

# **The Optimal Control Model for Supreme Power**

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## **Abstract**

The U.S. dollar has been institutionalized as the world's key currency in the Bretton Woods Conference in 1944 which naturally put the United States in a relatively freer position from the balance of payments constraints. It was one of backdrops that the United States emerged as a supreme power with unparalleled productivity and power. She has been engaged in geopolitical activities with dominant influence, deference, strategic depth, and sacrifice which was unprecedented in the world's history before the World War II. The special status has been, however, rarely reflected in the analytical model, so this study will fill this gap by introducing integral constraint for the supreme power in the optimal control model. This study will be a starting point for the model of supreme power, inviting us to the unexplored frontier.

**Keywords:** Key currency, Optimal control model, Supreme power, Integral constraint.

**JEL Classifications:** E42, F33, F50

## **I. Introduction**

This paper suggests an optimal control model for supreme power, which bears more relevance to the country issuing the world's key currency. In 1944, the Bretton Woods arrangements established the U.S. dollar as the world's key currency. Even after the shift to the floating exchange rate system in 1971, the U.S. dollar has been a bedrock of global liquidity taking various titles implying the three functions, such as vehicle currency bringing out the nuance for a medium of exchange (Devereux and Shi 2013), invoice currency or anchor currency connoting

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a unit of account (Kashiwagi 2014), and reserve currency implying a store of value (Jiang *et al.* 2021). We term all as *the key currency* in this study.

The concept of supreme power came into being since the end of the World War II in 1945, since then, American power has been unsurpassed and even gained the image of the custodian of universal values. Before 1945, the world history has been mixed with intermittent contacts and conflicts among regional powers that have not posed sustainable influences on a global scale. Obviously, America has had a relatively thriving economy and her military domination firmly established American supremacy. These two existential factors of military and economy in building up American dominance are inter-related in crafting security and economic policies. For instance, the U.S. Navy has been guaranteed *the safety of trade flows* since the United States annexed the Kingdom of Hawaii in 1898, and nowadays the United States and her alliance countries regularly conduct in the vicinity of the Paracel Islands and the Spratly Islands under the name of Freedom of Navigation Operation. Relatedly, the benefit of issuing the world's key currency is clear, raising capital that could finance even persistent and huge current account deficits so long as the value of the world's key currency is recognized. Then, it is necessary that the supreme power continues to be dominant, which entail sacrifices and risks like geopolitical activities. However, this extraordinary presence of supreme power like the United States since the World War II issuing the key currency, has not been explicitly spelled out within the economic models. So, it is worth developing the model of supreme power *per se* reflecting its distinctive features appearing in the balance of payments statistics that reflects overall financial constraint of an economy.

We see that the introduction of integral constraint for supreme power leads to different growth path and the steady state. Further, it is expected to bring us more unexplored serendipity in the future research. The remaining part of this paper is organized as follows. Section II will review the background and related literature, section III presents the model after comparing the balance of payments statistics in the United States and the rest of the world, and section IV will conclude.

## **II. Background and Literature Review**

Adam Smith (1776) pointed that the source of national power is the production capacity formed by free market forces and invisible hand rather than tangible precious metals from possessive

desires and interventions, in his classical book, “*An Inquiry into the Nature and Causes of the Wealth of Nations*”<sup>2</sup>. Capturing the essential points in the market mechanism was ingenious and unprecedented in his era when Mercantilism and strong control of the state were prevailing and much easier to be accepted by the monarchs. For this reason, Adam Smith emerged as an early luminary on the Classical School and his thoughts have been a guiding light such as what we revisit and reflect problems arising from every field of economic affairs. To some extent, Mercantilism also resonates today because a store of precious metals and government intervention are understood in the modern context of accumulating foreign reserves that most of countries need in order to prevent foreign exchange crisis (Lee 2014). The sea power and competent administration are what Adam Smith and Mercantilism have shared for states, which are still valid for the contemporary states.

Beckley(2018) argues that the power of nations should be a function of net resources, not of gross resources that would exaggerate the wealth and military capabilities for poor and populous countries by taking historical examples of competing rivals since the 19th century. It is forced to consider costs to keep production, welfare, and security. Then, it becomes a question of political and economic regime to which extent the regime allows efficiency.

American naval supremacy controls the world’s ocean and the annexation with the Kingdom of Hawaii was a stretching moment to the last patch of land in the Pacific Ocean. Since 1947, the U.S. Indo Pacific Command headquarter has started being stationed in Hawaii. Further, the formation of NATO in 1949 has also strengthened the American hegemony over the North Atlantic and the Mediterranean ocean, which had served against Soviet Union during the cold war. The global clout of American military force, i.e., the ultimate source of exercising national power has been unchallenged, and she ranks the world’s first both in military power and military spending. As recent figures, America keeps troops and service members numbered more than 220,000 persons abroad across 173 countries, operating special forces and clandestine units (Defense Manpower Data Center, as of 31 March 2021).

In the sense that the freedom is one of the core values in modern civilization, she has also gained deference; America opened its market without requiring equal access to markets from allied nations, from Europe to Asia such as Japan, South Korea, and Taiwan that were utterly in need

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<sup>2</sup> It is commonly referred to “*The Wealth of Nations*.”

of sustainable economic growth from the debris of wars in the 20<sup>th</sup> century; she has granted benefits to as many states as possible for not joining another alliance hostile to American power and even directly provides national security by stationing army in the Asian countries such as Japan and the Republic of Korea.

Unlike the geopolitical and historical context that both Adam Smith and Mercantilism have reverberated, Caballero *et al.* (2008) has a neoclassical model where the source of capability discrepancy among countries comes from the capacity to generate financial assets from real investments. Similarly, countries show heterogeneous productivity and an asymmetry in financial development in Ikeda and Phan (2019), so capital flows into emerging economies have been predominantly short-term, making them pay more to hedge macroeconomic risk (Bonizzi, Kaltenbrunner, and Powell 2019).

The neoclassical literature often posed the question how U.S. monetary policy shocks have become spillovers to emerging market economies, such as volatile capital flows, credit cycles, exchange rates, and investor sentiments with U.S. monetary policy<sup>3</sup>. More importantly, they often presume a non-hierarchical world economy as consisting of equal states that have policy sovereignty in a Westphalian sense (Murau and Van't Klooster 2020) like the Mundell-Fleming model (Mundell 1960, Fleming 1962).

In this regard, this paper has different perspectives, i.e., we pay more attention to the U.S. economy that has been relatively free from balance of payments constraint for a considerably long time, which would mean both blessing and responsibility. Institutionally, America is supposed to provide emergency U.S. dollar liquidity from the Federal Reserve to non-U.S. central banks (Murau *et al.* 2021). The balance of payments statistics can be a lens for observing currency hierarchy<sup>4</sup>, international asymmetry (Cohen 1998), original sin (Eichengreen and Hausmann 2005), monetary subordination (Kaltenbrunner 2015), and can be a window to see the world's modern history since 1945. The presence of a balance of payments constraint would slow productivity growth (Thirwall 1997).

Apparently, the balance of payments disequilibrium under the gold standard was not corrected

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<sup>3</sup> Lee 2014, Farhi and Maggiori 2018, Brauning and Ivashina 2020, Mohanty and Banerjee 2021, Meulendyke 1998, Georgiadis 2015, Akinci and Queralto 2021.

<sup>4</sup> Strange 1971, Kindleberger 1974, Terzi 2006, Andrade and Prates 2013, Kaltenbrunner 2015, Fritz, Paula and Prates 2018; Bonizzi, Kaltenbrunner, Powell 2019.

by a counter-flow gold as expounded by the classical price-specie-flow mechanism claimed by David Hume and led to the end of the Bretton Woods system as predicted by Triffin's dilemma<sup>5</sup>. Since 1971, the dollar no longer needs to have a strict value to gold, but deeply integrated global financial markets and the world's recognition of the supreme power have sustained global imbalances. Since then, the world has rather accommodated demands of global liquidity as one observes from quantitative easing and tapering that has been more accentuated since the global financial crisis started in 2008 and the corona pandemic crisis started in 2020.

### III. The Model

#### 1. The Time Horizon from Balance of Payments Statistics

Equation is a succinct way of expressing economic relationships, but it often awaits interpretation as seen from one of the well-known basic equations in macroeconomics like equation (1). There is a subtle distinction between equilibrium and identity that has not always been clearly understood as Turnovsky (1996, p.18) mentions. Indeed, we suppose that the source of that subtlety is *time*, that is, as we frequently use the expression, *ex ante* and *ex post*, in a way to explain the distinction. Basically, there is a *timing difference*. A timing difference between planning of economic activity *ex ante* and accounting entry *ex post*, is obvious.

This study is also concerned with the subtle difference about time, to be more specific, *the time horizon*. Before discussing the time horizon, we are going to observe the balance of payments statistics linking its meaning to macroeconomics. Equation (1) describes how the national income earned is disposed of, i.e.,

$$Y = C + I + G + EX - IM \quad (1)$$

where  $Y$  represents national income earned or national production or national expenditure,  $C$

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<sup>5</sup> It is named after Robert Triffin who wrote the book, *Gold and the Dollar Crisis: The Future of Convertibility*.

denotes consumption expenditure,  $I$  does investment expenditure,  $G$  does government spending,  $EX$  does the amount of exports, and  $IM$  does the amount of imports.<sup>6</sup>

By rearranging equation (1), one has equation (2) as Obstfeld and Rogoff (1996, p.60) where the current account balance has the inter-temporal nature, as including the time changes of foreign assets acquired, i.e.,  $\dot{B} = \frac{dB}{dt}$ .

The rising export in the current account is recorded as the increase in external asset or the decline in external debt in the financial account ( $\dot{B} > 0$ ). It could lead to capital outflows, and thus the economy being world's creditor. In the same manner, growing import in the current account is recorded as the decrease in external asset or the increase in external debt in the financial account ( $\dot{B} < 0$ ). It could result in capital inflows, and thus the economy being world's debtor. In the balance of payments data, the term,  $\dot{B}$  corresponds to the financial account of which statistics have been released from the International Monetary Fund.<sup>7</sup>

The current account appears to be more natural to us than the financial account because it involves resource flows oriented to the current period while the financial account measures how the net lending to or borrowing from nonresidents is financed. The financial account is full of sub-accounts influencing future periods recording net acquisition and disposal of financial assets and liabilities. They are supposed to contribute to production and income generation, changing in values and volumes due to revaluations.

$$EX - IM = \text{Current account} = S - I = (Y + \xi B - C - G) - I = \dot{B} = \frac{dB}{dt} \quad (2)$$

where  $S$  is national savings that equals to the sum of terms,  $(Y + \xi B - C - G)$ ,  $B$  denotes net holdings of bonds issued by foreign countries,  $\xi B$  is interest earned on foreign assets acquired previously, and  $\xi$  is the world's nominal interest rate that prevails an open economy. As

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<sup>6</sup> Equation (1) has the only term involving transactions with foreign residents,  $EX - IM$  that this paper considers as current account even though  $EX - IM$  is also regarded as a narrower category like the trade balance of goods and services.

<sup>7</sup> Currently, the statistics follows the 6<sup>th</sup> edition of Balance of Payments and International Investment Position Manual (BPM6 henceforth). BPM6 was revised in 2008, with major themes of reflecting globalization and financial innovation.

shown in equation (2), current account summarizes a country's intertemporal saving and investment decisions, hence decomposing savings and investments will invite us to many intertemporal issues.

Formally, the sum of the balances on the current account and the capital account represents the net lending or net borrowing by the economy with the rest of the world and this amount is expected to be equal to the net balance of the financial account.<sup>8</sup> The capital account represents credit and debit entries for non-produced, non-financial assets and capital transfers such as land, leases, and licenses, although the size is normally much smaller than current and financial accounts. So, the hypothesis that the sum of the current account and capital account are equal to the financial account balance, can be tested to see whether there is any difference between supreme power and the rest of the world. The null hypothesis for the data from 1970 to 2020 is,<sup>9</sup>

$$H_0: \text{Current account} + \text{Capital account} - \text{Financial account} = 0$$

For the United States, this null hypothesis is accepted with  $t - \text{value} = -1.4987$ ,  $p - \text{value} = 0.1402$ . For the data points in the rest of the world, the null hypothesis is rejected with  $t - \text{value} = 10.295$ ,  $p - \text{value} = 2.2e - 16$ . The results confirm that the huge current account deficit in the United States has been well financed. On the other hand, the rest of the world rejected the null hypothesis and the average amount of *Current account + Capital account - Financial account*, has turned out to be 1.27 billion U.S. dollars, possibly for keeping it positive to prevent financial crisis. It shows the different reality that the United States has faced in the balance of payments constraint.

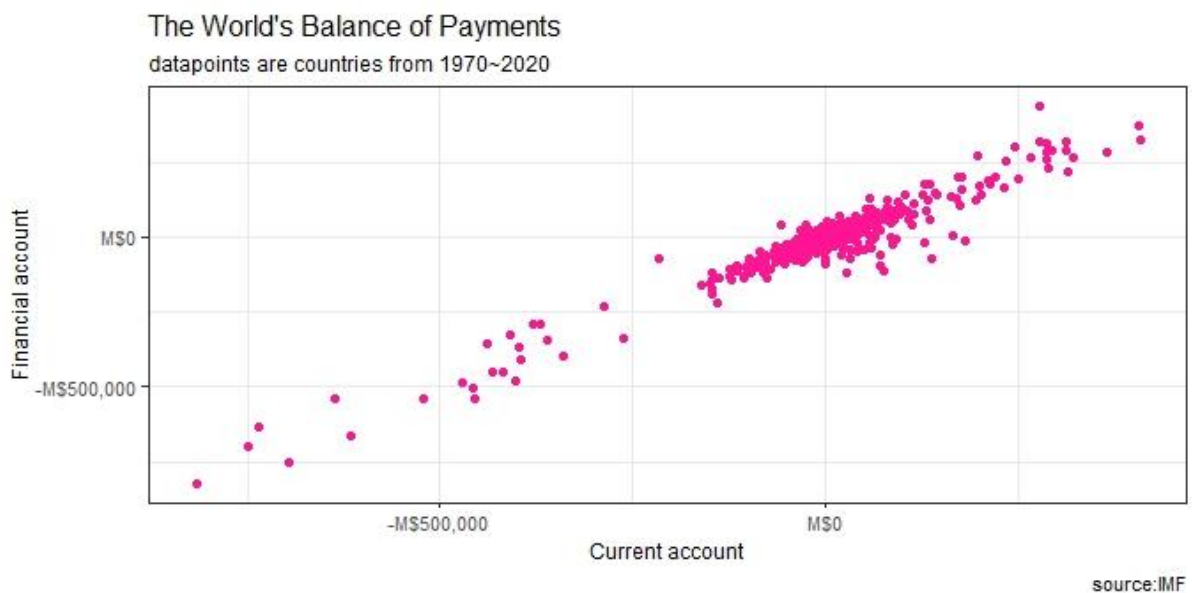
As seen in Figures 1 and 2, the United States has shown persistent and even greater current and

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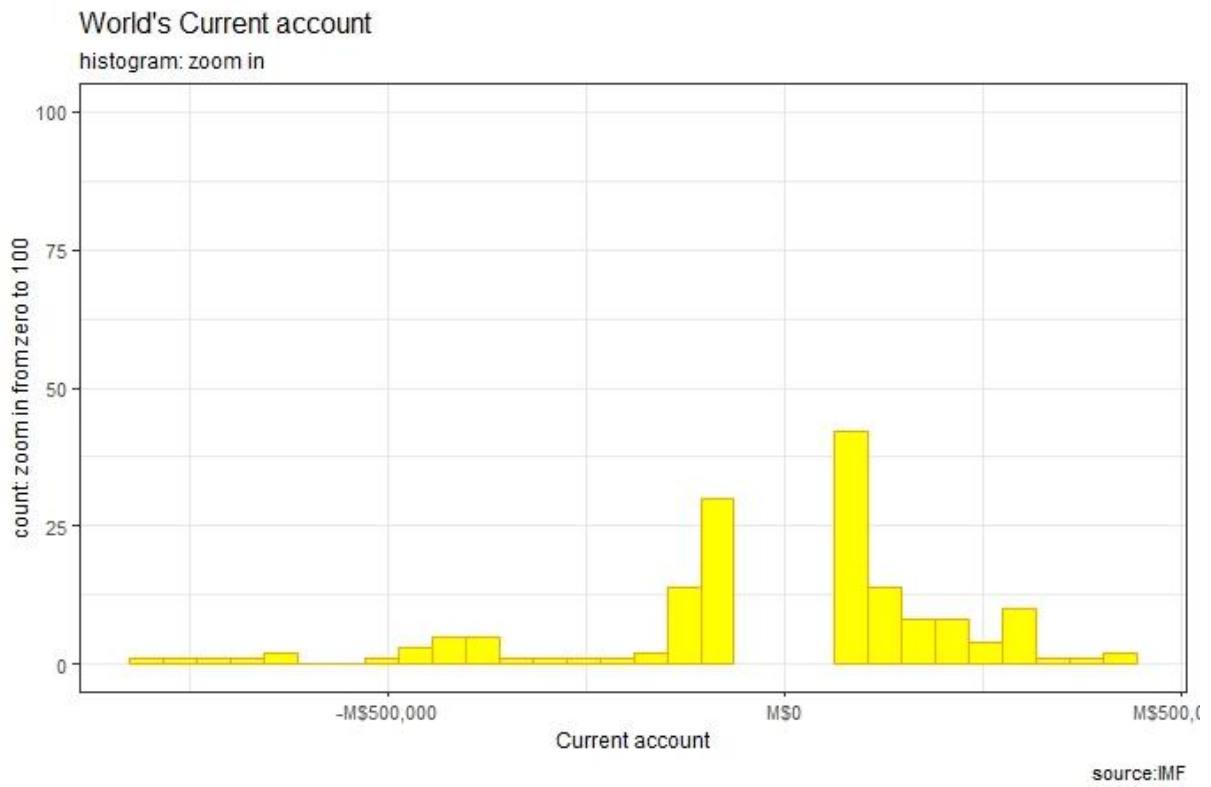
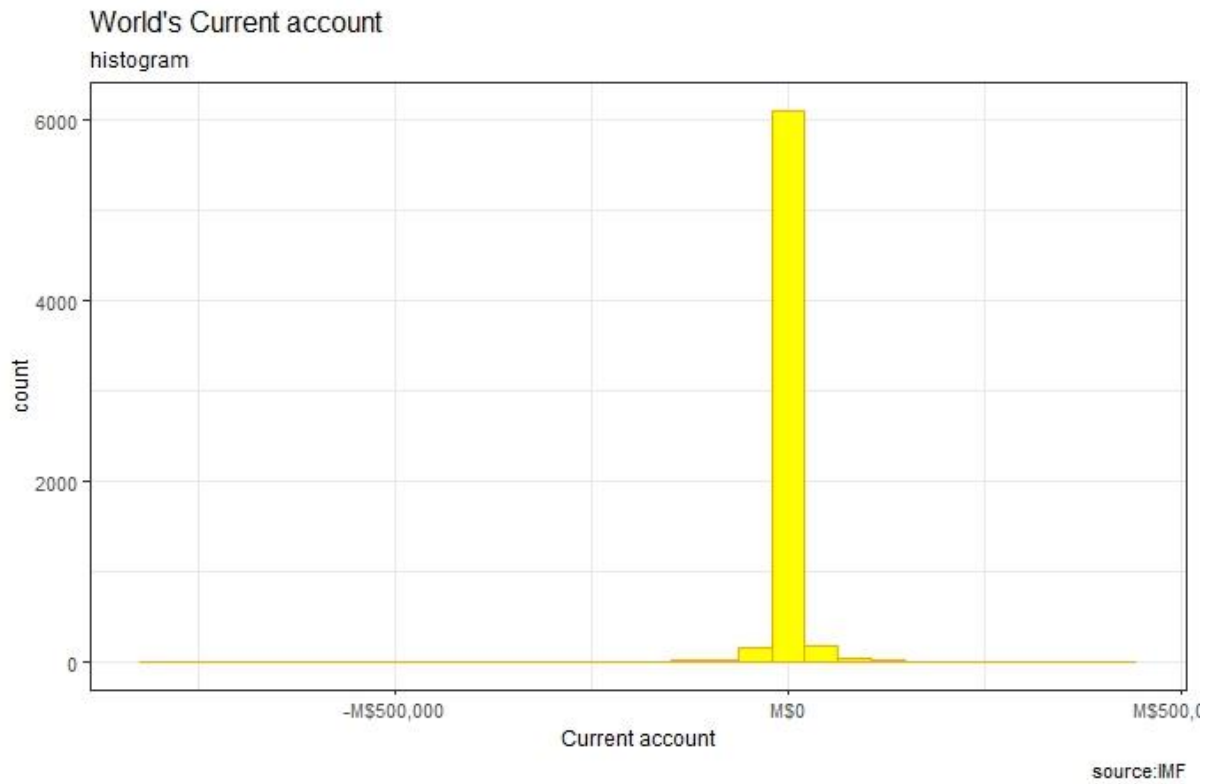
<sup>8</sup> This is also the test about errors and omissions being equal to zero. Regarding the hypothesis, the IMF defines errors and omissions as coming from imperfections in source data and compilation. It is derived residually as net lending/net borrowing and can be derived from the financial account minus the same item derived from the current and capital accounts.

<sup>9</sup> This part often shows confusing statements in the textbooks and websites that are saying, the financial account balance plus the current account balance cancelled out or the sum of them are equal to zero in equilibrium.

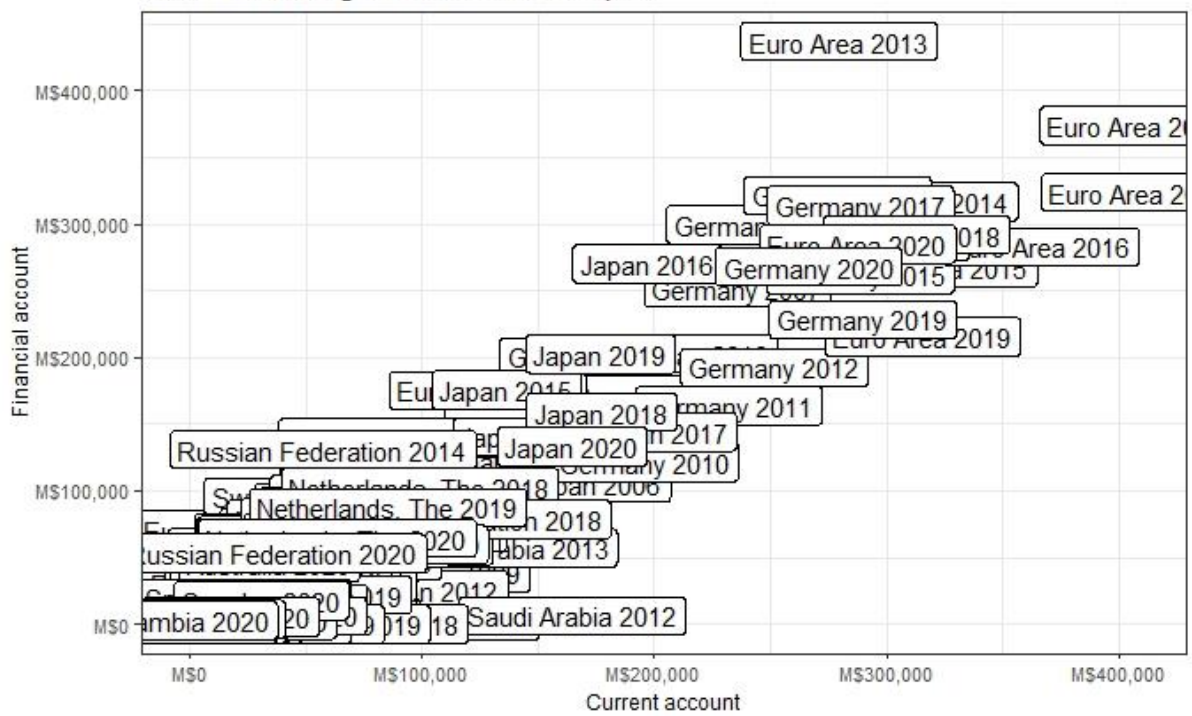
financial account deficits since the collapse of Bretton Woods system in 1971. Huge deficits in both current account and financial account that the United States has recorded, has not been available options for the rest of the world as witnessed from a number of financial crises, which underlined that the supreme power is relatively free from the balance of payments constraint. It means enjoying the Trinity, i.e., relatively stable exchange rate with capital flows that investors regard the U.S. dollar denominated assets as safe. Also, the Federal Open Market Committee's leading role in setting monetary policy that is more than monetary policy independence. It awaits our attempts to reflect within an analytical model.





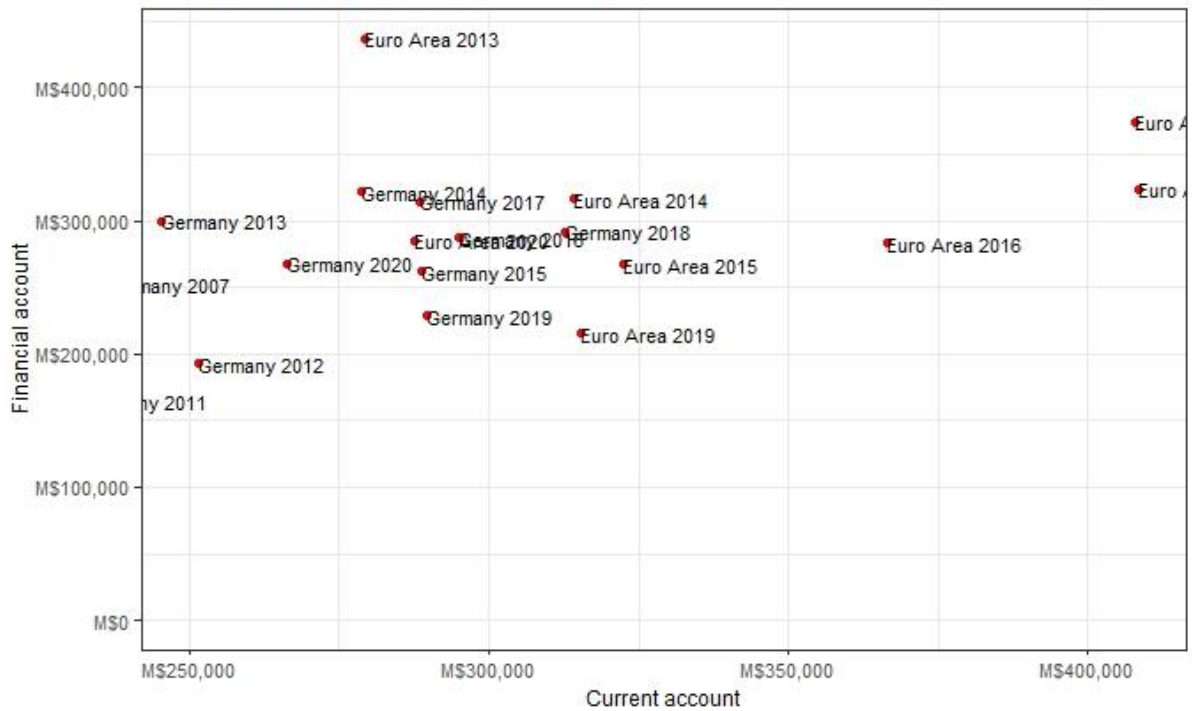


Countries running both accounts in surplus



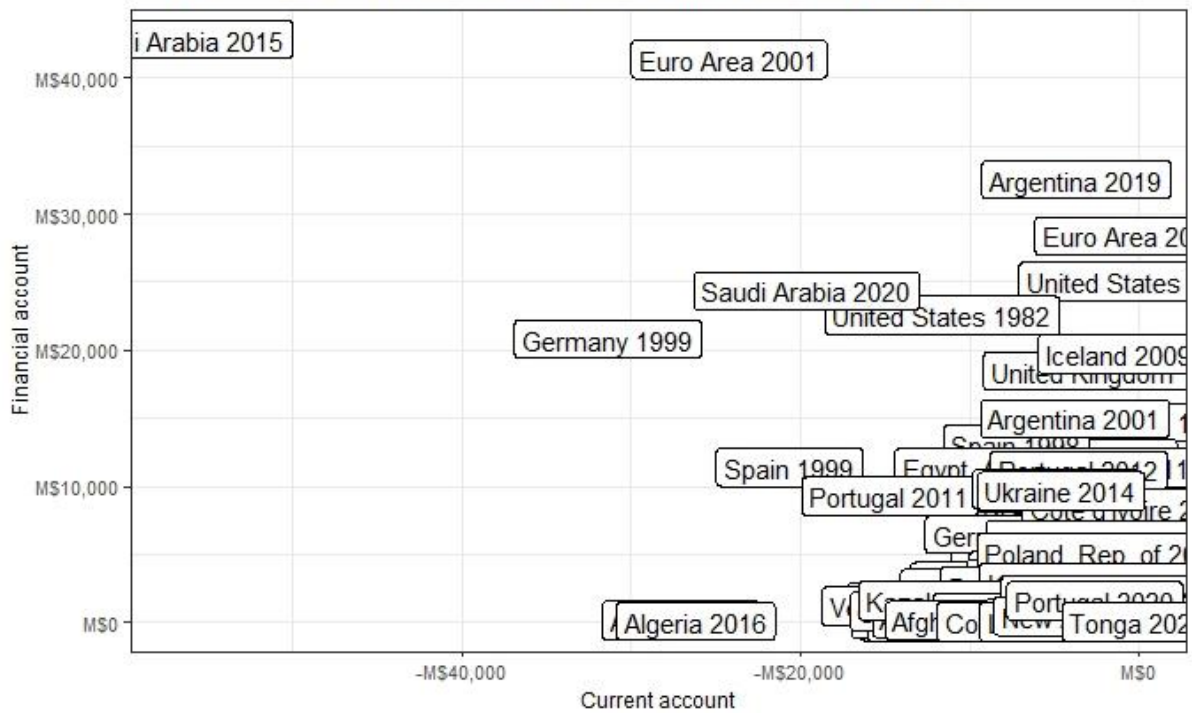
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Countries running both accounts in surplus



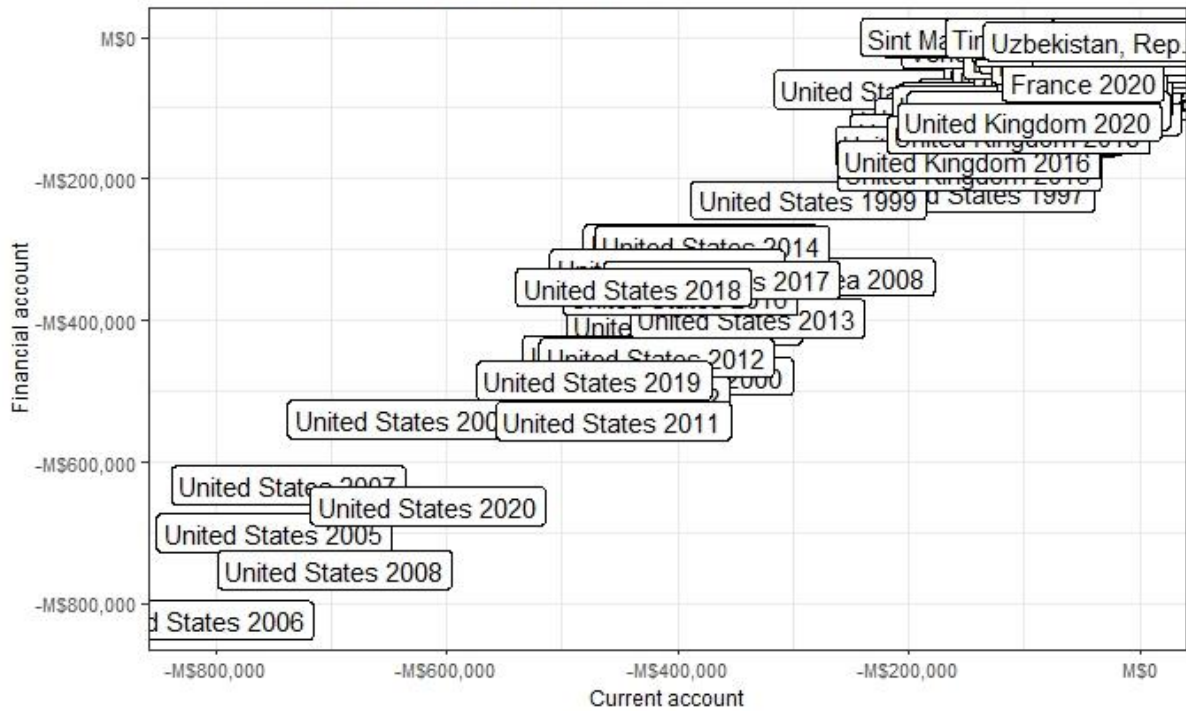
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Countries running current account deficit and financial account surplus



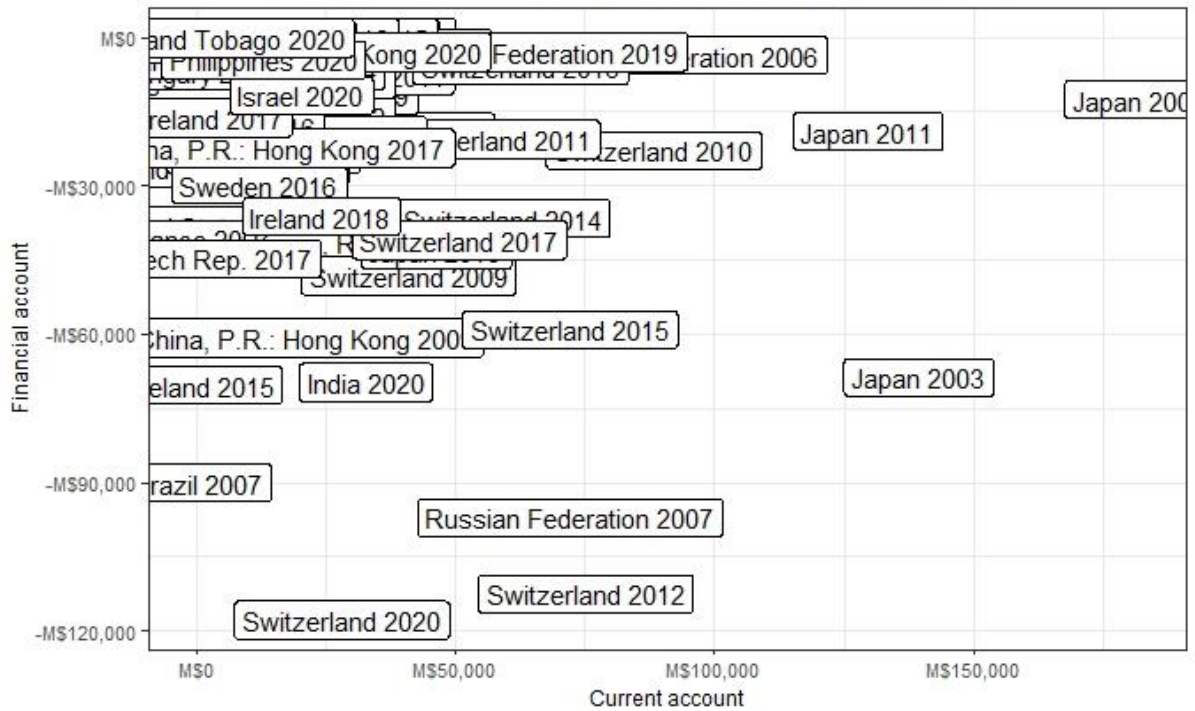
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Countries running both accounts in deficit



source:IMF

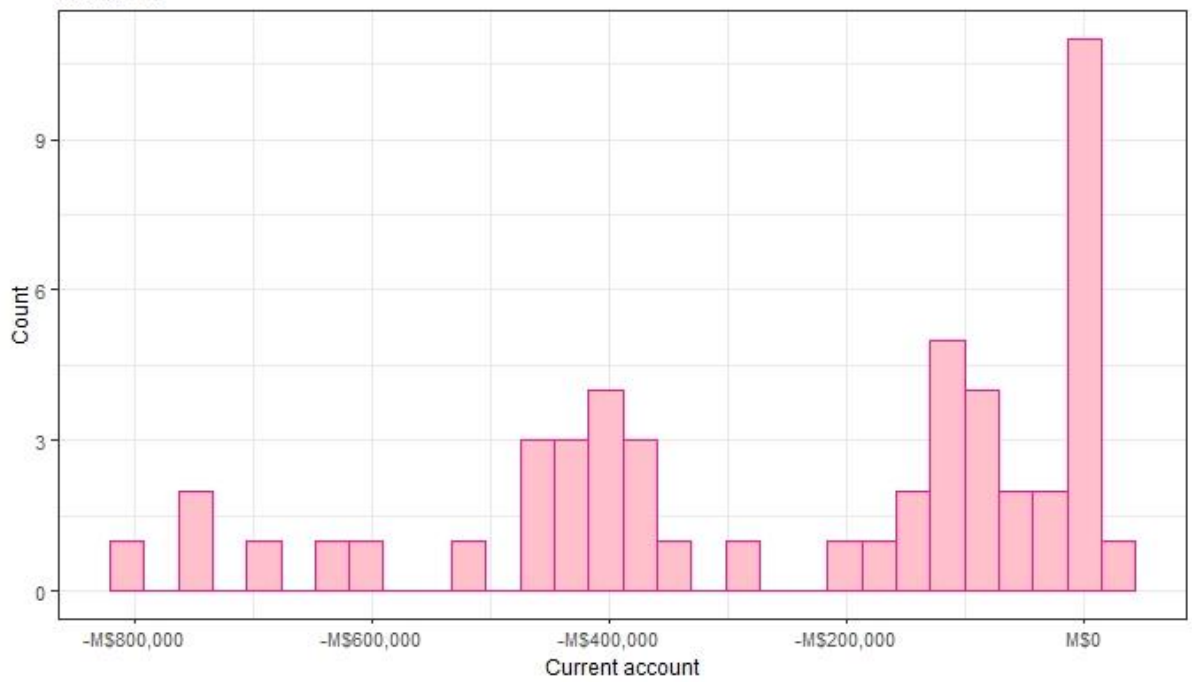
Countries running current account surplus and financial account deficit



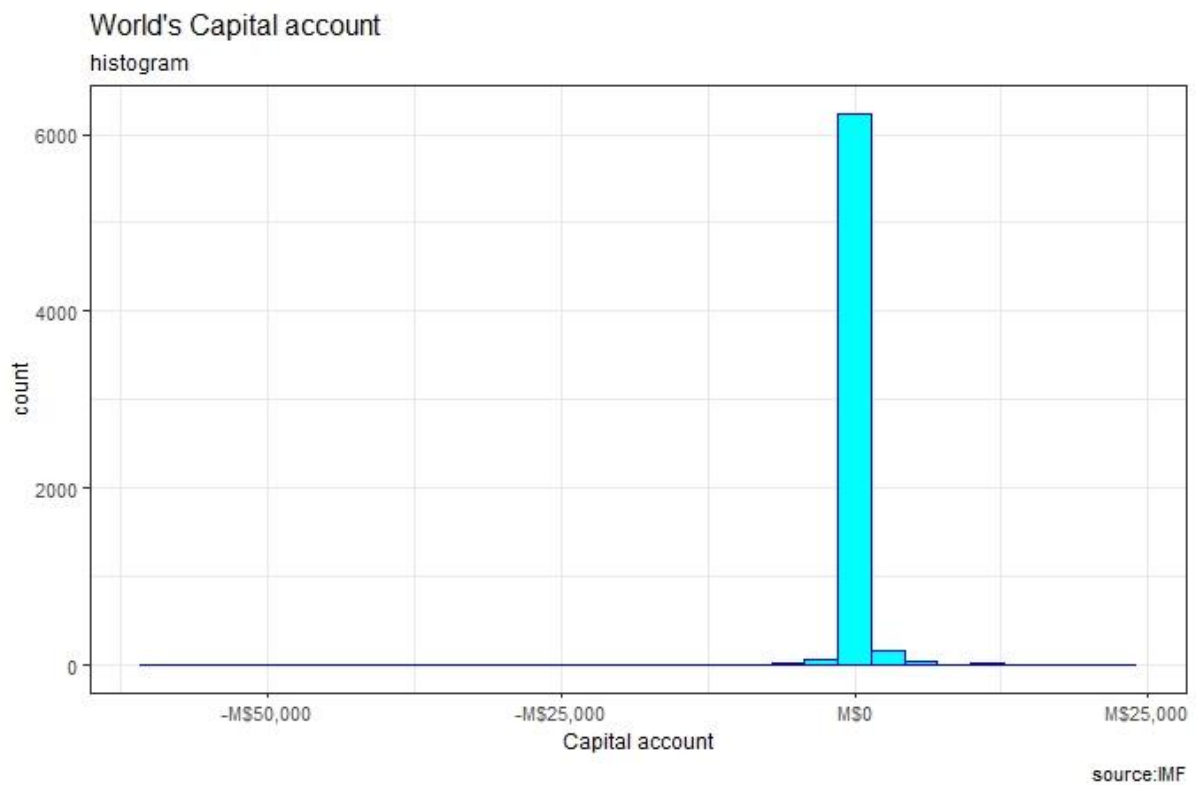
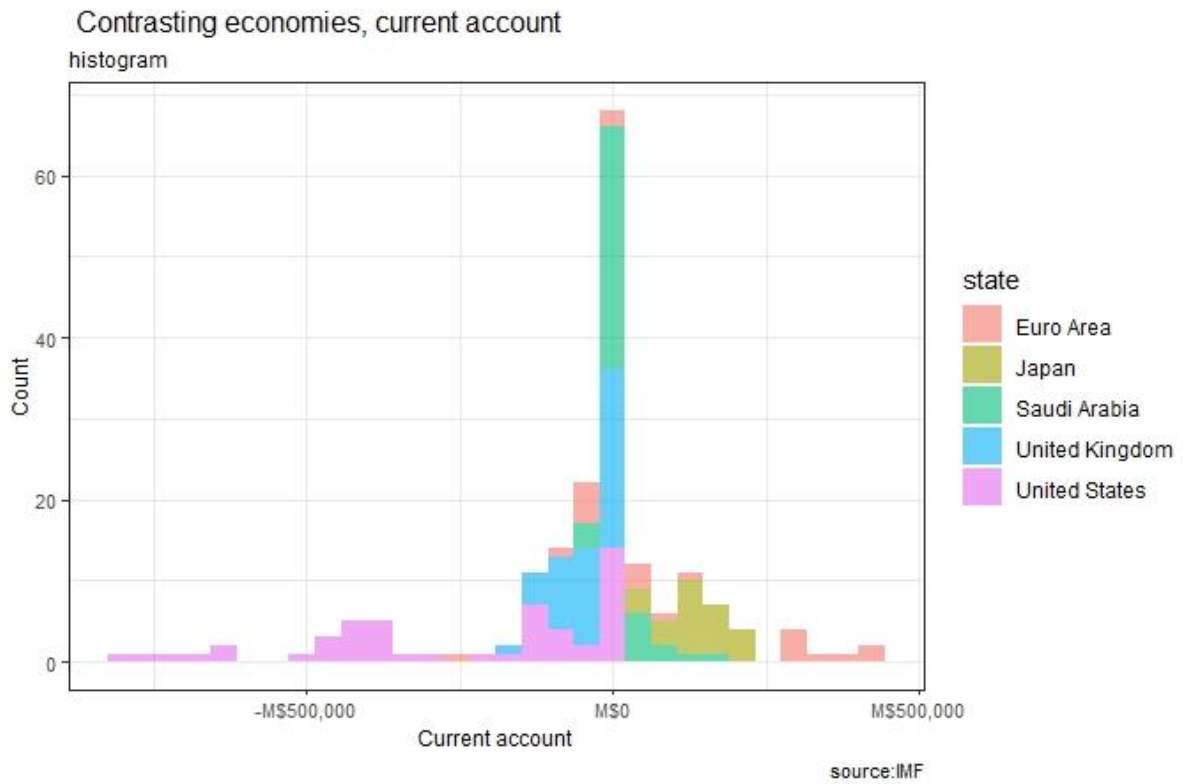
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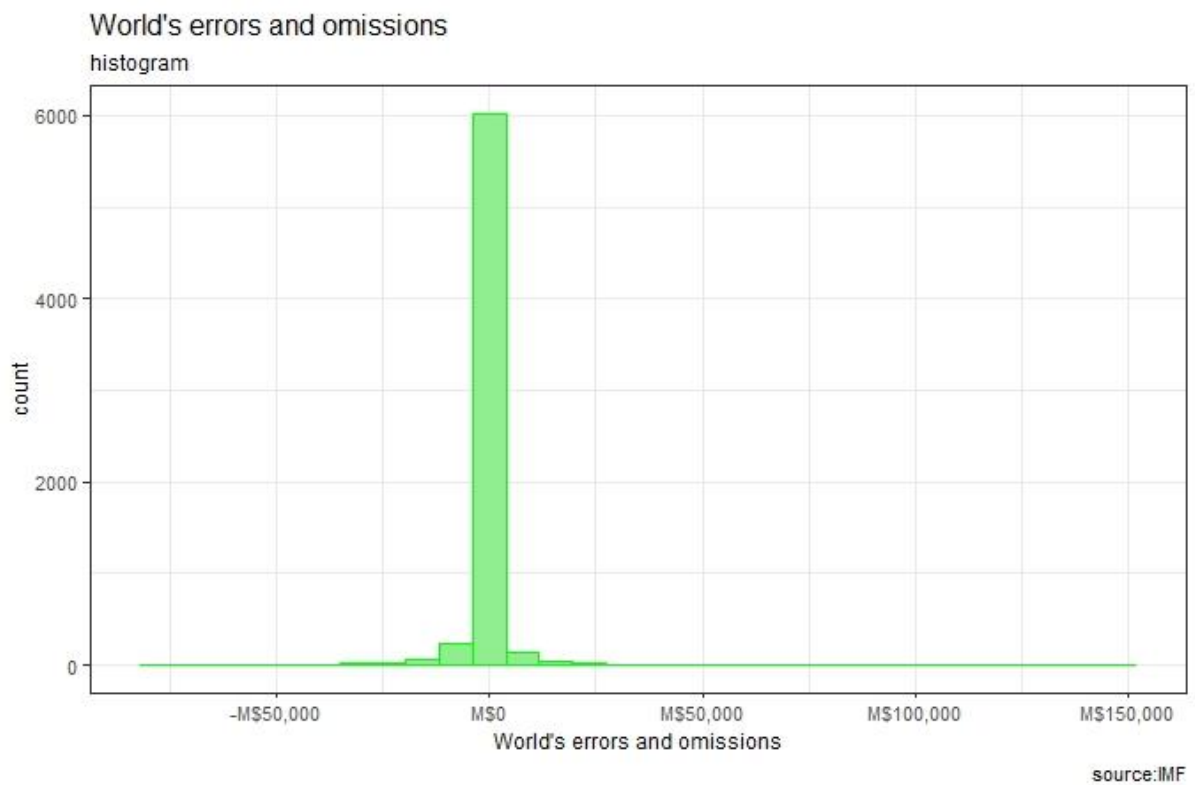
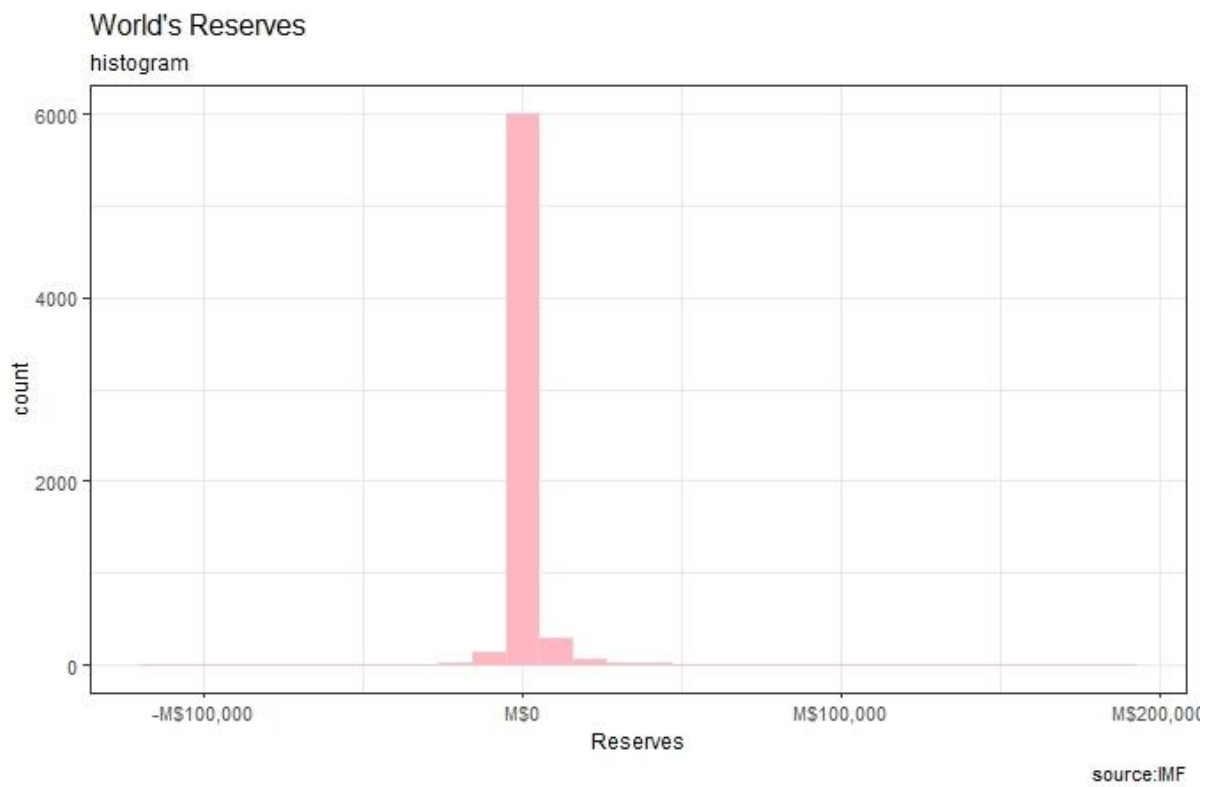
# USA, Current account

histogram

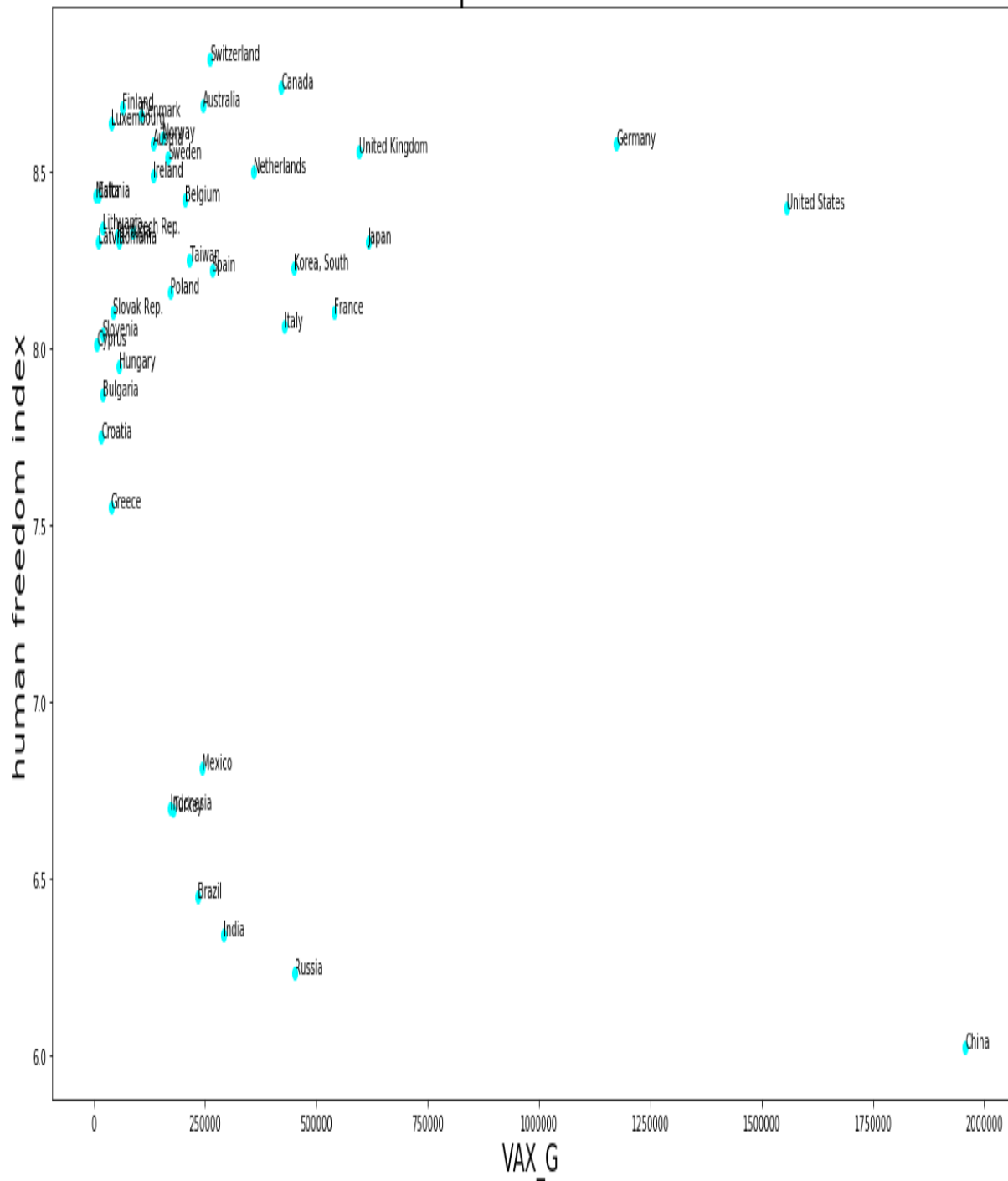


source:IMF





## Value added Exports and Human Freedom



### More explanations

*To be written*



## 2. The Model

Once the state is recognized as a supreme power, there is little concern about period-specific current account balance that most of countries needs to care in practice. Given this reality, we suggest the introduction of integral constraint for the supreme power. After dividing the equation (2) by the price level,  $P$ , we get the familiar integrand budget constraint in real terms as,

$$F(k) - c - \dot{k} - g + (\xi - p)b - \dot{b} - pm - \dot{m} = 0 \quad (3)$$

where real output is  $\frac{Y}{P} = y = F(k)$ , real consumption is  $\frac{C}{P} = c$ , real investment is  $\frac{I}{P} = i = \dot{k}$ , real government spending is  $\frac{G}{P} = g$ , real stock of bond is  $\frac{B}{P} = b$ , real stock of money is  $\frac{M}{P} = m$ , and the inflation rate is  $p = \frac{\dot{P}}{P}$ .

From here, the integral budget constraint can be written as,

$$\int_0^\infty (F(k) - c - \dot{k} - g + (\xi - p)b - \dot{b} - pm - \dot{m})dt = 0. \quad (4)$$

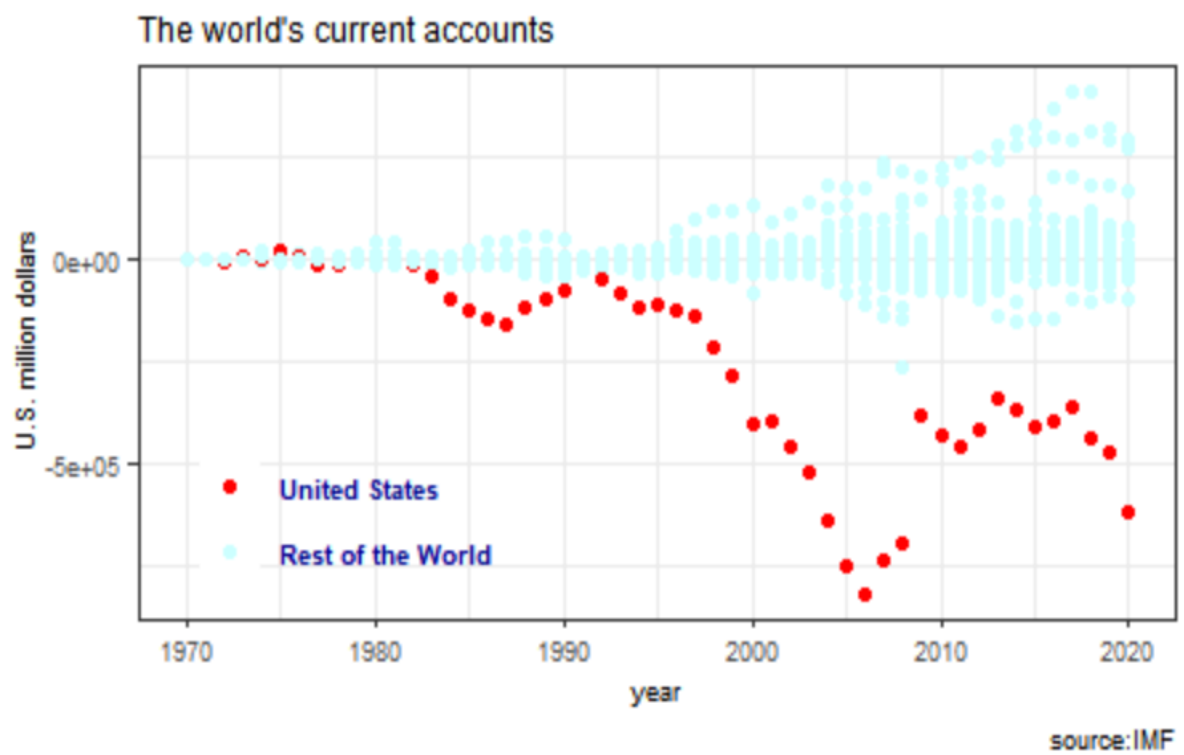
This integral constraint has dramatically relaxed the constraint because current account deficit at certain point is expected to be repaid at any other points in the future, not within a specific period. This study suggests the integral constraint in a way of expressing the supreme power's ability to repay and the world's recognition for it.<sup>10</sup>

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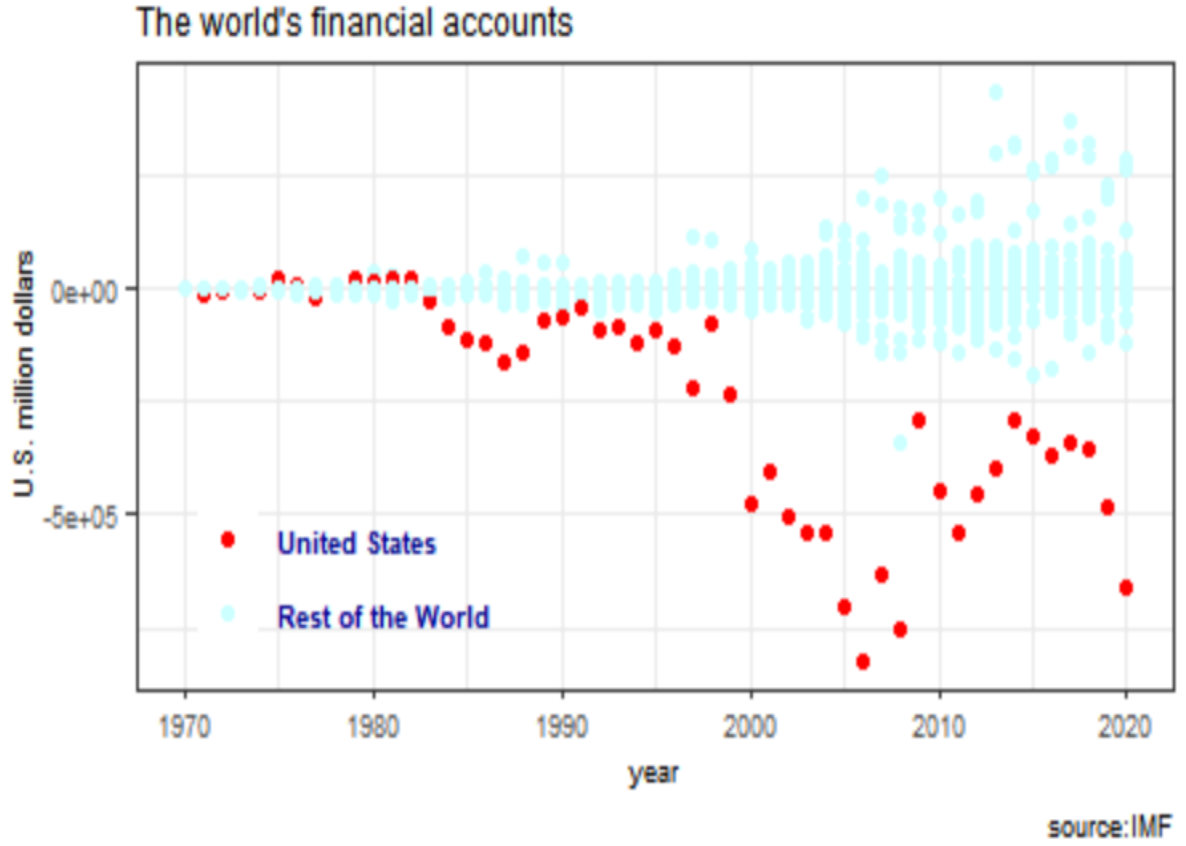
<sup>10</sup> The fundamental result stays the same when the infinite horizon is changed into the finite one in the integral constraint.

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**Figure 1. The World's Current Accounts**



**Figure 2. The World's Financial Accounts**



Suppose that the supreme power is assumed to maximize its representative utility as,

$$\max_c \int_{t=0}^{\infty} U(c) e^{-\rho t} dt$$

$$\text{subject to } \int_0^{\infty} (F(k) - c - \dot{k} - g + (\xi - p)b - \dot{b} - pm - \dot{m})dt = 0.$$

$$b(0) = b_0 \quad k(0) = k_0 \quad m(0) = m_0$$

(5)

We introduce a new state variable  $\Gamma(t)$  into the problem such that the integral constraint can be replaced by a condition in terms of  $\Gamma(t) = -\int_0^t (F(k) - c - \dot{k} - g + (\xi - p)b - \dot{b} - pm - \dot{m})dt = 0$ . Also, we can replace the given integral constraint by a terminal condition on

the  $\Gamma(t)$  variable as suggested by Chiang(1992), so

$$\begin{aligned}\Gamma(0) = 0, \quad \Gamma(\infty) &= -\int_0^\infty (F(k) - c - \dot{k} - g + (\xi - p)b - \dot{b} - pm - \dot{m})dt = 0 \quad \text{and} \\ \dot{\Gamma}(t) &= -(F(k) - c - \dot{k} - g + (\xi - p)b - \dot{b} - pm - \dot{m})\end{aligned}\tag{6}$$

Now, the problem requires finding optimal time path for a state variable,  $\Gamma(t)$  and a control variable,  $c$  that is assumed to have an interior solution. Then, we maximize the following value function as,

$$\begin{aligned}\text{Maximize } V &= \int_0^\infty U(c)e^{-\rho t} dt \\ &\quad - \mu \left[ \int_0^\infty (F(k) - c - \dot{k} + (\xi - p)b - \dot{b} - pm - \dot{m} + \dot{\Gamma}(t)) dt \right] \\ &= \int_0^\infty [H(t, c, \Gamma, \mu) - \mu \dot{\Gamma}] dt\end{aligned}$$

$$\text{where Hamiltonian, } H = U(c)e^{-\rho t} - \mu[F(k) - c - \dot{k} + (\xi - p)b - \dot{b} - pm - \dot{m}]\tag{7}$$

If we perturb the optimal consumption,  $c^*(t)$  path with a perturbing curve  $p_c(t)$ , we can generate neighboring control paths as,

$$c(t) = c^*(t) + \varepsilon p_c(t)\tag{8}$$

Similarly,  $\Gamma(t)$  can be considered around the optimal  $\Gamma^*(t)$  path as,

$$\Gamma(t) = \Gamma^* + \varepsilon q_\Gamma(t)\tag{9}$$

Then, one can express the value function as the function of perturbation  $\varepsilon$  as,

$$V(\varepsilon) = \left[ \int_0^{T(\varepsilon)} H(t, c^* + \varepsilon p_c(t), \Gamma^* + \varepsilon q_\Gamma(t), \mu) + \dot{\mu}(\Gamma^* + \varepsilon q_\Gamma(t)) dt \right] - \mu(T)\Gamma(T) + \mu(0)\Gamma(0) \quad (10)$$

We now apply the condition,  $\frac{dV}{d\varepsilon} = 0$  to reach the optimal values,  $c^*$  and  $\Gamma^*$ , then

$$\begin{aligned} \frac{dV}{d\varepsilon} = & \left[ \int_0^{T(\varepsilon)} \left\{ \frac{\partial H}{\partial c} p_c(t) + \left( \frac{\partial H}{\partial \Gamma} + \dot{\mu}_t \right) q_\Gamma(t) \right\} dt \right] + \\ & [H(t, c^* + \varepsilon p_c(t), \Gamma^* + \varepsilon q_\Gamma(t), \mu)]_{t=T} \triangle T - \mu(T)\Gamma(T) = 0 \end{aligned} \quad (11)$$

From the optimal condition,  $\frac{dV}{d\varepsilon} = 0$ , one can deduce two conditions, known as the maximum principle, i.e.,

$$\frac{\partial H}{\partial c} = 0 \quad - \frac{\partial H}{\partial \Gamma} = \dot{\mu}_t = 0 \quad (12)$$

From  $-\frac{\partial H}{\partial \Gamma} = \dot{\mu}_t = 0$ , we find that  $\mu_t$  is constant.

Also, from the term,  $[H(t, c^* + \varepsilon p_c(t), \Gamma^* + \varepsilon q_\Gamma(t), \mu)]_{t=T} \triangle T = 0$  and a variable terminal time  $T$ , one has

$$\lim_{t \rightarrow \infty} H = 0. \quad (13)$$

It is general transversality condition for infinite horizon as explained in Michel (1982), being interpreted as the leftovers after national spending in this case, has been taken advantage of as  $t \rightarrow \infty$ .

From the term,  $\mu(T)\Gamma(T)=0$ ,  $\Gamma(T)$  is not necessarily zero for free terminal state, so

$$\lim_{t \rightarrow \infty} \mu(t) = 0 \quad (14)$$

becomes a transversality condition for the infinite horizon. In the economic terms, it implies that the shadow price for current account balance preceded by minus sign is zero as  $t \rightarrow \infty$  for supreme power. Alongside with  $\lim_{t \rightarrow \infty} \mu(t) = 0$ , and the result that  $\mu_t$  is constant as confirmed from equation (12),

$$\mu_t = 0 \quad (15)$$

When applying the maximum principle, an optimal path for  $(\dot{\Gamma}, c_t)_0^\infty$  will satisfy a continuous function  $\mu(t)$  such that for all  $t \geq 0$ . The transversality condition in equation (14) is held, then the following is also held, that is,

$$\lim_{t \rightarrow \infty} \mu(t) \dot{\Gamma}_t = 0$$

where

$$\dot{\Gamma}(t) = -(F(k) - c - \dot{k} - g + (\xi - p)b - \dot{b} - pm - \dot{m}) \quad (16)$$

It is what Groth (2016) and Barro and Sala-i-Martin (1995) regard as transversality condition concluding the optimal path from other admissible paths. The intuition is that optimizing agents do not want to have any valuable assets be left over at the end. The transversality condition says that for  $t \rightarrow \infty$ , the shadow value of capital flows to finance the current account deficit, should go to zero. So, the shadow value of current account balance should go to zero, implying the reality for the supreme being in a free position from the balance of payments constraint.

It reminds us of the No-Ponzi Game(NPG) condition being satisfied with strict equality. (Groth 2016, p. 407, line 12~15). Once transversality condition is met, then there is no conflict with the NPG condition, either.<sup>11</sup>

From the Hamiltonian,  $H = U[c]e^{-\rho t} - \mu[F(k) - c - \dot{k} + (\xi - p)b - \dot{b} - pm - \dot{m}]$ ,

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<sup>11</sup> The NPG condition attracts researchers, for example, Tamegawa(2013) finds that the No-Ponzi Game condition is used to guarantee an existence of optimal solutions and the solution attains the finite intertemporal utility.

when one differentiates one of the first order conditions in equation (12), i.e.,

$$\frac{\partial H}{\partial c} = U_c e^{-\rho t} + \mu = 0$$

with respect to time, then we get the basic condition for choosing consumption over time as,

$$\frac{\dot{c}}{c} = -\frac{u_c}{u_{cc}c} [0 - \rho] = -\frac{1}{\left(\frac{u_{ccc}}{u_c}\right)} [0 - \rho]$$

(17)

By comparison, Ramsey (1928) has the familiar intertemporal condition as,

$$\frac{\dot{c}}{c} = -\frac{u_c}{u_{cc}c} (\text{real interest rate} - \rho) = -\frac{1}{\left(\frac{u_{ccc}}{u_c}\right)} (\text{real interest rate} - \rho) \quad (18)$$

In Ramsey (1928), the steady state is where the marginal product of capital is equal to the rate of time preference as in equation (18).

From our equation (17),  $\frac{\dot{c}}{c} = -\frac{u_c}{u_{cc}c} [0 - \rho] = -\frac{1}{\left(\frac{u_{ccc}}{u_c}\right)} [0 - \rho]$ , the supreme power can be on a possible bliss point with  $u_c = 0$  in the steady state. The supreme power's relative freedom from balance of payments constraint puts its economy to be in a position of being decoupled from the production side once the supreme power is recognized as having virtually supreme power-like productivity and power. It is not necessarily true for non-supreme powers in their steady state where real interest rate appears in equation (18) reflecting production side and constraint.

Also, if the time preference rate,  $\rho$  is zero in equation (17), then the supreme power reaches the steady state. In practice, the time preference rate is supposed to be normally greater than zero and less than one, but Ramsey has thought that zero discounting is ethical for future generation and a British philosopher, Derek Antony Parfit (1947~2017) considers that the low



time preference rate can be interpreted as caring neighbors (Hong p.436 line10-21). In whichever way the interpretation is made, the time preference rate equal to zero, can be one possibility to characterize the supreme power's steady state.

The role of willingness to intertemporal substitution or the relative risk aversion, i.e.,  $-\frac{u_{cc}c}{u_c}$ , stays the same in equations (17) and (18). Francis Bacon (1561~1642) had once mentioned the role of "courageous people" and suggested to make the courageous aristocracy take risky projects for the prosperity of kingdom, in his book, *The Essays* published in 1625. The attitude to risk still remain to affect the economic growth for the supreme power and the rest of the world.

## IV. Conclusions

This study covers the history-long topic of economics, i.e., the world's key currency and the optimal control model. We suggest introducing integral constraint for the supreme power. Once being recognized by the world, the supreme power would be free from balance of payments constraint. Integral constraint certainly leads to different optimal conditions and in the long run steady state, the supreme power's time preference rate is not pinned down to real interest rate as the rest of the world. Rather, the supreme power could be on the bliss point in the long run steady state. Taking no real interest rate part in the growth equation is natural because supreme power's relative freedom from balance of payments constraint puts its economy decoupled from production and constraint once the supreme power is recognized as having virtually supreme power-like productivity and power. Integral constraint would be highly operational, which will bring unexplored developments in the future research.

## Other Implications

*To be written*

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