

# Labor Market Volatility: A Macro-Finance View

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August 31, 2019

## Abstract

We study the impact of financial market incompleteness on the outcome of Kalai (1977)-Rawls (1971)  $\eta$ -egalitarian wage bargaining within the structure of a parsimonious DSGE macroeconomic model. Following Guvenen (2009) and Basak and Cuoco (1998), firm owners trade stock and default-free bonds while non-stockholder-workers trade only bonds. When imposed on the egalitarian bargaining regime, the partial income insurance provided by stockholders to non-stockholder-workers arising from this financial structure, and the pattern of stockholder consumption it implies, leads to countercyclical worker bargaining power, a very stable real wage and a flat yield curve in environments of high wealth inequality. Each of these phenomena is characteristic of the U.S. economy at present. They also result in high cyclical volatility of both vacancies and unemployment as well as their negative correlation at business cycle frequencies, statistical realities of the US labor market emphasized in Shimer (2005) and Hall (2005). Our results also lend conditional support to Hall (2017) who highlights the investment nature of vacancy postings. We further find that as wealth inequality grows, the ability of bond trading to promote risk sharing is diminished, while, in its place, the wage-setting mechanism endogenously creates a new “semi-fixed wage asset” to assist non-stockholders in their consumption stabilization needs.

*Keywords:*  $\eta$ -egalitarian wage bargaining, Nash wage bargaining; business cycles; unemployment volatility; limited participation; market incompleteness; vacancy volatility

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\*This paper has benefited from discussions with Marc Giannoni, Chris Moser, Andreas Mueller, Bruce Preston, Paolo Siconolfi, Stephanie Schmitt-Grohe, and Martin Uribe. The usual disclaimer applies.

# 1. Introduction

Within the framework of standard production DSGE (Dynamic Stochastic General Equilibrium) models, replicating the basic stylized facts of the labor market has proved especially challenging. A recent body of research (e.g., Hall (2005) and Shimer (2005)) argues that the by-now-standard labor market search paradigm of Diamond (1982), Mortensen (1992) and Pissarides (1988, 1990) (hereafter DMP) with Nash (1950) wage bargaining cannot easily account for the cyclical movement of key labor market variables when placed in a standard real business cycle environment. In particular, the high cyclical volatility of vacancies and unemployment, and their negative correlation at business cycle frequencies, are statistical regularities that are difficult to replicate.

The present paper argues that these regularities can be reproduced in an otherwise standard DSGE model that exhibits limited-participation financial market incompleteness in the presence of Kalai (1977)-Rawls (1971)  $\eta$ -egalitarian wage determination, a concept that reduces to standard Nash (1950) wage bargaining for typical bargaining parameter values when market completeness is restored. In particular, it is demonstrated that high wealth inequality, within the model framework, leads to a very stable wage. The present business cycle expansion is one characterized by high wealth inequality and real wage variation detached from labor productivity, phenomena replicated in the model's equilibrium.

A standard real business cycle model with a single persistent productivity shock and capital adjustment costs is the foundation on which we build. But rather than emphasizing the influence of labor market arrangements on a passive financial market (as in a complete markets representative agent setup), we focus on Guvenen (2009)-style limited financial market participation: there are two types of agents, stockholders and non-stockholders.<sup>1</sup> The former have full access to financial markets, namely the stock and (default-free) bond markets. The latter group, who comprise the majority of households, do not participate in the stock market but trade only in the bond market. Their wages are set under a regime of  $\eta$ -egalitarian wage bargaining subject to the same search and matching frictions as in Merz (1995), Andolfatto (1996), and Shimer (2005). As we demonstrate, the equilibrium outcome of the  $\eta$ -egalitarian wage determination process is substantially influenced by the assumed form of financial market incompleteness, and the characteristics of the stockholders' stochastic discount factor (SDF)

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<sup>1</sup> Basak and Cuoco (1998) employ the same financial market structure but in an exchange setting.

thereby implied.<sup>2</sup>

The equilibrium pattern of bond trading under restricted financial participation has the consequence of stockholders providing workers with *partial* income insurance against their labor income variation (see Guvenen (2009))<sup>3</sup>. Accordingly, the ratio of stockholder to non-stockholder-worker marginal utility of consumption conforms to a particular stochastic process, one with particularly useful properties. Since it reflects the time variation in the equilibrium consumption distribution across our two agent types, this variation in the ratio of marginal utilities is referred to as ‘distribution risk.’

In equilibrium, distribution risk will prove to be countercyclical and to affect the economy’s labor market in two important ways. First, it will be shown that effective non-stockholder-worker bargaining power is directly proportional to the distribution risk measure, making the former countercyclical as well.<sup>4</sup> With worker bargaining power stronger in low productivity states (recessions) and weaker in high productivity states (expansions), firms cannot lower wages to the competitive level in downturns and need not raise them to the competitive level in expansions. In our Baseline case, the result is wage volatility unrelated to productivity and high unemployment volatility, as seen in post-Great Recession data. By stabilizing the wage in this way, workers endogenously create a “semi-fixed wage-asset” to supplement the risk-sharing effectiveness of bond trading, particularly in environments of high wealth inequality. We refer to this wage stabilization mechanism as the “Guvenen (2009) channel,” as it built upon the asymmetric trading opportunities of non-stockholder-workers vis-à-vis stockholders proposed in his earlier framework.

Second, countercyclical distribution risk arising from incomplete risk sharing leads to a stockholder discount rate that is countercyclical and highly volatile. Hall (2017) argues persuasively that vacancy postings should be evaluated as investments and that the cyclical nature of vacancy postings and their acute diminution at business cycle downturns must be largely attributable to the countercyclical behavior of firms’ (stockholders’) discount rates, and,

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<sup>2</sup> The assumption of limited asset market participation is empirically reasonable: it is well documented that more than two thirds of US households owned no stock prior to the 1990s and that households in the top 20% of the wealth distribution owned more than 98% of stocks during the 1990s despite the stock market participation rate having increased substantially during this period (Mankiw and Zeldes (1991) and Poterba (2000)).

<sup>3</sup> The key in what follows is not simply that financial markets are incomplete, but the manner of their incompleteness. Many of the most important articles in this area assume extreme incompleteness: workers trade no financial instruments whatsoever. See, for example, Lansing (2015). They choose not to exploit the consequences of partial equilibrium risk sharing that undergirds the present model formulation, and thus are less useful for a study of the labor market.

<sup>4</sup> The flow of partial insurance payments effectively endogenizes and makes variable the non-stockholder-worker’s bargaining power.

in particular, to their high value at business cycle troughs. Confirming Hall’s (2017) intuition, vacancy posting in the model is both highly volatile and highly negatively correlated with the economy wide discount rate at business cycle frequencies. We refer to this discount rate behavior as “Hall’s (2017) discount channel.” Neither of the indicated “channels” is new to the literature, nor is their underlying motivation and intuition. What we find encouraging, however, is that they act in concert to allow an otherwise plain-vanilla real business cycle model with capital adjustment costs to replicate a wide range of business cycle and labor market stylized facts.<sup>5</sup>

The structure of this paper is as follows. Section 2 positions the present paper within the recent dynamic macro literature that focuses on the analysis of the labor market. Section 3 outlines the model. Section 4 discusses model parameter choices while Section 5 presents the baseline results, with special attention to labor market quantities. Section 6 explores the consequences of greater/lesser equilibrium wealth inequality on the volatility of macro aggregates and financial quantities while Section 7 provides concluding remarks. In contrast to most of the existing DSGE literature, rather than focusing exclusively on the influence of assumed labor market phenomena on asset returns, we emphasize the reverse-impact of specific financial market arrangements on the behavior of important labor market variables.

## 2. Literature Review and Positioning<sup>6</sup>

We first review the most pertinent dynamic macro literature cum search and matching frictions.<sup>7</sup> Merz (1995) and Andolfatto (1996) were the first to incorporate search and matching frictions into a DSGE model. In doing so they demonstrated that these features allow hours that are much more volatile than wages and a low correlation of hours and productivity. Each of these

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<sup>5</sup> From a stockholder’s perspective, the analysis will suggest that ‘distribution risk’ can be also be viewed as serving the role of Shimer’s (2005) hypothesized Nash bargaining ‘wage shock’, and, as such, can be viewed as suggesting micro-foundations for the latter device. (The more general wage determination of this paper reduces to Nash bargaining with time varying bargaining power in Shimer’s (2005) setting.) In this sense we can interpret the model as providing a direct answer to the question posed in Shimer (2005): “It seems plausible that a model with a combination of wage and labor productivity shocks could generate the observed behavior of unemployment, vacancies and real wages...the unanswered question is what exactly a wage shock is.” The present framework suggests one possible answer: the wage shock follows from bargaining shocks that arise in the conduct of  $\eta$ -egalitarian wage bargaining as the result of stockholder-non-stockholder relative consumption variation deriving from the incomplete income insurance.

<sup>6</sup> The sluggish response of wage income to output variation over the business cycle also leads to a stable aggregate wage bill. The sluggish wage bill in turn constitutes a form of ‘operating leverage’ because, like financial leverage, it increases stockholder dividend uncertainty, one pillar of successful financial fact replication. Together with the countercyclical discount factor, the stable wage bill allows a reasonable replication of a wide range of financial market stylized facts. We deal with these implications more fully in Donaldson and Kim (2017).

<sup>7</sup> An excellent literature review of search and matching in the dynamic macro context is Yashiv (2007). Hornstein et al. (2005) provide an excellent survey of the intuition and issues involved in introducing search and matching frictions into dynamic business cycle models. For an excellent survey of the related dynamic asset pricing literature see, e.g., Favilukis and Lin (2015) or Kaltenbrunner and Lochstoer (2010).

aggregate regularities is difficult to replicate in simple DSGE models. Despite these advances, Shimer (2005) noted that his critique remains valid; in his words: “Neither paper can match the negative correlation between unemployment and vacancies, and both papers generate real wages that are too flexible in response to productivity shocks.” (Shimer (2005, page 45.)). Indeed, the Andolfatto (1996) model does not display adequate vacancy volatility. The Merz (1995) model, however, is hard to reject on this basis alone. With fixed search intensity it generates a very sluggish wage and highly volatile vacancies. Both models generate a negative correlation between unemployment and vacancies albeit one that is only weakly negative. The relative success of the Merz (1995) model in generating realistic labor market statistics depends not only on wage stickiness, however, but also on the absence of hours variation at the intensive margin. If the Merz (1995) model were to admit this latter type of variation, its ability to explain labor market volatility might be significantly compromised: the representative firm could then substitute between hours per incumbent and hiring new workers, a feature allowed by Andolfatto (1996) with the indicated counterfactual consequences.

An important literature has arisen from these seminal works. Constantin and Reiter (2008), in a model very similar to Shimer’s (2005), show that high unemployment volatility or the weak response of unemployment to changes in unemployment insurance can be individually reproduced in their framework, but not both simultaneously. Hagedorn and Manovskii (2008) are able to replicate the basic labor market stylized facts, but at a cost of assuming the ‘replacement rate’ – the ratio of unemployment benefits to average wages – is .98, which seems implausibly high. Shimer (2005) assumes a lower ratio of .4. As reported in Hornstein et al. (2005), the OECD estimates the US average replacement rate to be only .2. Either of these values compromises the Hagedorn and Manovskii (2008) results. Hornstein et al. (2005) points out that these models also require a counterfactually high wage share, and a correspondingly counterfactually low profit share.

Two other leading real business cycle models with search and matching frictions are Christoffel and Kuester (2008) and Gertler and Trigari (2009). Both can account well for the observed volatility in the key labor market variables emphasized in Shimer (2005) and Hall (2005). Gertler and Trigari (2009) embed the standard Nash wage bargaining into the framework of Calvo style staggered multi-period wage contracting. Their wage contract takes the form of a fixed wage over an exogenously given horizon, a feature that lowers average wage volatility. The Gertler and Trigari (2009) model is quite successful in accounting for overall labor market volatility when the contract length is assumed to be one year. It is silent, however, regarding variations at the intensive margin, or how the introduction of such variation might affect the

model's quantitative validity; in other words, their model abstracts from variable hours, and its success may be sensitive to that feature. Christoffel and Kuester (2008) incorporate search frictions into a New Keynesian framework characterized by price rigidities in the goods market. Although they focus on the relationship of wage increases to overall inflation rates, their model is also able to account for the observed variations in the key indicators of labor market activity, including vacancies and unemployment. The Christoffel and Kuester (2008) model relies on (i) multiple shocks including exogenous productivity shocks, monetary policy shocks, government spending shocks and a risk premium shock and (ii) exogenously specified fixed costs of maintaining an existing job. Without the latter two features, in particular, their results are compromised. We favor parsimony and adopt a single source of uncertainty. More recently Christiano et al. (2016) explore similar issues within the setting of a New-Keynesian style model where real wages are set under "alternative offer bargaining" in contrast to simple Nash wage bargaining. As in the present model, their formulation creates substantial "wage inertia" which for Christiano et al. (2016) means a real wage which is both highly persistent and relatively immune to shocks. As they assume a representative agent complete markets setting, their mechanism for wage stability differs substantially from the one considered here.<sup>8</sup>

There is also a wide literature that considers how financial frictions affect the propagation of shocks that have their origin elsewhere in the economy. Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) are the early progenitors. In this credit channel literature, some exogenous shock forces firms to reduce borrowing which, in turn, reduces their ability to invest and hire workers. More recent contributions include Jermann and Quadrini (2012), Monacelli et al. (2011), Chugh (2013), Petrosky-Nadeau (2014), Petrosky-Nadeau and Wasmer (2013), Kehoe et al. (2014), and Garvin (2015). The present paper has no explicit credit channel or firm credit constraints, but relies on differential financial market access and the resulting consumption inequality to influence the equilibrium wage determination mechanism. Rudanko (2011) takes another extreme position: risk-averse workers are excluded from participating in any asset market and must rely solely on the form of their labor contract for any consumption smoothing. In this implicit contract setup, workers respond very modestly to wage increases during economic upturns, because of their desire for smooth consumption. Firms are thus forced to increase vacancies in upturns in order to increase the probability of a match and hiring a worker as wages remain relatively flat.

Lastly, two important papers that explicitly bridge the macro-labor-finance divide are

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<sup>8</sup> Christiano et al. (2016) also deals with a variety of government policy issues, something we eschew.

Guvenen (2009) and Petrosky-Nadeau et al. (2017). Petrosky-Nadeau et al. (2017) explicitly analyze the asset pricing characteristics of a complete markets dynamic model with Nash wage bargaining cum search and matching, but focus principally on the ability of their model to generate endogenous output ‘disasters.’ Guvenen’s (2009) model relies on incomplete participation in the financial markets, just as in the present model, but has only the most parsimonious labor market modeling: there is a simple labor-leisure choice with no search and matching feature.

Two other related strands of literature focus on heterogeneous agent models, broadly defined, and, more specifically models with uninsurable income risk (Aiyagari (1994), Huggett (1993)), both features of the present formulation. Some of this literature focuses alone on exploring the evolution of income, wealth and consumption inequality; see, for example, Krusell and Smith (1998), Quadrini (2000) and Kreuger and Perri (2006). Heathcote and Perri (2018) study a monetary version where increases in precautionary savings in response to perceived increased unemployment risk may lead to self-fulfilling recessions when household wealth is low. Broer et al. (2018), Den Haan et al. (2018) and Ravn and Sterk (2018) focus on New Keynesian contexts where uninsurable unemployment risks can lead to “deflationary spirals.” These authors emphasize the importance of assumed real wage rigidities and an assumed selection of critical nominal rigidities including a “zero lower bound.” In contrast, the present paper postulates no rigidities a priori but rather endogeneously creates them in an environment of high wealth inequality.

Our efforts provide two ancillary benefits. First, the model is shown to be compatible with the basic DSGE constructs of many of the various perspectives mentioned above (and their relative successes), while also generalizing them. In the case of Petrosky-Nadeau et al. (2017), we generalize their model to one with capital and investment; in the case of Lansing (2015), we make endogenous the variation in factor shares. The attractive results in Merz (1995) and Andolfatto (1996) are enhanced along the dimension of vacancy volatility (cf. Andolfatto 1996) and by allowing labor variation at the intensive margin.

Second, the incorporation of ‘distribution risk’ allows, in some cases, more reasonable parameter choices than are assumed in related models; e.g., a much lower unemployed utility than is assumed in Hagedorn and Manovskii (2008) or Petrosky-Nadeau et al. (2017) and a lower risk aversion level than is assumed in Greenwald et al. (2014). We next consider the model itself.

### 3. The Model

We consider a discrete-time infinite horizon economy with two distinct infinitely-lived

agent types, “stockholders” and “non-stockholders-workers.” These groups are uniformly distributed, respectively, on sets of Lebesgue measure  $\mu_s$  and  $\mu_n$  normalized to  $\mu_n = 1$ . Since both groups supply labor in the model, we refer to the non-stockholder-workers simply as “non-stockholders” to emphasize their distinguishing feature.

### 3.1 Stockholders

Following Guvenen (2009), a stockholder, endowed with one unit of time, supplies labor services to the (representative) firm and trades securities -- both equity claims to the firm's net income stream, and a one-period default-free real bond (henceforth referred to simply as a “bond”). Being an owner of the firm, a stockholder is assumed to have a permanent relationship with it and to trade her labor services in an exclusive stockholder labor market. This market is characterized by employment adjusting only along the intensive margin; i.e., the labor income risk of a stockholder originates entirely from fluctuations in hours worked.<sup>9</sup> Given her information set  $\Omega_0^s$ , the representative stockholder maximizes her lifetime expected utility as given by:

$$V^s(\Omega_0^s) = \max_{\{h_t^s, e_t^s, e_{t+1}^s, b_{t+1}^s\}} E_0 \sum_{t=0}^{\infty} \beta^t [u^s(c_t^s - \chi^s \mathbf{c}_{t-1}^s, h_t^s)] \quad (1)$$

$$\text{s.t.} \quad c_t^s + p_t^e e_{t+1}^s + p_t^b b_{t+1}^s \leq w_t^s h_t^s + (p_t^e + d_t) e_t^s + b_t^s \quad (2)$$

where  $u^s(\cdot)$  denotes her period utility function,  $c_t^s$  her period  $t$  consumption,  $h_t^s$  her period  $t$  labor hours, and  $\chi^s$  the stockholder's habit parameter. The expression  $\mathbf{c}_{t-1}^s$  represents the average consumption level across the entire stockholder group in the previous period:

$$\mathbf{c}_{t-1}^s \equiv \frac{1}{\mu_s} \int c_{t-1}^s d\kappa$$

with  $\kappa$  standing for the measure of stockholders. In addition,  $d_t$  represents the period  $t$  dividend payment by the firm and  $e_t^s$  and  $b_t^s$  denote, respectively, the stockholder's period  $t$  stock and bond holdings. The corresponding period  $t$  equilibrium prices of these securities are represented as  $p_t^e$  and  $p_t^b$ . Lastly,  $w_t^s$  is the stockholder's period  $t$  wage, while  $E_t^s \equiv E(\cdot | \Omega_t^s)$  denotes her expectations operator conditional on her information set  $\Omega_t^s$ . The parameter  $\beta$  is the economy-wide subjective discount factor, identical for all agents. Stockholders regard all

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<sup>9</sup> Our set-up differs from Guvenen (2009) where the hours of both agent types are added together to form the labor input to production.



prices,  $p_t^e$ ,  $p_t^b$  and  $w_t^s$  as exogenous.

Stockholders are characterized by a form of GHH (Greenwood, Hercowitz and Huffman (1988)) utility:

$$u^s(c_t^s - \chi^s c_{t-1}^s, h_t^s) = u^s(c_t^s - \chi^s c_{t-1}^s - H(h_t^s))$$

where  $H(\cdot)$  denotes the disutility of labor hours in units of consumption. This specification of the period utility function combines standard GHH preferences with “catching up with the Joneses” (Abel (1990)).<sup>10</sup>

Conditional upon her period  $t$  information set,  $\Omega_t^s$ , the recursive formulation of the stockholder's problem may be represented as:

$$V^s(\Omega_t^s) = \max_{\{c_t^s, h_t^s, e_{t+1}^s, b_{t+1}^s\}} \left[ \begin{array}{c} u^s(c_t^s - \chi^s c_{t-1}^s, H(h_t^s)) \\ + \lambda_t^s [w_t^s h_t^s + (p_t^e + d_t^e)e_t^s + p_t^b b_t^s - c_t^s - p_t^e e_{t+1}^s - b_{t+1}^s] \\ + \beta E(\tilde{V}^s(\Omega_{t+1}^s) | \Omega_t^s) \end{array} \right] \quad (4)$$

where  $\lambda_t^s$  is the Lagrange multiplier associated with budget constraint (2).

The solution to problem (4) is characterized by three necessary and sufficient first order conditions

$$w_t^s = H_1(h_t^s) \quad (5)$$

$$p_t^e = \beta E_t[\tilde{\Lambda}_{t,t+1}^s (p_{t+1}^e + d_{t+1}^e)] \quad (6)$$

$$p_t^b = \beta E(\tilde{\Lambda}_{t,t+1}^s | \Omega_t^s) \quad (7)$$

where  $\Lambda_{t,t+1}^s = \lambda_{t+1}^s / \lambda_t^s = u_1^s(c_{t+1}^s - \chi^s c_t^s - H(h_{t+1}^s)) / u_1^s(c_t^s - \chi^s c_{t-1}^s - H(h_t^s))$  denotes the non-stockholder's intertemporal marginal rate of substitution (IMRS), to be determined in equilibrium. In what follows we denote  $\beta \tilde{\Lambda}_{t,t+1}^s$  by  $\tilde{M}_{t,t+1}$ . It represents the economy-wide stochastic discount factor (SDF) for all valuation purposes.

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<sup>10</sup>If  $\chi^s = 0$ , the preference function specified above is reduced to the standard GHH utility function widely employed in the investment-shock literature. It is well known that for the GHH class of preferences, the Hicksian wealth effect of a real wage increase on hours worked is zero. As such, the labor supply is determined independently of intertemporal consumption-savings choice and thus the effect of the intertemporal consumption substitution on the labor supply is completely eliminated. Accordingly, the GHH class of preferences features a marginal rate of substitution between consumption and labor supply that depends only on the labor supply itself:

$$-\frac{u_2^s(c_t^s - X_t, h_t^s)}{u_1^s(c_t^s - X_t, h_t^s)} = H_1(h_t^s).$$

For more detail, see Jaimovich and Rebelo (2009). Guvenen (2009) also employs GHH utility although in an overall Epstein-Zin (1989) intertemporal preference context.

### 3.2 Non-stockholders

Non-stockholders differ from stockholders in their investment opportunity sets, job opportunity sets, and consumption-smoothing motives. First, as the name suggests, non-stockholders are restricted from participating in the equity market, although they can freely trade bonds. Second, non-stockholders trade their services exclusively in a separate labor market with two distinct characteristics: (1) the non-stockholder's labor market is characterized by variation in employment at both the extensive and intensive margins, and (2) firms and non-stockholders Kalai (1977)-Rawls (1971)-Nash (1950) (hereafter KRN)  $\eta$ -egalitarian wage bargain in a context of search and matching frictions. As we show, the outcome of this wage bargaining is endogenously influenced by the asymmetric security trading opportunities and the consequent imperfect income insurance implicitly provided by the stockholders.

Following Merz (1995), each non-stockholder is viewed as a large extended family which contains a continuum of family members uniformly distributed on a set of Lebesgue measure one. Each family consists of the employed and unemployed, who pool their financial and labor incomes (perfect risk-sharing within the family) before choosing per-capita consumption and bond holdings. Accordingly, given the information set  $\Omega_0^n$ , the representative non-stockholder family solves<sup>11</sup> :

$$V^n(\Omega_0^n) = \max_{\{h_t^n, c_t^{n,e}, c_t^{n,u}, b_{t+1}^n\}} E_0 \left( \sum_{t=0}^{\infty} \beta^t \left[ n_t u^n(c_t^{n,e} - \chi^n \mathbf{c}_{t-1}^{n,e} - L(h_t^n)) + (1-n_t) u^n(c_t^{n,u} - \chi^n \mathbf{c}_{t-1}^{n,u} - L(0)) \right] \right) \quad (8)$$

$$\text{s.t.} \quad n_t c_t^{n,e} + (1-n_t) c_t^{n,u} + p_t^b b_{t+1}^n \leq w_t^n h_t^n n_t + b(1-n_t) + b_t^n + T_t, \text{ and} \quad (9)$$

$$n_{t+1} = (1-\rho) n_t + s_t(1-n_t). \quad (10)$$

In the above problem,  $u^n(\cdot)$  denotes a representative non-stockholder's period utility function,  $c_t^{n,e}$  and  $c_t^{n,u}$ , respectively, his period  $t$  consumption when employed and when unemployed,  $\mathbf{c}_{t-1}^{n,e}$  and  $\mathbf{c}_{t-1}^{n,u}$  their average values, respectively, in the prior period,  $L(\cdot)$  his

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<sup>11</sup>A more "structural" form of the contemporaneous utility is to introduce search effort per worker seeking employment:  $u^n(c_t^n - n_t L(h_t^n) - (1-n_t)L(e_t))$ , where  $e_t$  is period  $t$  search effort. However, empirical studies show that search effort is negligible. Kruger and Mueller (2011) estimate, for example, that formal search activities typically consume less than 10% of an unemployed person's week days (8 hours). Therefore, without loss of too much generality, we simplify and assume that  $L(e_t) = L(0) = 0$ .

disutility of labor function,  $\chi^n$  his habit parameter, and  $h_t^n$  his period  $t$  labor hours supplied when employed. The expression  $b_t^n$  denotes the family's period  $t$  bond holdings;  $w_t^n$  is the non-stockholder's wage determined through the bargaining process, while  $b$  represents unemployment benefits and  $T_t$  is the lump sum tax levied on non-stockholders by the government to finance these benefits. Accordingly, equation (9) is the representative non-stockholder family's budget constraint. The  $n_t$  term represents the measure of non-stockholders actually at work in period  $t$ , while  $E_t \equiv E(\cdot | \Omega_t^n)$  is the expectation operator conditional on their information set  $\Omega_t^n$ . Equation (10) describes the evolution of the fraction of non-stockholders who are employed, as a function of the exogenous separation rate  $\rho$  and,  $s_t$ , the (exogenous from the non-stockholder's perspective) fraction of unemployed non-stockholders matched to the firm in period  $t$ . As with stockholders, non-stockholders take all prices as exogenous to their decision problem.

We adopt the same form of GHH preferences for the representative non-stockholder's period utility. Conditional upon  $\Omega_t^n$ , the recursive formulation of the non-stockholder's problem can be represented as:

$$V^n(\Omega_t^n) = \max_{\{c_t^n, b_{t+1}^n, h_t^n\}} \left[ \begin{aligned} &u^n(c_t^n - \chi^n \mathbf{c}_{t-1}^n - n_t L(h_t^n)) \\ &+ \lambda_t^n (b_t^n + w_t^n h_t^n n_t + b(1 - n_t) - p_t^b b_{t+1}^n - c_t^n) \\ &+ \beta E(\tilde{V}^n(\Omega_{t+1}^n) | \Omega_t^n) \end{aligned} \right], \quad (11)$$

where  $c_t^n = n_t c_t^{n,e} + (1 - n_t) c_t^{n,u}$ , and  $\lambda_t^n$  is the Lagrange multiplier associated with the non-stockholder's budget constraint (9).<sup>12, 13</sup> The solution to problem (11) is characterized by the necessary and sufficient first-order conditions:

$$u_1^n(c_t^n - \chi^n \mathbf{c}_{t-1}^n - n_t L(h_t^n)) = \lambda_t^n \quad (12)$$

$$w_t^n = L_1(h_t^n) \quad (13)$$

$$p_t^b = \beta E \left( \frac{u_1^n(\tilde{c}_{t+1}^n - \chi^n \mathbf{c}_t^n - n_{t+1} L(\tilde{h}_{t+1}^n))}{u_1^n(c_t^n - \chi^n \mathbf{c}_{t-1}^n - n_t L(h_t^n))} | \Omega_t^n \right). \quad (14)$$

Note that non-stockholders' hours are supplied under the condition that the (hourly) wage equals the marginal rate of substitution of consumption for leisure. By analogy,  $\mathbf{c}_t^n = \int c_t^n d\omega$  is average

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<sup>12</sup>  $\Omega_t^n = \{w_t, n_t, s_t, b_t^n, p_t^b, \mathbf{c}_{t-1}^n\}$ .

<sup>13</sup> The transition from (8) – (10) to (11) is not obvious. See Part B of the Appendix.

worker consumption in period  $t$  and  $\omega$  the measure of workers.

We next describe the functioning of the labor market and its wage determination process.

### 3.3 Search in the labor market for non-stockholders

Since stockholders are permanent employees, Section 3.3 focuses on the structure of the non-stockholder's labor market. There is one infinitely lived representative firm that behaves competitively.<sup>14</sup> The firm hires  $n_t$  non-stockholders, and posts  $V_t$  vacancies in order to attract new non-stockholders for its period  $t+1$  production. The total number of unemployed non-stockholders who search for a job in period  $t$ , is  $u_t$ , where:

$$u_t \equiv 1 - n_t.$$

Following basic DMP search theory, we postulate the following matching technology in the labor market for non-stockholders:

$$MA(v_t, 1 - n_t) = \sigma_m v_t^\sigma (1 - n_t)^{1-\sigma},$$

where  $m_t \equiv MA(v_t, 1 - n_t)$  represents "matches," the number of newly hired non-stockholders, and  $\sigma_m$  is a scale parameter. The exponents  $\sigma$  and  $(1 - \sigma)$  describe, respectively, the elasticity of matches with respect to vacancies and unemployment.

The probability that the firm fills a vacancy in period  $t$ ,  $q_t$ , is given by

$$q_t = \frac{MA(v_t, 1 - n_t)}{v_t} = \frac{m_t}{v_t},$$

while the probability that a searching outsider finds a job in period  $t$ ,  $s_t$ , is given by

$$s_t = \frac{MA(v_t, 1 - n_t)}{1 - n_t} = \frac{m_t}{u_t}.$$

The tightness of the labor market,  $\theta_t$ , is measured by  $\theta_t = v_t/u_t$ . Both quantities,  $q_t$  and  $s_t$ , are assumed exogenous from the perspectives of both the firm and an individual non-stockholder. Employment relationships between the firm and non-stockholders may dissolve for exogenous reasons in each period  $t$ , as represented by the invariant probability of separation  $\rho$ . Equation (10) can thus also be written as

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<sup>14</sup>Equivalently, it can be assumed that there is a continuum of infinitely lived identical competitive (in the product market) firms distributed on the unit interval  $[0, 1]$ .

$$n_{t+1} = (1 - \rho)n_t + m_t. \quad (15)$$

The dual specifications that the job separation rate is constant while the job finding probability is variable are broadly consistent with evidence presented in Hall (2005) and Shimer (2005): they report that while the job finding probability is indeed cyclical, the separation rate is substantially less so.

### 3.4 The Firm

Each period, the firm produces output,  $y_t$ , according to the following aggregate production function:

$$y_t = f(k_t, \mu_s h_t^s, n_t h_t^n) z_t$$

where  $z_t$ ,  $k_t$ ,  $\mu_s h_t^s$ , and  $n_t h_t^n$  denote, respectively, the period  $t$  aggregate productivity shock, capital stock, aggregate labor (hours) supplied by the stockholders, and the aggregate labor hours supplied by the working non-stockholders. Non-stockholder employment at the firm evolves according to (15).

The firm owns the (physical) capital stock,  $k_t$ , which depreciates each period at the rate of  $\delta$  while being supplemented by new investment  $\dot{i}_t$ . Costs of adjusting the firm's capital stock and its labor force of non-stockholders are next introduced. We adopt the capital-accumulation technology specification employed in Jermann (1998) and Kaltenbrunner and Lochstoer (2010):

$$k_{t+1} = (1 - \delta)k_t + G\left(\frac{\dot{i}_t}{k_t}\right)k_t$$

where the adjustment cost function  $G(\cdot)$  is given by

$$G\left(\frac{\dot{i}_t}{k_t}\right) = \frac{a_1}{1 - \frac{1}{\xi}} \left(\frac{\dot{i}_t}{k_t}\right)^{1 - \frac{1}{\xi}} + a_2,$$

where  $a_1$  and  $a_2$  are chosen so that  $G(\delta) = \delta$ , and  $G_1(\delta) = 1$ .<sup>15</sup>

Second, a cost of adjusting employment is introduced. These costs influence the rate at which the firm adds new workers to its existing labor force. As in Gertler and Trigari (2009), the

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<sup>15</sup> With these identifications, the elasticity parameter  $\xi \equiv -\frac{1}{G_{11}(\delta)\delta} > 0$  is independent of the determination of the model's steady-state equilibrium; i.e. the steady state is not affected by the positive value  $\xi$ ;  $\xi = \infty$  corresponds to the benchmark case of no adjustment costs. This specification enables Tobin's  $q$  to vary by differentiating between the (shadow) prices of the installed capital and the new investment good prices.

standard assumption of fixed costs of posting a vacancy is replaced with quadratic labor adjustment costs. Define the hiring rate,  $x_t$ , as the ratio of new hires  $q_t v_t$  to the existing pool of employed non-stockholders:  $x_t \equiv \frac{q_t V_t}{n_t} = \frac{\text{new hires}}{\text{existing work force}} \equiv \text{hiring rate}$ . The quadratic adjustment cost to altering the employment level of non-stockholders within the firm is then given by:

$$\frac{\kappa}{2} x_t^2 n_t,$$

where  $\kappa$  denotes a vacancy cost parameter. The (financial) capital structure of the representative firm consists of one perfectly divisible equity share, price  $p_t^e$ , and one-period default-free bonds which it issues at the price  $p_t^b$ . The total supply of corporate bonds is assumed constant over time and equals a fraction  $\phi$  of the steady state capital stock owned by the firm. In each period, the firm makes net interest payments  $(\phi \bar{k} - p_t^b \phi \bar{k})$  to bondholders where the  $\bar{\cdot}$  above  $k$  denotes its certainty steady state value.<sup>16</sup>

The firm's decision problem is to maximize its pre-dividend stock market value  $d_t + p_t^e$  on a period-by-period basis given its information set  $\Omega_t^f = \Omega^f(k_t, z_t, q_t, n_t)$ :

$$\max_{\{i_t, h_t^s, x_t\}} d_t + p_t^e \equiv d_t + E(\tilde{M}_{t,t+1}(p_{t+1}^e + d_{t+1}) | \Omega_t^f) \quad (16)$$

$$\text{s.t. } d_t \equiv f(k_t, \mu_s h_t^s, n_t h_t^n) z_t - i_t - \mu_s w_t^s h_t^s - w_t^n h_t^n n_t - \frac{\kappa}{2} x_t^2 n_t - \phi \bar{k} + p_t^b \phi \bar{k}$$

$$k_{t+1} = (1 - \delta) k_t + G\left(\frac{i_t}{k_t}\right) k_t$$

$$n_{t+1} = (1 - \rho) n_t + q_t v_t. \quad ^{17}$$

In the above problem,  $\tilde{M}_{t,t+1}$  is the inter-temporal SDF of the stockholders,  $w_t^s$  denotes their competitive wage, and  $w_t^n$  is the KRN  $\eta$ -egalitarian bargaining wage for non-stockholders (to be specified).

Defining  $V^f(\Omega_t^f) \equiv d_t + p_t^e$ , the recursive representation of the firm's problem may be

<sup>16</sup>The Miller-Modigliani Theorem is not guaranteed to hold in the present model context because the financial markets are not complete. Telmer (1993) argues, however, that the trading of one period default free debt makes it “near-to-complete.” Accordingly, the Miller-Modigliani Theorem approximately holds.

<sup>17</sup> Note that in problem (16) to choose the hiring rate  $x_t$  is to choose the number of vacancies  $v_t$ .

written as:

$$V^f(\Omega_t^f) = d_t + E(\tilde{M}_{t,t+1} V^f(\Omega_{t+1}^f) | \Omega_t^f).$$

The necessary and sufficient first-order condition for the firm's optimal investment decision is given by:

$$i_t : (-1) + E(\tilde{M}_{t,t+1} V_{k_{t+1}}^f | \Omega_t^f) \frac{\partial k_{t+1}}{\partial i_t} = 0.$$

By the envelope theorem,

$$\frac{\partial V^f(\Omega_t^f)}{\partial k_t} = f_1(k_t, \mu_s h_t^s, n_t h_t^n) z_t + E(\tilde{M}_{t,t+1} V_{k_{t+1}}^f | \Omega_t^f) \frac{\partial k_{t+1}}{\partial k_t} = 0.$$

The investment Euler equation is thus represented as:

$$1 = E(\tilde{M}_{t,t+1} G_1(\frac{i_t}{k_t}) [f_1(k_{t+1}, \mu_s h_{t+1}^s, n_{t+1} h_{t+1}^n) z_{t+1} + \frac{(1-\delta) + G(\frac{i_{t+1}}{k_{t+1}})}{G_1(\frac{i_{t+1}}{k_{t+1}})} - \frac{i_{t+1}}{k_{t+1}}] | \Omega_t^f). \quad (17)$$

The first-order condition for the firm's optimal hiring decision for stockholder hours is given by

$$h_t^s : w_t^s = f_2(k_t, \mu_s h_t^s, n_t h_t^n) z_t, \quad (18)$$

while the first-order condition for the firm's optimal hiring rate for non-stockholders is given by

$$x_t : \kappa x_t = E_t \tilde{M}_{t,t+1} J_{t+1} \quad (19)$$

where  $J_t \equiv \frac{\partial V^f(\Omega_t)}{\partial n_t}$  is the firm's shadow value of hiring one additional non-stockholder (to be characterized shortly).<sup>18</sup>

### 3.5 Characterizing the KRN $\eta$ -egalitarian wage bargaining problem

We now formalize the wage bargaining process between the firm and the non-stockholders and show that its solution can be constructed in a tractable way. In particular, the firm's matching surplus and the non-stockholder's employment and unemployment values can each be defined in terms of current consumption so as to make them consistent, respectively, with the firm's shadow value of one added non-stockholder and his value of becoming employed.

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<sup>18</sup> Here and in what follows we economize on notation by using the symbol  $E_t[\cdot]$  to represent  $E_t[\cdot | \Omega_t^i]$ ,  $i \in \{s, f, n\}$  when there is no ambiguity.

What emerges from this representation of the KRN  $\eta$ -egalitarian bargaining problem in terms of current consumption is a tractable form of wage determination which nests, as a special case, the standard Nash bargaining wage of the representative agent analogue. First, three critical quantities are defined and measured.

### 3.5.1 Firm's shadow value of hiring one additional non-stockholder

The structure of  $J_t = \partial V^f(\Omega_t^f)/\partial n_t$ , the value to the firm of hiring one additional non-stockholder in period  $t$ , is first specified. From (19)

$$J_t = h_t^n f_3(k_t, \mu_s h_t^s, n_t h_t^n) z_t - w_t^n h_t^n - \frac{\kappa}{2} x_t^2 + (1 - \rho) E_t \tilde{M}_{t,t+1} J_{t+1}$$

where  $h_t^n f_3(k_t, \mu_s h_t^s, n_t h_t^n) z_t$  defines the “extensive marginal product of the non-stockholder’s labor.”<sup>19</sup>

The first-order condition for the hiring rate equates the marginal cost to the firm of adding a non-stockholder with the discounted marginal benefit:

$$\kappa x_t = E_t \tilde{M}_{t,t+1} J_{t+1}, \quad (20)$$

which is identical to the equation defining the firm's optimal hiring decision for outsiders, equation (19).

Using the definition of  $J_t$ , and recursively using (20) to substitute out  $J_{t+1}$ , the following equivalent (relative to (19)) optimality condition governing the hiring of outsiders is derived:

$$\kappa x_t = E_t \tilde{M}_{t,t+1} [h_{t+1}^n f_3(k_{t+1}, \mu_s h_{t+1}^s, n_{t+1} h_{t+1}^n) z_{t+1} - w_{t+1}^n h_{t+1}^n - \frac{\kappa}{2} x_{t+1}^2 + (1 - \rho) \kappa x_{t+1}].$$

### 3.5.2 Non-stockholder's shadow value

The period  $t$  present discounted value to a non-stockholder of employment in terms of period  $t$  consumption, denoted  $EP_t$ , is defined recursively by:

$$EP_t = w_t^n h_t^n + (1 - \rho) \beta E_t \tilde{\Lambda}_{t,t+1}^n EP_{t+1} + \rho \beta E_t \tilde{\Lambda}_{t,t+1}^n U_{t+1},$$

where  $\Lambda_{t,t+1}^n \equiv \lambda_{t+1}^n / \lambda_t^n$  is the non-stockholder's IMRS, and  $U_t$  denotes the present discounted value to a non-stockholder of unemployment in terms of current consumption in period  $t$ . In like fashion,  $U_t$  is recursively defined by the corresponding relationship:

<sup>19</sup>In the matching labor market for outsiders, we distinguish between the “extensive marginal product of outsiders’ labor” and the “intensive marginal product of outsiders’ labor,” the latter being the  $MPL_{h_t^n}$ , as defined by

$$\frac{\partial y_t}{\partial h_t^n} = n_t z_t f_3(k_t, \mu_s h_t^s, 1, h_t^n \cdot n_t).$$



$$U_t = L(h_t^n) + b + s_t \beta E_t \Lambda_{t,t+1}^n EP_{t+1} + (1-s_t) \beta E_t \Lambda_{t,t+1}^n U_{t+1}.$$

Here, the value of being unemployed is the sum of the non-stockholder's current disutility of supplying hours,  $L(h_t^n)$ , his unemployment benefit  $b$ , and the discounted values of being employed or unemployed next period, weighted by their relative likelihoods where an unemployed non-stockholder has a probability  $s_t$  of finding a new job. Each of these quantities is foregone when the unemployed non-stockholder accepts employment and each is measured in terms of final goods consumption. Accordingly, the non-stockholders' matching shadow value is then defined as the difference  $EP_t - U_t$ .

### 3.5.3 Distribution risk

In equilibrium, the extent of partial risk sharing that results from stockholders and non-stockholders interacting in the bond market will influence the outcome of the  $\eta$ -egalitarian wage bargaining process and will in turn be affected by it. To measure the aggregate effect the ratio between the stockholder's and non-stockholder's marginal utilities,

$$\phi_t \equiv \frac{u_1^s(c_t^s - \chi^s \mathbf{c}_{t-1}^s - H(h_t^s))}{u_1^n(c_t^n - \chi^n \mathbf{c}_{t-1}^n - n_t L(h_t^n))} = \frac{\lambda_t^s}{\lambda_t^n}, \quad (21)$$

is introduced as characterizing the extent of risk-sharing between these two groups. Going forward, we refer to  $\phi_t$  as “distribution risk.” If  $\phi_t$  is constant across time and across all states, relation (21) coincides with the efficient risk-sharing condition. Alternatively, suppose that  $\phi_t$  is constant across period  $t$  states for each  $t$ , but is time-varying.<sup>20</sup> In this event, a larger  $\phi_t$  is evidence of a greater share of aggregate income to non-stockholders, while a smaller  $\phi_t$  suggests a greater share to stockholders. Suppose, in addition, that  $\phi_t$  is time-varying *and* countercyclical over the business cycle. This countercyclical behavior means that when a high-productivity state is realized, a smaller  $\phi_t$  ensues and stockholders reap most of the benefits from that high productivity state; alternatively, when a low-productivity state is realized, a greater share of aggregate income goes to non-stockholders; i.e., the normally low payment to capital owners (stockholders) is further reduced by labor's priority claim on output. Accordingly, the countercyclical nature of  $\phi_t$  captures the idea that the shares of income going to labor and capital are not equally risky and that stockholders, via the institution of the firm, partially insure

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<sup>20</sup> Here the *optimal* contract is not necessarily optimal in the Pareto sense. In this case, relation (26) is reduced to the optimality condition of the Boldrin-Horvath (1995) type *optimal* contract.

non-stockholders. “Distribution Risk” (variation in  $\phi_t$ ) is thus largely borne by owners of the firm.<sup>21, 22</sup> It is assumed to be uninsurable.

No *a priori* assumption concerning either the source or cyclicity of distribution risk; rather, distribution risk (defined as per (21)) is generated entirely endogenously. In the present model, however, it is indeed also *countercyclical* over the business cycle. Furthermore, the bargained wage contract between stockholders and non-stockholders precisely identifies distribution risk  $\phi_t$  as influencing the balance of “bargaining power” between them. As such, a structural specification for the source of distribution risk is provided.

### 3.5.4 Distribution risk and $\eta$ -egalitarian wage bargaining

Before formalizing the  $\eta$ -egalitarian bargaining wage contract between stockholders and non-stockholders, note that in the model environment there is no agency problem between firm owners and managers. Accordingly, the firm's matching surplus can be identified with the marginal benefit to the representative stockholder of adding one non-stockholder. In other words, the firm's matching surplus in utility terms accruing to stockholders, denoted  $V_{n_t}^s$ , can be formulated as:

$$V_{n_t}^s \equiv \frac{\partial V_t^s}{\partial n_t} = \lambda_t^s J_t$$

where  $V_t^s \equiv V^s(\Omega_t^s)$  denotes the value function of a representative stockholder.

Similarly, a non-stockholder's matching surplus in utility terms,  $V_{n_t}^n$ , can be readily identified with the marginal benefit (to the non-stockholder family) of one additional non-stockholder being hired:

$$V_{n_t}^n \equiv \frac{\partial V_t^n}{\partial n_t} = \lambda_t^n (EP_t - U_t).$$

Identifying each matching surplus with its marginal benefit to the corresponding agent is reasonable in a situation where two heterogeneous agents with different attitudes toward risk

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<sup>21</sup> Empirically, the labor income share is much less risky than the share going to capital; labor's claim on output is largely fixed and negotiated prior to the actual realization of the output.

<sup>22</sup> In an earlier paper, Danthine and Donaldson (2002) posit that the observed variations in factor income shares are the result of exogenous changes in this ratio  $\phi_t$  which they refer to as *distribution risk*. We adopt the same terminology. They view  $\phi_t$  as capturing the relative bargaining power of the two parties at the time the contract is negotiated. The assumed countercyclicity of this distribution risk guarantees that labour's share is much less risky than the share going to capital. In contrast to Danthine and Donaldson (2002), our distribution risk measure is endogenous and different in origin.

bargain over the wage. Indeed, the existing game theory literature holds that the division of the joint bargaining surplus can be significantly affected by heterogeneity in the agents' risk aversion coefficients<sup>23</sup>. It is therefore reasonable to define the matching surplus in this environment in terms of marginal benefits in a manner that captures the nontrivial effect of risk aversion on bargaining. Unfortunately, the axiom of scale invariance does not apply in this context.

To remedy this shortcoming, the present analysis postulates the bargaining outcome as a KRN  $\eta$ -egalitarian solution; that is, the non-stockholder's wage,  $w_t^n$ , negotiated period-by-period, satisfies:

$$\eta \left( V_t^s(\Omega_t^s) - \bar{V}_t^s \right) = (1 - \eta) \left( V_t^n(\Omega_t^n) - \bar{V}_t^n \right), \quad (22)$$

where  $\bar{V}_t^n$  and  $\bar{V}_t^s$ , respectively, denote time-varying but non-stochastic disagreement points and  $\eta$  denotes the exogenously given bargaining parameter. There are two arguments in favor of this choice:

(1) For agents in a long term relationship, as is the case with stockholders and non-stockholders in their association with the firm, it is reasonable to expect that bargaining would evolve to respect the “equal gains principle.” The  $\eta$ -egalitarian solution satisfies this principle when  $\eta = 1/2$ , a parameter choice we will later adopt.

(2) In the analogous model with financial market completeness, the  $\eta$ -egalitarian-bargaining solution is Pareto optimal and coincides with the Nash-bargained solution when the Hosios (1990) condition is satisfied. Proposition 3.1 formalizes this assertion.

**Proposition 3.1:** There exists bargaining parameter  $\eta$  for which equilibrium in the analogous complete markets economy under  $\eta$ -egalitarian bargaining coincides with the equilibrium under Nash wage bargaining.

Proof: Consider the complete markets, representative agent-social planning version of the present model, decentralized as per Andolfatto (1996) or Donaldson and Kim (2018), with Nash real-wage bargaining between households and the firms they own. If the Hosios (1990) conditions are satisfied, then equilibria, subject to search and matching frictions, are Pareto optimal.

In the period-by-period Nash bargaining problem let  $y_1, y_2$ , respectively, denote the contract payoffs to households and firms, with  $v_1, v_2$  their respective disagreement points. By Myerson (1991) Theorem 8.2, there exists a bargaining allocation that maximizes the Nash

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<sup>23</sup>For greater detail, see Roth and Rothblum (1982).

product  $(y_1 - v_1)(y_2 - v_2)$  if and only if there exist numbers  $(\lambda_1, \lambda_2)$ , such that the  $(\lambda_1, \lambda_2)$ -egalitarian and  $(\lambda_1, \lambda_2)$ -utilitarian solutions coincide. Furthermore, the allocation that maximizes  $(y_1 - v_1)(y_2 - v_2)$  also maximizes  $\frac{1}{2} \ln(y_1 - v_1) + \frac{1}{2} \ln(y_2 - v_2)$  and, equivalently,  $(y_1 - v_1)^{1/2} (y_2 - v_2)^{1/2}$ . By concavity this allocation is unique. Thus, if  $\eta = \sigma = \lambda_1 = \lambda_2 = \frac{1}{2}$ , the  $\eta$ -egalitarian bargained and Nash-bargained allocations cum search and matching frictions coincide and are unique in the case of complete financial markets. ■

In the simulation results to follow, parameter values  $\eta = 1/2$  and  $\sigma = 1/2$  are specified.

Problem (22) takes into account that in each period, non-stockholders' hours worked are then determined competitively according to the following condition:

$$MRS_{c,l}^n = w_t^n \quad (23)$$

where  $MRS_{c,l}^n$  represents the non-stockholder's marginal rate of substitution for leisure vs. consumption.

The wage  $w_t^n$  which solves the bargaining problem (22) must satisfy the following optimality condition:

$$\eta V_{n_t}^s = (1 - \eta) V_{n_t}^n, \text{ or,} \quad (24)$$

$$\eta \lambda_t^s J_t = (1 - \eta) \lambda_t^n (EP_t - U_t).^{24} \quad (25)$$

A standard calculation based on the condition (25) guarantees that the  $\eta$ -egalitarian bargained wage between the two groups is given by:

$$w_t^n = \frac{(1 - \eta) \frac{1}{\phi_t}}{(1 - \eta) \frac{1}{\phi_t} + \eta} \frac{[L(h_t^n) + b - F_t^n]}{h_t^n} + \frac{\eta}{(1 - \eta) \frac{1}{\phi_t} + \eta} \frac{[h_t^n f_3(k_t, \mu_s h_t^s, h_t^n n_t) z_t - \frac{\kappa}{2} x_t^2 + F_t^s]}{h_t^n} \quad (26)$$

where  $F_t^n \equiv \beta(1 - \rho - s_t) E_t \frac{\tilde{\lambda}_{t+1}^n}{\lambda_t^n} (EP_{t+1} - U_{t+1})$  and  $F_t^s \equiv \beta(1 - \rho) E_t \frac{\tilde{\lambda}_{t+1}^s}{\lambda_t^s} J_{t+1}$  denote,

respectively, the future net expected welfare benefits to the non-stockholder-family and to the stockholders from one additional employed non-stockholder. From the presence of the  $\phi_t$  term in expression (26), it is apparent that the financial market structure influences Nash wage determination. Define the “effective” bargaining power of the non-stockholder group as:

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<sup>24</sup>This condition is called the *constant surplus sharing rule*.

$$\eta_t \equiv \frac{\eta}{(1-\eta)\frac{1}{\phi_t} + \eta} . \quad (27)$$

Equation (26) can then be rewritten as:

$$w_t^n = (1-\eta_t) \frac{[L(h_t^n) + b - F_t^n]}{h_t^n} + \eta_t \frac{[h_t^n f_3(k_t, \mu_s h_t^s, h_t^n n_t) z_t - \frac{\kappa}{2} x_t^2 + F_t^s]}{h_t^n} . \quad (28)$$

From (28) we see that the “distribution risk adjusted”  $\eta$ -egalitarian-bargained non-stockholder wage reduces to the standard Nash-bargained wage in a representative agent regime in that special case: in a representative-agent construct, markets are complete and  $\phi_t$  is equal to one. This observation highlights the significant role of limited asset market participation in generating variable distribution risk  $\phi_t$ , and thus variable  $\eta_t$ .<sup>25</sup>

Proposition 3.2 summarizes the prior discussion.

**Proposition 3.2:** Within the present model framework, KRN  $\eta$ -egalitarian wage bargaining is equivalent to Nash wage bargaining with time varying bargaining power. ■<sup>26</sup>

It can furthermore be shown that up to a first-order approximation,

$$\hat{\eta}_t = (\text{constant}) \cdot \hat{\phi}_t . \quad (29)$$

Since distribution risk will prove to be countercyclical in this model, by (29) the same will be true of non-stockholder bargaining power. In turn, this latter property will play a key role in generating unemployment with the coveted properties: the countercyclical behavior of non-stockholder bargaining power creates excessively smooth wages that enhance the observed volatility of the key labor market variables of interest.<sup>28</sup>

Equation (28) may also be written as

$$w_t^n h_t^n = (1-\eta_t) [b + (c_t^{n,e} - c_t^{n,u} - \chi^n (\mathbf{c}_{t-1}^{n,e} - \mathbf{c}_{t-1}^{n,u}))] - \left( \frac{u^n (c_t^{n,e} - \chi^n \mathbf{c}_{t-1}^{n,e} - L(h_t^n)) - u^n (c_t^{n,u} - \chi^n \mathbf{c}_{t-1}^{n,u})}{\lambda_t^n} \right)$$

<sup>25</sup> It is in this sense that we endogenize the relative bargaining power.

<sup>26</sup> Variable bargaining power guarantees endogenous factor share variation, something that is generally believed to be important for successful asset pricing. See Lansing (2015) and Favilukis and Lin (2015). Drautzburg et al. (2017) generate factor share variation by placing an exogenous process on the Nash bargaining parameter calibrated to reflect policy changes.

<sup>27</sup> A  $\hat{\cdot}$  on a variable denotes log deviations from the corresponding steady-state value. The latter values are distinguished by a  $\bar{\cdot}$  above them. See the Appendix, Part D for the derivations of equations (26) – (28).

<sup>28</sup> Our sense of distribution risk thus suggests itself as a candidate for the Nash bargaining power shock Shimer proposed without invoking its source (Shimer, 2005).

$$+\eta_t \left[ h_t^n f_3(k_t, \mu_s h_t^s, n_t h_t^n) z_t - \frac{\kappa}{2} x_t^2 + \kappa x_t s_t \right] \quad (30)$$

where  $w_t^n$ ,  $h_t^n$ ,  $\lambda_t^n$ ,  $c_t^{n,e}$ ,  $c_t^{n,u}$  and  $b$  are as previously defined. The first term in (30),

$$\zeta_t = \left[ b + \left( c_t^{n,e} - c_t^{n,u} - \chi^n (\mathbf{c}_{t-1}^{n,e} - \mathbf{c}_{t-1}^{n,u}) \right) + \left( \frac{u^n (c_t^{n,u} - \chi^n \mathbf{c}_{t-1}^{n,u}) - u^n (c_t^{n,e} - \chi^n \mathbf{c}_{t-1}^{n,e} - L(h_t^n))}{\lambda_t^n} \right) \right],^{29}$$

represents an employed non-stockholder's dynamic outside option (reservation) value (of being unemployed). It consists of three components: (i)  $b$ , the exogenously given unemployment benefit, (ii)  $(c_t^{n,e} - c_t^{n,u}) - \chi^n (\mathbf{c}_{t-1}^{n,e} - \mathbf{c}_{t-1}^{n,u})$ , the difference in consumption when employed vs. unemployed, and (iii) the utility benefit of not supplying hours when not working. Under GHH

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<sup>29</sup> Chodorow-Reich and Karabaronis (2016) (hereafter CRK) estimate the components of  $\zeta_t$  from data and find the following: (1)  $\zeta_t$  is strongly procyclical due to the strongly procyclical right-most term, (2)  $(c_t^{n,e} - c_t^{n,u})$  is nearly acyclical, and (3)  $b_t$  is countercyclical. However,  $b_t / (\zeta_t - b_t)$  is sufficiently small so as not to reverse the overall procyclical property of  $\zeta_t$ . Accordingly, their results suggest that there is little net incentive for workers to assume jobs in expansions or to surrender them in contractions. As a result unemployment volatility should be low, from which the Shimer (2005) puzzle follows. Equivalently, since  $\pi_{n_t}$  is strongly procyclical, by relationship (31),  $w_t^n h_t^n$ , the wage bill per worker, must then also be strongly procyclical which would also destroy the operating leverage mechanism of the present paper.

In the present model, GHH preferences guarantee that

$$\frac{u^n (c_t^{n,u} - \chi^n c_{t-1}^{n,u}) - u^n (c_t^{n,e} - \chi^n c_{t-1}^{n,e} - L(h_t^n))}{\lambda_t^n},$$

the prime procyclical component, is zero. In the calibration we further set  $b_t \equiv 0$ , leaving the cyclical nature of  $\zeta_t$  to be determined exclusively by the  $(c_t^{n,e} - c_t^{n,u})$  term, which CRK (2016) find to be empirically weakly procyclical, as it is in the present model. Since  $\hat{\eta}_t = (\text{constant}) \hat{\phi}_t$ , expression (34) can be rewritten as

$$\widehat{w_t^n h_t^n} = c' \hat{\phi}_t + d' \hat{\pi}_{n_t} + e' \hat{\zeta}_t$$

where  $c'$ ,  $d'$ , and  $e'$  are suitable constants. Since  $\hat{\phi}_t$  is countercyclical and  $\zeta_t$  only mildly procyclical in the present model, these features together allow it to generate a stable wage bill per worker.

preferences and the representative family model for non-stockholders, this third term is zero, with the non-stockholder's dynamic outside option, henceforth denoted  $\zeta_t$ , correspondingly simplified to:

$$\zeta_t = b + (c_t^{n,e} - c_t^{n,u}) - \chi^n (\mathbf{c}_{t-1}^{n,e} - \mathbf{c}_{t-1}^{n,u}) = b + (L(h_t^n) - h(0)).$$

The expression  $\pi_{n_t} = \left[ h_t^n f_3(k_t, \mu_s h_t^s, n_t h_t^n) z_t - \frac{\kappa}{2} x_t^2 - \kappa x_t s_t \right]$  represents the match related benefit to the firm (stockholders) of one marginally added worker. Equation (30) can then be expressed as:

$$w_t^n h_t^n = (1 - \eta_t) \pi_{n_t} + \eta_t \zeta_t \equiv \eta_t^{firm} \zeta_t + \eta_t^{non-stockholder} \pi_{n_t}. \quad (31)$$

We will use (29) – (30) later on to make more explicit the effects of countercyclical distribution risk on the stability of the non-stockholder wage bill.

### 3.6 Equilibrium

In this economy, market clearing requires that for all  $t$ ,

$$\begin{aligned} e_t &= e_t^s d\chi = 1, \\ \phi \bar{k} &= \int b_t^s d\chi + \int b_t^n d\omega, \\ c_t &= \int c_t^s d\chi + \int c_t^n d\omega, \\ y_t &= c_t + i_t + \frac{\kappa}{2} x_t^2 n_t, \end{aligned}$$

where  $\chi$  and  $\omega$  stand, respectively, for the measure of stockholders and the measure of nonstockholders. Lump sum employment taxes are levied on non-stockholders to balance the government's budget constraint:

$$T_t + (1 - n_t)b = 0.$$

Internal consistency also requires that  $\mathbf{c}_t^n = c_t^n$  and  $\mathbf{c}_t^s = c_t^s$  for all  $t$ .

Equilibrium is defined as follows:

**Definition 1** *Under the above market-clearing conditions, a decentralized stationary recursive equilibrium is defined as a set of decision rules*

$\{c_t^s(\cdot), c_t^n(\cdot), h_t^s(\cdot), h_t^n(\cdot), e_{t+1}(\cdot), i_t(\cdot), b_{t+1}^s(\cdot), b_{t+1}^n(\cdot), v_t(\cdot)\}$ , *and a set of wage and price functions*

$\{w_t^s(\cdot), w_t^n(\cdot), p_t^e(\cdot), p_t^b(\cdot)\}$  given the information set of aggregate states  $\Omega_t = \{k_t, n_t, b_t^n, z_t\}$  such that (i)  $\{c_t^s(\cdot), h_t^s(\cdot), e_{t+1}(\cdot), b_{t+1}^s(\cdot)\}$  solve the intertemporal problem (1) given the information set  $\Omega_t^s = \{e_t, b_t^s, w_t^s, p_t^e, p_t^b, c_{t-1}^s\}$ , (ii)  $\{c_t^n(\cdot), h_t^n(\cdot), b_{t+1}^n\}$  solve the non-stockholder family's intertemporal problem (11) its information set  $\Omega_t^n = \{b_t^n, w_t^n, p_t^b, s_t, c_{t-1}^n\}$ , (iii)  $\{w_t^n(\cdot)\}$  satisfies the optimality condition (25), (iv)  $\{i_t(\cdot), x_t(\cdot)\}$  solve the firm's intertemporal problem (12) given the information set  $\Omega_t^f = \{k_t, z_t, q_t, n_t\}$ , (v)  $w_t^s(\cdot)$  satisfies condition (14), (vi)  $\{p_t^e(\cdot), d_t(\cdot)\}$  satisfy the Lucas (1978a) asset pricing equation (6), while  $\{p_t^b(\cdot)\}$  satisfies equation (7), (vii) the economy follows two laws of motion:  $k_{t+1} = (1 - \delta)k_t + G(i_t/k_t)k_t$  and  $n_{t+1} = (1 - \rho)n_t + q_t v_t$ . Rational expectations are assumed for all agents.

A critical aspect of the steady state (stochastic or non-stochastic) is the equilibrium distribution of wealth between agents, which has important consequences for the consumption risk facing the stockholders, not only in terms of the dividend stream they receive, but also in terms of the pattern of the distribution risk they experience. Let  $W^n$  and  $W^s$  denote, respectively, the wealths in terms of consumption of a representative non-stockholder and a representative stockholder. We summarize consumption and wealth inequality by their respective Gini coefficients, the former being determined by the latter. Of special relevance is the steady-state Gini coefficient for wealth  $\bar{G}^W$ . In the present two agent economy,

$$\bar{G}^W = 1 - \left( \frac{\mu_s}{1 + \mu_s} + \frac{\bar{W}^n}{\bar{W}^s + \bar{W}^n} \right) \quad (32)$$

where

$$\bar{W}^s = \mu_s \bar{b}^s + \bar{k} \quad ,$$

$$\bar{W}^n = \bar{b}^n \quad , \text{ and}$$

$$\mu_s \bar{b}^s + \bar{b}^n = \beta \phi \bar{k} \quad .^{30}$$

A  $\bar{\cdot}$  above a variable indicates its steady-state value. In particular, the magnitude of  $\mathcal{X}^s$ , relative to  $\mathcal{X}^n$ , will be a critical determinant of the model's equilibrium steady state  $\bar{G}^W$  and  $\bar{G}^C$

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<sup>30</sup> More generally,  $\mu_s b_{t+1}^s + b_{t+1}^n = q_t^b \phi k_t$ . At any time  $t$ , the  $G_t^W = 1 - \left( \frac{\mu_s}{1 + \mu_s} + \frac{W_t^n}{W_t^s + W_t^n} \right)$ . See the Technical Appendix for details.



(consumption Gini coefficient).<sup>31</sup>

Model calibration and the selection of functional forms follows.

## 4. Calibration

The assumed functional forms, as well as most parameter value choices, are commonplace in the literature. The remaining parameters (such as  $\chi^n$ ,  $\chi^s$ ) are determined endogenously within the model in order to generate realistic stationary wealth distributions. The model is then solved using log-linearization about the certainty steady state, a methodology widely employed in the business cycle literature (see Uhlig (1999) and Campbell (1994)). Reported statistics are averages based on 1,000 independent runs each 1,000 periods in length. Log-normal formulae are applied to compute the relevant asset returns (see Jermann (1998), or the Technical Appendix).<sup>32</sup>

For all simulation runs, the production function employed is the customary Cobb-Douglas specification:

$$z_t f(k_t, h_t^s \cdot 1, h_t^n \cdot n_t) = z_t A k_t^\alpha ((\mu_s h_t^s \cdot 1)^\mu (h_t^n \cdot n_t)^{1-\mu})^{1-\alpha}.$$

where  $\mu = \mu_s / (1 + \mu_s)$  and  $1 - \mu$  are, respectively, the normalized measures of stockholder and non-stockholders. Following Lansing (2015), the Baseline model's stock market participation rate,  $\mu_s$ , is set to be 10 percent, so that  $\mu$  equals 0.091. The extent of debt financing is determined by choosing  $\phi = .4$ .<sup>33</sup> The parameter  $A$  serves as a scale parameter chosen to guarantee the debt-financing is default free, and to guarantee a uniformly positive dividend in all states of nature; in particular,  $A = 1.25$ .

When factor markets are competitive, the parameter  $\alpha$  is typically calibrated to reproduce the observed share of capital in total value added. While the labor market in the present model is not competitive, the most commonly used value, 0.36, is nevertheless retained. The time interval in the model corresponds to three months. Accordingly, the subjective discount

<sup>31</sup> See the Technical Appendix for details on the steady state characterization.

<sup>32</sup> Petrosky-Nadeau et al. (2017) argue persuasively that log-linearization procedures for solving models featuring DMP Nash wage bargaining are fundamentally inaccurate since they lead to periodic endogenous disasters where equilibrium employment levels are far from the steady state. This conclusion rests on two assumptions: a high flow value of unemployment activities (in our model a high “b”) and a high fixed cost to posting vacancies. The present model (the parameterized, simulated version) has neither of these features, while possessing a quadratic labor adjustment cost. In all our simulations, we observe no tendency for a disaster state to develop. Our methodology parallels that in Christiano et al. (2016). In addition, our equilibrium wage determination mechanism is not “pure Nash.”

<sup>33</sup> This level is proposed in Kandel and Stambaugh (1991) and reported and employed in Rouwenhorst (1995).

factor  $\beta$  is fixed at  $\beta = 0.99$ , corresponding to a steady state annualized return on capital of 4%. Following Kydland and Prescott (1982), and Kaltenbrunner and Lochstoer (2010), the quarterly capital depreciation rate  $\delta$  is chosen as 0.020. Following Guvenen (2009) we fix the cost of the adjustment parameter  $\xi$  at  $\xi = .45$ .

The productivity shock  $\tilde{z}_t$  evolves according to the law of motion:

$$\log \tilde{z}_{t+1} = 0.95 \log z_t + \tilde{\varepsilon}_{t+1}$$

where the  $\{\tilde{\varepsilon}_t\}$  are distributed i.i.d. normal, with mean zero and standard deviation  $\sigma_\varepsilon = .712\%$  (see Cooley and Prescott (1995)).<sup>34</sup>

Since the model economy assumes that search and matching frictions characterize the labor market for non-stockholders, we calibrate it using standard parameters for labor market search and matching. The empirical literature provides several estimates of the US worker separation rate. We follow Davis et al. (1996) and fix the quarterly separation rate  $\rho$  at .10 percent.<sup>35</sup> According to Petronglo and Pissarides (2001), the elasticity of matches to unemployment of outsiders,  $1 - \sigma$ , falls within the range of plausible values of 0.5 to 0.7. We set  $1 - \sigma$  to be 0.5. The existing literature mostly adopts the bargaining power parameter  $\eta = 0.5$ ; as per the comments in Section 3.5.4, we follow suit.

The period utility function of the representative stockholder is postulated as

$$u^s(c_t^s - \chi^s c_{t-1}^s - H(h_t^s)) = \frac{(c_t^s - \chi^s c_{t-1}^s - B_s(h_t^s)^{\psi_s})^{1-\gamma_s} - 1}{1 - \gamma_s}$$

where  $\gamma_s$  is her coefficient relative risk aversion;  $\psi_s$  is the parameter which controls her Frisch elasticity of hours supplied, and  $H(h_t^s) = B_s(h_t^s)^{\psi_s}$ , where  $B_s$  is a disutility parameter.

The utility of the non-stockholder is postulated similarly:

$$u^n(c_t^n - \chi^n c_{t-1}^n - n_t L(h_t^n)) = \frac{(c_t^n - \chi^n c_{t-1}^n - n_t B_n(h_t^n)^{\psi_n})^{1-\gamma_n} - 1}{1 - \gamma_n}$$

where  $\gamma_n$ ,  $\psi_n$ , and  $B_n$  have the same analogous interpretations as in the stockholder case. For all simulations,  $\gamma_s = \gamma_n \equiv \gamma$ , and  $\psi_s = \psi_n \equiv \psi$  are assumed. With these identifications, none of

<sup>34</sup> All reported statistics are computed on the basis of 1000 independent simulation runs each 1000 periods in length and represent averages of the statistics computed across the individual runs themselves.

<sup>35</sup> With  $\rho = .10$ , the expected duration of employment of an outsider-nonstockholder before separation is approximately 12 quarters or three years.

the results cited below can be attributed to a priori differential risk aversion across agents.<sup>36</sup> As in Jaimovich and Rebelo (2009),  $\psi = 1.4$  for all cases, which implies a Frisch elasticity of labor supply of  $\frac{1}{(1.4-1)} = 2.5$ .

The parameter  $b$  is set to  $b = 0$ . The logic is as follows: In the Andolfatto-Mortensen-Pissarides search cum real business cycle model (Andolfatto (1996)), the steady state pure value of leisure normalized by output,

$$\frac{\frac{1}{\bar{\lambda}^n} \left( B_n (\bar{h}^n)^\psi \right)}{\bar{y}},$$

is .9, which seems high. Nevertheless, Andolfatto's (1996) model is unable to generate adequate unemployment volatility, an outcome in contrast to the results of the structurally very different model of Hagedorn and Manovskii (2008) where  $b = .95$ , a value allowing the unemployment volatility puzzle to be resolved. As the present formulation is closer to Andolfatto (1996) than it is to Hagedorn and Manovskii (2008), a similarly high value is placed on leisure. The choice of  $b = 0$  represents an attempt to bias the model against an easy replication of the observed level of unemployment volatility. Anticipating the steady-state discussion to follow, Table 1 below compares the normalized value of leisure across various models. The Baseline calibration gives values in the lower range.

**Table 1: Value of Being Unemployed Relative to Per Capita Output and Labor Productivity per Employed Outsider Non-Stockholder in the Steady State: Various Models**

	Baseline <sup>(iii)</sup>	Shimer (2005) <sup>(iv)</sup>	Hagedorn & Manovskii (2005) <sup>(v)</sup>	Hall (2005) <sup>(vi)</sup>	Andolfatto (1996) <sup>(vii)</sup>
$\frac{\bar{\zeta}}{\bar{y} / (1 + \mu_s)}$ <sup>(i)</sup>	.50	.40	.95	.40	.90
$\frac{\bar{\zeta}}{(1 - \alpha)(1 - \mu) \bar{y} / \bar{n}}$ <sup>(ii)</sup>	.70	.40	.95	.40	.98

<sup>(i)</sup>  $\frac{\bar{\zeta}}{\bar{y} / (1 + \mu_s)}$ : steady-state value of being unemployed as a fraction of steady state per capita output.

<sup>(ii)</sup> Steady-state value of being unemployed relative to labor productivity of an employed outsider.

Note: For case (iv), (v) and (vi),  $\bar{\zeta} = b$ , while (ii) and (v) consider only the endogenous value of labor.

<sup>36</sup>This being said, we acknowledge that habit formation causes the insider-stockholder to be *effectively* more risk averse than the outsider-nonstockholder.

The free parameters  $B_s$  and  $B_n$  are selected to match the following wage and hours ratios:

$$\bar{w}^s / \bar{w}^n = 1.57, \text{ and } \bar{h}^s / \bar{h}^n = .59.$$

The hours supply ratio and the wage ratio are closely related. In particular, without an hours ratio less than one, the stockholder's wage rate may be less than that of the workers. Heathcote et al. (2010) report a male college wage premium of 1.4 in 1980, and college-educated persons are much more likely to be stockholders; the chosen wage ratio thus roughly approximates the Heathcote et al. (1980) estimate. The hours ratio follows accordingly (in the Baseline case to be reported,  $\bar{h}^s = .2$  and  $\bar{h}^n = .34$ ).<sup>37</sup>

The Baseline version of the model corresponds to the case where the top 10% of the population owns 90% of all financial assets (Poterba and Samwick (1995), Guvenen (2009), Lansing (2015)); i.e.,  $\bar{\Omega}^W = .9$ , which corresponds to a Gini coefficient of  $\bar{G}^W = .80$  via (32). In turn, the corresponding Gini coefficient for consumption is  $\bar{G}^C = .12$ , a value so much less than the Gini coefficient for wealth because the consumption of both agents is mostly financed by their wage incomes which are large relative to their capital incomes and much less dissimilar. Reasonable values of  $\bar{G}^C$  require stockholders to receive wage income.<sup>38</sup>

Under the assumed model structure, an agent's habit parameter  $\chi$  influences the extent of his precautionary savings, with greater habit parameter values leading to greater wealth accumulation, at least for values not too close to one.<sup>39</sup> Accordingly, with stockholders possessing the majority of the wealth  $\chi^s$  must exceed  $\chi^n$ . In fact, there is a one-to-one mapping between the choice of habit parameters  $\chi^n$  and  $\chi^s$  according to the steady state relationship:

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<sup>37</sup> Lansing (2015), in contrast, exhibits an hours supply ratio of .225 in order to achieve a wage ratio of 2 between capitalists and workers. In Lansing (2015), workers hold no bonds, however (zero wealth).

<sup>38</sup> The provision of hours by stockholders is included precisely for this reason.

<sup>39</sup> In a stylized two period model of financial income uncertainty, a greater habit will proportionally increase the extent of the agent's precautionary savings. There is, however, an upper bound for habit parameter values  $\chi^*$ , where  $0 < \chi^* < 1$ , beyond which the extent of the agent's precautionary savings declines. This latter result is particularly relevant to an incomplete-markets heterogeneous agent setting: the existence of such an  $\chi^*$  effectively guarantees a stationary wealth distribution with finite consumption. Otherwise, with a common subjective discount factor  $\beta$  across all agents (as in the present model), it is well known that any incomplete markets heterogeneous agent model without habits may be susceptible to the Chamberlain and Wilson (2000) observation: when  $\beta^{-1}$  is equal to the steady-state interest rate  $1 + \bar{r}$ , each agent in the aforementioned environment will accumulate unbounded assets resulting in unbounded consumption. When  $\beta^{-1} = 1 + \bar{r}$  habit parameter heterogeneity seems appropriate. See the online Technical Appendix, Part G.

$$\chi^s = 1 + \frac{1}{\bar{\phi}^{\frac{1}{\gamma}} \left( \frac{\bar{c}^s}{\bar{c}^n} \right)} (\chi^n - 1) + \frac{1}{\bar{c}^s} \left[ \frac{\bar{n} \bar{\Gamma}^n}{\bar{\phi}^{\frac{1}{\gamma}}} - \bar{\Gamma}^s \right] \quad (33)$$

where  $\bar{\Gamma}^s = B_s (\bar{h}^s)^\psi$ ,  $\bar{\Gamma}^n = B_n (\bar{h}^n)^\psi$  and  $0 \leq \chi^n \leq .299$ .<sup>41</sup> This is of the form  $\chi^s = a + b\chi^n$  for constants  $a$  and  $b$ . Note that  $\bar{\Omega}^C$  (and its antecedent  $\bar{\Omega}^W$ ) is present in the form of the terms  $\bar{c}^s / \bar{c}^n$  and  $\bar{\phi}$  in the formula. In order for the wealth distribution to be stationary, however,  $\chi^s$  cannot be too large; elementary calculations suggest a value in the range  $[\cdot 5, \cdot 7]$ . Consistent with  $\bar{\Omega}^W$ , relationship (33) and these latter considerations we choose  $\chi^s = .61$  and  $\chi^n = .08$  for the Baseline case.<sup>42</sup>

Table 2 presents the full set of critical parameter choices for the cases we consider.

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<sup>40</sup> Note that the second term in (33) is negative.

<sup>41</sup> If  $\chi^n$  exceeds this range, the argument of the worker's utility function becomes negative due to the presence of negative work disutility.

<sup>42</sup> Qualitatively, this particular choice of habit parameters enjoys substantial theoretical support. Hornstein and Uhlig (2000) emphasize the self-selection of agents: agents who become accustomed to a high consumption level; i.e., have habit formation preferences, are more likely to build up large precautionary capital stocks (i.e., become stockholders) than agents who do not. In a classic study, Becker (1980) shows that if heterogeneity across households takes the form of differential subjective discount factors, then the household with the lowest rate of discount (i.e., the most patient household, the one with the highest  $\beta$ ) owns all the capital and earns wage income in the long-run steady state, while all other households receive only wage income. This study suggests that an unequal wealth distribution has its origin in preference heterogeneity. More recently, Diaz et al. (2003) show that in a heterogeneous agent economy, identical habit formation preferences encourage a more uniform wealth distribution relative to standard preferences, suggesting that a skewed wealth distribution will obtain only if heterogeneity in habit parameters (differential habits) is allowed. Lastly, Fuhrer (2000) shows that to be consistent with the VAR finding of a hump-shaped response of consumption to income, the aggregate consumption function should derive from two distinct groups of agents; i.e., a group of agents with habit-formation preferences and a group who lives "for the moment" (low habit). This is the Baseline calibration. We also consider cases of both higher and lower consumption and wealth inequality, and compare these results with those of our Baseline calibration. Lower and higher extremes of wealth inequality can be achieved by varying  $\mu_s$  and especially the  $\chi^s$  and  $\chi^n$ , as noted above.

**Table 2: Capital Ownership Concentration Parameters<sup>(i)</sup>**

	$\mu$	$\gamma$	$\chi^s$	$\chi^n$	$\bar{\phi}$	$\bar{\eta}$	$\bar{G}^W$ <sup>(i)</sup>	$\bar{G}^c$
Data							(.75–.81) <sup>(ii)</sup>	(.23–.26) <sup>(iii)</sup>
High Wealth Inequality								
	.075	2	.68	.08	.82	.45	.83	.12
Baseline								
	.10	2	.61	.08	.8	.44	.8	.12
Low Wealth Inequality								
	.20	2	.44	.08	.71	.41	.7	.104

(i)  $\bar{G}^W = .80$  corresponds to  $\bar{\Omega}^W = .90$ .

(ii) The  $\bar{G}^W$  range is taken from Quadrini (2000), Krusell and Smith (1998), and Favilukis (2013)

(iii) The  $\bar{G}^c$  range is reported in Kreuger and Perri (2006) and Favilukis (2013)

## 5. Results and Interpretation: Baseline Case

Table 3, Panel A, reports the second moments of endogenous aggregate and labor market variables as implied by the Baseline model, namely unconditional standard deviations, and their contemporaneous correlation with output, alongside the moments implied by the data. Panel B reports cross-correlations of various financial and macroeconomic variables.

**Table 3: Baseline Case**  
**Panel A: Macro and Labor Market Statistics**

	Data: 2008.1 – 2015.2			Data: 1959.2 – 2007.4			Baseline Case		
	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)
$y$	1.30			1.59	-	-	1.55	-	-
$c^{(i)}$	1.27	.98	.99	1.23	.77	.83	1.50	.97	.97
$c^s$							2.61	1.69	.98
$c^n$							1.56	1.01	.96
$i$	5.20	4.00	.93	4.87	3.06	.91	1.94	1.26	.92
$k$							.18	.12	.48
$n$							1.08	.71	1.00
$h^{tot (ii)}$	1.02	.78	.90	.69	.43	.92	1.39	.90	.85
$h^s$							1.11	.71	1.00
$h^n$							.35	.23	.99
$a^{(iii)}$	1.00	.77	.38	1.30	.82	.76	.82	.53	.45
$w_t^n^{(iv)}$	1.11	.85	-.35	.96	.60	.23	.14	.09	.77
$w^A^{(v)}$							.21	.14	.99
$n$	1.78	1.37	.69	1.42	.89	.70	1.08	.70	.99
$u$	6.17	4.75	-.78	11.01	6.92	-.87	9.73	6.29	-.84
$v$	8.22	6.32	1.00	13.15	8.27	.91	13.27	8.58	.98
$\theta$	13.61	10.47	.96	21.66	13.62	.90	21.63	13.97	.99
$\phi$	10.72	8.25	-.95	-			12.61	8.15	-.90
$l^s (BEA)^{(vi)}$	1.30	1.00	.05	1.10	.69	-.37	.90	.58	-.28
$l^s (BLS)$	1.20	.92	.00	1.10	.69	-.21	-	-	-
$d$	11.69	8.99	.43	8.52	5.36	.71	17.32	11.19	.74

(a) Standard deviation (x), (b) S.D. (x) / S.D. (y), (c) corr (x, y) for all series "x". Actual and model data H.P. filtered with smoothing parameter 1600.

**Panel B: Correlations of Macroeconomic and Financial Quantities  
(Hall's (2017) Discount Channel)**

	Data		
	(2008.1 - 2015.3)	(1959.2 - 2007.4)	Baseline
$\text{corr}(\tilde{y}_t, \tilde{V}_t^f)^{(vii)}$	.23	.70	.83
$\text{corr}(\tilde{y}_t, E_t[\tilde{M}_{t,t+1}]^{-1})^{(viii)}$	-.24	-.54	-.38
$\text{corr}(\tilde{y}_t, E_t[\tilde{r}_{t,t+1}^e - \tilde{r}_{t,t+1}^b])$	N/A	N/A	-.39
$\text{corr}(\tilde{y}_t, \tilde{p}_t^e)^{(ix)}$	.38	.89	.83
$\text{corr}(\tilde{v}_t, \tilde{V}_t^f)$	.13	.52	.89
$\text{corr}(\tilde{v}_t, E_t[\tilde{M}_{t,t+1}]^{-1})$	-.24	-.43	-.25
$\text{corr}(\tilde{v}_t, E_t[\tilde{r}_{t,t+1}^e - \tilde{r}_{t,t+1}^b])^{(x)}$	N/A	N/A	-.40
$\text{corr}(\tilde{v}_t, \tilde{p}_t^e)$	.27	.77	.89
$\text{corr}(\tilde{v}_t, \tilde{d}_t)$	.60	.18	.97
$\text{corr}(\tilde{d}_t, E_t[\tilde{M}_{t,t+1}]^{-1})$	N/A	N/A	-.90
$\text{corr}(\tilde{v}_t, \tilde{i}_t)$	.80	.95	.96

(i)  $c_t = \mu_s c_t^s + \mu_n [n_t c_t^{n,e} + (1 - n_t) c_t^{n,u}]$

(ii)  $h_t^{\text{tot}} = \mu_s h_t^s + h_t^n$

(iii)  $a_t = \text{average labor productivity} = y_t / (\mu_s + n_t)$ .

(iv)  $w_t^n$  denotes the egalitarian bargained wage. We believe that the model's egalitarian-bargained wage best corresponds to BLS wage data.

(v)  $w_t^A$  denotes the average wage:  $w_t^A = (\mu_s / (\mu_s + n_t)) w_t^s + (n_t / (\mu_s + n_t)) w_t^n$ . BEA average wage data is not computed as an average of the wages of stockholders vs. non-stockholders as there is no such distinction in the data. We thus do not present average wage data.

(vi)  $\ell^s$  denotes the labor share of income in the model; it is computed as  $\ell_t^s = \frac{w_t^n h_t^n n_t}{w_t^n h_t^n n_t + \mu_s w_t^s + r_t k_t}$ .

(vii)  $V_t^f$  represents the aggregate ex dividend value of firms (debt and equity); we extend the data set constructed in Merz and Yashiv (2007).

(viii)  $(M_{t,t+1})^{-1}$  is estimated as  $(1 - \tau) r_{t,t+1}^{\text{commercial paper}} - \inf_{t,t+1}$ . Similar results are obtained using the definition of discount rates presented in Fama (1999). In the model analysis  $E_t[\tilde{M}_{t+1}]$  is computed as in Jermann (1998); see the Technical Appendix

(ix) In the data  $p_t^e$  represents the value of the S&P 500 portfolio.

(x)  $\tilde{r}_{t,t+1}^e, \tilde{r}_{t,t+1}^b$  are, respectively, the return on equity and the return on default free debt from  $t$  to  $t+1$ . Accordingly,

$E_t[\tilde{r}_{t,t+1}^e - \tilde{r}_{t,t+1}^b]$  is the conditional expected equity premium. It has no data counterpart.



## 5.1 Macro-aggregates and prices

We first focus our discussion on the extended, and statistically more robust, data period 1959.2 – 2007.4. The extremes of wealth inequality underlying the Baseline case were not generally seen during this time interval except in perhaps its final ten years.

In general, the Baseline case displays excessive consumption and hours volatility and insufficient investment volatility. The most basic characterization of the business cycle, that investment is more volatile than output which, in turn, is more volatile than consumption is, however, preserved. We attribute both the excessive aggregate consumption variation and the insufficient investment volatility to the capital cost of adjustment feature of the model (see Jermann (1998)).<sup>43</sup> <sup>44</sup> Stockholder consumption volatility exceeds that of non-stockholders, a property first confirmed in Mankiw and Zeldes (1991). It reflects the high dividend volatility arising from the “operating leverage” effect of the wage-bargaining-induced stable wage bill. If we look behind these consumption levels to the corresponding annualized growth rates, we find a similar pattern: stockholder consumption growth volatility,  $\sigma_{\tilde{g}^{cs}}$ , is roughly twice that of non-stockholders,  $\sigma_{\tilde{g}^{cn}}$ : 2.01% versus 1.16%, estimates in line with those reported in Malloy et al. (2009) (see Table 11 in Section 6.2). Labor share volatility statistics have roughly the correct relative magnitudes, and the mild negative correlation of the labor share with real output reflects the data as well.

In the more recent historical period (2008.1 – 2015.2), output volatility has declined with aggregate consumption and investment volatility largely unchanged. On these dimensions the model performs no better than for the expanded historical period – and for the same reasons. Hours are substantially more volatile, however, something that goes hand-in-hand with the dramatic transition to “downward wage rigidity” manifest as a negative correlation of output and the real wage ( $\text{corr}(w^n, y) = -.35$ ). While the present model is unable to replicate this negative correlation, which we interpret as an indication of downward wage rigidity, overall wage rigidity is a natural outcome:  $\sigma_{w^n} = .14$  and  $\sigma_{w^n} / \sigma_y = .09$ . This essential wage acyclicity will be

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<sup>43</sup> The stockholder’s habit formation, per se, works in the opposite direction: lower consumption variation and investment volatility. For the present formulation, capital adjustment costs dominate.

<sup>44</sup> Increased investment volatility is enhanced in similar models by adding firm specific investment shocks unrelated to productivity shocks. See, e.g., Fahri and Gourio (2018).

seen as a natural model implication when wealth inequality is high.<sup>45</sup> During this period U.S. wealth inequality was high by historical standards. We expand upon this discussion in the next section. For either data source (BLS or BEA), the labor share becomes more volatile in this period, and nearly acyclical. While these phenomena depart from their model-generated counterparts, they can be replicated by increasing the level of wealth inequality as analyzed in Section 6.<sup>46</sup>

### 5.1.1. An Empirical Measure of Distribution Risk

Distribution risk is a quantity entirely endogenous to the model economy. It is nevertheless of interest to find some proxy or instrumental variable to serve as an empirical counterpart in order to assess if the model generated distribution risk volatility and correlation with output have any reasonable empirical counterpart. A first choice stand-in for distribution risk would be some measure of the relative growth rates of stockholder and non-stockholder consumption, but such data is unavailable at quarterly frequencies. Since  $\tilde{\phi}$  reflects variation in relative non-stockholder consumption (and, more fundamentally, in wealth and income inequality), we hypothesize that a reasonable proxy would be an index that measured social concern for “inequality.” Google Trends Data on Income and Wealth allows the construction of such an index, and it is proposed as the empirical proxy for  $\tilde{\phi}$ .

Specifically, it is assumed that variation in consumption inequality is reflected in public awareness/concern as measured by the number of persons who access the Google Trends Data on its Income and Wealth Website. This daily data was averaged within a quarter and then subject to the HP filter in the exact same manner as for all the other series presented in Table 3. It is of interest (though likely coincidental) that the statistical characterizations of model data and the Google Trends proxy for distribution risk are nearly identical (see Table 3). See also Figure 1 and Table 4 below which further compare model generated and Google Trends data along a number of other dimensions.

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<sup>45</sup> Later in the paper we explore the low correlation of wages and labor productivity as an alternative sense of wage rigidity.

<sup>46</sup> These labor share “anomalies” can also be reproduced by altering the agent’s coefficient of relative risk aversion in the context of “confidence shocks.” See Donaldson and Kim (2019), Table 5, Panel A, for the results when  $\gamma = 2.5$ .

**Table 4: Distribution Risk Correlations: Model Generated and Google Data<sup>(i)</sup>**

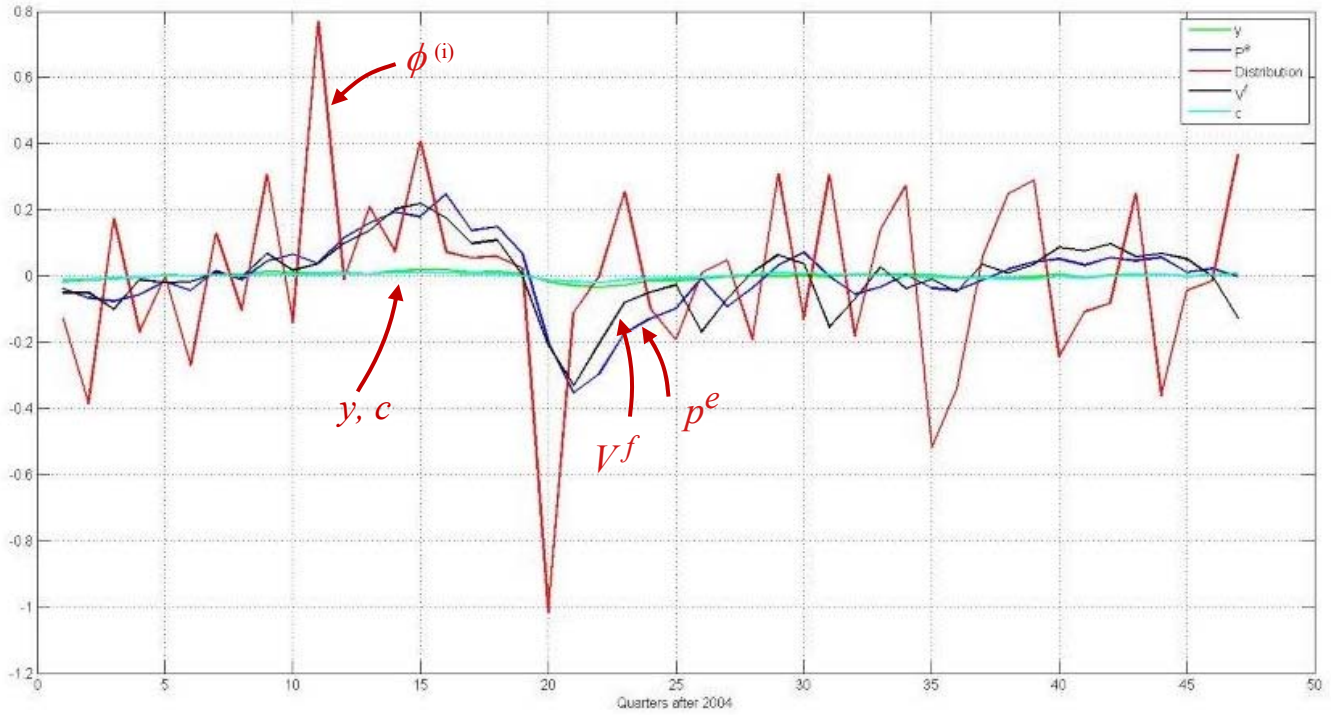
	$\rho(\tilde{\phi}, \tilde{y})^{(ii)}$	$\rho(\tilde{\phi}, \tilde{c})$	$\rho(\tilde{\phi}, \tilde{p}^e)^{(iii)}$	$\rho(\tilde{\phi}, \tilde{V}^f)^{(iv)}$
Data: 2004.1-2015.2	-.28	-.36	-.33	-.26
Data: 2008.1-2015.2	-.10	-.23	-.22	-.10
Baseline Model	-.90	-.77	-.97	-.97

(i) All series H-P filter detrended with detrending parameter  $\lambda=1600$ .

(ii)  $\tilde{\phi}$  represents the number of persons accessing Google Trends Data on Income and Wealth.

(iii)  $\tilde{p}^e$  denotes the value of the S&P 500 index.

(iv)  $\tilde{V}^f$  denotes the aggregate ex dividend of all firms inclusive of debt and equity; we extend the data set constructed in Merz and Yashiv (2007) to this later period.



(i)  $\phi$  measured by Google Trends data.

**Figure 1**

Time Series of  $\phi^{Google}$ ,  $y$ ,  $c$ ,  $V^f$  and  $p^e$  since 2004.

Note that model-generated and Google-generated correlation patterns are of the same

sign, although the “single focus” Baseline model correlations are much more negative. The data suggests that in cyclical expansions, when jobs are plentiful or increasingly so, public concern for “inequality” diminishes. Cyclical expansions are also usually accompanied by a rising stock market.

Another empirical cross check on the plausibility of the endogenous distribution risk measure  $\tilde{\phi}$  comes from Drautzberg et al. (2017). These authors impose an exogenous AR-1 process on the bargaining parameter  $\tilde{\eta}$  in a complete markets version of the present model, similar to that of Andolfatto (1996). Using a detailed statistical procedure based on data across a number of countries that focuses on policy changes of relevance to income inequality, they estimate the parameters of their exogenous (to the model) process, deriving an AR-1 persistence parameter of  $\rho = .95$ , and an innovation SD,  $\sigma_{\tilde{\varepsilon}} = .128$ . While the origin of our distribution risk shock is endogenous, it has a similar effect on worker bargaining power. The AR-1 process on worker bargaining power they derive ( $\tilde{\eta}$  in the present model) is characterized by  $\rho = .77$  and  $\sigma_{\tilde{\varepsilon}} = .125$ .

## 5.2 Labor Market Volatility

The most striking result overall in Table 3 is the Baseline model’s low reported wage volatility. This fact is directly attributable to the strongly countercyclical nature of the endogenous distribution risk ( $\text{corr}(\tilde{y}_t, \tilde{\phi}_t) = -.90$ ), which strengthens non-stockholder bargaining power in recessions (low productivity states) and weakens it in expansions (high productivity states). As a result the  $\eta$ -egalitarian-bargained wage,  $w_t^n$ , hardly varies at all. The average wage,  $w_t^A$ , varies more, as it includes the stockholder wage which is competitively determined. We believe that the BLS provided wage data best corresponds to the model’s egalitarian-bargained wage. With extremely low wage volatility, hours volatility is thus correspondingly higher in the Baseline case.

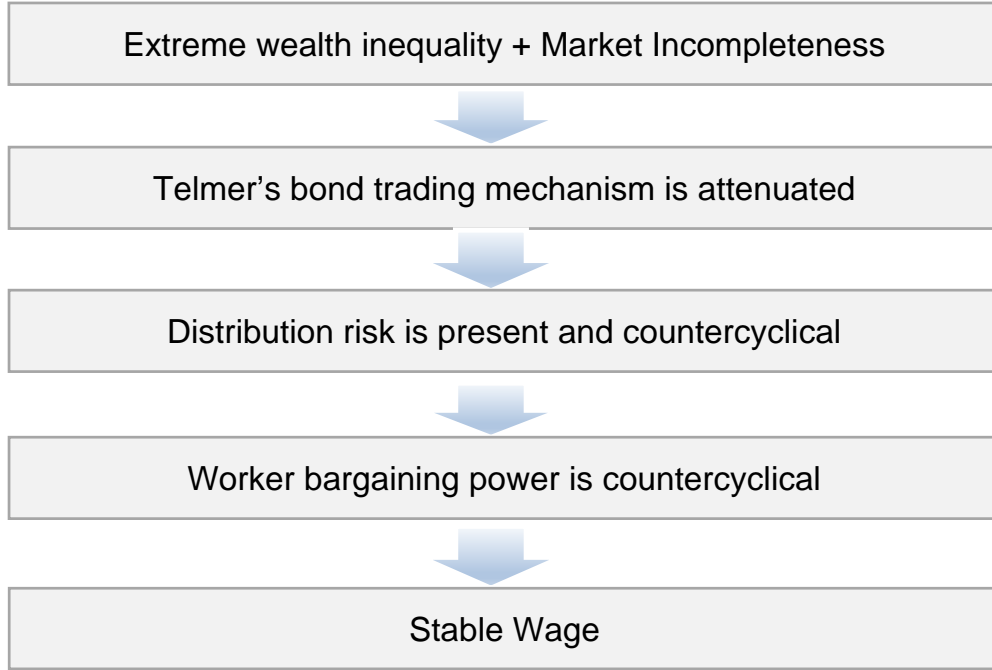
Before going on, we explore the origins of these results in more detail, first focusing on the low equilibrium wage volatility. Following Guvenen (2009), the extent of income insurance provided by bond trading is partial in the sense that business cycle risks arising from productivity shocks  $\{\tilde{z}_t\}$  are not equiproportionately shared across all states, especially under circumstances of high-wealth inequality, which the Baseline case presumes. In the present model, extreme

wealth inequality means that stockholders who already own the capital stock also own a preponderance of the bonds, although they receive little income from them due to low rates (to be discussed in Section 5.3). In these circumstances, stockholders are reluctant to buy bonds from workers, especially in low productivity states, something necessary for worker consumption to be more completely stabilized. These are the states where stockholder consumption is comparatively low and marginal utility high, due to the combination of financial leverage ( $\phi=.4$ ) and operating leverage (arising from the stable wage bill whose origins are presently being considered). With high stockholder consumption marginal utility, distribution risk is high and worker bargaining power is high in the low productivity states. The reverse is true in high productivity states, so that distribution risk manifests itself as supplemental worker bargaining power as per relationship (27). As a result, workers' wages decline relatively less (vis-à-vis the competitive alternative) in low output states and increase relatively less in high output states (firm bargaining power increases). Accordingly, the  $\eta$ -egalitarian bargained wage and the wage bill become very stable thereby enhancing the operating leverage mentioned earlier. The wage bill's stability and its large magnitude relative to investment together allow the dividend to be procyclical. This is in contrast to more standard business cycle models, where highly procyclical investment causes the dividend to be countercyclical.

Via the mechanism of  $\eta$ -egalitarian wage bargaining, risk averse non-stockholders thus effectively create an alternative (to Telmer's (1993) bond trading) form of consumption insurance by stabilizing their wage income, particularly in low productivity states. As such, "distribution risks" are the residual risks that bond trading cannot eliminate, but end up being contained via the wage-setting mechanism.<sup>47</sup> Under high wealth inequality (Baseline case), KRN  $\eta$ -egalitarian wage bargaining effectively creates an endogenous "wage asset," which takes the form of a semi-fixed wage contract leading to a countercyclical non-stockholder labor income share. This wage contract, and the countercyclical share it implies, is effectively a "semi-safe" asset from the perspective of non-stockholders. Figure 2 presents the aforementioned logic schematically.

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<sup>47</sup> Levine and Zame (2002) present examples where risk sharing in an incomplete markets set-up similar to the one considered here breaks down under high wealth (endowment) inequality due to price effects. Their context is an exchange setting which bars endogenous asset creation of the type considered here, however. Philosophically, it is perhaps not surprising that under the equal gains principle, which leads to more egalitarian consumption allocations than pure Nash wage bargaining, agents would creatively introduce new risk sharing mechanisms.



**Figure 2: The Logic of Wage Stability**

As presented in Table 3, the Baseline model is also able substantially to replicate the volatility in the other key labor market variables found in the data emphasized by Shimer (2005) and Hall (2005) including unemployment,  $u$ , vacancies,  $v$ , and the market tightness measure  $\theta = \frac{v}{u}$  (both absolute levels and relative to output).<sup>48, 49</sup> The prior summary is especially reflective of the 1959.2 – 2007.4 expanded data period; for the more recent historical period, comparative model volatilities are about 25% too high. All three variables,  $u$ ,  $v$ , and  $\theta$ , have the

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<sup>48</sup> Since the  $u^{rate} = \frac{u}{1 + \mu_s}$ , the statistical properties of the unemployment rate  $u^{rate}$  and the level of unemployment

$u$  are identical under our log-linearization methodology.

<sup>49</sup> For the U.S. historical period 1964:1 - 2002:1, Chéron and Langot (2004) report that  $\text{corr}(w, y) = .28$ , a much lower value than we report in Table 3 ( $\text{corr}(w, y) = .76$ ). In order to achieve a wage-output contemporaneous correlation this low these authors employ a Rogerson and Wright (1988) utility specification of the form

$$\left( c_t^n - n_t L(h_t^n) \right)^{1-\gamma} / (1-\gamma) + a c_t^n, \quad a > 0.$$

They work, however, with a representative agent formulation similar to Andolfatto (1996). We suspect that this modification of worker preferences would, in our context, work towards the same goal. It has the added feature that if the constant  $a > 0$  is properly chosen, the utility of the non-stockholder workers who are employed will exceed that of their unemployed family members.

correct sign as regards their correlation with output;  $\text{corr}(u, y)$  is also of the correct magnitude;  $\text{corr}(v, y)$  and  $\text{corr}(\theta, y)$  are somewhat too positive, relative to both data periods). The correlation between unemployment and vacancies in the Baseline model (not reported in Table 3) is  $\rho(u, v) = -0.76$ , which approximates the data (-0.88) reasonably well.

The model-generated high vacancy volatility (which, in turn, contributes to high match and unemployment volatility) is next considered. The origin of this phenomenon can be found in Panel B and, in particular, the countercyclical behavior of the firm's (stockholders') stochastic discount rate:  $\text{corr}\left(y_t, E_t\left[\tilde{M}_{t,t+1}\right]^{-1}\right) = -0.38$ .<sup>50</sup> These are the rates at which the firm discounts its match benefits, with the implication that in recessions the benefits are deeply discounted, leading to very few vacancy postings at that time, and conversely in expansions. As a result, vacancy postings become highly volatile and extremely procyclical as in the data. As such, the mechanism becomes an illustration of Hall's (2017) perspective that the high volatility of vacancy postings and their extreme diminution in cyclical downturns must be attributable to high discount rates in recessions, as confirmed by  $\text{corr}\left(v_t, E_t\left[\tilde{M}_{t,t+1}\right]^{-1}\right) = \text{corr}(v_t, r_t) = -0.25$ . Since free cash flows from physical investment and firm valuations use the same discount rates, these quantities are both also highly procyclical (Panel B), as in the data, especially for the 2008.1 – 2015.2 period. In the case of investment, the capital adjustment cost suppresses where would otherwise be high investment volatility. This same logic dictates that investment and vacancies should also be highly correlated. This is the case both for the model and in the data; see Table 3, Panel B, final entry.

Tables 5, 6, and 7 provide additional dimensions along which labor market behavior can be measured. Table 5, in particular, considers the elasticity of the egalitarian-bargained non-stockholder wage with respect to labor productivity  $a$ ,  $\varepsilon_{w^n, a}$ , where

$$\varepsilon_{w^n, a} = \frac{\partial \log w_t^n}{\partial \log a_t},$$

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<sup>50</sup> In contrast, standard DSGE models have difficulty in replicating a countercyclical discount rate. In particular, a standard Real Business Cycle model with DMP Nash wage bargaining generates a highly procyclical discount rate. See Donaldson and Kim (2019), Table 5, Panel D.

a lower value being associated with greater wage stability.<sup>51</sup> The corresponding values for a selection of other models are also presented. By this measure, the real wage of the non-stockholders is extremely stable in the Baseline model; in contrast, wages are much too responsive in Shimer's (2005) interpretation of the DMP model. The correlation of the log of the non-stockholder real wage with the log of labor productivity is also reported and, as regards this measure, the Baseline model is somewhat extreme but is nevertheless far from insignificant in magnitude. It is also of the correct sign and better reflects the data than Shimer's (2005) complete markets formulation. We further explore these latter measures of wage rigidity in Section 6.1.

Table 6 considers labor share statistics where the non-stockholder's share of total income,  $\ell_t^s$ , is defined to be

$$\ell_t^s = \frac{w_t^n h_t^n n_t}{w_t^n h_t^n n_t + \mu_s w_t^s + r_t k_t}.$$

It is well known that the labor share is countercyclical in the data, and Table 5 captures this phenomenon in three measures:  $\varepsilon_{\ell_t^s, a_t}$ ,  $\text{corr}(\log \tilde{\ell}_t^s, \log \tilde{a}_t)$  and  $\text{corr}(\log \tilde{\ell}_t^s, \log \tilde{y}_t)$ .<sup>52</sup> Note that the Baseline model replicates Table 6's selection of data reasonably closely, but is somewhat inferior to the Gertler and Trigari (2009) model except along the  $\text{corr}(\log \tilde{\ell}_t^s, \log \tilde{y}_t)$  dimension. Furthermore, note also that Shimer's (2005) DMP model yields signs that are correct, but the measured quantities (elasticities and correlations) are counterfactually extreme.

Table 7 presents the model-inspired Beveridge curve, the correlation of contemporaneous unemployment levels and vacancy levels at various leads and lags. Unlike the Merz (1995) and Andolfatto (1996) models, the Baseline model is able to capture the full correlation structure, especially as regards the signs of the correlations.

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<sup>51</sup> The elasticity  $\varepsilon_{w^n, a}$  is obtained as the OLS estimator  $\hat{\beta}$  in the regression

$$\log \tilde{w}_t^n = \beta \log a_t + \tilde{\varepsilon}_t, \quad t = 1, 2, \dots, N$$

where the  $\{\tilde{\varepsilon}_t\}$  are error terms and  $N$  the length of the data series.

<sup>52</sup> In a like fashion to Table 4,  $\varepsilon_{\ell_t^s, n}$  is identified with the OLS estimator  $\hat{\beta}$  in the regression

$$\log \tilde{\ell}_t^{s, n} = \beta \log a_t + \tilde{\varepsilon}_t, \text{ etc.}$$



**Table 5: Wage Statistics: Wages and Productivity**

	U.S. Data <sup>(i)</sup>	DMP <sup>(iii)</sup>	Baseline	Rudanko <sup>(iv)</sup>	Gertler and Trigari
$\partial \log w_t^n / \partial \log a_t$ <sup>(ii)</sup>	.44	.95	.026	-17	.50
$\text{corr}(\log \tilde{w}_t^n, \log \tilde{a}_t)$	.56	1.00	.19	N/A	.63

<sup>(i)</sup> Statistics computed using BLS data for the period 1951.1 – 2015.1.

<sup>(ii)</sup>  $a_t = y_t / (n_t + \mu_s)$

<sup>(iii)</sup> Adapted from Shimer (2005) when  $b = .40$ .

<sup>(iv)</sup> Rudanko (2011) identifies this negative wage elasticity with equilibrium wage “rigidity.”

**Table 6: Labor Share Statistics**

	U.S. Data <sup>(i)</sup>	DMP <sup>(ii)</sup>	Baseline	Gertler and Trigari
$\partial \log \ell_t^s / \partial \log a_t$	-.56	-.02	-.90	-.50
$\text{corr}(\log \tilde{\ell}_t^s, \log \tilde{a}_t)$	-.65	-1.00	-.84	-.63
$\text{corr}(\log \tilde{\ell}_t^s, \log \tilde{y}_t)$	-.23	-1.00	-.28	-.56

<sup>(i)</sup> All statistical quantities computed using BLS data, 1951.1 – 2015.1.

<sup>(ii)</sup> Basic DMP model as adapted in Shimer (2005);  $b = .4$ .

**Table 7: The Beveridge Curve**

$\text{corr}(\tilde{u}, \tilde{v})$  <sup>(i)</sup>

	$\rho(\tilde{u}_{t-3}, \tilde{v}_t)$	$\rho(\tilde{u}_{t-2}, \tilde{v}_t)$	$\rho(\tilde{u}_{t-1}, \tilde{v}_t)$	$\rho(\tilde{u}_t, \tilde{v}_t)$	$\rho(\tilde{u}_{t+1}, \tilde{v}_t)$	$\rho(\tilde{u}_{t+2}, \tilde{v}_t)$	$\rho(\tilde{u}_{t+3}, \tilde{v}_t)$
US data, 59.1 - 88.2	-.357	-.607	-.824	-.954	-.928	-.769	-.535
Merz (1995) (variable s)	.197	-.365	-.476	.322	.263	.224	2.00
Merz (1995) (fixed s)	-.400	-.590	-.824	-.153	-.045	.035	.094
Andolfatto (1996)	-.65	-.73	-.65	-.19	.05	.17	.24
Baseline	-.13	-.32	-.56	-.77	-.96	-.95	-.82

<sup>(i)</sup> Quarterly frequency; Hagedorn and Manovskii (2008, Table 4) report only  $\rho(\tilde{u}_t, \tilde{v}_t) = .724$ , which is highly counterfactual; Gertler and Trigari (2009) do not report these statistics.

In summary, the distribution risk mechanism arising through KRN  $\eta$ -egalitarian wage bargaining dampens wage variation, creating substantial wage stability, operating leverage, and vacancy volatility by a variety of measures.

### 5.3 Financial statistics

For the basic return statistics related to the equity premium -- the short rate (or one-period default-free rate), the return on equity, and the premium itself (all averages) -- the Baseline model provides a reasonable match of theory to data (see Table 8). In particular, the average default-free return is low, as is its volatility in contrast to many models with capital costs of adjustment (Jermann (1998)). We attribute the low mean return as arising from non-stockholder demand: it is the only asset available to non-stockholders for consumption smoothing purposes. At the same time, the relative habit (precautionary savings) parameters and the attendant extreme wealth inequality reveals that workers hold relatively few bonds. It follows that bond prices are high. The return on equity and its volatility are also too low relative to the data, but not trivially so. At 3.12%, the equity premium (and its volatility) is also too low relative to the period studied by Mehra and Prescott (1985). For the expanded period 1871-1993, however, Campbell and Cochrane (1999) report a U.S. equity premium of 3.9%, which is closer to the Baseline model generated counterpart. The Sharpe ratio is a respectable .275.<sup>53</sup>

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<sup>53</sup> Recently, Ai and Bansal (2018) document empirical findings that stock returns around pre-scheduled announcement dates, such as the employment and FOMC reports, account for 55% of the market equity premium; i.e., to use their term, the macroeconomic announcement premium accounts for 3.36% of the 6.19% overall equity premium (Ai and Bansal (2018), Table 1). Accordingly, the non-announcement premium resulting from non-diversifiable macroeconomic systematic risk should account only for 2.82%, which is close to the corresponding Baseline result. Furthermore, the announcement premium cannot be replicated in a habit model formulation.

**Table 8: Baseline Case  
Financial Statistics**

<b>Panel A: Equity-Related Financial Statistics<sup>(i)</sup></b>							
	$E\tilde{r}^e$	$\sigma_{\tilde{r}^e}$	$E\tilde{r}^b$	$\sigma_{\tilde{r}^b}$	$E\tilde{r}_p$	$\sigma_{\tilde{r}_p}$	$E\tilde{r}_p / \sigma_{\tilde{r}^p}$
Data <sup>(ii)</sup>	6.98	16.54	.80	5.67	6.18	16.67	.33
Baseline	3.61	11.78	.50	1.80	3.12	11.32	.275

<b>Panel B: Term Structure of Default Free Rates</b>					
	U.S. Data <sup>(iii)</sup>			Baseline	
Maturity <sup>(iv)</sup>	Mean		SD	Mean	SD
4	1.06		1.61	.44	1.84
8	1.39		1.37	.52	1.67
12	1.69		1.23	.62	1.49
16	1.95		1.15	.72	1.36
20	2.16		1.09	.80	1.24

(i) All rates in percent, annualized.

(ii) Data from Mehra and Prescott (1985).

(iii) McCullough data as reported in Piazzesi and Schneider (2006); Sinha (2016) confirms these results using an expanded TIPS data set. Using extensive U.K. inflation indexed security returns data Sinha (2016) also derives an upward sloping real term structure for that nation as well.

(iv) Maturity measured in terms of quarters.

The generally acceptable performance of the model along the various equity return-dimensions can be attributed to wage stabilization. Countercyclical non-stockholder bargaining power and the stable wage and wage bill that follow from it increase the firm's operating leverage which, in turn, generates procyclical dividends and high dividend volatility. This increased dividend risk confronts a stockholder who not only desires a smooth consumption stream due to habit-formation utility but who also is hindered in securing it by the cost of adjusting both the labor input and the capital stock. As a result, stockholders require a substantial premium to hold stocks vis-à-vis default free bonds.

As regards the term structure, the model term structure lies substantially below its data-based counterpart, yet both are increasing. In a reversal, the term structure of standard deviations for the Baseline lies somewhat above its empirical counterpart, but not dramatically so. Both, however, are downward sloping. In general, the model performs adequately on the term structure dimensions. See Donaldson and Kim (2019) for a more detailed discussion of the model's financial implications, especially as wealth inequality increases.

## 6 Changing Levels of Wealth Inequality

In this section we explore the consequences of changing wealth inequality on the dynamic properties of the economy, with special attention to the labor market, and the partial risk-sharing mechanism it provides. Tables 9 – 12 provide alternative perspectives.

**Table 9: Changing Wealth Inequality<sup>(i)</sup>**

<b>Panel A: Macro and Labor Market Statistics</b>									
	Extreme Wealth Inequality $\mu = .075, \bar{G}^W = .83, \bar{G}^C = .12$			Baseline $\mu = .10, \bar{G}^W = .80, \bar{G}^C = .12$			Low Wealth Inequality $\mu = .20, \bar{G}^W = .70, \bar{G}^C = .10$		
	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)
$y$	1.64			1.55	-	-	1.68		
$c$	1.77	1.08	.96	1.50	.97	.97	1.61	.96	.99
$c^s$	3.18	1.94	.98	2.61	1.69	.98	2.09	1.25	.97
$c^n$	2.03	1.25	.90	1.56	1.01	.96	1.65	.99	1.00
$i$	1.86	1.13	.87	1.94	1.26	.92	1.94	1.16	.96
$k$	.19	.12	.55	.18	.12	.48	.17	.10	.37
$n$	1.70	1.03	.97	1.07	.69	.99	.51	.30	.92
$h^{tot}$	2.11	1.29	.83	1.39	.90	.85	1.22	.73	.98
$h^s$	1.17	.71	1.00	1.11	.71	1.00	1.21	.71	1.00
$h^n$	.98	.60	.11	.36	.23	.77	.91	.54	.99
$a$	1.19	.73	-.10	.82	.53	.45	1.39	.83	.96
$w^n$	.39	.24	.11	.14	.09	.77	.35	.21	.99
$w^A$	.51	.29	.19	.21	.14	.99	.44	.26	.98
$u$	15.29	9.34	-.98	9.73	6.29	-.84	4.52	2.70	-.66
$v$	20.74	12.68	.87	13.27	8.58	.98	6.30	3.76	.99
$\theta$	33.98	20.78	.97	21.63	13.97	.99	10.05	5.99	.92
$\phi$	21.27	13.01	-.65	12.61	8.15	-.90	4.82	2.88	-.99
$l^s$	1.63	.99	.11	.90	.58	-.28	.26	.18	-.48
$d$	62.44	38.17	.46	17.32	11.19	.74	38.23	22.80	.74

**Panel B: Discount Rate Statistics (Hall's Discount Channel)**

	Extreme Wealth Inequality $\mu = .075, \bar{G}^W = .83, \bar{G}^C = .12$	Baseline $\mu = .10, \bar{G}^W = .80, \bar{G}^C = .12$	Low Wealth Inequality $\mu = .20, \bar{G}^W = .7, \bar{G}^C = .104$
$\text{corr}(y_t, V_t^f)$	.70	.83	.94
$\text{corr}(y_t, E_t[\tilde{M}_{t+1}]^{-1})$	-.35	-.38	-.59
$\text{corr}(\tilde{y}_t, \tilde{p}_t^e)$	.70	.83	.93
$\text{corr}(\tilde{v}_t, \tilde{V}_t^f)$	.94	.89	.90
$\text{corr}(\tilde{v}_t, E_t[\tilde{M}_{t,t+1}]^{-1})$	.15	-.25	-.53
$\text{corr}(\tilde{v}_t, \tilde{p}_t^e)$	.94	.92	.90
$\text{corr}(\tilde{v}_t, \tilde{d}_t)$	-.02	.64	.66
$\text{corr}(\tilde{d}_t, E_t[\tilde{M}_{t,t+1}]^{-1})$	-.99	-.90	-.98
$\text{corr}(\tilde{v}_t, \tilde{i}_t)$	.99	.96	.95
$\text{corr}(\tilde{i}_t, \tilde{p}_t^e)$	.73	.97	.98
$\text{corr}(\tilde{i}_t, \tilde{V}_t^f)$	.73	.97	.98

(i) All interpretive notes for Table 9 are identical to those for Table 3.

**Table 10: The Term Structure of Interest Rates <sup>(i)</sup>**

Maturity <sup>(ii)</sup>	Data <sup>(iii)</sup>		$\bar{G}^W = .83, \mu = .075$		$\bar{G}^W = .8, \mu = .10$		$\bar{G}^W = .7, \mu = .20$	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
4	1.06	1.61	-4.00	2.28	.44	1.84	3.15	2.17
8	1.39	1.37	-4.12	1.73	.52	1.67	3.24	.76
12	1.69	1.23	-4.05	1.53	.62	1.49	3.30	1.56
16	1.95	1.15	-3.94	1.38	.72	1.36	3.34	1.41
20	2.16	1.09	-3.83	1.25	.82	1.24	3.38	1.28
40	--	--	-3.41	.83	1.10	.83	3.51	.88
Term Premium			-.21		.61		.52	

(i) All rates are annualized.

(ii) Maturity measured in quarters.

(iii) McCullough data as reported in Piazzesi and Schneider (2006); Sinha (2016) confirms these results using an expanded TIPS data set.

(iv) The bond term premium is defined to be  $R^{(40)} - R^{(1)}$ ; i.e., the ten year rate less the quarterly rate.

The transition to higher wealth inequality is accomplished by progressively reducing the measure of stockholders (see Table 2). As  $\mu_s$  declines from  $\mu = .20$  (low inequality) to  $\mu = .10$  (Baseline), for example, the per capita wealth of each stockholder roughly doubles with  $\bar{G}^W$  increasing from  $\bar{G}^W = .7$  to  $\bar{G}^W = .8$ , and the corresponding  $\bar{G}^C$  increasing from .10 to .12. Note that the increase in wealth inequality effected as  $\mu_s$  decreases from  $\mu = .10$  to  $\mu = .075$  is proportionately much less than that resulting from the aforementioned decrease –  $\bar{G}^W = .83$  vs.  $\bar{G}^W = .8$  – with nearly identical consumption inequality measures.<sup>54</sup> In this sense, both the Baseline case and the  $\bar{G}^W = .83$  case are “high wealth inequality” ones.

## 6.1 Risk Sharing and the Labor Market

Since a theme of this paper will be the changing form of risk-sharing we begin by focusing on this topic. In Tables 9, 10 and 11 there is substantial evidence that risk sharing via bond trading is enhanced by lower wealth inequality, while the provision of worker income insurance via wage stabilization follows the reverse pattern. While these mechanisms can be complementary risk-sharing devices, the degree of income and wealth inequality influences their relative power.

Prima facie evidence of the increasing power of bond trading alone to share risk as wealth inequality declines is found in Table 9: the SD of distribution risk,  $\sigma_{\tilde{\phi}}$ , declines monotonically from  $\sigma_{\tilde{\phi}} = 21.27$  (high inequality) to  $\sigma_{\tilde{\phi}} = 4.82$  (low inequality); relative to output, the proportional reduction is approximately the same. At the same time (see Table 12),  $\text{corr}(\tilde{c}_t^s, \tilde{c}_t^n)$  increases from .85 ( $\bar{G}^W = .83$ ; high inequality) to 1.00 ( $\bar{G}^W = .7$ ; low inequality). Other circumstantial evidence comes from Table 10: bond prices are very high and the term structure uniformly negative when  $\bar{G}^W = .83$ , rising to a flat and positive term structure in the neighborhood of 3-3.5% when  $\bar{G}^W = .7$ . While the former level suggests a reluctance on the part of stockholders to sell bonds or otherwise trade bonds actively,<sup>55</sup> the latter level is similar to what would be expected in a complete-markets DSGE environment where the economy’s

<sup>54</sup> The increase in the consumption Gini coefficient is slight because most consumption is financed by wage income, as noted earlier.

<sup>55</sup> In a sense, there is a “shortage” of bonds.

quarterly discount rate is 1%, as is the case here ( $\beta = .99$  for both agent types). Telmer's (1993) observation that bond trading alone is able to achieve near-to-perfect risk sharing is thus confirmed when wealth inequality is low. Overall, the pattern of term structure shifts confirms a movement from bond "scarcity" ( $\bar{G}^W = .83$ ) to one where bonds are abundant and freely traded ( $\bar{G}^W = .70$ ), suggesting enhanced stockholder willingness to sell bonds.<sup>56</sup>

At the same time, there is evidence that non-stockholder income insurance via wage stabilization (with origins in countercyclical distribution risk) is weakened as wealth inequality declines. First and foremost, the non-stockholder's wage is vastly more procyclical: when  $\bar{G}^W = .83$ ,  $\text{corr}(\tilde{w}_t^n, \tilde{y}_t) = .11$ ; when  $\bar{G}^W = .7$ ,  $\text{corr}(\tilde{w}_t^n, \tilde{y}_t) = .99$ . Another manifestation of the same effect is the reduction in non-stockholder labor share volatility:  $\sigma_{\tilde{l}^s} / \sigma_{\tilde{y}}$  declines from .99 in the  $\bar{G}^W = .83$  case to .18 in the case of  $\bar{G}^W = .70$ . Note that absolute labor share variation similarly declines. Taken together, these observations suggest that non-stockholder wage behavior is becoming more "competitive-like" under reduced inequality: at the competitive extreme where labor and capital are paid their marginal products, the labor and capital shares would exhibit no variation at all. From the perspective of this paper, a distinguishing feature of the financial crisis (and its aftermath) is the resulting transition to higher wealth inequality resulting in the a-cyclical behavior of both wage and labor income share variation.<sup>57</sup>

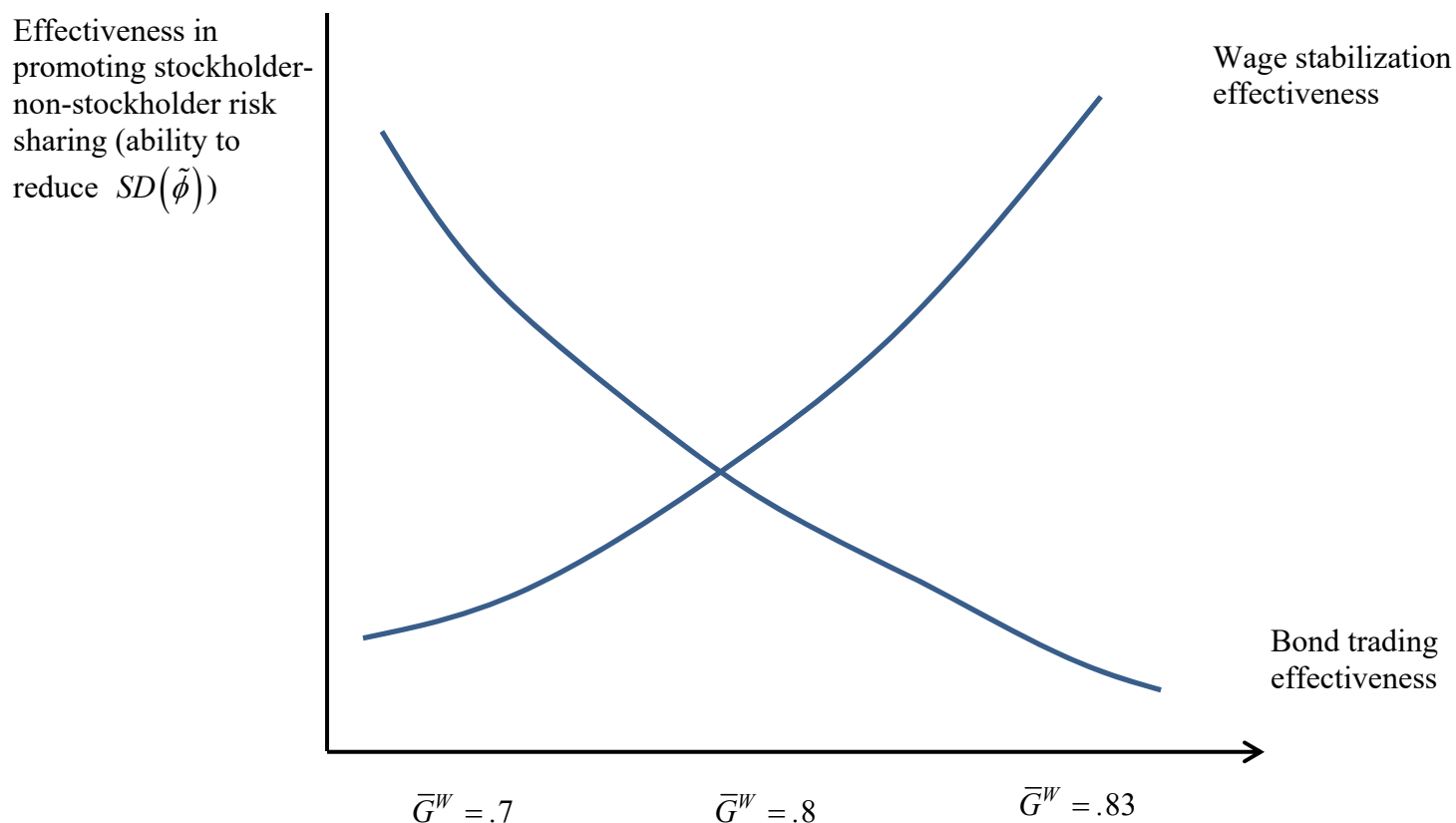
This can be explained as follows: while in all three cases non-stockholder bargaining power remains countercyclical, its power is dramatically attenuated by the enhanced bond trading: the reduced  $\sigma_{\tilde{\phi}}$  that bond trading entails implies comparatively weaker non-stockholder effective bargaining power in low productivity states (the corresponding value of  $\tilde{\phi}$  is less), with the reverse being true in high productivity ones. Ceteris paribus, the non-stockholder wage becomes more volatile. This is most apparent when we compare the  $\bar{G}^W = .7$  and  $\bar{G}^W = .8$

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<sup>56</sup> Why this reluctance on the part of stockholders to trade bonds with and thus provide income insurance to non-stockholders when wealth inequality is high? For one thing, stockholder consumption volatility is already high due to the enormous dividend volatility brought on by the high operating leverage. This is compounded by the sheer relative measure of non-stockholders – the ratio of non-stockholders to stockholders is approximately 12 to 1, a fact that magnifies the insurance consequences for stockholders.

<sup>57</sup> In particular,  $\text{corr}(y_t, \ell_t^s) = \text{corr}(y_t, w_t^n) = .11$ ; see Table 9.

cases.<sup>58</sup> Enhanced bond trading thus diminishes the power of countercyclical distribution risk to stabilize the wage. Figure 3 below represents a qualitative attempt to summarize this discussion.



**Figure 3**  
**Relative Effectiveness of Bond Trading vs. Wage Stabilization for Different Wealth Inequality Levels**

Viewing the two aforementioned risk-sharing mechanisms as counterbalancing one another (as one strengthens in power it causes the other to weaken) suggests the possibility of a range of wealth inequality in which the two effects are least “destructive” of one another or, indeed, perhaps reinforcing of one another as regards the overall risk sharing in the economy available to non-stockholders. We are unable to characterize this region precisely. The cases

<sup>58</sup> For  $\bar{G}^W = .83$ , wage volatility again increases due to the enormous increase in distribution risk as bond trading largely breaks down.



presented in Table 9, however, suggest that the Baseline  $\bar{G}^W = .80$  case is within the region of reinforcement, at least relative to the other two as evidenced by the fact that the measures  $\sigma_{\tilde{w}^n}, \sigma_{\tilde{w}^n} / \sigma_{\tilde{y}}, \sigma_{\tilde{c}^n}$  (Table 9) and  $\sigma_{\Delta \tilde{c}^n}$  (Table 12) all assume their least values in the Baseline case.

In the entirety of the above discussion, wage rigidity is largely measured against output variation. If wage rigidity is measured differently, however, the effects are more stark. Rudanko (2011), for example, measures wage rigidity relative to average productivity with a more rigid wage being one less related to it. In particular, Rudanko (2011) considers the measure  $\partial \log w_t^n / \partial \log a_t$  with a lower value indicating greater rigidity. Table 11 presents this measure as well as other related statistics for the three cases of Table 9.<sup>59</sup>

**Table 11: Wage Rigidity and Labor Productivity**

	Extreme Wealth Inequality	Baseline	Low Wealth Inequality
	$\mu = .075, \bar{G}^W = .83$	$\mu = .10, \bar{G}^W = .8$	$\mu = .20, \bar{G}^W = .7$
$\partial \log w_t^n / \partial \log a_t$	-1.09	.026	.254
$\text{corr}(\log \tilde{w}_t^n, \log \tilde{a}_t)$	-.80	.19	.99
$\text{corr}(\log y_t, \log a_t)$	-.10	.74	.96
$\text{corr}(\log z_t, \log a_t)$	.70	.99	1.00
$\text{corr}(\log z_t, \log y_t)$	.55	.84	.99

As is evident from the Table, higher wealth inequality leads to progressively more rigid wages by Rudanko's (2011) measure. Under extreme wealth inequality, furthermore, labor productivity becomes almost acyclical.

<sup>59</sup> See also Table 5.

## 6.2 Other Labor Market Consequences of Changing Wealth Inequality

Employment,  $\tilde{n}$ , vacancy,  $\tilde{v}$ , unemployment,  $\tilde{u}$ , and tightness,  $\tilde{\theta}$ , all experience significant monotonic volatility reductions, both absolute and relative to output, as wealth inequality declines. One interpretation of this fact is simply to observe that distribution risk is also declining (and dramatically) so that  $\eta$ -egalitarian wage bargaining better approximates the standard Nash bargained outcome. As a result, the volatilities when  $\bar{G}^W = .70$  more closely resemble those reported by Andolfatto (1996) whose model assumes Nash bargaining alone, but is otherwise similar to the one presented here.

Since hours are competitively determined, with non-stockholder wage volatility the least when  $\bar{G}^W = .80$ , it is to be expected that hours volatility,  $\sigma_{\tilde{h}^n}$ , is also the least in this case, as observed. Non-stockholder hours and wages are identically correlated with output, confirming their close association. Since non-stockholders supply the vast majority of hours worked, it further follows that output and dividend volatility should also be least when  $\bar{G}^W = .80$ , as is the case. These conclusions are confirmed in part because capital stock shows little variation across the three cases due to the adjustment costs. As wealth inequality declines, stockholder consumption volatility declines monotonically, reflecting the wage becoming less rigid: stockholders are providing less wage-related insurance and thus are experiencing less operating leverage. Worker consumption volatility is least when  $\bar{G}^W = .80$ , since the two income stabilization mechanisms (bond trading and wage stabilization) appear to conflict least in that case.

Due to the countercyclical pattern of distribution risk in all three cases, Hall's (2017) discount channel remains active, and the stochastic discount rate remains countercyclical across the board (Table 9, Panel B). Yet it is less volatile, with less wealth inequality leading to less volatile vacancy postings, as observed, and thus less match volatility. By equation (15), employment volatility consequently declines. Unemployment volatility thus declines as well.<sup>60</sup>

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<sup>60</sup> The behavior of stockholders as regards bond trading is complex in this model. Under high wealth inequality, stockholder consumption volatility is already high due to operating leverage effects. This discourages them from selling bonds to non-stockholders in high output states and paying them off in low output states, which is what non-stockholders crave, because it further destabilizes their consumption flow. Yet they are willing to sell some bonds to non-stockholders as they are per capita extremely wealthy in these states so that the proportionate increase in consumption volatility they assume as a result is not large, and they receive a high return on the ones they sell (rates

Another confirmation of the discount channel's continued presence is the high correlation of both investment and vacancies, both with output and with one another (see Table 9, Part B).

**Table 12: Cross-Agent Correlations and SDs: Various Models**

	Extreme Wealth Inequality	Baseline	Low Wealth Inequality
	$\bar{G}^W = .83$ $\mu = .075$	$\bar{G}^W = .8$ $\mu = .10$	$\bar{G}^W = .7$ $\mu = .20$
$\text{corr}(\tilde{c}_t^s, \tilde{c}_t^n)$	.85	.99	1.00
$\text{corr}(\tilde{b}_{t+1}^n, \tilde{y}_t)$	.36	-.35	-.78
$\text{SD}(\tilde{b}_{t+1}^n) / \text{SD}(\tilde{y}_t)$	.94	.65	.89
$\text{SD}(\Delta \tilde{c}^s)^{(i)}$	2.15	2.01	2.10
$\text{SD}(\Delta \tilde{c}^n)$	2.52	1.16	2.19
$\text{SD}(\Delta \tilde{c}^s) / \text{SD}(\Delta \tilde{c}^n)^{(ii)}$	.85	1.73	.96

(i)  $\Delta \tilde{c}^s$  represents the annualized growth rate of stockholder consumption and analogously for  $\Delta \tilde{c}^n$ .

(ii) Malloy et al. (2009) report  $\text{SD}(\Delta \tilde{c}^s) / \text{SD}(\Delta \tilde{c}^n) = 1.63$ , Mankiw and Zeldes (1991) report a figure of 1.60.

### 6.3 Financial Relationships

We considered the bond market earlier (Table 10), and observed that greater bond trading, and the reduced scarcity it implies, lead to an upward shift in the term structure. In the tryptich of cases, this shift amounts to as much as a 7% increase at the “short end” of the term structure (comparing the  $\bar{G}^W = .83$  and  $\bar{G}^W = .70$  cases) to more than a 5% increase at the “long end.” These are very large changes. Otherwise, all three cases exhibit term structures with the appropriate qualitative properties: the term structures are upward sloping and the return standard deviations declining with maturity. As is evident in Table 10 one exception to this

are negative!) Thus  $\text{corr}(\tilde{b}_{t+1}^n, \tilde{y}_t)$  is positive.

Under low wealth inequality, stockholders' consumption is much more stable but they are also per capita much less wealthy and the marginal impact of bond trading on their consumption is greater. In equilibrium they are more willing to sell bonds, but the pattern is different:  $\text{corr}(\tilde{b}_{t+1}^n, \tilde{y}_t) < 0$ .

statement is that the  $\bar{G}^W = .83$  case exhibits a negative term premium of -.21%, which is evidence of an “inverted yield curve,” something of concern to policy makers.<sup>61</sup> Table 10 also reports that the entire term structure of real rates is negative under extreme wealth inequality.

Equity market relationships are presented in Table 13.

Table 13  
Equity Return Statistics<sup>(i)</sup>

	$E\tilde{r}^e$	$\sigma_{\tilde{r}_e}$	$E\tilde{r}^b$	$\sigma_{\tilde{r}_b}$	$E\tilde{r}_p$	$\sigma_{\tilde{r}_p}$	$E\tilde{r}_p / \sigma_{\tilde{r}_p}$
$\bar{G}^W = .83$	.27	7.44	-3.20	.50	3.48	5.24	.66
$\bar{G}^W = .8$	3.61	11.78	.50	1.80	3.12	11.32	.275
$\bar{G}^W = .7$	4.94	14.76	2.99	3.75	1.95	15.84	.12

(i) All numbers in percent, annualized except  $E\tilde{r}_p / \sigma_{\tilde{r}_p}$ , the Sharpe ratio

Note first that the equity premium declines as wealth inequality diminishes while the returns on both stock and bonds increase, viz., with greater wealth equality asset prices fall. Many of the reasons for these effects have already been discussed; our present remarks are thus brief.

When  $\bar{G}^W = .83$ , stockholders are reluctant to trade bonds, even in the face of high non-stockholder demand since their consumption is highly volatile. As a result bond prices are high and rates are low (negative). Low discount rates and portfolio rebalancing considerations cause equity prices to rise: in the  $\bar{G}^W = .83$  case, despite enormous dividend volatility (Table 9, Panel A), average stock returns are a measly .27%. With average bond returns at -3.20%, the resulting premium is a robust 3.48%.

When wealth inequality is low, the reverse is true: enhanced bond trading by stockholders allows supply to increase, prices to decline, and returns to be higher. With higher discount rates, stock prices decline and then average returns rise. Overall, the premium declines since stockholder consumption volatility is so much lower due to diminished operating leverage: the premium they thus expect from stock declines.

Note that the summary statistics for the  $\bar{G}^W = .7$  case closely resemble those of a

<sup>61</sup> Research by, e.g., Bauer and Mertens (2018) (San Francisco Federal Reserve Bank), suggests that an inverted yield curve can be a strong signal of an impending economic downturn. In contrast, the model generated negative yield curve under high wealth inequality is *unconditional*: the econometrician would estimate a negative term premium with increasing frequency, independent of cyclical downturns.

complete markets standard DSGE model. The equity premium is as large as it is because of two specific features of the model: (1) habit formation, resulting in relatively high stockholder effective risk aversion, and (2), costs of adjusting both labor and capital which deter stockholders from directly smoothing their consumption via investment and vacancy posting manipulation. Default free rate volatility is low, despite the presence of capital adjustment costs because of the very low sensitivity of the SDF to productivity disturbances,<sup>62</sup> a fact that is consistent with active bond trading when wealth inequality is not extreme.

We conclude this section by noting that the pattern illustrated in Table 13 roughly mirrors the U.S. experience in the post Great Recession period when wealth inequality grew. Risk free rates were low to negative, and equity prices extremely high, as in the  $\bar{G}^W = .83$  case. It is also apparent from the model's implications that enormous capital gains would be earned by stockholders in an economy passing from low to high wealth distributions, something that is also consistent with the U.S. experience since the early 1990s.<sup>63</sup>

#### 6.4 The SDF: A Macro-Finance Perspective

In this final section we explore the equilibrium reaction of the model economy's SDF (the price of a one-period bond) to changes in several of its state variables. As will be argued, each model variant ( $\bar{G}^W = .83, .80$  and  $.70$ ) has a "signature" set of "reaction functions." The changes (across variants) inform us of the changing nature of risk sharing as wealth inequality declines. In order better to contrast the present model with others in the literature, the same analysis is extended to include the rigid-wage model of Shimer (2010) and an RBC version with complete markets as per Andolfatto (1996).

Perturbations of  $z_t$ ,  $n_t$ , and  $b_t^n$ , where relevant, are explored. In all cases the sign of the slope of the reaction function ( $\partial SDF / \partial x = \partial p_t^b / \partial x$ ) where "x" is the variable being perturbed)

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<sup>62</sup> The reasons for this are discussed extensively in Donaldson and Kim (2019).

<sup>63</sup> Closely related to our extreme wealth inequality case is the "unemployment trap" *monetary* phenomenon of Ravn and Sterk (2018). They show that depending upon the degree of wage rigidity, there can occur a transition from a "normal-times" steady state to a "crisis" steady state which they identify as an "unemployment trap." In the unemployment trap aggregate demand is depressed to a level at which it is no longer profitable for firms to invest in vacancies; hiring also declines to a minimum, which perpetuates high unemployment risk and hence low demand. The zero lower bound binds. In contrast, our polar case represents a *real* macro-finance phenomenon: a transition from the Baseline case to one of extreme wealth inequality can generate the phenomenon of "secular stagnation," without resorting to presumed real wage rigidity and nominal rigidities including the typical zero lower bound. As higher wealth inequality restricts risk-sharing due to enhanced precautionary-savings motives, (implicit) portfolio choices between bonds and bargained wages as semi-fixed assets will emerge in equilibrium, reinforcing the endogenous rigidity of the bargained wage. See Section 6.4 for a more extensive discussion of this transition.

determines the cyclical nature of the bond price while the magnitude of the slope determines its volatility. The *SDF* intercept is associated with the mean risk free rate, in the sense that if the intercept (at  $\bar{b}^n$ ) exceeds 1, the mean risk free rate is negative and vice versa. Figure 4's Subfigures A, B, C, D, and E portray the above relationships for the documented model variants. In what follows, each is briefly discussed, with differences highlighted and interpreted.

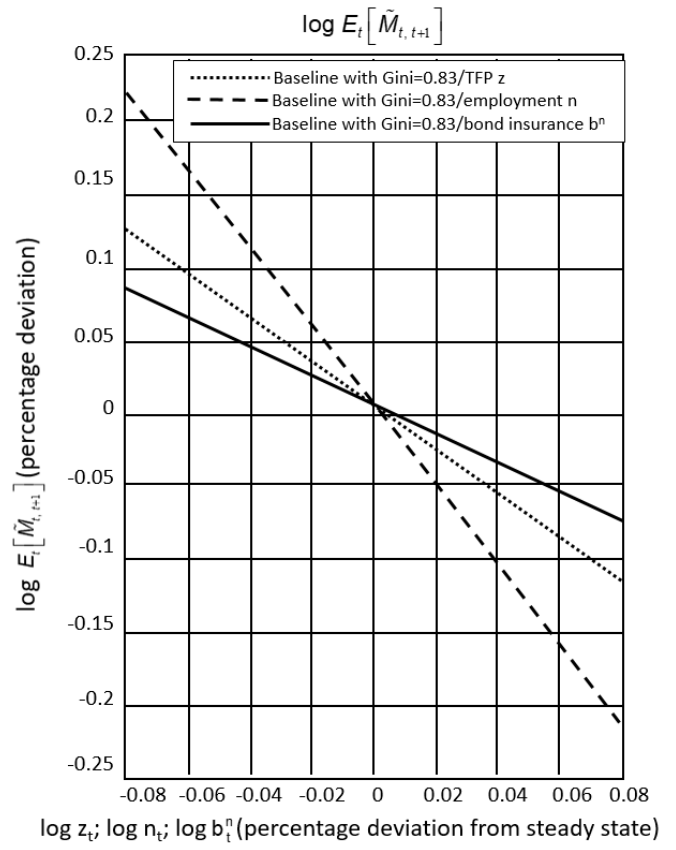
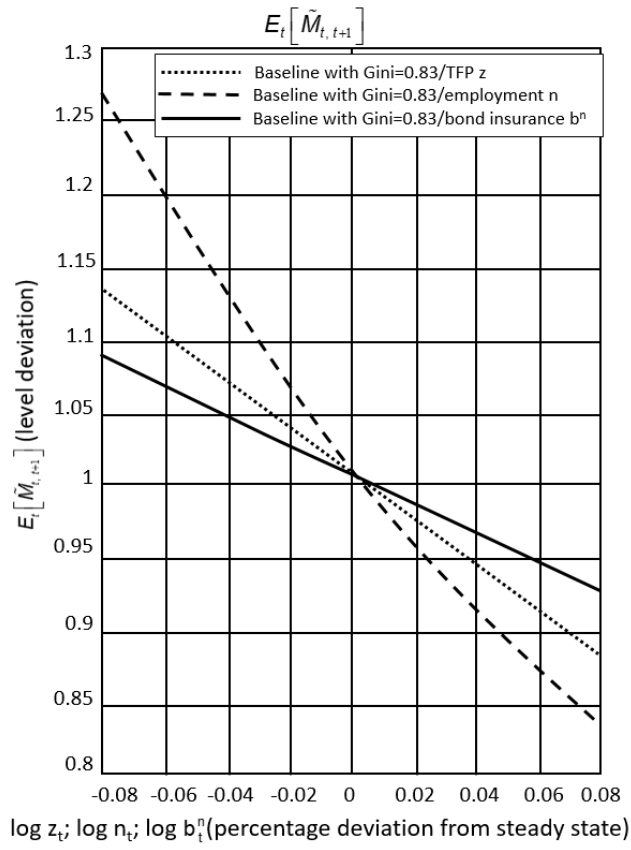


Figure 4, Part A;  $\bar{G}^W = .83$

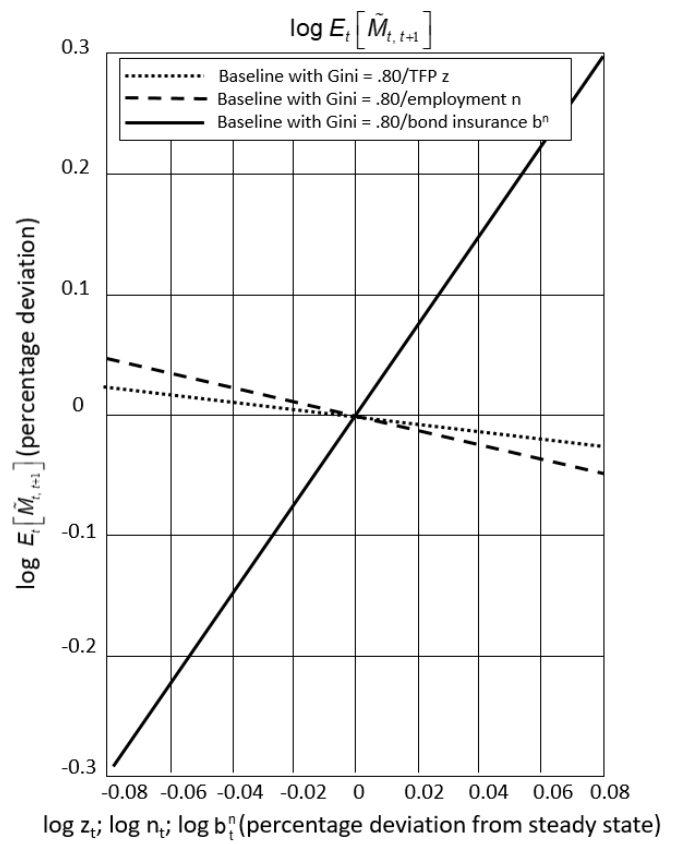
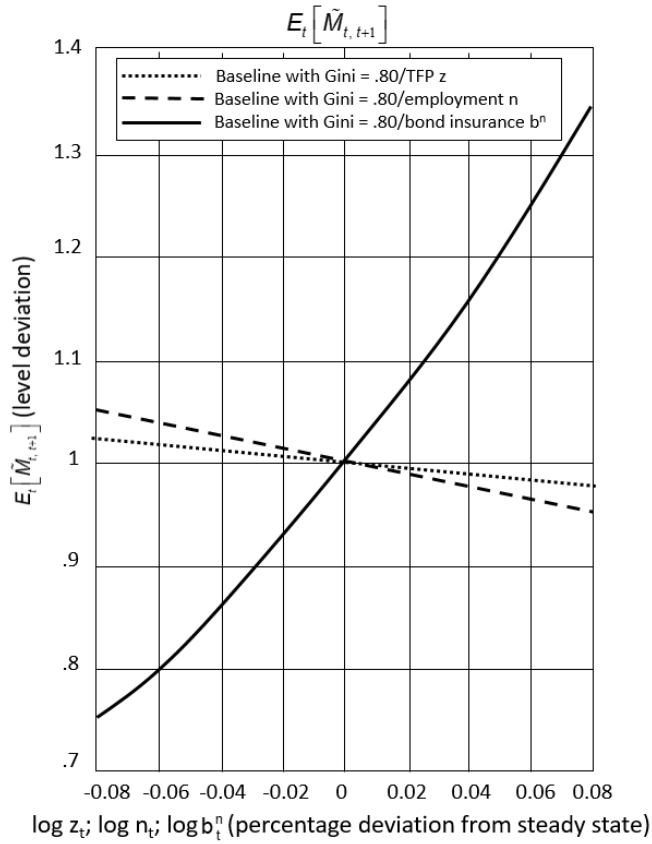


Figure 4, Part B;  $\bar{G}^W = .80$

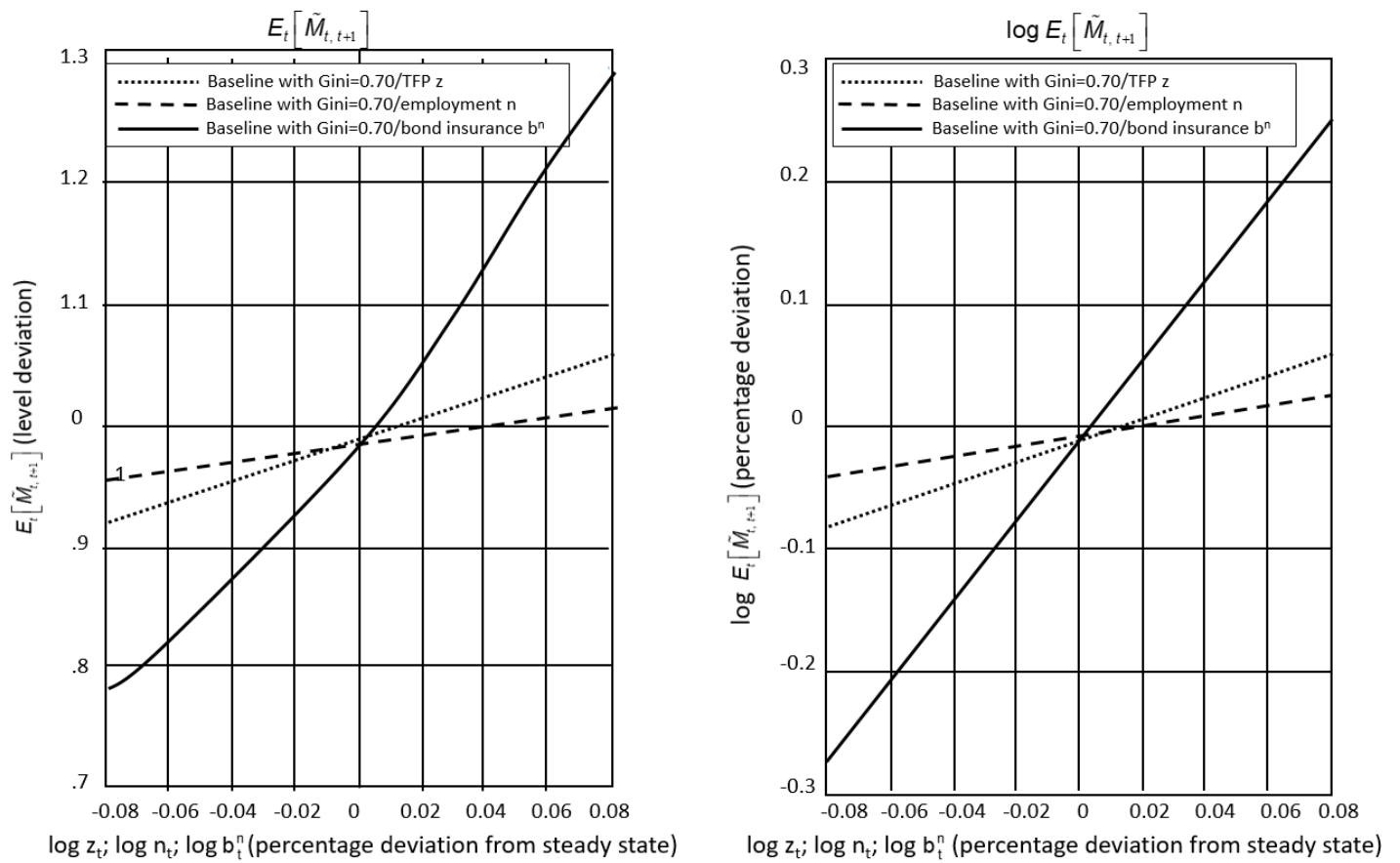


Figure 4, Part C;  $\bar{G}^W = .70$

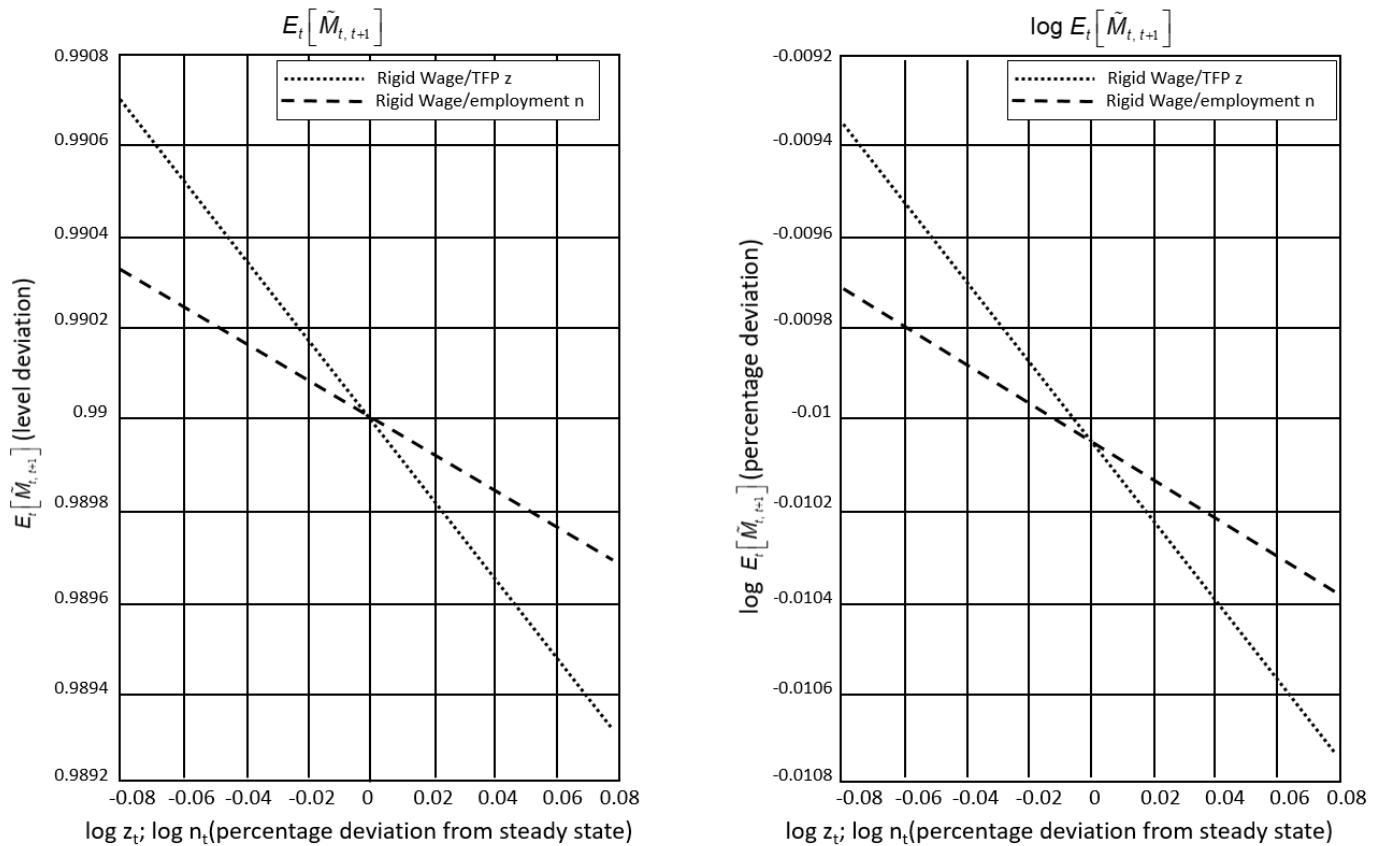
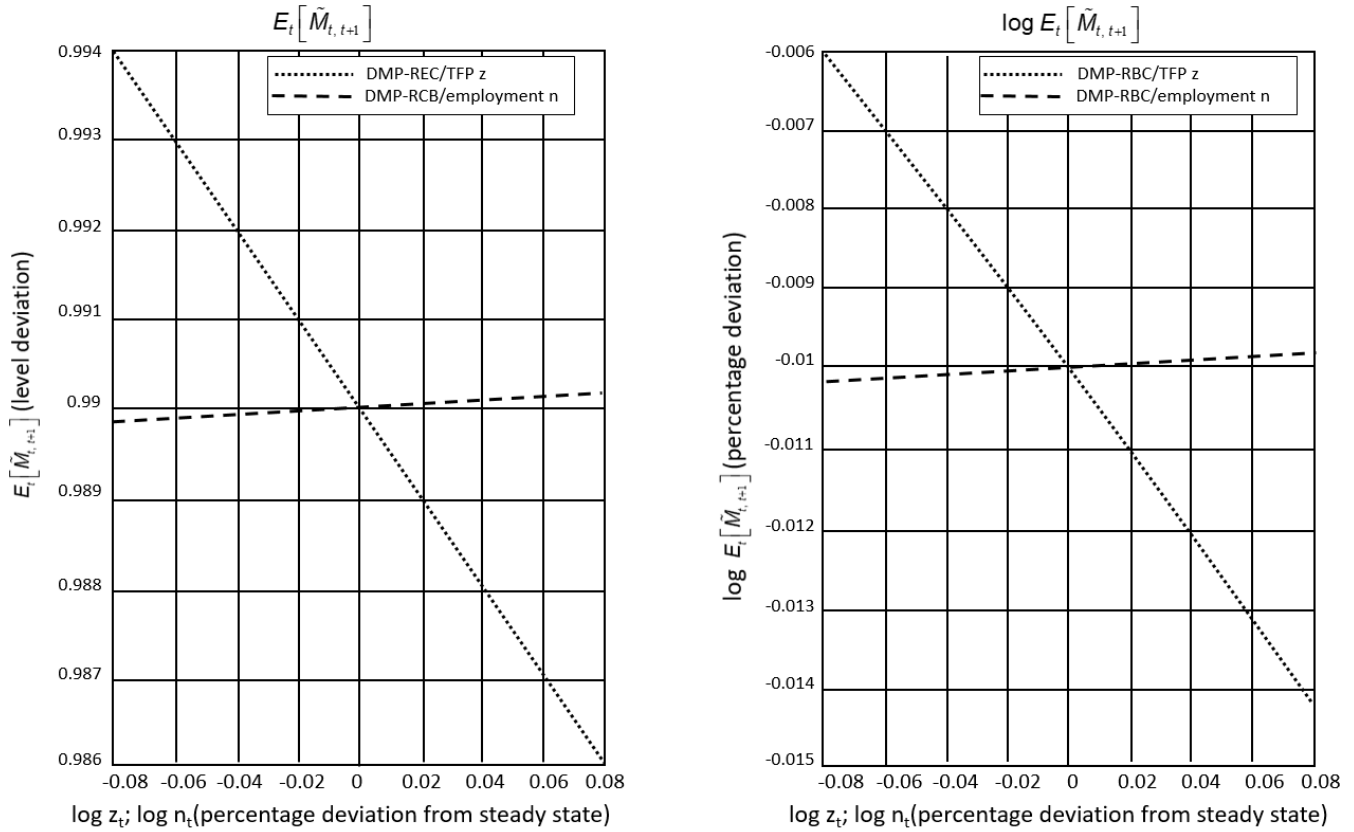


Figure 4, Part D; Shimer's (2010) Rigid Wage Model





**Figure 4, Part E; RBC-NWB (Andolfatto (1996))**

a. Figure 4A,  $\bar{G}^W = .83$ , indicates that when productivity is low ( $z_t < \bar{z}$ ), bond prices are high (above average,  $p_t^b > \bar{p}^b$ ), suggesting a high cost to bond purchases (precautionary insurance) in persistent recession states. Furthermore, as the recession deepens ( $z_t$  further declines), bond prices rise substantially (the slope of the bond price reaction curve is comparatively steep). The reverse is true when  $z_t > \bar{z}$  (above average productivity): bond prices are low and “precautionary savings” are cheap – when they are not so acutely needed. More significantly, if  $b_t^n < \bar{b}^n$  (non-stockholders hold fewer bonds on average than they would like to hold), prices are high suggesting a disadvantage to reaching the desired goal, and vice versa. Note that the average bond price exceeds one, confirming the negative rate scenario for this case. Cyclical changes in non-stockholder employment are seen to have the greatest overall price impact. Taken together, these observations suggest limited effectiveness of bond trading for the purposes of consumption stabilization, as the earlier commentary on the extreme wealth

inequality  $\bar{G}^W = .83$  case suggested.

As the slopes of all the reaction functions are negative vis-à-vis the respective  $z_t$ ,  $n_t$  and  $b_t^n$  shocks, the mechanisms underlying the  $\bar{G}^W = .83$  case departs from Hall's (2017) discount channel. Indeed, the departure from Hall's (2017) perspective is confirmed in Table 9, Figure B: despite their high volatility, vacancy postings are seen to be typically unrelated to high discount rates in recessions as confirmed by  $\text{corr}\left(\tilde{v}_t, E_t\left(\tilde{M}_{t,t+1}\right)^{-1}\right) = .15$ .<sup>64</sup> Accordingly, the observed high vacancy volatility despite the absence of a countercyclical discount rate is prima-facie evidence of the wage setting mechanism endogenously creating a “semi-fixed wage asset” to replace bond trading as the principal device for promoting worker income insurance: in this same environment, the wage paid to workers is nearly acyclical ( $\text{corr}(\tilde{y}_t, \tilde{w}_t^n) = .11$ ) and of low volatility ( $\sigma_{\tilde{w}^n} / \sigma_{\tilde{y}} = .24$ ). The fact that its relative volatility exceeds that of the Baseline case ( $\sigma_{\tilde{w}^n} / \sigma_y = .14$  in the Baseline while  $\sigma_{\tilde{w}^n} / \sigma_{\tilde{y}} = .24$  in the  $\bar{G}^W = .83$  case) is solely due to the enormous variation in (countercyclical) worker bargaining power present in these circumstances.

b. Figure 4B,  $\bar{G}^W = .80$ , indicates a diminished reaction of bond prices to productivity and employment shocks relative to the  $\bar{G}^W = .83$  case, although the cyclical pattern is unchanged. Bond price reactions to changes in non-stockholder bond holdings change dramatically, however, in the direction more favorable to precautionary savings: when  $b_t^n < \bar{b}^n$ , bond prices are low facilitating bond accumulation in those circumstances, and vice versa. Taken together, these effects suggest more effective bond trading with diminished price effects of outside events ( $z_t$ ,  $n_t$  changes), and enhanced ability of non-stockholders to alter their bond holdings to more desirable levels whatever they may be. This is our Baseline intermediate case: both bond trading and wage stabilization work together to promote non-stockholder consumption stability. The average bond price is slightly less than one so rates are low but positive.

c. Figure 4C,  $\bar{G}^W = .70$  suggests a fully functioning bond market, congenial to non-

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<sup>64</sup> As the bond market is “malfunctioning” bargained wages are becoming “semi-fixed assets” in place of bonds. This replacement in turn reinforces the “operating leverage” effects, thereby preserving the countercyclical property of the across-the-board discount rate ( $\text{corr}\left(\tilde{y}_t, E_t\left[\tilde{M}_{t,t+1}\right]^{-1}\right) = .35$ ).

stockholders' precautionary insurance needs: the slopes of all three reaction functions are positive. When  $z_t < \bar{z}$  and, under the present formulation, is likely to persist in that state, bond prices are low ( $p_t^b < \bar{p}^b$ ) facilitating their accumulation. In high productivity states bond prices are high. Accordingly, when non-stockholders are enjoying relatively high wage income and need fewer precautionary bonds, they may reduce their holdings at favorable prices. The same pattern holds for employment shocks: in "bad times" ( $n_t < \bar{n}$ ), workers can increase their precautionary savings relatively inexpensively. The reaction (sign and magnitude of the slope) of bond prices to non-stockholder bond holdings departing from the steady state value is largely the same as in the  $\bar{G}^W = .80$  case, befitting the same interpretation. Taken together the reaction of bond prices in this case to provocations in  $z_t$ ,  $n_t$  or  $b_t^n$  is exactly the opposite of what is observed under high wealth inequality (the corresponding slopes are each of different signs). The steady state bond's price is less than one, befitting the more-typically observed positive interest rate pattern. Taken together, these observations suggest even more active bond trading than was observed in the  $\bar{G}^W = .80$  case, confirming our earlier remarks.

The  $\bar{G}^W = .70$  case also advocates for Hall's (2017) discount perspective, squaring with low bond prices and high discount rates in cyclical downturns and vice versa. Nevertheless, it fails to replicate the high volatility of labor market variables, most especially vacancies, characteristic of the Baseline case. This observation can be attributed to the fact that under lower wealth inequality, distribution risk ( $\sigma_{\tilde{\phi}} / \sigma_{\tilde{y}}$ ) is declining, so that the  $\eta$ -egalitarian wage bargaining better approximates the Nash bargained outcome. As a result, labor market volatilities when  $\bar{G}^W = .70$  resembles those reported by Andolfatto (1996) whose model assumes Nash wage bargaining alone.

Figures 4D and 4E discussed below present the analogous bond price reactions to changes in  $z_t$  and  $n_t$  for the Shimer (2010) and Andolfatto (1996) models. Both are representative agent models with no-trade equilibria in the (complete) securities markets. They are offered as evidence that the KRN  $\eta$ -egalitarian wage setting mechanism cum financial market incompleteness differs fundamentally from other "rigid wage" assumptions.

d. Figure 4D presents the bond-price consequences of employment and productivity shocks for Shimer's (2010) "rigid wage" (exogenously specified) complete financial markets

model.<sup>65</sup> Notice first that the corresponding slopes are of identical sign to those in the  $\bar{G}'' = .83$  case (Figure 4A), indicating effects in the same general direction, where comparable.

Nevertheless, an examination of the vertical scale reveals slope magnitudes that are very modest, suggesting much weaker underlying volatility mechanisms. To express the same thought differently, market-incompleteness-inspired wage behavior creates more powerful endogenous links from the financial markets to the labor market than simple wage rigidity alone.<sup>66</sup>

e. Figure 4E presents the basic reaction functions for Andolfatto's (1996) complete markets model: an otherwise standard Real Business Cycle formulation but with simple Nash wage bargaining. From the spacing of the scale lines on the vertical axis, it is apparent that employment shocks have almost no effect on bond prices, and productivity shocks only very modestly so.<sup>67</sup> The pattern of slopes differs from all the cases considered thus far, further confirming the distinct perspective of market-incompleteness-inspired wage rigidity.

In summary, the observations made in this section confirm those made in the earlier subsections of Section 6.

## 7. Concluding Remarks

In response to the unsatisfying empirical performance of the conventional model of unemployment dynamics due to Mortensen and Pissarides, a recent body of studies (e.g., Merz (1995), Andolfatto (1996), Gertler and Trigari (2009) and Christiano et al. (2016)) have emphasized the importance of the degree of “wage inertia” in accounting for observed volatility in variables characterizing labor market activity over the business cycle. In this paper, we extend the Mortensen and Pissarides model to one with  $\eta$ -egalitarian wage bargaining to an environment where the asset market is incomplete and where perfect risk-sharing between capital owners and workers cannot be guaranteed because of high wealth inequality. In addition, we adopt Hall's

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<sup>65</sup> In an otherwise conventional RBC model, Shimer (2010) posits that the wage evolves exogenously according to:  $\log w_t = (1 - r) \log w_t^{NWB} + r \log w_{t-1}$  with  $r = .95$ . This simple “wage-rigidity” model achieves high volatility in key labor market variables; e.g.,  $\sigma_{\hat{w}} / \sigma_{\hat{y}} = 11.207$ .

<sup>66</sup> The intention behind this statement is as follows: the very modest reaction of bond prices to employment and productivity shocks in the Shimer (2005) model suggests very little discount rate variation. As a consequence his model displays insufficient vacancy and unemployment variation (regarding employees as assets).

<sup>67</sup> As a consequence we may conclude that Nash wage bargaining when imposed on an otherwise standard RBC does not improve its ability to explain the financial stylized facts.

(2015) perspective that a firm's decision to invest in physical capital and its decision to undertake new vacancy postings are "two sides of the same coin," in the sense of both decisions being dependent on the properties of the representative stockholders' SDF. In the setting of this paper, the effect of market incompleteness is to introduce a new form of risk, distribution risk, which is countercyclical and generates countercyclical bargaining power on the part of workers. As a result, the  $\eta$ -egalitarian bargained wage and the aggregate wage bill become very sluggish, and vacancy postings highly volatile. Through their interactions in the labor market firms and workers create an endogenous low risk "wage asset" which helps to stabilize worker consumption. These features increase the operating leverage of the firm, and the cash flow risk borne by its stockholders. A reasonable calibration of the resulting model not only replicates the basic financial statistics, but also accounts well for aggregate fluctuations in unemployment and vacancies and their negative correlation at business cycle frequencies, and also for the observed wedge between variations at the intensive margin (hours per worker) and at the extensive margin (total hours) over the business cycle.

Critical to these results is a high level of wealth inequality. If wealth inequality diminishes, bond trading takes over as the principal risk-sharing device, and wages become less stable. Within the present framework bond trading and wage-stabilization are offsetting mechanisms: when the effectiveness of one is impaired the other steps in to take its place.

Many years of research on the properties of DSGE models suggest that it is the allocation of risks across the various economic participants that determines the ability of models to explain jointly the financial stylized facts and the basic properties of macro-aggregates. With respect to the latter, the replication of labor market-related statistics has historically proven to be generally the most challenging. Clearly, the assignment of risks cannot be separate from the financial market structure confronting a model's economic agents. It is in this spirit that we have elected to impose upon a DSGE model with well-understood labor market features (search and matching cum Nash wage bargaining) an empirically realistic incomplete financial market structure. In doing so we claim that a careful understanding of the observed behavior in the labor market cannot be separated from a careful understanding of the financial structure context in which individuals and firms make their labor market decisions.

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