Outside Intervention of Public Information and Welfare in a Beauty Contest Environment*

Jooyong Jun† Byoung-Ki Kim‡

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Abstract

This paper studies the welfare effects of reflecting outside opinion to public information in a beauty contest environment à la Morris and Shin (2002). A public information provider receives two signals: (i) private information about the true state, and (ii) outside opinion of the higher office (e.g., the president's or prime minister's office), which includes bias and noise. The public information provider then releases public information by combining these two signals with differing weights. This paper characterizes the optimal weights between the two signals from the public information provider's perspective, and thereby draws some understanding of the credibility of public information and the welfare effect in a beauty contest environment. The optimal weight in equilibrium exhibits a bang-bang property, implying that even a public information provider with the power to veto intervention still fully reflects the outside opinion to the public message it releases in some situations, and not at all in others. The equilibrium outcome does not change with the bias in the outside opinion, given that it is stochastic and its distribution is publicly known.

Key Words: public information, outside intervention, beauty contest.

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†Associate Professor of Economics, Dongguk University, Seoul, Korea. Email: jooyong@dongguk.edu.

‡Corresponding Author. Bank of Korea, 55 Namdaemun-Ro, Jung-Gu, Seoul, Republic of Korea. Email: bkkim@bok.or.kr.

1 Introduction

Governments and other public agencies, referred to as public information providers (PIPs), release public messages to provide information about the state of the economy, society, and/or so on for example and enhance coordination among people. Perhaps, most well-known example is the "irrational exuberance" comment by Alan Greespan in 1996, which, in turn, caused downward moves in stock markets worldwide.[1] The ministry of economy and finance of Korea issued 867 press releases, including 33 press releases regarding real estate, in 2021 when markets were overly hot.[2] Consider, for example, public administrations and agencies, the central bank, and the disease control agency. These agencies require expertise in their area and, thus, are allowed to operate with some degree of independence and accountability so that the public information released by these agencies can be trusted.

Although PIPs' objectives are usually supposed to be aligned with social welfare by law, their public messages are never free from external intervention. For example, a PIP may be biased in interpreting the collected data and releasing its message to the public. A more plausible scenario is that political pressure or public sentiment may create a short-term policy need for elected officials in higher office (e.g., the president's or prime minister's office). Since all economic agents can use public information to guess other agents' behavior, form their expectations on the true state of the world, and coordinates their actions, the higher office may want to intervene and add their opinion, which includes bias and noise to the message to affect the expectations formation of the public and nudge it to a specific direction.[3] Thus, the value of the PIP's public message might be prone to external intervention.

At first glance, this kind of external intervention in the public message is harmful to society. The objective of the PIP, social welfare, does not depend upon the deviation from the average nor the coordination of the agents while the positive weight on the external, non-expertise opinion would inevitably deteriorate the precision of the public

[1] "Immediately after he said this, the stock market in Tokyo, which was open as he gave this speech, fell sharply, and closed down 3%. Hong Kong fell 3%. Then markets in Frankfurt and London fell 4%. The stock market in the US fell 2% at the open of trade." Shiller (2005) (source: http://www.irrationalexuberance.com/definition.htm)

[2] Numbers are from the search at https://www.moef.go.kr/nw/nes/nesdta.do?bbsId=MOSFBBS_000000000028&menuNo=4010100. However, their effects on the real estate were little, in the sense that the real estate market bubble lasted longer, quite different from the case of Greespan's comment.

[3] One of the most recent incidents is Japan's overstating of some construction order data from 2013 until 2020, allegedly 'corrected' in 2021. (source: https://www.reuters.com/world/asia-pacific/japan-ministry-overstated-construction-orders-data-years-asahi-2021-12-15/)

information. However, Morris and Shin (2002) (MS hereafter) show that there exist cases in which more precise public information is detrimental to social welfare in a beauty contest environment. In their model, private agents try to form expectations, making use of private and public information, appropriate to the economic fundamentals and also close to the average expectation of the whole population. This complementarity in expectations generates a coordination motive among agents and increases reliance on public information with respect to private information. More precise public information heightens coordination among agents but, at the same time, can make the agents' expectations more inappropriate with respect to economic fundamentals. The latter leads to an adverse effect on social welfare.

We give a little tweak and view the above environment from the PIP's perspective. Specifically, we investigate the welfare effects of a PIP's public message if it is combined with an additional external signal reflecting the short-term policy need of the higher office in a beauty contest setting à la Morris and Shin (2002). For example, the higher office might want to send public information to stimulate or suppress the real estate demand of households and add its opinion to the message of the department/ministry of finance (or even the central bank in some countries) which observes private signals about the true state of the real estate market.

In the model, the PIP may need to combine two signals: (i) its own signal about the true state of the world of agents' interest, and (ii) a top-rank governmental office's (the higher office) opinion. It then releases a public message by linearly combining these two signals, with possibly different weights. Each agent has her own private information about the true state of the world as well as public information, which is common knowledge, and forms an expectation of the true state by combining private and public information. Beauty contest setting settles into the model in the form that each agent cares not only about the distance between their own expectation and the true state of the economy but also about the average distance between their own expectation and all other agents' expectations as well. Social welfare, however, is defined as the average distance between the economic agents' expectations and the true state of the economy, which means that coordination among agents is not socially valuable.

We characterize, from the PIP's perspective, the optimal weights for combining the two signals and thereby draw some understanding on the effect of the external intervention of public information on social welfare in a beauty contest environment. The optimal weight that the PIP puts in equilibrium exhibits a bang-bang property. That is, the PIP may have the power to refuse the intervention (e.g., the central bank) or not (e.g., small governmental agencies), but in some situations, even the PIP with the power to veto an outside intervention fully reflects the external opinion to the public message it releases, and not at all in others. Moreover, we find that if necessary and possible, a PIP releases a public message if and only if reflecting the external opinion enhances social welfare. We also find that the bias in the higher office's opinion does not alter the equilibrium outcome due to the stochastic, not strategic, nature of the outside opinion.

While it is better for a public information provider to be open to outside opinions, our results do not justify outside intervention in general, but rather emphasize the importance of the independence of public information provider. For example, if a PIP can reject the outside intervention if necessary, the welfare will be enhanced, specifically if the PIP's mandate includes regular release of public information.

Related Literature

This paper is closely related to the literature exploring social value of public information in a beauty contest setting. After the seminal paper by Morris and Shin (2002), there is a series of debate regarding the adverse effect of more precise public information. Svensson (2006) argues that the condition warranting the adverse effect is empirically hard to satisfy. Morris et al. (2006) replies that the plausibility of the condition is still an empirical issue by showing that correlation between private and public information enhances the plausibility. Cornand and Heinemann (2008) shows that the adverse effect disappears if the public information is released to sufficiently small portion of agents. Kim (2010) removes the adverse effect by allowing agents to share private information locally with other agents. Angeletos and Pavan (2007) analyzes social value of private and public information in more general environment with economic externalities, strategic complementarity or substitutability and shows that the adverse effect of more precise public information appears when the equilibrium degree of coordination is higher than the socially optimal one. Further, they show that an increase in the precision of private information can be detrimental to social welfare when the equilibrium degree of coordination is lower than the socially optimal one. Recently, some studies including Angeletos and Werning (2006), Morris and Shin (2018), and Rondina and Shim (2015), to name a few, extends the environment to cover endogenous formation of public information through market prices, especially in financial markets. This paper analyzes a similar model to MS but focuses not on the adverse effect but on the manipulation and credibility of public information.

This paper is also related to literature on central bank transparency and reputation. The above studies on social welfare of public information can be naturally extended to discussions on the PIP's transparency and reputation. The focus of the literature has been given to central bank. To see this line of research, please refer to, for example, Demertzis and Hallett (2007), Van der Cruijsen et al. (2010), and Duffy and Heinemann (2021), and references therein.

Since this paper covers information manipulation and credibility, this paper is related, although remotely, to game theoretic approaches to the subject. To name a few, Sobel (1985), and Ettinger and Jehiel (2010). They study credibility and deception under a two-player repeated game environment in which opponents' payoff or strategy is uncertain. This paper studies manipulation incentive from the PIP's perspective in a beauty contest environment populated by one PIP and a continuum of agents, and draws some lessons on the credibility of public information.

2 Model

The model starts from the setting of MS. There is a continuum of agents in $[0,1]$. The public information provider's (PIP) objective is to maximize social welfare (defined below). Agent i chooses an action $a_i \in \mathbb{R}$, and $\mathbf{a}$ denotes the action profile over all agents. The payoff function for agent i is given by

$$u_i(a,\theta) \equiv -(1-r)(a_i-\theta)^2 - r(L_i - \bar{L}), \tag{1}$$

where θ is the fundamental state of the economy and $r \in (0,1)$ is a constant, and

$$L_i \equiv \int_0^1 (a_j - a_i)^2 dj, \qquad \bar{L} \equiv \int_0^1 L_j dj.$$

The first term of the payoff function represents a standard quadratic loss in the distance between the underlying state θ and the agent's action a_i: it is higher the closer the action is to the state of the economy. The second term corresponds to Keynes' beauty contest example: it is higher the closer the action is to the average action of the whole population. The constant r is the weight that each agent puts on the beauty contest term.

Social welfare is the average payoff of the agents (normalized by $1-r$), given by

$$W(a,\theta) \equiv \frac{1}{1-r}\int_0^1 u_i(a,\theta)di = -\int_0^1 (a_i-\theta)^2 di. \tag{2}$$

Note that the beauty contest term is canceled out at the social level, so that social welfare depends only on the average distance between the action and the fundamental state of the economy. As pointed out by Angeletos and Pavan (2007), coordination is not socially valuable in this model. There may therefore be a conflict between individual decisions and the socially optimal solution.

Each agent i receives a private signal

$$x_i = \theta + \epsilon_i \text{ with } \epsilon_i \sim N(0, 1/\beta), \tag{3}$$

where β denotes the precision of private information. No aggregate uncertainty is assumed:

$$\int_0^1 x_i di = \theta. \tag{4}$$

Departing from MS, the PIP now receives two signals: (i) one about the true state of the world, denoted by z, and (ii) the outside opinion from the higher office about the true state, denoted by θ^* defined as

$$z = \theta + \eta_1 \text{ with } \eta_1 \sim N(0, 1/\alpha_1), \tag{5}$$

where α_1 denotes the precision of the signal z, and

$$\theta^* = \eta_2 \text{ with } \eta_2 \sim N(0, 1/\alpha_2), \tag{6}$$

where α_2 denotes the precision of the signal θ^*.

Here, the opinion θ^* is supposed to reflect the higher office's shorter-term policy needs, as mentioned before, and to be regarded as a stochastic, not strategic, value. Note that θ^* is independent of z and follows a Normal distribution with *zero* mean. We will check later whether the agents would still behave in a similar fashion if the opinions were biased (i.e., $E(\theta^*) \neq q$).

Public message y is a linear combination of the two signals with different weights h and $1 - h$, respectively, as the following form:

$$y = hz + (1-h)(z + \theta^*) = z + (1-h)\theta^* \text{ with } y \sim N(\theta, 1/\alpha), \tag{7}$$

where $\alpha = \frac{\alpha_1 \alpha_2}{\alpha_2 + (1-h)^2 \alpha_1}$ denotes the precision of the public signal. As the weight on the PIP's private information h increases to *one*, the portion of the outside opinion θ^* in public information y shrinks to *zero*. We first assume that the PIP is such an independent organization that it can *choose* the value h. We later check that this assumption does not alter the equilibrium outcomes.

As in MS, specifications of distributions related to private and public information are known to private agents and the PIP, but not the realized signals except the ones that each agent directly observes. For a moment, we assume that the PIP cannot opt out of the mandatory public information provision requirement: it must release the public information y, and cannot release the weight h it chooses.[4]

At first glance, the PIP might naturally choose $h = 1$ given that social welfare is solely determined by the distance of underlying economic fundamentals and actions of agents. This is so because combining the outside opinion with the PIP's private signal will deteriorate the precision of the public information. However, there is a situation, as shown by MS, in which more precise public message is detrimental. This leaves room for the PIP to reflect the outside opinion before releasing the public message.

Similarly to MS, the optimal action of agent i is derived by differentiating Equation (1) with respect to a_i.

$$a_i^* = (1-r)E_i(\theta) + rE_i(\bar{a}), \tag{8}$$

where $\bar{a} = \int_0^1 a_j dj$ denotes average action of the whole population. Note that agent i's expectations on θ and other agent j's realization are the same:

$$E_i(\theta|x_i, y) = \frac{\beta x_i + \alpha y}{\alpha + \beta}, \tag{9}$$

$$E_i(x_j|x_i, y) = \frac{\beta x_i + \alpha y}{\alpha + \beta}. \tag{10}$$

As the same procedure used in MS, the unique linear equilibrium action a_i^* is derived as

$$a_i^* = \frac{\alpha y + \beta(1-r)x_i}{\alpha + \beta(1-r)}, \text{ where } \alpha = \frac{\alpha_1 \alpha_2}{\alpha_2 + (1-h)^2 \alpha_1}. \tag{11}$$

which is the same as Equation (13) of MS except for the value of α.

Now we turn to social welfare. First note

$$\begin{aligned} a_i^* - \theta &= \frac{\alpha(y-\theta) + \beta(1-r)(x_i - \theta)}{\alpha + \beta(1-r)} \\ &= \frac{\alpha[z + (1-h)\theta^* - \theta] + \beta(1-r)(\theta + \epsilon_i - \theta)}{\alpha + \beta(1-r)} \\ &= \frac{\alpha[\theta + \eta_1 + (1-h)\theta^* - \theta] + \beta(1-r)\epsilon_i}{\alpha + \beta(1-r)} \\ &= \frac{\alpha[\eta_1 + (1-h)\eta_2] + \beta(1-r)\epsilon_i}{\alpha + \beta(1-r)}. \end{aligned} \tag{12}$$

[4]In Section 3, this assumption will be relaxed. In a nutshell, the PIP does not release public information if its precision is lower than some threshold.

Next derive expected social welfare given the optimal action profile of all agents:

$$
\begin{aligned}
E[W(\mathbf{a}^*,\theta)] &= -E\left[\int_0^1 (a_i^* - \theta)^2 di\right] \\
&= -\frac{1}{[\alpha+\beta(1-r)]^2}\left\{\alpha^2 E[(\eta_1 + (1-h)\theta^*)^2] + [\beta(1-r)]^2 E[\epsilon^2]\right\} \\
&= -\frac{\alpha^2\left[\frac{1}{\alpha_1} + (1-h)^2\frac{1}{\alpha_2}\right] + \beta(1-r)^2}{[\alpha+\beta(1-r)]^2} \\
&= -\frac{\alpha^2\left[\frac{\alpha_2+(1-h)^2\alpha_1}{\alpha_1\alpha_2}\right] + \beta(1-r)^2}{[\alpha+\beta(1-r)]^2} \\
&= -\frac{\alpha+\beta(1-r)^2}{[\alpha+\beta(1-r)]^2}.
\end{aligned} \tag{13}
$$

To see the effect of more precise private information on social welfare, take a derivative of social welfare with respect to β.

$$
\frac{\partial E[W(a^*,\theta)]}{\partial\beta} = \frac{(1-r)[\alpha(1+r)+\beta(1-r)^2]}{[\alpha+\beta(1-r)]^3} > 0. \tag{14}
$$

Note that an increase in the precision of private information always leads to heightened social welfare. Turn to the effect of more precise public information on social welfare.

$$
\frac{\partial E[W(a^*,\theta)]}{\partial\alpha} = \frac{\alpha-\beta(2r-1)(1-r)}{[\alpha+\beta(1-r)]^3}. \tag{15}
$$

$$
\begin{aligned}
\frac{\partial E[W(a^*,\theta)]}{\partial\alpha_1} &= \frac{\partial E[W(a^*,\theta)]}{\partial\alpha}\cdot\frac{\partial\alpha}{\partial\alpha_1} \\
&= \frac{\alpha-\beta(2r-1)(1-r)}{[\alpha+\beta(1-r)]^3}\cdot\frac{\alpha_2^2}{[\alpha_2+(1-h)^2\alpha_1]^2}.
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
\frac{\partial E[W(a^*,\theta)]}{\partial\alpha_2} &= \frac{\partial E[W(a^*,\theta)]}{\partial\alpha}\cdot\frac{\partial\alpha}{\partial\alpha_2} \\
&= \frac{\alpha-\beta(2r-1)(1-r)}{[\alpha+\beta(1-r)]^3}\cdot\frac{(1-h)^2\alpha_1^2}{[\alpha_2+(1-h)^2\alpha_1]^2}.
\end{aligned} \tag{17}
$$

An increase in the precision of public information strictly enhances welfare provided that $\alpha - \beta(2r-1)(1-r) > 0$. The same condition applies to the precision of signal about the true state of the world the PIP receives, that is α_1. For the precision of the outside opinion, α_2, the same condition applies if $h \neq 1$. When $h = 1$, the opinion is discarded and its precision has no effect on the precision of the public message and social welfare. This analysis of the effect of an increase in the precision of private and public information is consistent with MS.

3 Analysis

This section analyzes the PIP's strategy and characterizes the optimal weight h. The PIP chooses an optimal weight h^{**} that maximize the social welfare.[5] A partial derivative with respect to h yields

$$\begin{aligned}\frac{\partial E[W(a^*,\theta)]}{\partial h} &= \frac{\partial E[W(a^*,\theta)]}{\partial \alpha}\cdot\frac{\partial \alpha}{\partial h}\\ &= \frac{\alpha-\beta(2r-1)(1-r)}{[\alpha+\beta(1-r)]^3}\cdot\frac{2\alpha_1^2\alpha_2(1-h)}{[\alpha_2+(1-h)^2\alpha_1]^2}.\end{aligned} \tag{18}$$

Note

$$\begin{aligned}\alpha-\beta(2r-1)(1-r) &= \frac{\alpha_1\alpha_2}{\alpha_2+(1-h)^2\alpha_1}-\beta(2r-1)(1-r)\\ &= \frac{1}{\alpha_2+(1-h)^2\alpha_1}\Big[\alpha_1\alpha_2-\alpha_2\beta(2r-1)(1-r)\\ &\qquad -(1-h)^2\alpha_1\beta(2r-1)(1-r)\Big].\end{aligned} \tag{19}$$

Hence the first order condition $\frac{\partial E[W(a^*,\theta)]}{\partial h}=0$ implies

$$\begin{aligned} h^* &= 1 \text{ or}\\ (1-h^*)^2 &= \frac{\alpha_2\left[\alpha_1-\beta(2r-1)(1-r)\right]}{\alpha_1\beta(2r-1)(1-r)}.\end{aligned} \tag{20}$$

Note that $(1-h^*)^2\geq 0$ holds if and only if $\alpha_1\geq\beta(2r-1)(1-r)$. And checking the condition for $(1-h^*)^2\leq 1$ shows

$$\begin{aligned}&\alpha_2\left[\alpha_1-\beta(2r-1)(1-r)\right]\leq\alpha_1\beta(2r-1)(1-r)\\ &\Leftrightarrow \alpha_1\alpha_2\leq(\alpha_1+\alpha_2)\beta(2r-1)(1-r)\\ &\Leftrightarrow \alpha_0\leq\beta(2r-1)(1-r),\ \text{where } \alpha_0\equiv\frac{\alpha_1\alpha_2}{\alpha_2+\alpha_1}.\end{aligned} \tag{21}$$

It is noticeable that

$$\alpha_0\leq\alpha\leq\alpha_1 \text{ for } h\in[0,1]. \tag{22}$$

In Equation (22), first inequality holds with equality if and only if $h=0$, and second inequality holds with equality if and only if $h=1$. Note, therefore, that $\alpha_0<\alpha_1$ always holds.

Now taking second derivative of social welfare with respect to h to see the second

[5] In this section, h^* and h^{**} denote point of local maxima and global maxima, respectively.

order condition yields

$$\begin{aligned}\frac{\partial}{\partial h}\left(\frac{\partial E[W(a^*,\theta)]}{\partial h}\right) &= \frac{\partial}{\partial h}\left(\frac{\partial E[W(a^*,\theta)]}{\partial \alpha}\cdot\frac{\partial \alpha}{\partial h}\right)\\ &= \frac{\partial}{\partial h}\left(\frac{\partial E[W(a^*,\theta)]}{\partial \alpha}\right)\cdot\frac{\partial \alpha}{\partial h} + \frac{\partial}{\partial h}\left(\frac{\partial \alpha}{\partial h}\right)\cdot\frac{\partial E[W(a^*,\theta)]}{\partial \alpha}\\ &= -\frac{2[\alpha-\beta(3r-1)(1-r)]}{[\alpha+\beta(1-r)]^4}\cdot\frac{2\alpha_1^2\alpha_2(1-h)}{[\alpha_2+(1-h)^2\alpha_1]^2}\\ &\quad -\frac{2\alpha_1^2\alpha_2[\alpha_2-3\alpha_1(1-h)^2]}{[\alpha_2+(1-h)^2\alpha_1]^3}\cdot\frac{\alpha-\beta(2r-1)(1-r)}{[\alpha+\beta(1-r)]^3}. \end{aligned} \tag{23}$$

Equation (18) implies that if there exists $h \in [0,1)$ then $\alpha = \beta(2r-1)(1-r)$. Hence, evaluating Equation (23) at $h^* \in [0,1)$ gives

$$\frac{\partial}{\partial h}\left(\frac{\partial E[W(a^*,\theta)]}{\partial h}\right)\Bigg|_{h=h^*} = -\frac{2[\alpha-\beta(3r-1)(1-r)]}{[\alpha+\beta(1-r)]^4}\cdot\frac{2\alpha_1^2\alpha_2(1-h^*)}{[\alpha_2+(1-h^*)^2\alpha_1]^2}. \tag{24}$$

Note that the sign of Equation (24) is determined by the sign of $-[\alpha-\beta(3r-1)(1-r)]$. Meanwhile, evaluating the second derivative at $h=1$ yields

$$\frac{\partial}{\partial h}\left(\frac{\partial E[W(a^*,\theta)]}{\partial h}\right)\Bigg|_{h=1} = \frac{-2\alpha_1^2}{[\alpha_1+\beta(1-r)]^3\alpha_2}[\alpha_1-\beta(2r-1)(1-r)]. \tag{25}$$

The sign of $-[\alpha_1-\beta(2r-1)(1-r)]$ determines the sign of the second derivative evaluated at $h=1$. To derive optimal weight $h \in [0,1]$, it needs to find out critical points falling in the support $[0,1]$ and then sort out points of local and global maxima by examining the social welfare is increasing or decreasing, and concave or convex in the support. To do so, it would be convenient to divide cases as follows.[6]

Case I : $\alpha_1 \leq \beta(2r-1)(1-r)$

First, consider the case in which the Case I holds with strict inequality ($\alpha_1 < \beta(2r-1)(1-r)$). By Equation (25) the sign of the second derivative of social welfare with respect to h evaluated at $h=1$ is positive, which means that social welfare achieves its local minimum at $h=1$. Note, by Equation (20), $(1-h^*)^2 < 0$ under Case I with strict inequality. This implies that there does not exist any other critical points except $h^*=1$ for $h \in [0,1]$. Meanwhile, Equation (18), under Case I with strict inequality, implies

$$\begin{aligned}\frac{\partial E[W(a^*,\theta)]}{\partial h}\Bigg|_{h=0} &= \frac{\alpha_0-\beta(2r-1)(1-r)}{[\alpha_0+\beta(1-r)]^3}\cdot\frac{2\alpha_0^2}{\alpha_2}\\ &< 0,\ \text{where } \alpha_0 \equiv \frac{\alpha_1\alpha_2}{\alpha_2+\alpha_1}. \end{aligned} \tag{26}$$

[6]Please see Appendix for graphs corresponding to each case below.

This is so because, as in Equation (22), $\alpha_0 \equiv \frac{\alpha_1\alpha_2}{\alpha_2+\alpha_1} < \alpha_1$ and hence $0 < \alpha_0 < \alpha_1 < \beta(2r-1)(1-r)$ under Case I with strict inequality. Putting all these together means that social welfare is strictly decreasing for $h \in [0,1]$ and hence achieves its global maximum at $h^{**} = 0$ under Case I with strict inequality.

Now consider the case in which Case I holds with equality. Then $\left.\frac{\partial E[W(a^*,\theta)]}{\partial h}\right|_{h=1} = 0$ by Equation (18), and $\left.\frac{\partial E[W(a^*,\theta)]}{\partial h}\right|_{h=0} < 0$ by Equation (26). Note also that by Equation (20), $(1-h^*)^2 = 0$ when Case I holds with equality. This implies that there does not exist any other critical points except $h^* = 1$ for $h \in [0,1]$. Therefore, social welfare still achieves its global maximum at $h^{**} = 0$. Note also that under the Case I more precise public information is detrimental to social welfare for all $h \in [0,1)$[7] because $\alpha < \alpha_1 \leq \beta(2r-1)(1-r)$ makes the sign of Equation (15) negative. The following proposition summarizes analysis of optimal weight h under Case I.

Proposition 1. *Suppose $\alpha_1 \leq \beta(2r-1)(1-r)$ holds. Then social welfare achieves its global maximum at $h^{**} = 0$. In other words, the PIP fully reflects the outside opinion in the sense that $y = h^{**}z+(1-h^{**})(z+\theta^*) = z+\theta^*$ because more precise public information is detrimental to social welfare for all $h \in [0,1)$ and the PIP keeps the precision of public information at its minimum.*

It remains the case with $\alpha_1 > \beta(2r-1)(1-r)$. Reflecting Equations (21) and (22), it is convenient to divide the remaining case into two incorporating α_0.

Case II : $\alpha_1 > \alpha_0 > \beta(2r-1)(1-r)$

By Equation (25), $\alpha_1 > \beta(2r-1)(1-r)$ implies the sign of the second derivative of social welfare with respect to h evaluated at $h = 1$ is negative. Therefore, social welfare achieves its local maximum at $h^* = 1$. Note also Equation (21) implies $(1-h^*)^2 > 1$, which means that there does not exist any other critical points except $h^* = 1$ for $h \in [0,1]$. Under Case II, Equation (18) implies

$$\left.\frac{\partial E[W(a^*,\theta)]}{\partial h}\right|_{h=0} = \frac{\alpha_0 - \beta(2r-1)(1-r)}{[\alpha_0+\beta(1-r)]^3} \cdot \frac{2\alpha_0^2}{\alpha_2} > 0. \tag{27}$$

All these implies that social welfare is increasing in $h \in [0,1]$ and achieves its global maximum at $h = 1$. Note also that, by Equation (15), an increase in the precision of public information enhances social welfare for all $h \in [0,1]$ because $\alpha \geq \alpha_0 > \beta(2r-1)(1-r)$

[7] If $\alpha_1 < \beta(2r-1)(1-r)$ then this holds for all $h \in [0,1]$.

where the equality holds if and only if $h = 0$, as given in Equation (22). The following proposition summarizes analysis of optimal weight h under Case II.

Proposition 2. *Suppose $\alpha_1 > \alpha_0 > \beta(2r-1)(1-r)$ holds. Then social welfare achieves its global maximum at $h^{**} = 1$. In other words, the PIP does not reflect, if possible, the outside opinion at all in the sense that $y = h^{**}z + (1-h^{**})(z+\theta^*) = z$. More precise public information enhances social welfare for all $h \in [0,1]$, and the PIP keeps the precision of public information at its maximum.*

Case III : $\alpha_1 > \beta(2r-1)(1-r) \geq \alpha_0$

Equation (25) indicates that social welfare achieves local maximum at $h^* = 1$. Note also $(1-h^*)^2 \in (0,1)$, meaning that there exists another critical point $h^* \in [0,1]$ except $h^* = 1$. At $h^* \in [0,1)$, $\alpha = \beta(2r-1)(1-r)$ holds, and Equation (24) tells us that the sign of the second derivative evaluated at $h^* \in (0,1)$ is determined by $-[\alpha - \beta(3r-1)(1-r)]$, which is strictly positive under Case III. This is so because $\beta(3r-1)(1-r) > \alpha = \beta(2r-1)(1-r)$. Hence, social welfare achieves its local minimum at $h^* \in (0,1)$. This means that social welfare achieves its maximum at $h = 0$ or at $h = 1$ depending on parameter values. That is,

$$E[W(a^*,\theta)]|_{h=0} = -\frac{\alpha_0 + \beta(1-r)^2}{[\alpha_0 + \beta(1-r)]^2} \gtreqless -\frac{\alpha_1 + \beta(1-r)^2}{[\alpha_1 + \beta(1-r)]^2} = E[W(a^*,\theta)]|_{h=1} \tag{28}$$

Indeed, social welfare increases with more precise public information for $h \in [0,1]$ that satisfies $\alpha > \beta(2r-1)(1-r)$ and the opposite happens for $h \in [0,1]$ that satisfies $\alpha < \beta(2r-1)(1-r)$. In particular,

$$\begin{aligned} &E[W(a^*,\theta)]|_{h=1} \gtreqless E[W(a^*,\theta)]|_{h=0} \text{ if and only if} \\ &\alpha_0\alpha_1 + \beta(1-r)^2[\alpha_1 + \alpha_0 - \beta(2r-1)] \gtreqless 0. \end{aligned} \tag{29}$$

The following proposition summarizes the analysis of optimal weight h under Case III.

Proposition 3. *Suppose $\alpha_1 > \beta(2r-1)(1-r) \geq \alpha_0$ holds. Then social welfare achieves its global maximum at either $h^{**} = 0$ or $h^{**} = 1$ or both when $\alpha_0\alpha_1 + \beta(1-r)^2[\alpha_1 + \alpha_0 - \beta(2r-1)]$ is less than or greater than or equal to zero. In other words, the PIP either fully reflects the outside opinion or does not at all. More precise public information enhances (reduces) social welfare for $h \in [0,1]$ that satisfies $\alpha > (<)\ \beta(2r-1)(1-r)$. The PIP keeps the precision of public information either at its maximum or at its minimum, depending on which one achieves greater social welfare.*

Extension

Propositions 1-3 mean that the PIP's optimal choice of weight h exhibits the bang-bang property. If the parameter values are given in such a way that more precision in public information enhances social welfare, then the PIP keeps the precision of public information at its maximum by rejecting the outside opinion completely. On the contrary, the opposite happens, then the PIP keeps the precision of public information at its minimum by fully reflecting the outside opinion. But, in any case, agents will anticipate the PIP's behavior and incorporate the PIP's solution into their expectations formation.

Based on this result, now we briefly check that the equilibrium outcome does *not* change even if $E(\theta^*) \neq 0$. As long as α_1, α_2, and β are commonly known, all agents know which case (I, II, or III) they are in. That is, they know whether the public message reflects the outside opinion of the higher office or not: $y = z + \theta^* + b$ where b is the average reflected bias of the higher office (case I and $\alpha < \beta(2r-1)(1-r)$ of case III), or $y = z$ (case II and $\alpha > \beta(2r-1)(1-r)$ of case III). Because the PIP's equilibrium choice $h^{**} \in \{0, 1\}$ shows the bang-bang property, they can correctly conjecture when the public message y includes b and derive $z + \theta^*$, although still the values of z and θ^* cannot be separately identified. That is, agents' choice of action $\mathbf{a}$ does not change. Expecting that b does not alter the agents' choice, the PIP does not change its welfare-maximizing choice of h^{**} in equilibrium, either. Note that this result stems from the assumption that the value of θ^* is stochastic.

What if the PIP is allowed to opt out of public information provision? As pointed out by Svensson (2006), a threshold for the precision of public information can be calculated which makes social welfare with the public information is equal to social welfare without.

$$E[W(a^*, \theta)]|_{\alpha=0} = -\frac{1}{\beta} = -\frac{\alpha + \beta(1-r)^2}{[\alpha + \beta(1-r)]^2}. \tag{30}$$

Solving for α yields the threshold $\overline{\alpha}$.

$$\overline{\alpha} = \beta(2r-1). \tag{31}$$

So, together with Equation (22), the PIP would release the public information if and only if

$$\alpha_1 \geq \overline{\alpha} = \beta(2r-1). \tag{32}$$

Equation (32) is the condition that must be satisfied in all Cases I-III, and hence it overrides the conditions given in each case. In Case I, the PIP would not release the public information. In Case II, the PIP would release the public information if and only if Equation (32) holds. And in Case III, the PIP would release the public information if and only if Equation (32) holds, and if it does, $h^{**} = 1$ by Equation (29). Next proposition summarizes the discussion so far.

Proposition 4. *Suppose the PIP can choose not to release a public message. Then the PIP releases the public information if and only if $\alpha_1 \geq \beta(2r-1)$ without reflecting the outside opinion at all (i.e., $h^{**} = 1$) in the sense that $y = h^{**}z + (1-h^{**})(z+\theta^*) = z$.*

Proposition 4 states that if the PIP has the power to choose not to release a public message, then the released public information is fully credible in the sense that it is not intervened at all. But we warn that not releasing public information, especially if it is supposed to be regular, can cause adverse effects and problems not considered in this paper such as panic.

4 Concluding Remarks

We study the optimal degree of reflecting outside opinion from higher office to public message from a public information provider's perspective under a beauty contest environment. We find that in some situations, reflecting outside opinion is better in the sense that it enhances social welfare. We also find that the optimal weight on reflecting outside opinion shows a bang-bang property in equilibrium. That is, for a sufficiently independent PIP, the optimal choice is either fully reflecting the outside opinion, or not at all. The results are critically dependent upon the beauty contest environment in which coordination among agents is not socially valuable, as pointed out by Angeletos and Pavan (2007).

While it is better for a public information provider to be open to outside opinions, our results do not justify outside intervention in general, but rather emphasize that the importance of the independence of public information provider. Its power to reject the outside intervention, if necessary, can enhance welfare, specifically if a PIP's mandate includes regular release of public information.

Note that the effects of outside intervention of public information on the welfare in Propositions 1-3 are dependent upon the criterion of welfare. Outside intervention, if executed, inevitably deteriorates the precision of public messages while enhancing social welfare. Then, if the outside intervention as defined here can be always regarded harmful if the precision of the public message is the PIP's criterion.

Our results owe a great deal not only to the beauty contest environment where the coordination is not socially valuable, but also to the feature in the agent's utility that the coordination of actions is strategic complement to each agent. It would be interesting, therefore, to modify the strategic complementarity of agents' actions, or to put the PIP's problem in more various and general environments in which, for example, coordination among agents is socially valuable.

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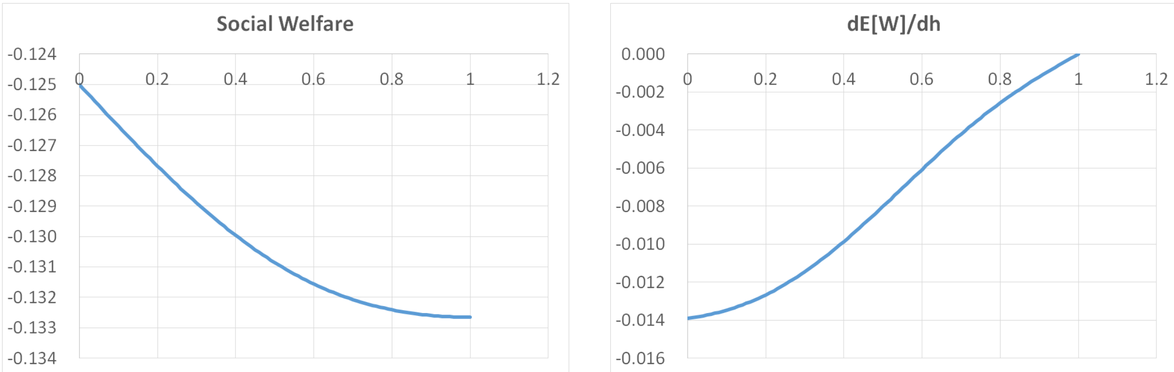
Appendix

This Appendix presents graphs under specific parameter values corresponding to Cases I-III . In each graph, x-axis represents h and y-axis represents either social welfare or $dE[W(a^*, \theta)]/dh$.

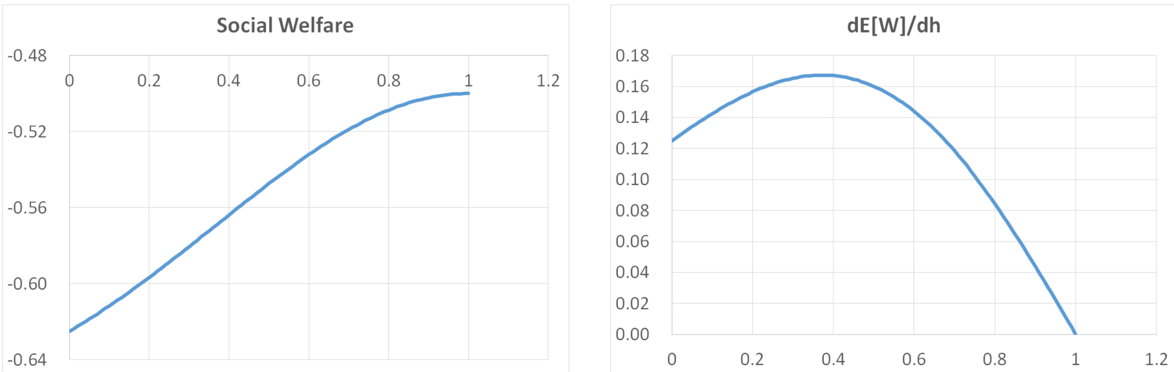
Case I: $r = 0.75, \alpha_1 = 1, \alpha_2 = 1, \beta = 10$



(a) (b)

Figure 1: Case I (Example)

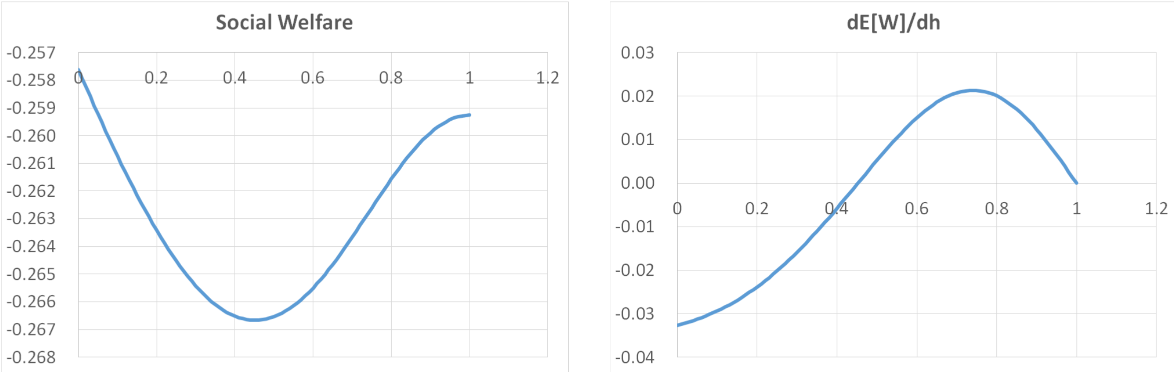
Case II : $r = 0.75, \alpha_1 = 1, \alpha_2 = 1, \beta = 2$



(a) (b)

Figure 2: Case II (Example)

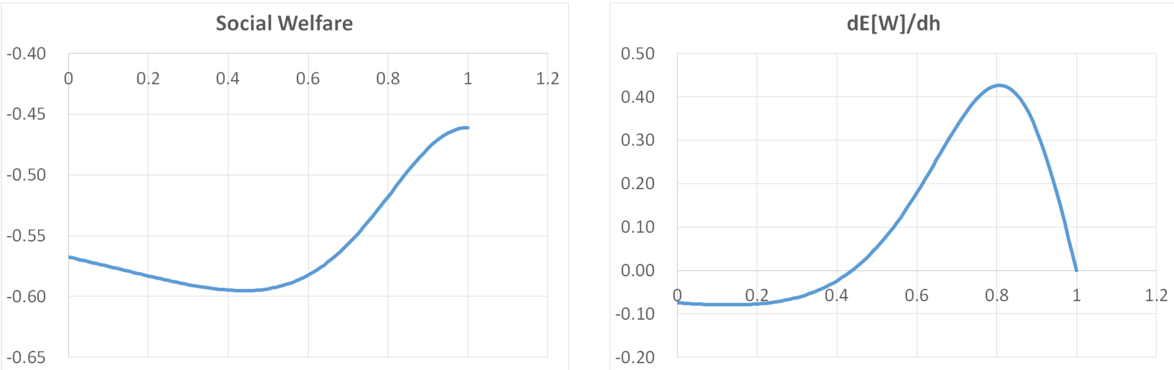
Case III : $r = 0.75, \alpha_1 = 1, \alpha_2 = 0.5, \beta = 5$



(a) (b)

Figure 3: Case III (Example 1)

Case III : $r = 0.75, \alpha_1 = 1, \alpha_2 = 0.1, \beta = 2$



(a) (b)

Figure 4: Case III (Example 2)