partial\Phi\left(\omega-\sigma\sqrt{T-\tau}\right)}{\partial\mathrm{D}}-\mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)}\Phi\left(\omega-\sigma\sqrt{T-\tau}\right)$
$=\mathrm{H}(\tau)\mathrm{e}^{-\delta(\mathrm{T}-\tau)}\frac{1}{2\sqrt{\pi}}\mathrm{e}^{-\frac{\omega^2}{2}}\frac{1}{\sigma\sqrt{T-\tau}}\left(-\frac{1}{D}\right)-D\mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)}\frac{1}{2\sqrt{\pi}}\mathrm{e}^{-\frac{\omega^2}{2}}\frac{\mathrm{H}(\tau)}{D}\mathrm{e}^{(\mathrm{r}-\delta)(\mathrm{T}-\tau)}\frac{1}{\sigma\sqrt{T-\tau}}\left(-\frac{1}{D}\right)$
$-\mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)}\Phi\left(\omega-\sigma\sqrt{T-\tau}\right)$
$=-\mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)}\Phi\left(\omega-\sigma\sqrt{T-\tau}\right)>-1.$
2) $\frac{\partial^2\mathrm{g}(\mathrm{D})}{\partial\mathrm{D}^2}=-\mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)}\{-\frac{\varphi\left(\omega-\sigma\sqrt{T-\tau}\right)}{D\sigma\sqrt{T-\tau}}\}>0,$
3) As $\lim_{\mathrm{D}\to0^+}\mathrm{H}(\tau)\mathrm{e}^{-\delta\mathrm{T}}\Phi(\omega_\tau)=\mathrm{H}(\tau)\mathrm{e}^{-\delta\mathrm{T}}$, $\mathrm{g}(\mathrm{D})=\mathrm{H}(\tau)\mathrm{e}^{-\delta\mathrm{T}}\Phi(\omega_\tau)<H(\tau)$, and
4) $\mathrm{g}(\mathrm{H}(\tau))>0$.

C) Proof of Corollary 1: comparative statics of D.

1) $\frac{\partial\mathrm{g}(\mathrm{D})}{\partial\delta}=-\mathrm{H}(\tau)(T-\tau)\mathrm{e}^{-\delta(\mathrm{T}-\tau)}\varphi(\omega)+\mathrm{H}(\tau)\mathrm{e}^{-\delta(\mathrm{T}-\tau)}\frac{\partial\Phi(\omega)}{\partial\delta}-D\mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)}\frac{\partial\Phi\left(\omega-\sigma\sqrt{T-\tau}\right)}{\partial\delta}$
$=-\mathrm{H}(\tau)(T-\tau)\mathrm{e}^{-\delta(\mathrm{T}-\tau)}\varphi(\omega)+\mathrm{H}(\tau)\mathrm{e}^{-\delta(\mathrm{T}-\tau)}\frac{1}{2\sqrt{\pi}}\mathrm{e}^{-\frac{\omega^2}{2}}\frac{\sqrt{T-\tau}}{\sigma}$
$-D\mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)}\frac{1}{2\sqrt{\pi}}\mathrm{e}^{-\frac{\omega^2}{2}}\frac{\mathrm{H}(\tau)}{D}\mathrm{e}^{(\mathrm{r}-\delta)(\mathrm{T}-\tau)}\frac{\sqrt{T-\tau}}{\sigma}$
$=-\mathrm{H}(\tau)(T-\tau)\mathrm{e}^{-\delta(\mathrm{T}-\tau)}\varphi(\omega)<0,$

2) $\frac{\partial\mathrm{g}(\mathrm{D})}{\partial\sigma}=\mathrm{H}(\tau)\mathrm{e}^{-\delta(\mathrm{T}-\tau)}\varphi(\omega)\frac{\partial\omega}{\partial\delta}-D\mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)}\varphi\left(\omega-\sigma\sqrt{T-\tau}\right)\left(\frac{\partial\omega}{\partial\delta}-\sqrt{T-\tau}\right)$
$=\mathrm{H}(\tau)\sqrt{T-\tau}\mathrm{e}^{-\delta(\mathrm{T}-\tau)}\varphi(\omega)$, and

3) $\frac{\partial\mathrm{g}(\mathrm{D})}{\partial\mathrm{r}}=\mathrm{H}(\tau)\mathrm{e}^{-\delta(\mathrm{T}-\tau)}\varphi(\omega)\frac{\partial\omega}{\partial\mathrm{r}}-D\mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)}\varphi\left(\omega-\sigma\sqrt{T-\tau}\right)\frac{\partial\omega}{\partial\mathrm{r}}+\mathrm{D}(T-\tau)\mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)}\Phi\left(\sigma\sqrt{T-\tau}-\omega\right)=\mathrm{D}(T-\tau)\mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)}\Phi\left(\sigma\sqrt{T-\tau}-\omega\right).$

D) The derivation of the value of a development option and its trigger value

Upon investment at τ, the developer receives D and $H(\tau) = \int_{\tau}^{\infty} e^{-\mu(t-\tau)} E_{\tau}\ \left(\delta H(t)\right) \mathrm{dt} = \mathrm{Y}(H(\tau), \tau, \mathrm{T}-\tau) + \int_{T}^{\infty} e^{-\mu(t-\tau)} E_{\tau}\ \left(\delta H(t)\right) \mathrm{dt} = \mathrm{D} - \mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)} \mathrm{D} * \Phi\left(\omega - \sigma\sqrt{T-\tau}\right) + \mathrm{H}(\tau) \mathrm{e}^{-\delta(\mathrm{T}-\tau)} \Phi(\omega)$, which implies that the developer has an option to default at the maturity of the chonsi contract.

From the value-matching codition, $\Omega_1 = (\mathrm{H}(\tau) - k)\mathrm{H}(\tau)^{-\beta}$ and $\frac{\partial \mathrm{F}(\mathrm{H}(\tau))}{\partial \mathrm{H}(\tau)} = -\mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)} \mathrm{D} * \varphi\left(\omega - \sigma\sqrt{(T-\tau)}\right) \frac{1}{\sigma\sqrt{(T-\tau)}} \frac{1}{\mathrm{H}(\tau)} + \mathrm{e}^{-\delta(T-\tau)} \Phi(\omega) + \mathrm{H}(\tau) \mathrm{e}^{-\delta(\mathrm{T}-\tau)} \varphi(\omega) \frac{1}{\sigma\sqrt{T-\tau}} \frac{1}{\mathrm{H}(\tau)}$.

Therefore, $\beta(\mathrm{H}(\tau) - k) = \mathrm{e}^{-\delta(T-\tau)} \Phi(\omega) \mathrm{H}(\tau) + \left[\mathrm{e}^{-\delta(\mathrm{T}-\tau)} \mathrm{H}(\tau) \varphi(\omega) \frac{1}{\sigma\sqrt{(T-\tau)}} - \mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)} \mathrm{D} * \varphi\left(\omega - \sigma\sqrt{T-\tau}\right) \frac{1}{\sigma\sqrt{T-\tau}}\right]$,

where $\mathrm{e}^{-\delta(\mathrm{T}-\tau)} \mathrm{H}(\tau) \varphi(\omega) \frac{1}{\sigma\sqrt{(T-\tau)}} - \mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)} \mathrm{D} * \varphi\left(\omega - \sigma\sqrt{T-\tau}\right) \frac{1}{\sigma\sqrt{T-\tau}} = 0$, which can be proven by

$$
\begin{aligned}
&\mathrm{e}^{-\delta(\mathrm{T}-\tau)} \mathrm{H}(\tau) \varphi(\omega) - \mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)} \mathrm{D} \varphi\left(\omega - \sigma\sqrt{T-\tau}\right) \\
&= \mathrm{e}^{-\delta(\mathrm{T}-\tau)} \mathrm{H}(\tau) \frac{1}{\sqrt{2\pi}} e^{-\frac{\omega^2}{2}} - \mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)} \mathrm{D} e^{-\frac{\left(\omega - \sigma\sqrt{T-\tau}\right)^2}{2}} \\
&= \mathrm{e}^{-\delta(\mathrm{T}-\tau)} \mathrm{H}(\tau) \frac{1}{\sqrt{2\pi}} e^{-\frac{\omega^2}{2}} - \mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)} \mathrm{D} e^{-\frac{\omega^2 - 2\omega\sigma\sqrt{T-\tau} + \sigma^2(T-\tau)}{2}} \\
&= \mathrm{e}^{-\delta(\mathrm{T}-\tau)} \mathrm{H}(\tau) \frac{1}{\sqrt{2\pi}} e^{-\frac{\omega^2}{2}} - \mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)} \mathrm{D} e^{-\frac{\omega^2}{2}} e^{\ln\left(\frac{\mathrm{H}(\tau)}{\mathrm{D}}\right) + \left(\mathrm{r} - \delta + \frac{\sigma^2}{2}\right)(\mathrm{T}-\tau)} e^{-\frac{\sigma^2(T-\tau)}{2}} \\
&= \mathrm{e}^{-\delta(\mathrm{T}-\tau)} \mathrm{H}(\tau) \frac{1}{\sqrt{2\pi}} e^{-\frac{\omega^2}{2}} - \mathrm{e}^{-\mathrm{r}(\mathrm{T}-\tau)} \mathrm{D} e^{-\frac{\omega^2}{2}} \frac{\mathrm{H}(\tau)}{\mathrm{D}} \mathrm{e}^{(\mathrm{r}-\delta)(\mathrm{T}-\tau)} = 0.
\end{aligned}
$$

Therefore, we have $\beta(\mathrm{H}(\tau) - k) = \mathrm{e}^{-\delta(T-\tau)} \Phi(\omega) \mathrm{H}(\tau)$, which can be converted to $(\beta - \mathrm{e}^{-\delta T} \Phi(\omega)) * \mathrm{H}(\tau) = \alpha k$. As a result, we derive $\mathrm{H}(\tau) = \frac{\beta k}{\beta - \mathrm{e}^{-\delta(T-\tau)} \Phi(\omega)}$.

E) Proof of Proposition 2 and Proposition 3

From $\mathrm{H}(\tau) = \frac{\beta k}{\beta - \mathrm{e}^{-\delta(T-\tau)} \Phi(\omega)}$, we have $\frac{\mathrm{H}(\tau)}{k} = \frac{\beta}{\beta - \mathrm{e}^{-\delta(T-\tau)} \Phi(\omega)}$. Define $\mathrm{f(H)} = \frac{\mathrm{k}}{H}$, where $\mathrm{H}(\tau) = H$ and then we know $\frac{\partial \mathrm{f(H)}}{\partial H} < 0$ and $\frac{\partial^2 \mathrm{f(H)}}{\partial H^2} > 0$. Again define $\mathrm{g(H)} = \frac{\beta - \mathrm{e}^{-\delta(T-\tau)} \Phi(\omega)}{\beta}$, where $\frac{\partial \mathrm{g(H)}}{\partial H} = \frac{-\mathrm{e}^{-\delta(T-\tau)} \varphi(\omega)}{\beta} \frac{1}{\sigma\sqrt{(T-\tau)}} \frac{1}{\mathrm{H}} < 0$ and $\frac{\partial^2 \mathrm{g(H)}}{\partial H^2} = \frac{\mathrm{e}^{-\delta(T-\tau)} \varphi(\omega)}{\beta} \frac{1}{\sigma\sqrt{(T-\tau)}} \frac{1}{\mathrm{H}^2} - \frac{\mathrm{e}^{-\delta(T-\tau)}}{\beta} \frac{1}{\sqrt{2\pi}} e^{-\frac{\omega^2}{2}} (-\omega) \left(\frac{1}{\sigma\sqrt{(T-\tau)}} \frac{1}{\mathrm{H}}\right)^2 > 0$.

We know $\mathrm{f(k)} = 1$ and $\mathrm{g(k)} = \frac{\alpha - \mathrm{e}^{-\delta(T-\tau)} \Phi(\mathrm{w}_*)}{\alpha} < f(k)$ where $\omega_* = [\ln(\mathrm{k/D}) + (\mathrm{r} - \delta + \sigma^2/2)(\mathrm{T}-\tau)]/\sigma\sqrt{(T-\tau)}$, where $\mathrm{k} \geq \mathrm{D}$. In addition, $\mathrm{f}\left(\frac{\beta}{\beta-1}\mathrm{k}\right) = \frac{\beta - 1}{\beta}$ and $\mathrm{g}\left(\frac{\beta}{\beta-1}\mathrm{k}\right) = \frac{\beta - \mathrm{e}^{-\delta T} \Phi(\mathrm{w}_{**})}{\beta} > \mathrm{f}\left(\frac{\beta}{\beta-1}\mathrm{k}\right)$ where $\omega_{**} = \left[\ln\left(\frac{\beta}{\beta-1} * \mathrm{k/D}\right) + (\mathrm{r} - \delta + \sigma^2/2)(\mathrm{T}-\tau)\right]/\sigma\sqrt{(T-\tau)}$. Moreover, $\lim_{H\to\infty} f(H) = 0$ but $\lim_{H\to\infty} g(H) = \left[\beta - \mathrm{e}^{-\delta(T-\tau)}\right]/\beta$.

As a result, we know $\mathrm{k} < \mathrm{H}(\tau) < \frac{\beta}{\beta-1}\mathrm{k}$.

F) Proof of Corollary 2.

$\mathrm{f(H)} = \frac{\mathrm{k}}{H}$ and $\mathrm{g(H)} = \frac{\beta - \mathrm{e}^{-\delta(T-\tau)}\Phi\,(\omega)}{\beta}$

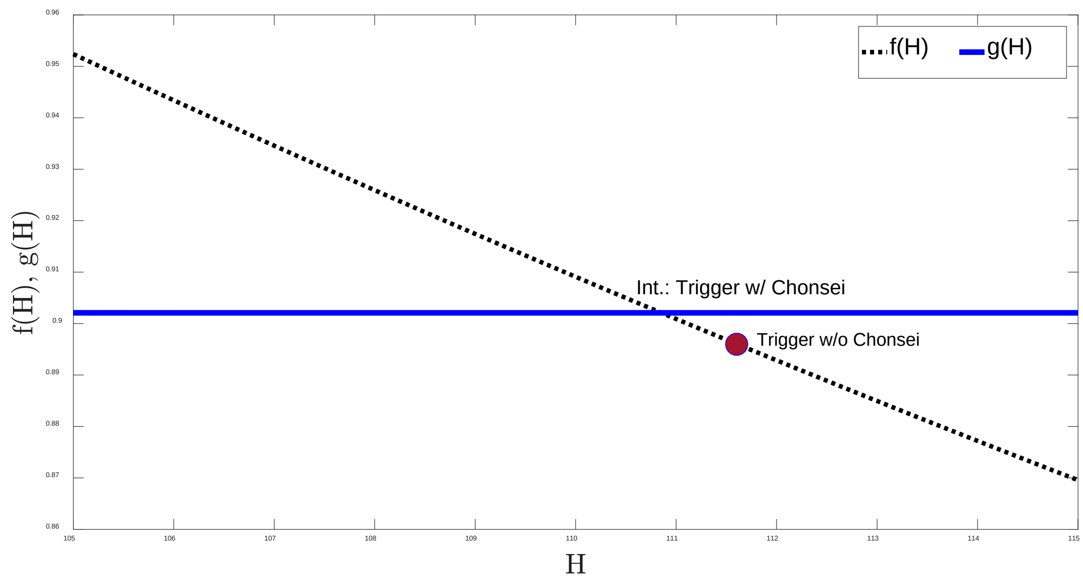
1) $\frac{\partial \mathrm{g(H)}}{\partial \delta} = \frac{\mathrm{e}^{-\delta(T-\tau)}\Phi\,(\omega)}{\beta^2}\frac{\partial \beta}{\partial \delta} + \frac{(T-\tau)\,\mathrm{e}^{-\delta(T-\tau)}\Phi\,(\omega)}{\beta} + \frac{\mathrm{e}^{-\delta(T-\tau)}\varphi(\omega)}{\beta}\frac{\sqrt{(T-\tau)}}{\sigma} > 0$ thus we know $\frac{\partial \mathrm{H}^*}{\partial D} < 0$.

2) $\frac{\partial \mathrm{g(H)}}{\partial r} = \frac{\mathrm{e}^{-\delta(T-\tau)}\Phi\,(\omega)}{\beta^2}\frac{\partial \beta}{\partial r} - \frac{\mathrm{e}^{-\delta(T-\tau)}\varphi(\omega)}{\beta}\frac{\sqrt{(T-\tau)}}{\sigma} < 0$, $\frac{\partial \mathrm{H}^*}{\partial r} > 0$.

3) $\frac{\partial \mathrm{f(H)}}{\partial K} > 0$ and $\frac{\partial \mathrm{g(H)}}{\partial K} = 0$ and thus, we know $\frac{\partial \mathrm{H}^*}{\partial K} > 0$.

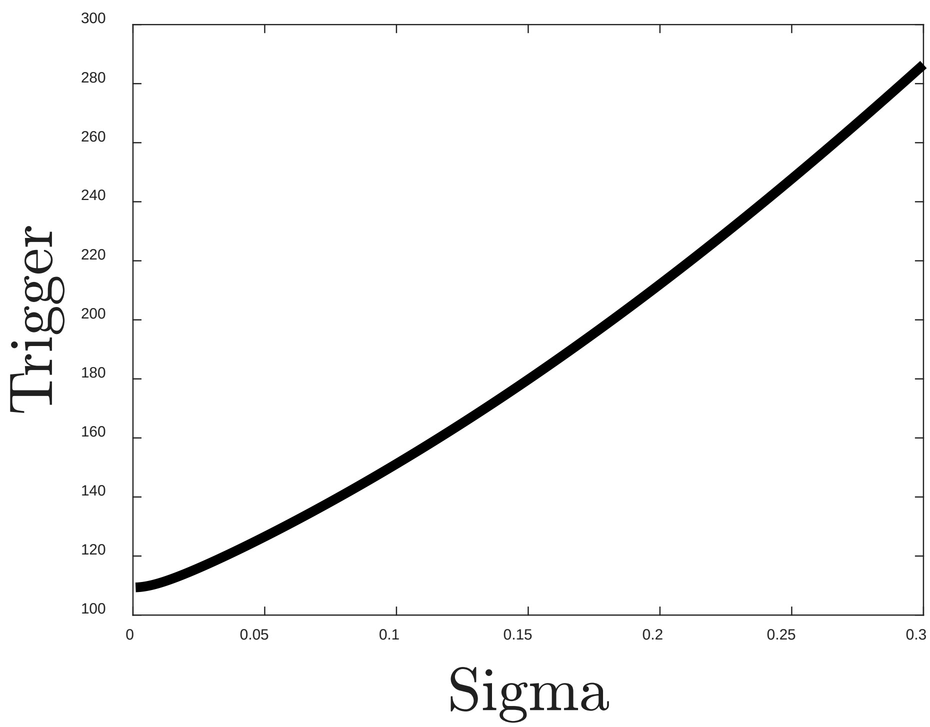
[Appendix 2] Numerical Analysis

A) Optimal Triggers under Chonsei Contracts



Note: r, δ, and σ are 3.31%, 3% and 1%, respectively. And k=100, which is financed by the chonsei deposit for 2 years.

B) Optimal Investment Trigger with respect to Sigma



Note: r and δ are 3.31% and 3%, respectively. k=100, which is financed by the chonsei deposit for 2 years.

[Appendix 3] $\hat{\sigma}_{\mathrm{lrhp}}$ **Estimation**

To measure the volatility of house prices, we adopt the simplest GARCH (1,1) model:

$$LRHP_t = \alpha_g + \beta_g LRHP_{t-1} + e_t,$$

$$\sigma_t^2 = c + \beta_\sigma e_{t-1}^2 + \beta_\vartheta \sigma_{t-1}^2,$$

where $e_t \sim N(0, \sigma_t)$, c is the constant error term, e_{t-1}^2 is the ARCH term and σ_{t-1}^2 is the GARCH term. The GARCH (1.1) process is weakly stable if and only if the stationary condition ($\beta_\sigma + \beta_\vartheta < 1$) holds (Teräsvirta, 2009: 20). For a non-negative variance, as in Nelson, and Cao (1992), Palm (1995), and Tsai and Chan (2008), $\sigma^2 = \omega/(1 - \beta_\sigma - \beta_\vartheta)$.

The estimated results are

$$\underset{}{LRHP_t} = \underset{(0.000)}{-0.001} + \underset{(0.006)}{0.928^{***}}\, LRHP_{t-1},$$

$$\sigma_t^2 = \underset{(1.15E06)}{2.97E06} - \underset{(0.003)}{0.105^{***}} e_{t-1}^2 + \underset{(0.002)}{1.070^{***}}\, \sigma_{t-1}^2, \quad \text{where } R^2 = 0.937.$$

The results satisfy the stationarity condition and thus $\hat{\sigma}^2_{\mathrm{lrhp}}$ can be estimated.

In addition, the constant term in the estimated AR(1) model is non-signicant. Therefore, we also tried AR(1) model with no constant term. With a new $\hat{\sigma}^2_{\mathrm{lrhp}}$, DOLS for Model 4 as reported in <Table 5> produced similar results, which are available upon request.