A Prepayment-Risk-Neutral Pricing Model for Korean Mortgage-Backed Securities

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Abstract

Korea Housing Finance Corporation is a state-run enterprise that provides long-term fixed rate mortgages and issues MBS of which underlying assets are the mortgages it provide. Most of the MBS issued by KHFC is in a type of CMO(Collateral Mortgage Obligation) with multiple tranches and KHFC began issuing another type of MBS, PT MBS, in November 2016. In this study, we try to analyze and price MBS with a simpler structure, of only one tranche, rather than of multiple tranches due to the complexity embedded in the pricing a multiple-tranche MBS. Using the data of KHFC that can reflect the characteristics of the Korean market, we want to examine the appropriate price and the proper spread in the coupon rate between CMO type MBS and PT type MBS in Korea.

1. Introduction

Korea Housing Finance Corporation(KHFC, henceforth) is a state-run enterprise, founded in 2004, that provides long-term fixed rate mortgages and issues MBS of which underlying assets are the mortgages it provide. This agency has regularly issued MBS almost twice a month, 20∼30 times a year since the global financial crisis. Most of the MBS issued by KHFC is in a type of CMO(Collateral Mortgage Obligation) as in the following table:

Table 1 presents the structure of a MBS issued by KHFC in June 2018. It consists of 8 tranches with different maturities to attract investors in Korea of different preference on maturities. Nevertheless, the tranches share a single underlying mortgage pool. There are also call options on the I-4 tranche and below, and these options are also supposed to be exercised sequentially starting from I-4 tranche. The fund available for the call option exercise are financed from the payment of the underlying mortgage pool. Therefore, because of this single underlying mortgage pool and its structure of multiple tranches, it is very difficult for investors to predict future cash flows, which makes it difficult to evaluate the value of KHFC MBS accordingly.

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Tranche	Amount	Maturity	Coupon	Interest Payout	Principal Payout	Call Option Exercise
I-1	87.9	1Y	2.005%	Quarterly	At maturity	
I-2	190	2Y	2.250%	Quarterly	At maturity	
I-3	160	3Y	2.415%	Quarterly	At maturity	
I-4	250	5Y	2.82%	Quarterly	At maturity	After 3M
I-5	140	7Y	2.91%	Quarterly	At maturity	After 2Y
I-6	100	10Y	2.956%	Quarterly	At maturity	After 3Y
I-7	50	15Y	2.936%	Quarterly	At maturity	After 4Y
I-8	10	20Y	2.916%	Quarterly	At maturity	After 5Y

Table 1: KHFC MBS 2018-13 Structure - CMO type

Besides, KHFC began issuing another type of MBS, PT MBS, in November 2016. The following table deliver the structure of KHFC MBS 2018-12, a PT MBS issued by KHFC.

Tranche	Amount	Maturity	Coupon	Interest Payout	Principal Payout	Pass-Through
I-1	153.4	2Y	2.318%	Quarterly	At maturity	
I-2	210	5Y	2.837%	Quarterly	At maturity	After 1-1 repayment
I-3	120	10Y	2.996%	Quarterly	At maturity	After 1-2 repayment
I-4	30	20Y	2.996%	Quarterly	At maturity	After 1-3 repayment

Table 2: KHFC MBS 2018-12 Structure - PT type

One can see that PT MBS has less number of tranches and different call option exercise scheme. KHFC transfers the cash flow received from the borrowers to the investors as it is. However, the peculiarity of KHFC PT MBS is that the cash flow to the I-1 tranche investors are the similar as the conventional PT MBS, but in case of I-2 tranche investors and above, they have to receive only interest and wait to receive the principal until earlier tranche(s) is fully repaid. Therefore, since KHFC PT MBS also has multiple tranches, it is not easy to evaluate the price.

In this study, we try to analyze MBS with a simpler structure, of only one tranche, rather than of multiple tranches due to the complexity embedded in the pricing a multiple-tranche MBS. Using the data of KHFC that can reflect the characteristics of the Korean marker, we want to examine the appropriate price and the proper spread in the coupon rate between CMO type MBS and PT type MBS in Korea, although they can be said to be virtual commodities.

2. The Model

2.1. The Cash Flows to the Investors

The following figure shows the structure of cash flows that the CMO MBS investors will receive.

When the borrowers pay the principal and interest on their loans, the intermediary KHFC uses the cash inflows to pay the MBS investors the scheduled principal and

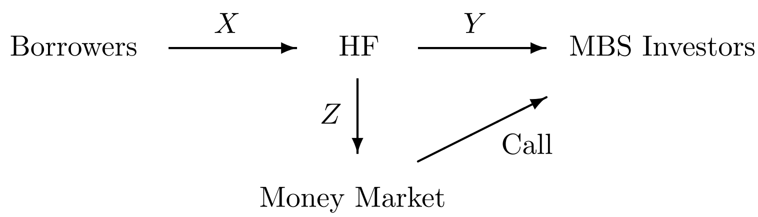

Figure 1: CMO MBS Cash Flows

interest and saves the remaining cash in the short-term money market accounts. X in Figure 1 denotes the cash flows from borrowers, Y, the scheduled cash flows to the investors, and Z, the saving amount of KHFC in the money market accounts. The equation $X = Y + Z$ naturally occurs here. HF manages the cash flows saved in the money market accounts and decides when and how many call options to exercise, that it can prepay a part of or the whole outstanding principal of MBS whenever KHFC has available cash in the short-term money market account. Since the cash available for the call option exercise must be financed in the corresponding trust account, the amount of (potential) call option exercise is determined by Z. Therefore, if Z occurs, i.e., additional savings occurs, then it is as if Z amount of the outstanding principal on the MBS turns into an American callable bond with face value Z. As a result, the value of CMO MBS is equal to the sum of the value of the scheduled interest and the value of the American callable bonds generated during the life of a bond, which can be written as follows:

$$V_c(0) = \mathbb{E}_Q\left[\sum_{t=1}^{T} P(0,t)\left[C(t,T)Z(t) + Y(t)\right]\right], \tag{1}$$

where $V_c(0)$ is the price of CMO MBS at time 0 and $P(0,t)$ is the price at time t of zero-coupon bond maturing at time t, and $C(t,T)$ is the price at time t of American callable bond maturing at time T. T is the expiration date of the MBS and $\mathbb{E}_Q[\cdot]$ is the expectation under the risk-neutral measure.

On the other hand, if investors purchase PT MBS instead, their cash flow income will appear as shown in the following figure:

Figure 2: PT MBS Cash Flows

KHFC hands over (passes-through) all the cash flows from the borrowers to the investors whenever it receives them, resulting in $X = Y$. The price of PT MBS, hence, can be written as follows:

$$V_p(0) = \mathbb{E}_Q\left[\sum_{t=1}^{T} P(0,t)X(t)\right], \tag{2}$$

where $V_p(0)$ is the price of PT MBS at time 0.

We now explain our model of the underlying mortgage pool and the corresponding MBS. Suppose that all the mortgages in the pool have the similar interest rate and maturity and they are all paid in monthly equal installment repayment of principal and interest basis. Then the cash flow process from borrowers can be modeled as follows:

$$X(t) = (c + p(t))\, Bal_{mortgage}(t-1) \tag{3}$$

where $X(t)$ is the amount of cash flow from borrowers at time t, c is the monthly repayment ratio of the pool, $p(t)$ is the prepayment rate at time t, and $Bal_{mortgage}(t)$ is the outstanding balance of the mortgage pool at the end of time t. If we assume that the payment method of all the mortgages are the monthly equal installment repayment of principal and interest, c is obtained as

$$c = \frac{WACdt}{1 - \left(\frac{1}{1+WACdt}\right)^{T/dt}} \tag{4}$$

where WAC is the interest rate of the mortgages, dt is the time interval between each payment, and T is the maturity of the mortgages.

In case of CMO MBS, the scheduled cash flow process to the investors can be described as follows:

$$Y(t) = Bal_{MBS}(t-1)ydt \tag{5}$$

where $Y(t)$ is the amount of cash flow to the investors at time t, $Bal_{MBS}(t)$ is the outstanding balance of the MBS at the end of time t, and y is the coupon rate of the MBS. Note that the principal of MBS is paid at maturity as in Table X. Now we can write $Z(t)$, the saving amount process of HF, as follows:

$$Z(t) = X(t) - Y(t) = (c + p(t))\, Bal_{mortgage}(t-1) - Bal_{MBS}(t-1)ydt \tag{6}$$

where $Z(t)$ is the saving amount of KHFC at time t.

In order to evaluate the price of the American callable bond, we apply the LSMC(least-squares Monte Carlo) approach suggested by Longstaff and Schwartz (2001) with a stochastic short-rate model to be discussed later. The continuation value for deriving the American callable bond price is assumed to follow a regression form such that

$$CV(t) = \alpha_0 + \alpha_1 r(t) + \alpha r(t)^2 \tag{7}$$

where $CV(t)$ is the continuation value of the American callable bond at time t, $r(t)$ is the interest rate level at time t, and αs are the regression parameters.

2.2. The Interest Rate and Prepayment Rate Model

The short-rate model for MBS valuation of this study is assumed to be a 1-factor Hull and White (1990) type model whose process is given by:

$$dr(t) = a\left(\theta_r(t) - r(t)\right) dt + \sigma_r dW_r(t) \tag{8}$$

where $r(t)$ is the short rate at time t, a is the reversion rate of the short rate process, σ_r is the interest rate volatility, $W_r(t)$ is the standard Brownian motion at time t under the risk-neutral measure. The time-dependent mean-reversion level $\theta_r(t)$ is computed to fit the initial interest rate term structure.

The prepayment model considered in this stury is a reduced-form prepayment model similar to the ones proposed by Kau et al. (2004) and Kolbe and Zagst (2008). In particular, we model the prepayment process $p(t)$ to have the form of a proportional hazard rate as follows

$$p(t) = e^{f(\mathbf{x}(t))+p_0(t)}, \tag{9}$$

$$f(\mathbf{x}(t)) = \beta_1 \arctan\left(\beta_2(spread(t)+\beta_3)\right) + \beta_4 age(t) + \beta_5 age(t)^2 + \beta_6 age(t)^3, \tag{10}$$

$$dp_0(t) = \kappa\left(\tilde{\theta}_p - p_0(t)\right)dt + \sigma_p d\widetilde{W}_p(t), \tag{11}$$

where $p(t)$ is the prepayment rate at time t, $f(\mathbf{x}(t))$ is a function of time-dependent covariate vector $\mathbf{x}(t)$ in a regression form as in (10), $spread(t)$ is the interest rate spread between the mortgage rate and the market interest rate at time t, $age(t)$ is the monthly age of the mortgage pool after issuance. We consider an arctangent function and a cubic term of $age(t)$ as in Kolbe and Zagst (2008). Also $p_0(t)$ is the baseline hazard process, basically captures the turnover components of prepayment, following a Vasicek (1977) process as in (11). κ is the mean-reversion speed of the baseline prepayment rate process, $\tilde{\theta}$ is the mean-reversion level, σ_p is the volatility, and $\widetilde{W}_p(t)$ is the standard Brownian motion at time t under the real-world measure. If we can find a constant λ_p for the risk-neutral adjustment of prepayment as in Kolbe and Zagst (2008), such that

$$d\widetilde{W}_p(t) = dW_p(t) - \lambda_p dt, \tag{12}$$

we can derive the baseline prepayment rate process under the risk-neutral measure as follows

$$dp_0(t) = \kappa\left(\theta_p - p_0(t)\right)dt + \sigma_p dW_p(t), \tag{13}$$

where $dW_p(t)$ is the standard Brownian motion at time t under the risk-neutral measure, and θ_p is defined as follows

$$\theta_p \equiv \tilde{\theta}_p - \frac{\lambda_p \sigma_p}{\kappa}. \tag{14}$$

The baseline prepayment rate process and the short rate process are independent of each other, and however, it can be seen that $f(\mathbf{x}(t))$ reflects the correlation between interest rate and prepayment rate through $spread(t)$. Notice that the two prepayment components (10) and (11) are incorporated in $p(t)$ in an exponential way to ensure that $p(t)$ is non-negative.

3. Model Calibration and Empirical Results

3.1. Calibration

We analyze primarily data of CMO MBS issued by KHFC since it has only just begun to issue PT MBS. HF started to issue PT MBS since 2016 November, and until the end of 2018 June, issued only 9 PT MBS. Given the trend that investors often require an additional spread for the initial market entry premium for these new products, the price data of PT MBS is likely to be distorted. Therefore, we use the data of CMO MBS for parameter calibration and applied the same parameters calibrated on the prepayment rate comparing the prices of CMO MBS and PT MBS in the later analysis.

The parametrization of the model is as in the following Table 3:

	Parameter	Estimate
Short-rate process	a	0.008430
	σ_r	0.005956
Regression parameters	β_1	3.961625
	β_2	93.266577
	β_3	-0.062259
	β_4	0.120272
	β_5	-0.002328
	β_6	0.000012
Baseline prepayment process	κ	2.197675
	$\tilde{\theta}_p$	0.014494
	σ_p	1.251989
	λ_p	-18.198175

Table 3: Baseline Parametrization

We used the time series of Korean Treasury yield curve for 2010∼2017 to calibrate the short rate process since KHFC uses 5-year Korean Treasury bond yield as the index rate for its MBS coupon rate and mortgage rate. By fitting the caplet price formula on the short rate process in (8) on the IRS caplet prices, we obtained $a = 0.008430$ and $\sigma_r = 0.005956$. When we simulate the prepayment rate process, we calculate the 5-year short rate from the simulated short rate process with the parameters in Table 3 and then compute $spread(t)$ in (10).

For the prepayment rate process, we first calibrated all the parameters in (10) and (11) with the real prepayment data in HF, and then calibrated the risk-neutral adjustment of prepayment, λ_p in (13), using the data of MBS price time series. We apply the prepayment data from 227 mortgage pools issued by KHFC in 2010∼2017 to calibrate $f(\mathbf{x}(t))$ in (10) by minimizing the sum of total squared errors. The coefficients for the refinancing component, β_2, and aging component, β_4 and β_6, are estimated to be positive, which is in accordance with general intuition. After deriving the time series estimates of $f(\mathbf{x}(t))$, the difference between the prepayment data and $f(\mathbf{x}(t))$ estimates, that is $\ln p(t) - f(\mathbf{x}(t))$ serves as the data set for calibrating the baseline prepayment process $p_0(t)$. Applying Jeon (2013)'s method, we obtain the parameter values in Table 3.

To estimate λ_p, we need the market price of KHFC MBS. Unfortunately, the trading volume of KHFC MBS in the Korean financial markets is too small to reflect the proper market valuation on the MBS. Instead, we apply the issuance data, which is fairly regularly priced by the initial buyers, so that λ_p presents the risk-neutral adjustment of issuance price. Therefore, one can say that our analysis mostly explains about the MBS issuance market in Korea. Furthermore, since the KHFC MBS consists of 8 tranches traded in the market, there exist 8 different MBS prices on one mortgage pool. We take the I-4 tranche, which has 5-year maturity, for the representative tranche of prepayment risk because it is the first tranche that can be exercised the call option. Then we assume the volume for the corresponding mortgage pool to be the size of the similar weight that the I-4 tranche has among the 8 tranches. We look for the values of each λ_p that satisfies the prices of each I-4 tranche issued in 2016∼2017, and compute the average to obtain $\lambda_p = -18.198175$.

3.2. Empirical Demonstrations and Analysis

The following figure presents the valuation result of MBS prices applied the model of this study issued in the first half of 2018.

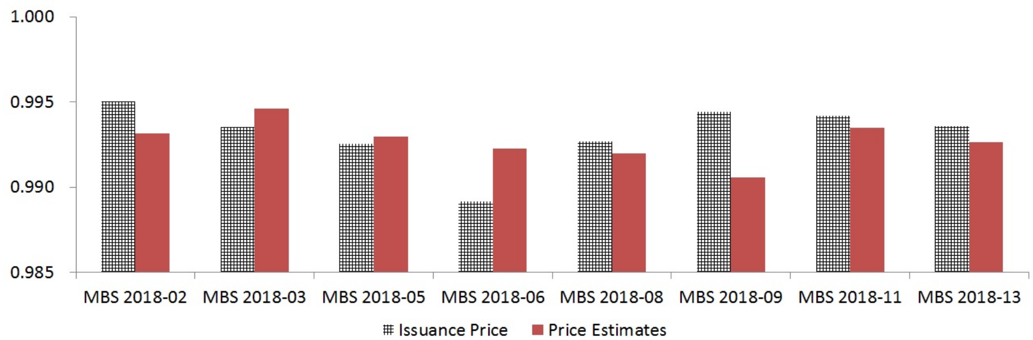

Figure 3: Price Estimation Results

As shown in the figure, our model generates a fairly close price evaluation result to the real price data. Also it can be seen that the estimated price is higher than the actual price in 1Q and vice versa for 2Q. This seems to be because the investors in the Korean market expected the interest rates to rise in the second quarter. Following Figure 4 shows several series of Korean treasury bond yields.

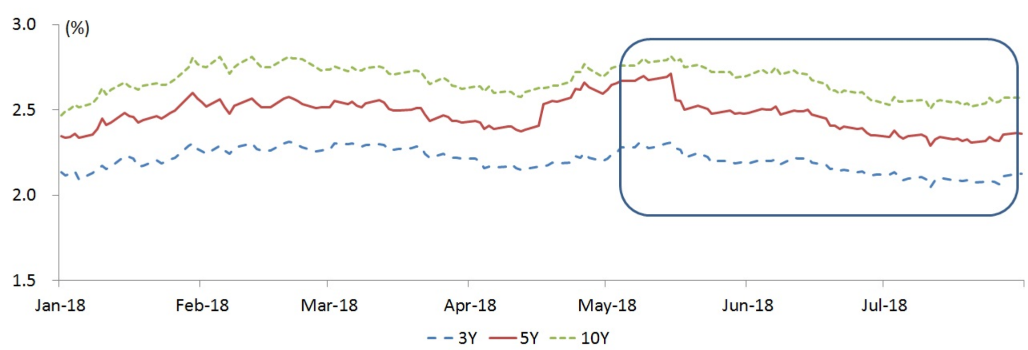

Figure 4: Korean Treasury Bond Yield

Now we move to the comparative analysis on the prices of CMO MBS and PT MBS, taking five PT MBS issued in the first half of 2018 as our analysis objectives. We also assume that the CMO MBS to be compared with PT MBS have the same maturities and coupon rates.

The following Figure 5 represents the estimated prices and fair coupon spread between the two types of MBS.

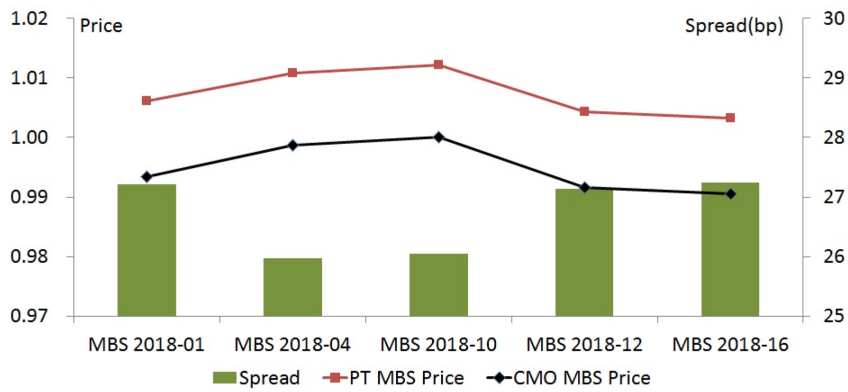

Figure 5: The Prices of CMO and PT MBS

The prices of PT MBS definitely are higher than CMO MBS's due to the value of call option embedded in CMO MBS. The difference between the prices of the two MBS is the call option value, looking around 1%. In addition, the amounts of fair spreads are also purely from the call option value, resulting from converting the option prices into spreads. The spread is the proper difference between the coupon rates of the two types of MBS when they have the same underlying mortgage poll and maturity, which is computed by dividing the difference of prices between two MBS by the duration of each MBS. In Figure 5, those spreads are likely to be around 25∼28 basis points.

We now do comparative statics on various parameters in Table 3 by deriving the prices of three MBS, KHFC MBS 2018-10, KHFC MBS 2018-12, KHFC MBS 2018-16 which are the most recently issued. Following Figure 6 demonstrates the fair spreads between the coupon rates of CMO MBS and PT MBS with respect to the interest rate volatility.

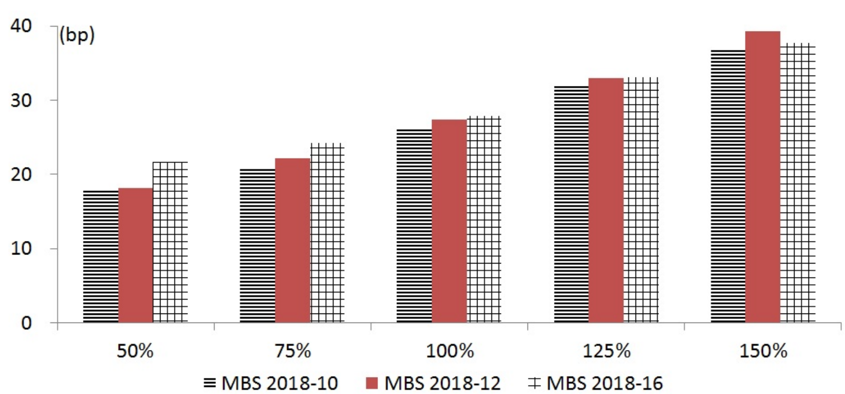

Figure 6: Fair Spreads with respect to Interest Rate Volatility

The % values on x-axis are the relative values to the volatility $\sigma_r = 0.005956$ in Table 3, i.e., 50% on x-axis in Figure 6 means $\sigma_r = 0.5 * 0.005956$. One can easily confirm that a higher interest rate volatility leads to a higher spread in Figure 6, since the callable bond price decreases with a higher interest rate volatility.

We also present the changes in the fair spreads according to the change in the refinancing sensitivity, β_2, in the following Figure 7.

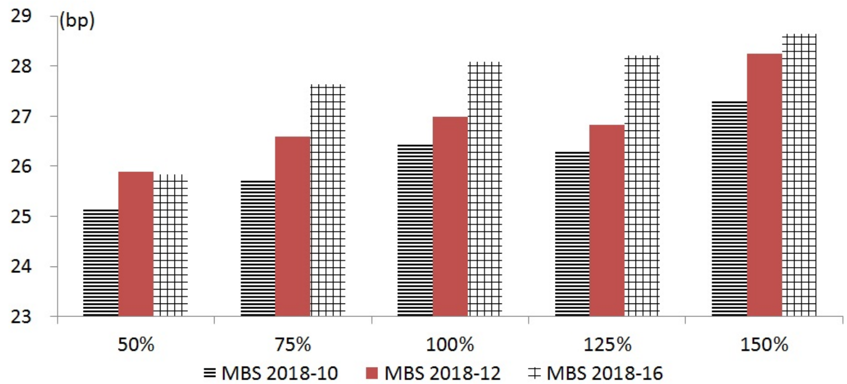

Figure 7: Fair Spreads with respect to Refinancing Sensitivity of the Prepayment

Also the % values on x-axis denote the relative values to $\beta_2 = 93.266577$ in Table 3. Since if β_2 is large, the rate of change in the prepayment rate is large for a change in interest rate of the same size, a borrower with a higher β_2 is able to be a refinance-sensitive borrower. As you can see from the figure, the more sensitive borrowers in the underlying mortgage pool, the more fair spread increases. This is because the cash flows from the mortgage pool are more volatile with more refinance-sensitive borrowers, resulting in the larger difference in the prices of CMO MBS and PT MBS. The risk of volatile cash flows are further amplified in the case of CMO MBS since the intermediary KHFC has the option of whether it passes through the cashes from borrowers.

The impact of the aging component of the prepayment rate on the fair spread is illustrated in the following Figure 8.

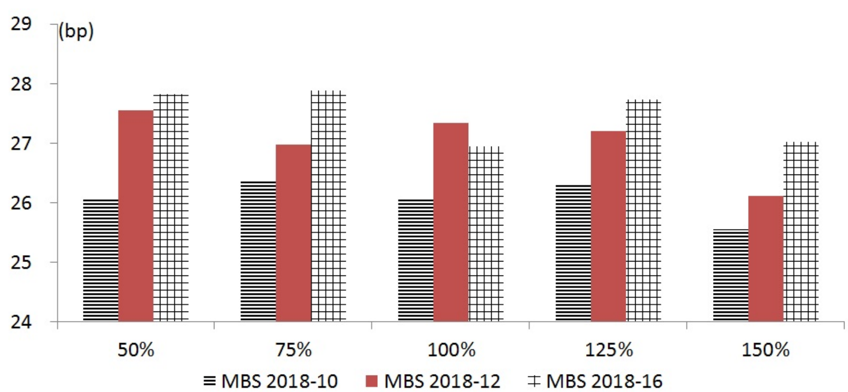

Figure 8: Fair Spreads with respect to Aging Sensitivity of the Prepayment

The values on x-axis are the relative values of aging sensitivity, $\beta_4 = 0.120272$ in Table 3. There is no obvious correlation in Figure 8. This is mostly since the aging motive in the prepayment rate process is a predictable factor that increases monotone without falling.

Following Figure 9 demonstrates the relation between the fair spreads and the long-term average of the baseline prepayment rate process, $\tilde{\theta}_p$ in Table 3.

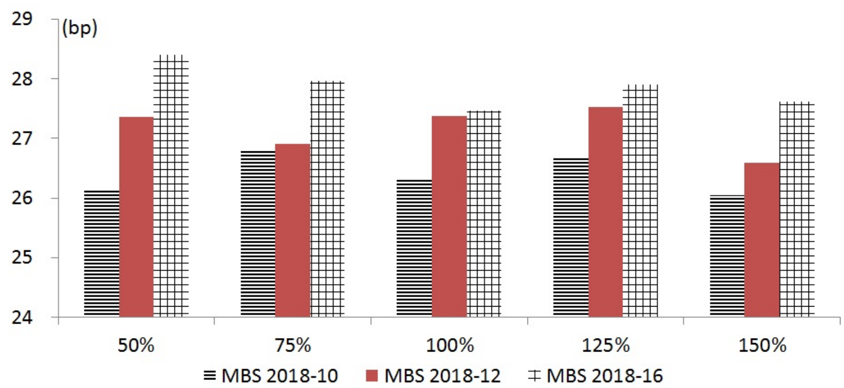

Figure 9: Fair Spreads with respect to Long-Term Average Prepayment Rate

The values on x-axis denote the relative values of the long-term average of the baseline prepayment rate process, $\tilde{\theta}_p = 0.0144944$ in Table 3. There seem to be no clear correlation between the fair spreads and $\tilde{\theta}_p$, probably since the value itself is a long-term average after eliminating the effects of refinancing and aging of prepayment rate, which is less impacting the cash flow volatility than other factors.

Now following Figure 10 represents the changes in fair spreads with respect to the level of the volatility of the baseline prepayment rate process, σ_p.

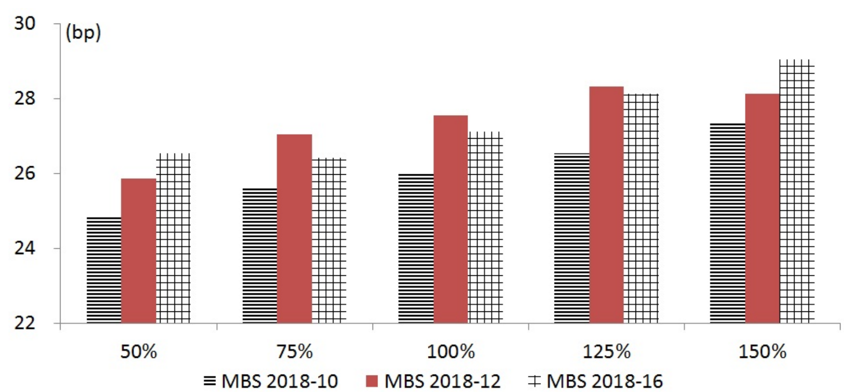

Figure 10: Fair Spreads with respect to the Volatility of Prepayment Rate Process

The % values on x-axis are the relative % levels of the volatility of the baseline prepayment rate process in Table 3. A higher volatility in the prepayment rate exposes the investors to a greater cash flow uncertainty, and this leads to a higher price of call option embedded in CMO MBS, and thus a higher fair spread as in Figure 10.

4. Conclusion

We investigated the implications in the pricing the two types of MBS of KHFC, CMO type and PT type, and showed that the volatilities of interest rate and prepayment rate are the key factor determining the fair spread between the coupon rates of the two MBS. Since the MBS studied in this paper is of a simpler structure with only on tranche due to the complexity embedded in the pricing a multiple-tranche MBS, the pricing of multiple-tranche MBS is definitely a great further research topic. We hope that this study provides an insightful view on the Korean MBS market and thus KHFC can support efficiently the welfare of mid- and low-income families in Korea seeking to buy a house.

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