A Study on the Nonlinearity in the Arbitrage Transactions of Crytocurrency

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Abstract

The relative price of a cryptocurrency, denominated in different currencies implies a multilateral exchange rate. If the multilateral exchange rate differs from the bilateral exchange rate, then there exists an opportunity for arbitrage, and hence the multilateral rate would not diverge much from the bilateral one. However, the pressure to return to the equilibrium is likely to depend on the level of the discrepancy, because cryptocurrency market is typically shallow and there are transaction costs for arbitrage. This study analyzes the nonlinearity in the convergence of multilateral exchange rate to bilateral exchange rate and finds that the two exchange rates are cointegrated and the cointegration relation is not linear.

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1 Introduction

Cryptocurrency is a digital asset produced by a public network which uses cryptography to make sure payments are sent and received safely.[1] It features with decentralized structure: No central authority controls or regulates it and transactions are processed through a peer-to-peer protocol. Cryptocurrency can be used as a method of consumer payment but most of the buyers hold it for investment purposes.[2]

Cryptocurrency can be converted into different currencies and hence there are chances of arbitrage. The relative price of a cryptocurrency, bitcoin for instance, denominated in different currencies implies a multilateral exchange rate. For example, the euro price of a bitcoin (€/₿) divided by the U.S. dollar price of a bitcoin ($/₿) implies the euro-U.S. dollar exchange rate (€/$). If the multilateral exchange rate differs from the bilateral one, then there could be arbitrage profits. Since traders would take advantage of these opportunities, the exchange rates would not diverge much.[3] However, bitcoin market observed unusual phenomenon between January 2016 and February 2018. During those period, bitcoins were traded at relatively higher price in Korea, which was called as "Kimchi premium." During the period of time, the average Kimchi premium was 4.73% and it reached as high as 54.48% in January 2018. Theoretically, such a relative price deviations are not supposed to persist as it would be immediately arbitraged away.[4]

Most of the economic studies on cryptocurrency are related to its *absolute* price level. For instance, Woo, Gordon, and Iaralov (2013), Bergstra and de Leeuw (2013), and Huhtinen (2014) estimate the potential size of cryptocurrency market. Garcia, Tessone, Mavrodiev, and Perony (2014) consider the energy costs in mining bitcoin to infer the fundamental

[1]The term "cryptocurrency" is widely used in academic literature, but there is no consensus yet. For instance, BIS (2015) calls it "digital currency," IMF (2016) does "virtual currency," and Financial Stability Board (FSB) does "crypto-assets."

[2]According to Meiklejohn, Pomarole, Jordan, Levchenko, McCoy, Voelker, and Savage (2013), among the bitcoins minted in 2009-2010, more than 60 percent remain unspent more than a year.

[3]Suppose bilateral euro-U.S. dollar exchange rate (€/$) is one. If the price of a bitcoin is 1,100 euros per bitcoin and 1,000 dollars per bitcoin, then the multilateral exchange rate (€/$) is 1.1. To take advantage of this discrepancy, a trader can buy 1,000 dollars for 1,000 euros and buy a bitcoin for the 1,000 dollars. After that transaction, if the bitcoin is sold at 1,100 euros, then the trader can make 100 euros of arbitrage profit. This type of trade would reduce euro price of bitcoin and increase dollar price of it, hence the multilateral exchange rate will converge to the bilateral one.

[4]Choi, Lehar, and Stauffer (2018) analyze the sources of Kimchi premium and conclude that capital control in Korea is the most important factor for the anomaly.

value of a bitcoin. MacDonell (2014) and Cheah and Fry (2015) raise the bubble issue in bitcoin prices. However, few studies focus on the *relative* price and arbitrage transaction using cryptocurrency.

This study tries to expound the nonlinearity in the convergence of multilateral exchange rate to bilateral one. Because of arbitrage opportunity, the two exchange rates are likely to be cointegrated. When two time series are cointegrated, we may utilize a vector error correction model (VECM), and measure the pressure to restore the long-run equilibrium relation. Recently, Smith (2016) shows that the multilateral exchange rate is highly cointegrated with the bilateral exchange rate and estimates a linear VECM to account for the causality of the two exchange rates. He concludes that the change in bilateral exchange rate affects multilateral rate but not vice versa. Smith (2016) implicitly assumes that even if the discrepancy in the nominal exchange rates may expand in the short run, it will linearly and immediately vanish through arbitrage transactions. However, it is suspected that the pressure to return to the equilibrium is likely to depend on the level of the discrepancy, because of the following two issues: First, cryptocurrency market is typically shallow. Therefore, an investor who wants to trade a large amount of cryptocurrency would not be able to do so without influencing the market price. Second, there are transaction costs for arbitrageurs. In general, verification of cryptocurrency transaction requires fees. On top of that, an exchange between conventional currencies is not free at all. Therefore, if the size of the discrepancy in the exchange rates is smaller than a certain threshold, then the adjustment is not likely to happen because the benefits could be less than the costs. On the other hand, if the discrepancy is larger than the threshold, then the arbitrage transaction will be active. If it is the case, then a linear VECM is not the appropriate model to account for the movements of the two exchange rates. Therefore, I first examine the cointegration relationship between the exchange rates and then test the relevance of a linear VECM for those variables. When it is found that there is nonlinearity in the pressure for returning to the equilibrium, a threshold VECM (TVECM) is utilized. Using the estimated TVECM, an inference about the timing of high-pressure periods for arbitrage and detection of potential bubble can be made.

The bitcoin prices denominated in the U.S. dollars, euros, British pounds, Japanese Yen, and Australian dollars are considered for the analysis. Using those prices, multilateral exchange rates relative to the U.S. dollar are calculated. The multilateral exchange rates of Korean won and Chinese yuan are not included for the analyses because they do not have unit roots and hence are not relevant for the model. According to the empirical analysis, the following results are found: First, each of the four pairs of multilateral and bilateral exchange rates has a cointegration relation. These results imply that traders are actively seeking arbitrage profits. Second, the null hypothesis that the linear VECM is the appropriate model is rejected. Third, by estimating the TVECM, the threshold values are identified. Also, the majority of the sample periods are classified as middle regime where the incentive for arbitrage transaction is limited. There are sporadic departures from the middle regime where the deviation and the pressure for arbitrage are substantially large. If these departures are prolonged for a while, then a bubble in a country's cryptocurrency market could be suspected.

The paper has the following organization: Section 2 explains the potential transaction costs for arbitrageurs using bitcoin platform as an example. In Section 3, linear and nonlinear models for the empirical analysis are briefed. In Section 4, empirical results including unit root test, cointegration test, and the TVECM estimation results are summarized. Utilizing the estimated threshold values, states of each periods are evaluated. Section 5 concludes.

2 Transaction Costs for Arbitrageurs

This section briefly reviews the reason why there are transaction costs for arbitrageurs using Bitcoin as an example.[5] Bitcoin is chosen because it is the first and most widely traded one among the variety of cryptocurrencies.[6] In-depth explanations on Bitcoin can

[5]Following the computer science litereature, uppercase name (e.g. Bitcoin) refers to a cryptocurrency platform and lowercase name (e.g. bitcoin) denotes the unit of account.

[6]In terms of market capitalization on January 29, 2018, the shares of Bitcoin, Ethereum, and Ripple in cryptocurrency market are 33.6%, 21.1%, and 8.8%, respectively.

be found at Böhme, Christin, Edelman, and Moore (2015).

Bitcoin is characterized by global and local platforms. Global platform is utilized when a person wants to wire transfer bitcoins to another. The sender and the recipient should have digital wallets, which are free and open-source software. Acquired bitcoins stay in a digital wallet on the user's computer.[7] The signal of wire transmission is sent to the network, which means that all transactions are public, although the information on involved parties is not known. At this point, so called "miners" step in and validate the transactions. Miners check the transactions to make sure no unauthorized transactions have been inserted, generating a linked sequence of verified records, or "block chain." These confirmation process is solving complex mathematical puzzles using high-powered computers. Miners are rewarded with newly minted bitcoins by the network when they find a solution. Recently, as bitcoin market becomes larger, it takes more time for miners to solve puzzles. Therefore, bitcoin senders offer transaction fees to miners for faster verification process. Transactions with higher fees are more likely to be included in a block chain with a shorter delay. An arbitrage transaction may occur through global platform by sending bitcoins in exchange for traditional currencies. In this case, the fees to a miner are significant transaction costs for the arbitrageurs.

Investors who want to make capital gain generally use the local platform of a currency exchange.[8] Since substantial delay is inevitable in the global platform, a currency exchange provides its own platform which does not require verification process and thus enables faster transaction. A currency exchange offers customer accounts similar to accounts with a stock broker. Customers log into their accounts to trade bitcoins between account holders at the same currency exchange and the transactions are recorded in the local platform, not on the block chain.[9] An account holder may transfer her bitcoins to private wallet out of exchange account, which will trigger an entry into the block chain

[7]As a result, there are risks that users may lose their cryptocurrency if they do not have adequate anti-hacking and back-up devices.

[8]Kraken, Hitbtc, and Bitfinx are well-known currency exchanges in the U.S.

[9]Since the records in a local platform is not public, the traders face risks of losing bitcoins. If a currency exchange is hacked and lose its bitcoin, then the customers of the currency exchange have to sue the currency eschange to recover their funds. These lawsuit resulted in the bankruptcy of Mt. Gox, which was the biggest currency exchange until early 2014.

and require substantial delay. Also, withdrawing fiat money out of currency exchange may take substantial time partly because commercial banks' refusal to deal with currency exchanges. An arbitrageur may consider sending fund to overseas to buy bitcoins at a foreign currency exchange. This transaction incurs fees for converting one currency to another. On top of that, some countries impose capital controls which limit the amount of funds that can be transferred abroad, or at least complicate the process, creating another friction for the arbitrage trade. Choi, Lehar, and Stauffer (2018) note that the capital control in Korea is the major factor which generated unusual rate of return in Korean bitcoin market.

Because of the microstructure of Bitcoin platform, traders have to deal with obstacles to arbitrage transaction. Also, bitcoin price is much more volatile than that of transitional assets, price risk can be another significant transaction costs for arbitrageurs.

3 Threshold Model of Exchange Rates

Bilateral exchange rate and multilateral exchange rate based on cryptocurrency prices are considered for this study. The two exchange rates would not diverge much because of arbitrage opportunities. If the two time series are both $I(1)$ and cointegrated, then the vector error correction model (VECM) can be the appropriate option to describe the dynamics of the two time series. Suppose that two exchange rates are combined into a (2×1) vector $\boldsymbol{y}_t$, then the linear VECM with p lags can be represented as:

$$\Delta \boldsymbol{y}_t = \boldsymbol{c} + \boldsymbol{\phi} s_{t-1} + \boldsymbol{A}_1 \Delta \boldsymbol{y}_{t-1} + \cdots + \boldsymbol{A}_p \Delta \boldsymbol{y}_{t-p} + \boldsymbol{\varepsilon}_t, \tag{1}$$

where (2×1) vector $\boldsymbol{\phi}$ on the error correction term s_t $(= y_{1,t} - \beta y_{2,t})$ indicates the pressure to return to the long run equilibrium. The cointegration relation is implied by parameter β and it is conjectured that $\beta \simeq 1$ because the two exchange rates are not supposed to diverge much. Therefore, the error-correction term reflects the spread between two exchange rates. By construction, the pressure to return to the equilibrium is a linear function of the error correction term in the linear VECM. However, the pressure to return to the equilibrium could differ by the level of the spread. This is a reasonable concern because of the following two issues: First, cryptocurrency market is typically shallow. It is reflected by the relatively low weekly trade volumes. Therefore, a trader who wants to buy or sell a large amount of cryptocurrency cannot do so quickly without affecting the market price. Second, there are transaction costs for arbitrageurs. As discussed in Section 2, verification of cryptocurrency transaction requires fees to miners. Also, there are fees for converting one currency to another currency. Some countries impose capital controls which limit the amount of funds that can be transferred abroad, or at least complicate the process. If it is the case, then the linear VECM may not be the appropriate model for the analysis of the exchange rates.

If the linear VECM is rejected, then the threshold VECM (TVECM) can be considered. In the TVECM, the coefficients change according to the level of the discrepancy in the two exchange rates. Consider the generalized threshold TVECM with M regimes. Equation (2) represents the shape of TVECM:

$$\Delta \boldsymbol{y}_t = \sum_{i=1}^{M} \Xi^{(i)} \boldsymbol{X}_t \mathbf{1}(\gamma_{i-1} < s_{t-1} \leq \gamma_i) + \varepsilon_t, \tag{2}$$

where $\boldsymbol{X}_t \equiv [1, s_{t-1}, \Delta \boldsymbol{y}'_{t-1}, \cdots, \Delta \boldsymbol{y}'_{t-p}]'$ and $\Xi^{(i)} \equiv (\boldsymbol{c}^{(i)}, \boldsymbol{\phi}^{(i)}, \boldsymbol{A}_1^{(i)}, \cdots, \boldsymbol{A}_p^{(i)})$ indicate the vectors of explanatory variables and coefficients in ith regime, respectively. Threshold values are denoted by $\{\gamma_0, \cdots, \gamma_M\}$ where $\gamma_0 = -\infty$ and $\gamma_M = \infty$. Indicator function $\mathbf{1}(\cdot) = 1$ when the condition in the parentheses is satisfied and $\mathbf{1}(\cdot) = 0$ otherwise. If $M = 1$, then the model is reduced to the linear VECM. When the threshold values are known, $\Xi^{(i)}$ can be estimated by the ordinary least squares method. However, the method is not applicable since $\{\gamma_1, \cdots, \gamma_{M-1}\}$ should be estimated and hence the indicator function is nonlinear. Therefore, concentrated least squares method based on grid search is utilized.[10]

To examine the null hypothesis that the linear VECM is the appropriate model for this study, the generalized Hansen and Seo (2002) test proposed by Larsen (2012) is employed. The generalized Hansen and Seo (2002) test works for the three-regime TVECM,

[10]A detailed explanation of this technique is found in Hansen and Seo (2002).

where there are two thresholds, while the prototypical Hansen and Seo (2002) test does for the two-regime model which has a threshold. When the movements of the spread in the exchange rates are plotted, it is found that that the fluctuations are quite symmetric around zero. Therefore, if the TVECM is the appropriate model, then there should be two thresholds, negative and positive ones. To check the robustness of the generalized Hansen and Seo (2002) test, Tsay (1998) test is implemented. This test is widely and conveniently used to evaluate the relevance of a linear model.

4 Empirical Analysis

4.1 Data

Since bitcoin commands the biggest market share and the largest sample size among cryptocurrencies, its prices are used to calculate multilateral exchange rates. The price data are obtained from data.bitcoinity.org. The bitcoin prices denominated in the U.S. dollars (USD), euros (EUR), British pounds (GBP), Japanese Yen (JPY), and Australian dollars (AUD) are considered. Using those bitcoin prices, multilateral exchange rates relative to the U.S. dollar are calculated. Bitcoin prices in Korean won (KRW) and Chinese Yuan (CHY) are excluded from the analysis because the multilateral exchange rate of those currencies are $I(0)$, which implies that cointegration with bilateral exhcnage rate is not applicable. It is conjectured that the stationarity of those multilateral exchange rates are due to the relatively strong capital controls. Bitcoin prices by different currency exchanges are averaged.[11] Mt. Gox, which was the largest currency exchange, was hacked in February 2014 and forced into bankruptcy. To remove the effects of the turmoil, the sample period is set from 5/14/2014 to 8/12/2018. Daily bilateral exchange rates against the U.S. dollar are obtained from fred.stlouisfed.org. Since foreign exchange market is closed on holidays and weekends, the daily data are averaged over a week. The sample size of the weekly data is 221. Following the literature, the natural logarithms of all

[11]Bitcoin prices differ by currency exchanges but the correlation coefficients are around 0.99. Moreover, since the prices are averaged over a week, the differences can be disregarded.

exchange rates are used for the analyses.

Table 1 presents the descriptive statistics of the bilateral and multilateral exchange rates. Mean, median, maximum, minimum, and standard deviation are calculated. All of the four pairs of exchange rates, which are EUR/USD, GBP/USD, JPY/USD, and AUD/USD, show that bilateral exchange rate and multilateral one have very similar descriptive statistics. These findings are reflected in Figure 1 - Figure 4 which show the historical movements of the two exchange rates. The upper panel is the level of the two exchange rates and lower one shows the spread between the two, calculated by multilateral exchange rate minus bilateral one. In general, it is easily noticeable that multilateral exchange rate represented by black dotted line has very similar fluctuations to that of bilateral exchange rate plotted in solid blue. As the lower panel shows, the spread of the exchange rates displays very stable movements around zero.[12] These plots imply that the two series are likely to be cointegrated due to arbitrage transactions. However, some fluctuations of the spread should be noted. Lower panel of Figure 1 shows that there is unusual spikes in EUR/USD spread in late 2014, mid-2015, and mid-2017. Figure 2 indicates that GBP/USD spread plunges around late-2017 and early-2018. In case of JPY/USD spread and AUD/USD spread, spikes are noticeable around mid-2017 and early-2018 as Figure 3 and Figure 4 indicate. The fluctuations of the spreads indicate a possibility that multilateral exchange rate might be adjusted nonlinearly to bilateral one.

4.2 Unit Root and Cointegration Tests

Following Balke and Fomby (1997), the existence of unit root is tested first. Once the data is concluded to be $I(1)$, then cointegration test will be conducted. To check the robustness of the test, five different test statistics are considered: Augmented Dickey and Fuller (1979) test, the GLS-detrended Dickey-Fuller test, Ng and Perron (2001) test, Phillips and Perron (1988) test, and the point optimal test of Elliott, Rothenberg, and Stock (1996). Table 2 presents the results of the unit root tests for the bilateral exchange rates, and multilateral exchange rate by bitcoin (BTC) price, and the spread between the

[12]This quite symmetric movement around zero is the reason why three-regime TVECM is employed.

two rates.

As the panel A and the panel B show, the bilateral and the multilateral exchange rates are well approximated by $I(1)$ processes over the sample. All of the test statistics are above critical value at ten percent significance level, implying that those data contain unit roots. Since traders will actively seek arbitrage profits, the two exchange rates are very close in level, hence the spread between the two are likely to be stationary. The panel C, which summarizes the test statistics of the spreads, indicates that the spreads are stationary, which implies the existence of cointegration relations.

Since it is concluded that the four bilateral exchange rates and the corresponding multilateral exchange rates by bitcoin price contain unit roots, Johansen cointegration rank test is conducted on the pairs of bilateral and multilateral rates. Table 3 summarizes the values of trace statistic, λ_{max} statistic, and critical values. As observed in the unit root test results, the trace statistic and the λ_{max} statistic indicate that the null hypothesis of cointegration rank $r = 0$ is strongly rejected. On the other hand, we could not reject the null hypothesis of $r \leq 1$ at the least 10 percentage of significance level. Hence, we conclude that the cointegration rank is $r = 1$.

Figure 5 supports the conclusion from the Johansen cointegration rank test. Left and right columns of the Figure represent the first and the second cointegration relations, respectively. The first cointegration relation looks stationary but the second one does not obviously, confirming the conclusion that the cointegration rank $r = 1$.

4.3 Tests for Threshold Cointegration

The null hypothesis that the linear VECM is the appropriate model is tested by the generalized Hansen and Seo (2002) test proposed by Larsen (2012). Basically, Hansen and Seo (2002) test uses LM statistic as follows:

$$LM(\beta, \gamma) = \hat{s}'[\widetilde{Var}(\hat{s})]^{-1}\hat{s} \tag{3}$$

where $\hat{s}$ and $\widetilde{Var}(\hat{s})$ denote the estimated coefficient vector of the TVECM and its co-variance matrix, respectively, given β and γ. The prototypical Hansen and Seo (2002) test considers two-regime VECM, where $\gamma = \gamma_1$. Larsen (2012) generalizes the test for three-regime VECM, where $\gamma = [\gamma_1, \gamma_2]$. As it is presented in Figure 1 - Figure 4, the movements of the spread in the exchange rates have fluctuations quite symmetric around zero. Therefore, the TVECM with two thresholds, negative and positive ones, is more reasonable choice. Since the $\sup LM$ statistic has a nonstandard asymptotic distribution, Hansen and Seo (2002) propose two bootstrapping methods for calculating the p-values: One is the fixed regressor bootstrap and the other is the residual bootstrap.[13] Those p-values are calculated out of 1,000 simulations. The null hypothesis of the linear VECM is rejected if the simulated p-values are smaller than the size chosen. The $\sup LM$ test statistics and p-values for the models with four different pairs of exchange rates are summarized in Table 4. In case of EUR/USD exchange rate pair, the $\sup LM$ statistic is calculated at 18.4 with p-values less than 0.1, so it is concluded that the linear VECM is rejected at the ten percent significance level. The test statistic of EUR/USD pair is 20.4, which imply that the TVECM is preferred model at the five percent significance level. The linear cointegrations for the pairs JPY/USD and AUD/USD are also rejected at the significance level of 0.1. Considering the test results, it is concluded that the threshold cointegration is more appropriate to account for the data.

The robustness of the generalized Hansen and Seo (2002) test is examined by Tsay (1998) test. This test is widely used to evaluate the relevance of a linear model. To impliment the test, the linear VECM is estimated by the recursive least square method. If the linear VECM is appropriate, then the residuals will be close to white noise. Therefore, we may easily use the estimated residual to test the relevance of the linear VECM. The results are summarized in Table 5. The size of initial sample (m_0) is chosen as $m_0 = cT^{(1/2)}$ where $c \in \{2, 3, 4, 5\}$ as Tsay (1998) suggests. Since there are 211 sample points, the selected initial sample is $m_0 = \{30, 45, 59, 74\}$. The table indicates that the p-values of the

[13]Fixed regressor bootstrap method proceeds with the assumption that β and γ are known. Therefore, the TVECM regressors are held fixed at their sample values. On the other hand, residual bootstrap only requires a complete specification of VECM and its residual vectors are used to simulate bootstrap distribution.

test statistic are very close to zero regardless of the size of the initial sample m_0. Therefore, the linear VECM can be easily rejected and the result from the generalized Hansen and Seo (2002) test are confirmed.

4.4 Estimation of TVECM

Since the relevance of the TVECM is verified by the generalized Hansen and Seo (2002) test and Tsay (1998) test, the three-regime TVECM is estimated.[14] Table 6 summarizes important parameter estimates, threshold values (γ_1 and γ_2), share of each regime, and SSR (sum of squared residuals). The cointegration parameter β is set at one, assuming that the two exchange rates are equalized in equilibrium.[15] Therefore, the error correction term is the spread between the two exchange rates.

Since there are three regimes in the model, the property of each regime need to be clarified. The middle regime can be regarded as a state of normal market functioning in which the spreads are close to zero. In this regime, arbitrage transactions are not likely to be active. Because it is the "normal" state, the majority of the sample is expected to belong to the middle regime. The low regime is characterized as the state where the multilateral exchange rate is substantially lower than bilateral exchange rate, which means that bitcoin price in the U.S. dollar is relatively higher than the price in the other currency. Therefore, arbitrageurs will actively sell bitcoins for the U.S. dollars and buy them for the other currency. The transactions will increase the multilateral exchange rate and there will be a shift into the middle regime. The high regime implies the opposite. When the market is in high regime, then bitcoin price in the U.S. dollar is significantly lower than the price in other currency, thus traders will buy bitcoins for the U.S. dollars and sell them for other currency. These transactions will reduce the exchange rate spread. The lower and upper threshold are represented by γ_1 and γ_2, respectively. In all of the four models, the estimated γ_1's are negative while γ_2's are positive, implying that the market is in the middle regime when the spread is around zero. The thresholds are not

[14] Lag length of TVECM is chosen by AIC. The criterion indicates that one period lag is optimal for the model.

[15] There is little difference in the estimation results when β is estimated.

always symmetric around zero. In case of EUR/USD model, upper thresholds are quite symmetric with respect to zero. In the model, the lower and upper threshold estimates are −0.0063 and 0.0069, respectively. The estimated thresholds in GBP/USD model are not as symmetric as those in EUR/USD model. The parameters γ_1 and γ_2 are estimated at −0.0283 and 0.0107. On the other hand, JPY/USD and AUD/USD model shows that upper thresholds have larger absolute values. The parameters γ_1 and γ_2 are estimated at −0.0048 and 0.0219 in JPY/USD model and −0.0114 and 0.0273 in AUD/USD model.

In general, the majority of the samples is located in the middle regime, as expected. In EUR/USD model, about 79 percent of the sample belongs to the middle regime while 12 percent and 8 percent of the sample are located as in the low and the high regime, respectively. The estimation results with GBP/USD model show that the middle, the low, and the upper regime have 83 percent, 7 percent, and 10 percent of shares, respectively. The share of the middle regime in AUD/USD model is 82 percent, and the low and the high regime have 6 percent and 12 percent of shares. One anomaly is JPY/USD model. According to the estimation results, 42 percent of the sample is located in the middle regime and 53 percent of it belongs to the low regime.

Additionally, Table 6 compares the SSR's of the TVECM and the VECM. In all of the four models, the SSR of the TVECM is consistently smaller than that of the VECM by more than ten percents, confirming the results of the generalized Hansen and Seo (2002) test and Tsay (1998) test that the TVECM is the more relevant model for this analysis.

Using the estimated threshold values, we may infer the pressure of arbitrage transactions. Figure 6 - Figure 9 show the classification of the sample periods by the regimes. The plots present the levels of error correction terms, which are virtually the spreads between the two exchange rates. Using the estimated threshold values, states can be classified into three regimes: The orange empty circles, the blue solid circles, and the red triangles represent the periods of the low, the middle, and the high regimes, respectively. The dotted horizontal lines indicate the estimated two threshold values. Figure 6 presents the regime classification in EUR/USD model. As mentioned at Table 6, the majority of sample is located in the middle regime. Most of the deviations from the middle regime are sporadic and do not last for a long time. However, a prolonged period of deviation is noticeable in mid-2015, when the spread plunges down to -0.025. This low regime lasts more than twelve weeks. The prolonged deviation is conjectured due to the appreciation of euro and the sluggish responses of arbitrageurs. After the period, deviations from middle regime occur, but they do not last as long as mid-2015 case. The error correction term of GBP/USD model is displayed in Figure 7. Again, the spread stays in the middle regime for the most of periods. From the end of 2017 to early 2018, there is a plunge in the spread down to -0.09 and the deviation from the normal state lasts about twenty weeks. These result is supposed to come from the relatively bigger hike in bitcoin price in the U.S. market. JPY/USD model is a bit of anomaly. As Figure 8 and Table 6 indicates, majority of period is classified as the low regime and there are few periods in the upper regimes. The error correction term of AUD/USD model is plotted in Figure 9. Most of the sample periods are classified as the middle state and prolonged deviations to the upper regime are observed in mid-2017 and late-2017.

5 Conclusion

Most of the cryptocurrency market participants aim at capital gains. Cryptocurrency transaction occurs through public network with great efficiency. Therefore, arbitrage transactions in this market are supposed to be active. The relative price of a cryptocurrency, denominated in different currencies implies an exchange rate. If the multilateral exchange rate differs from the bilateral exchange rate, then there exists an opportunity for arbitrage, and hence the multilateral rate would not diverge much from the bilateral one.

However, the pressure to return to the equilibrium is likely to depend on the level of the discrepancy, because cryptocurrency market is shallow and there are transaction costs for arbitrage. This study analyzes the nonlinearity in the convergence of multilateral exchange rate to bilateral exchange rate. According to the empirical analysis, it is found that the two exchange rates are cointegrated and the cointegration relations are not linear. Therefore, the threshold VECM is estimated to infer the nonlinear pressure to return to the equilibrium.

When sufficient sample of data is available, the threshold VECM can be estimated without high computation costs. Therefore, meaningful implications on the pressure in cryptocurrency market can be effectively derived. This model would be useful for traders and banking supervisors alike, since they may monitor cryptocurrency market's overheating in almost real time.

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Table 1: Descriptive Statistics

	EUR/USD		GBP/USD		JPY/USD		AUD/USD	
	B	M	B	M	B	M	B	M
Mean	-0.1423	-0.1421	-0.3486	-0.3514	4.7202	4.7254	0.2546	0.2655
Median	-0.1226	-0.1235	-0.3468	-0.3547	4.7187	4.7282	0.2699	0.2809
Maximum	-0.0423	-0.0373	-0.1965	-0.1877	4.8252	4.8221	0.3638	0.3818
Minimum	-0.3127	-0.3149	-0.5390	-0.5303	4.6089	4.6105	0.0588	0.0687
Std. Dev.	0.0656	0.0647	0.0937	0.0943	0.0576	0.0586	0.0711	0.0734

Notes: B and M denote bilateral and multilateral exchange rates, respectively.

Table 2: Unit Root Test

	ADF	DFGLS	NP	PP	ERS
(Panel A: Bilateral Exchange Rates)					
EUR/USD	-2.53	-0.39	-0.40	-2.51	37.75
GBP/USD	-1.48	0.32	0.34	-1.51	60.53
JPY/USD	-2.22	-1.01	-1.01	-2.11	12.41
AUD/USD	-2.48	-0.21	-0.20	-2.37	36.81
(Panel B: Multilateral Exchange Rates)					
EUR/USD by BTC	-2.56	-0.22	-0.22	-2.58	51.94
GBP/USD by BTC	-1.40	0.08	0.10	-1.46	39.44
JPY/USD by BTC	-2.71	-1.20	-1.20	-2.49	9.34
AUD/USD by BTC	-2.63	-0.32	-0.31	-2.57	30.83
(Panel C: Spreads)					
Spread EUR/USD	-6.77	-6.58	-5.50	-6.75	0.48
Spread GBP/USD	-4.26	-2.84	-2.77	-3.88	1.23
Spread JPY/USD	-5.67	-4.42	-4.07	-5.54	0.84
Spread AUD/USD	-4.95	-4.59	-4.20	-4.96	0.73
10% C.V.	-2.57	-1.62	-1.62	-2.57	4.35
5% C.V.	-2.87	-1.94	-1.98	-2.87	3.18
1% C.V.	-3.46	-2.58	-2.58	-3.46	1.92

Notes: ADF is the augmented Dickey and Fuller (1979) test statistic, DFGLS is the GLS-detrended Dickey-Fuller test statistic, NP is the t-type test of Ng and Perron (2001), and PP is the Phillips and Perron (1988) test, and ERS is the point optimal test statistic of Elliott, Rothenberg, and Stock (1996).

Table 3: Johansen Test

	$TRACE : r = 0$	$TRACE : r \leq 1$	$\lambda_{max} : r = 0$	$\lambda_{max} : r \leq 1$
EUR/USD pair	29.48	7.33	22.15	7.33
GBP/USD pair	16.00	2.43	13.57	2.43
JPY/USD pair	31.38	4.05	27.33	4.05
AUD/USD pair	32.51	5.86	26.66	5.86
10% C.V.	15.66	6.50	12.91	6.50
5% C.V.	17.95	8.18	14.90	8.18
1% C.V.	23.52	11.65	19.19	11.65

Note: Trace test sets the cointegration rank for the alternative hypothesis at $r_1 = r_0 + 1 = 2$. Maximum eigenvalue (λ_{max}) test sets $r_1 = 3$.

Table 4: Hansen-Seo Test

	SupLM	p-value	
		Fixed Reg	Res Boot
EUR/USD pair	18.9	0.072	0.098
GBP/USD pair	20.8	0.038	0.054
JPY/USD pair	18.7	0.087	0.101
AUD/USD pair	18.0	0.095	0.105

Notes: Fixed Reg and Res Boot represent fixed regressor bootstrap method and residual boostsrap method, respectively.

Table 5: Tsay Test

	$m_0 = 30$	$m_0 = 45$	$m_0 = 59$	$m_0 = 74$
EUR/USD pair	192.16	90.23	74.27	68.90
	(0.00)	(0.00)	(0.00)	(0.00)
GBP/USD pair	95.74	129.20	100.44	59.13
	(0.00)	(0.00)	(0.00)	(0.00)
JPY/USD pair	271.39	213.08	63.48	83.79
	(0.00)	(0.08)	(0.00)	(0.00)
AUD/USD pair	455.42	206.55	225.26	79.92
	(0.00)	(0.00)	(0.00)	(0.00)

Note: The statistic represents the threshold nonlinearity measure of Tsay (1998). The values in the parentheses are corresponding p-values. Lag lengths are selected based on AIC. m_0 represents the size of initial sample for RLS (recursive least square) estimation.

Table 6: TVECM Estimation Results

	$-\beta$	γ_1	γ_2	Share of Regimes			SSR	
				Low	Middle	High	TVECM	VECM
EUR/USD	-1.0000	-0.0063	0.0069	0.1233	0.7945	0.0822	0.0334	0.0385
GBP/USD	-1.0000	-0.0283	0.0107	0.0685	0.8311	0.1005	0.0552	0.0669
JPY/USD	-1.0000	-0.0048	0.0219	0.5297	0.4201	0.0502	0.0542	0.0616
AUD/USD	-1.0000	-0.0114	0.0273	0.0594	0.8174	0.1233	0.0602	0.0698

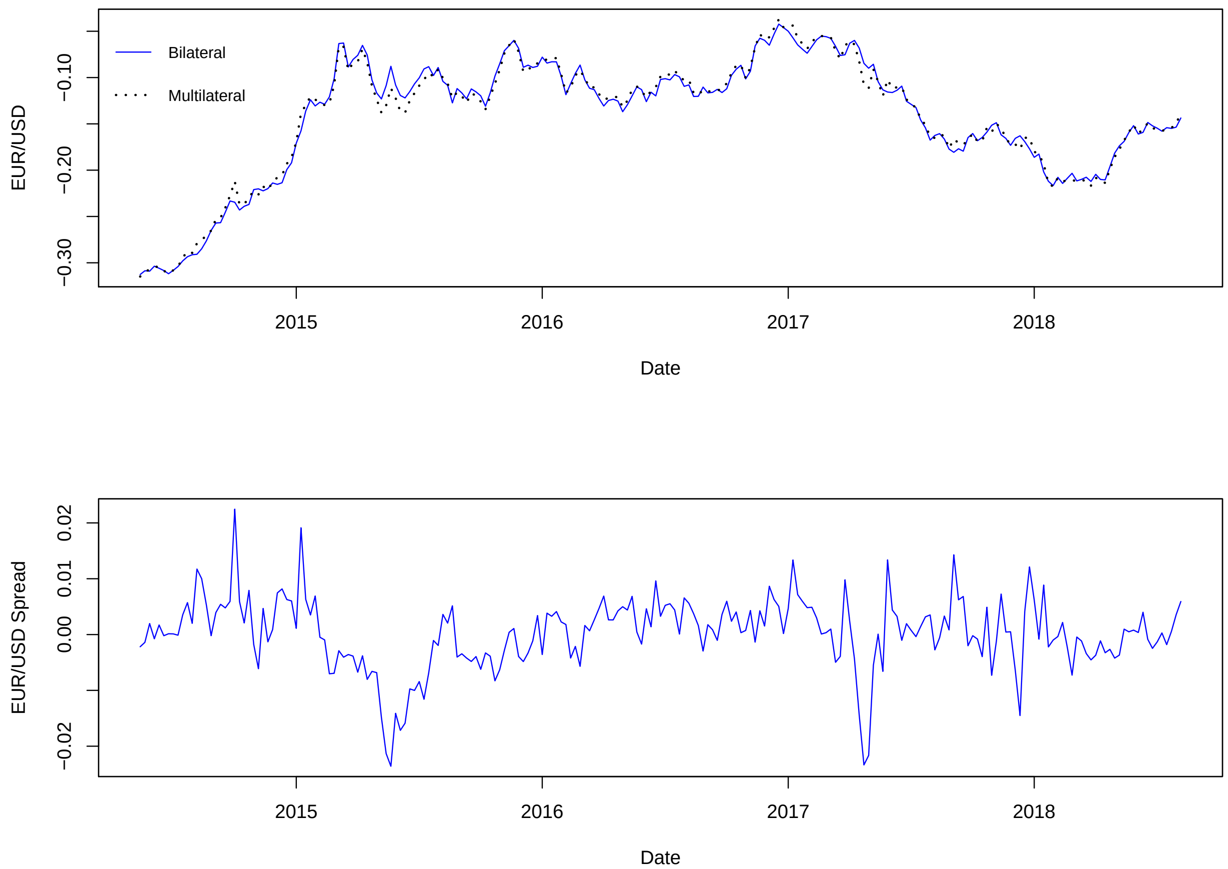

Figure 1: Bilateral and Multilateral Exchange Rates (EUR/USD)

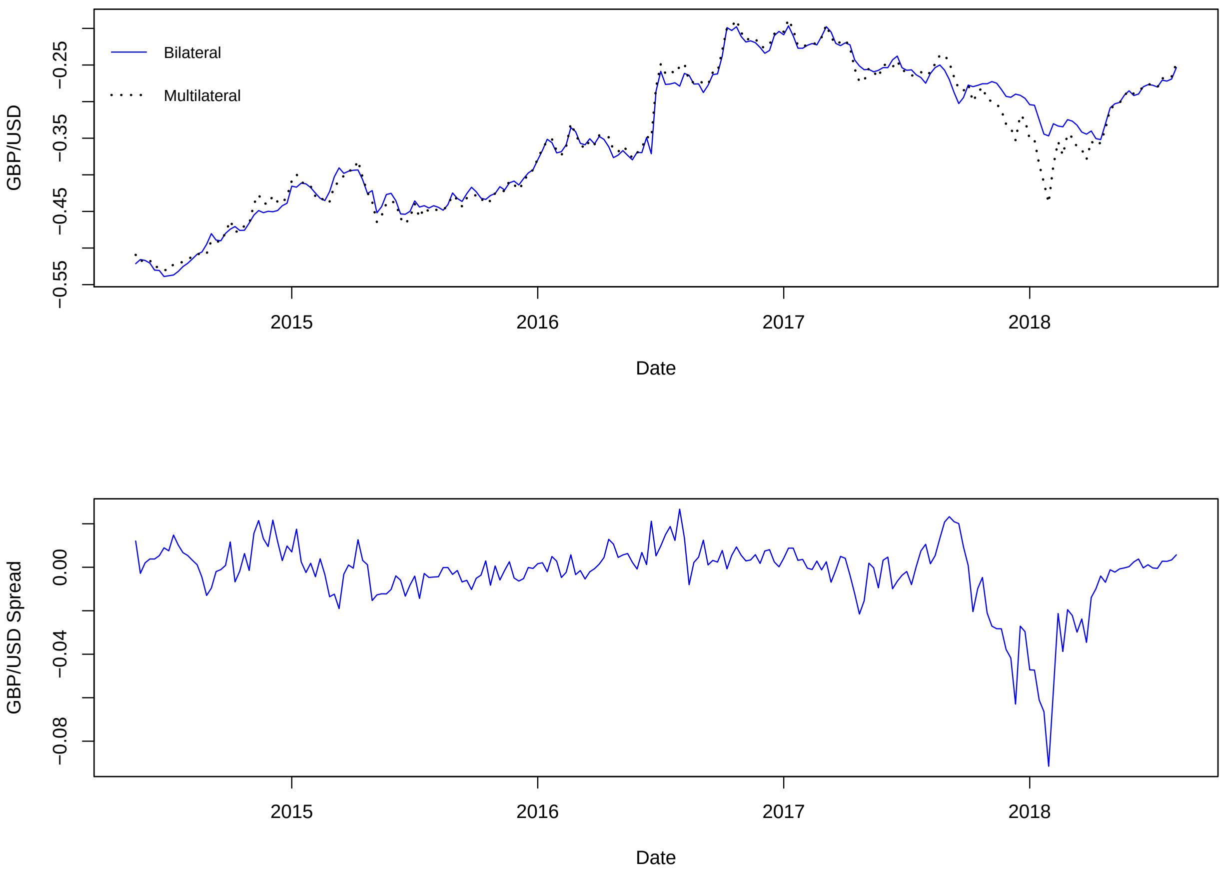

Figure 2: Bilateral and Multilateral Exchange Rates (GBP/USD)

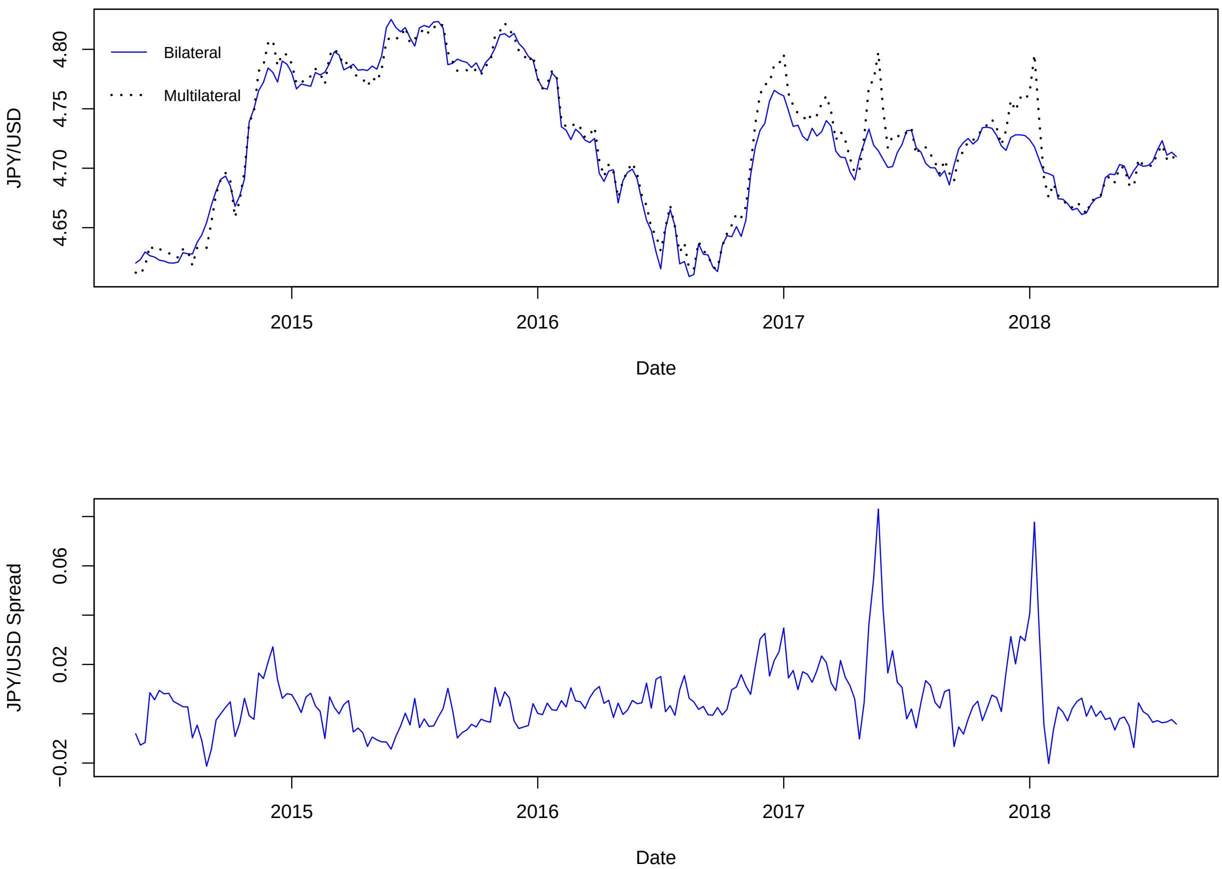

Figure 3: Bilateral and Multilateral Exchange Rates (JPY/USD)

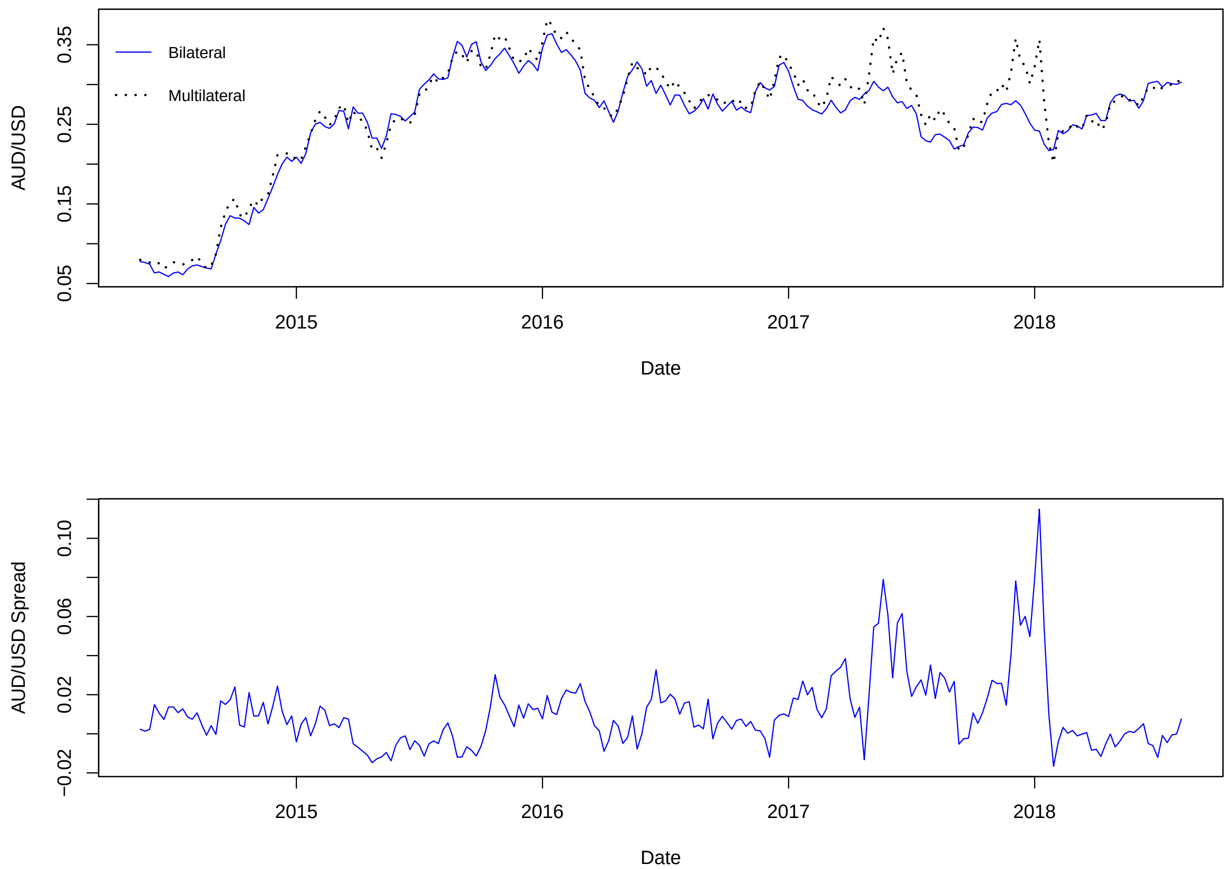

Figure 4: Bilateral and Multilateral Exchange Rates (AUD/USD)

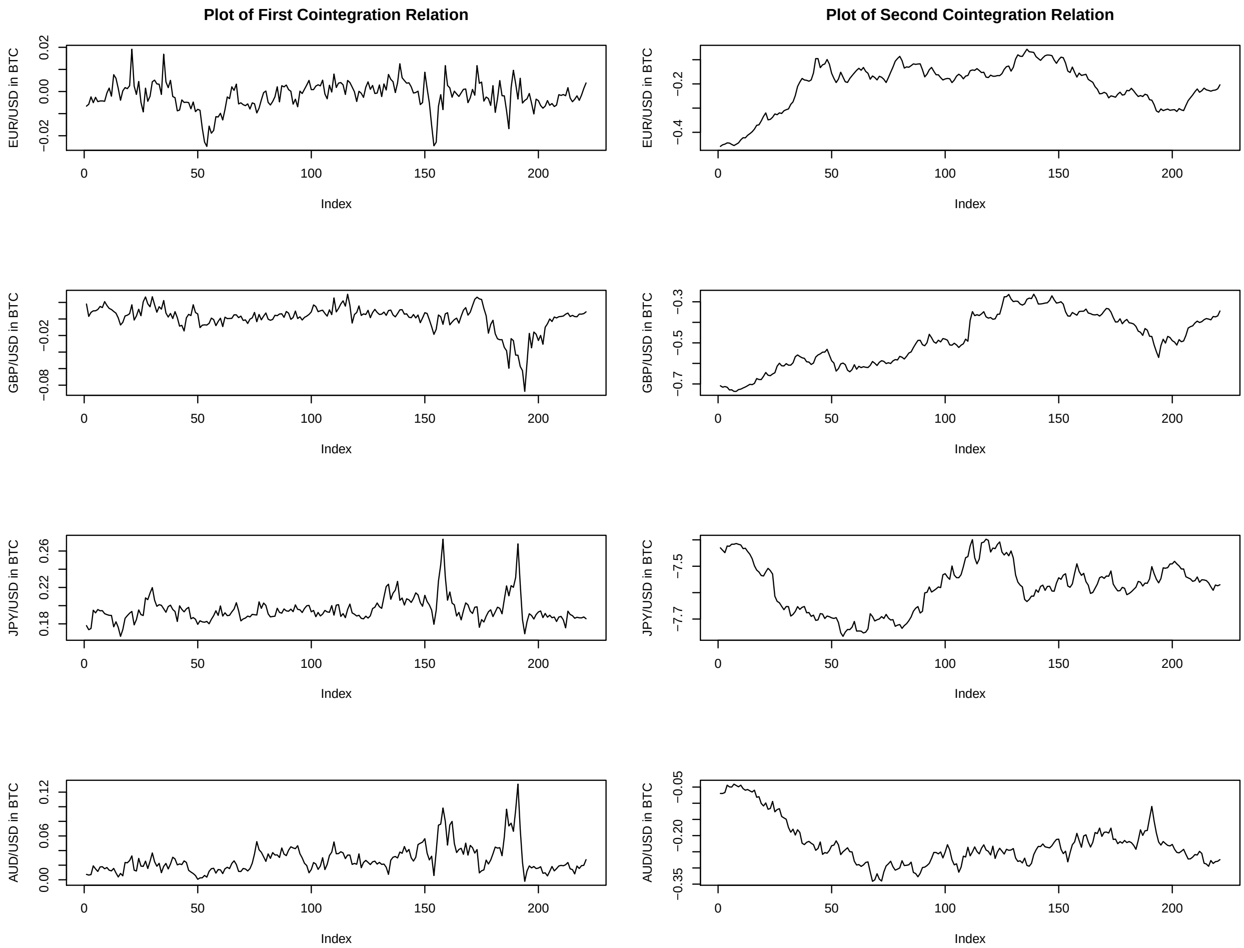

Figure 5: Cointegration Relations

Note: Two cointegration vectors are estimated. The left column shows the correlation relation $\boldsymbol{\beta}_1'\boldsymbol{y}$ and the right one does the correlation relation $\boldsymbol{\beta}_2'\boldsymbol{y}$.

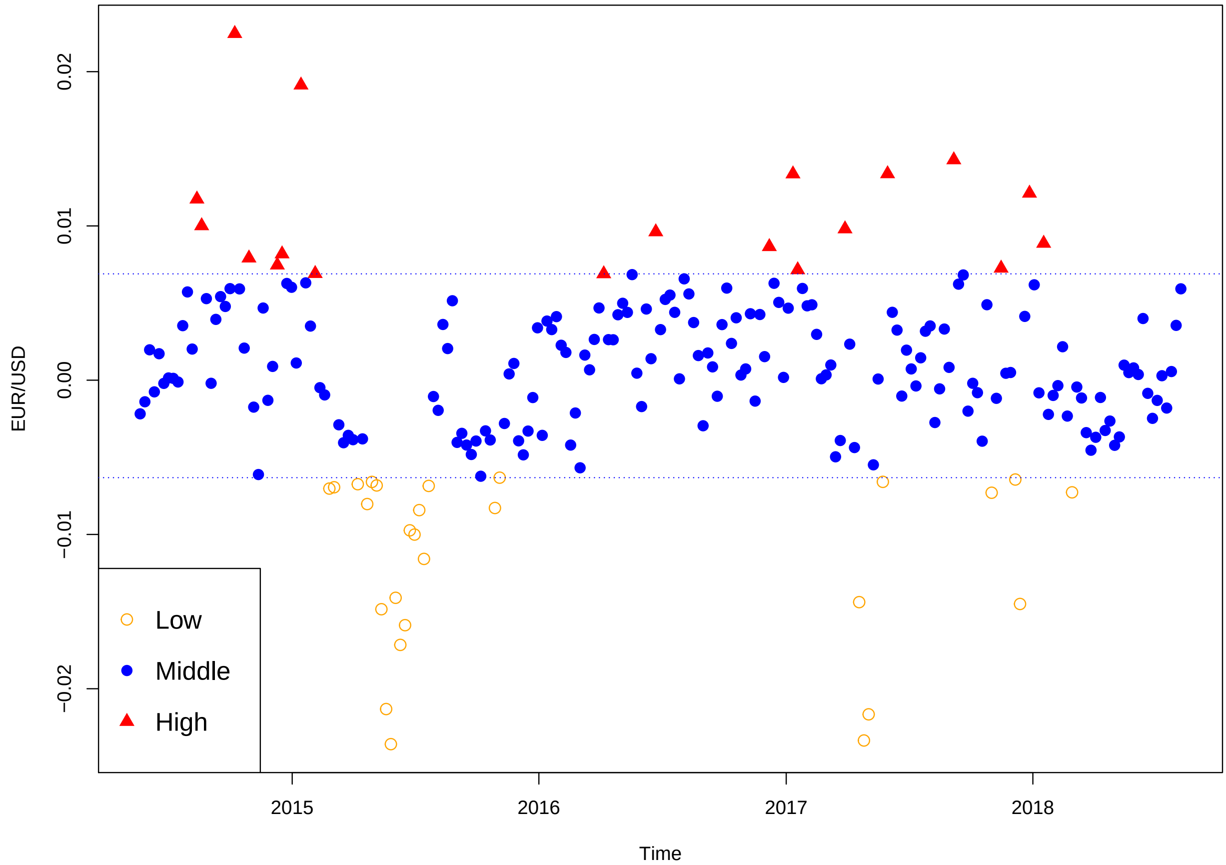

Figure 6: Error Correction Term and Threshold Values: EUR/USD

Note: Dotted horizontal lines represent the threshold values.

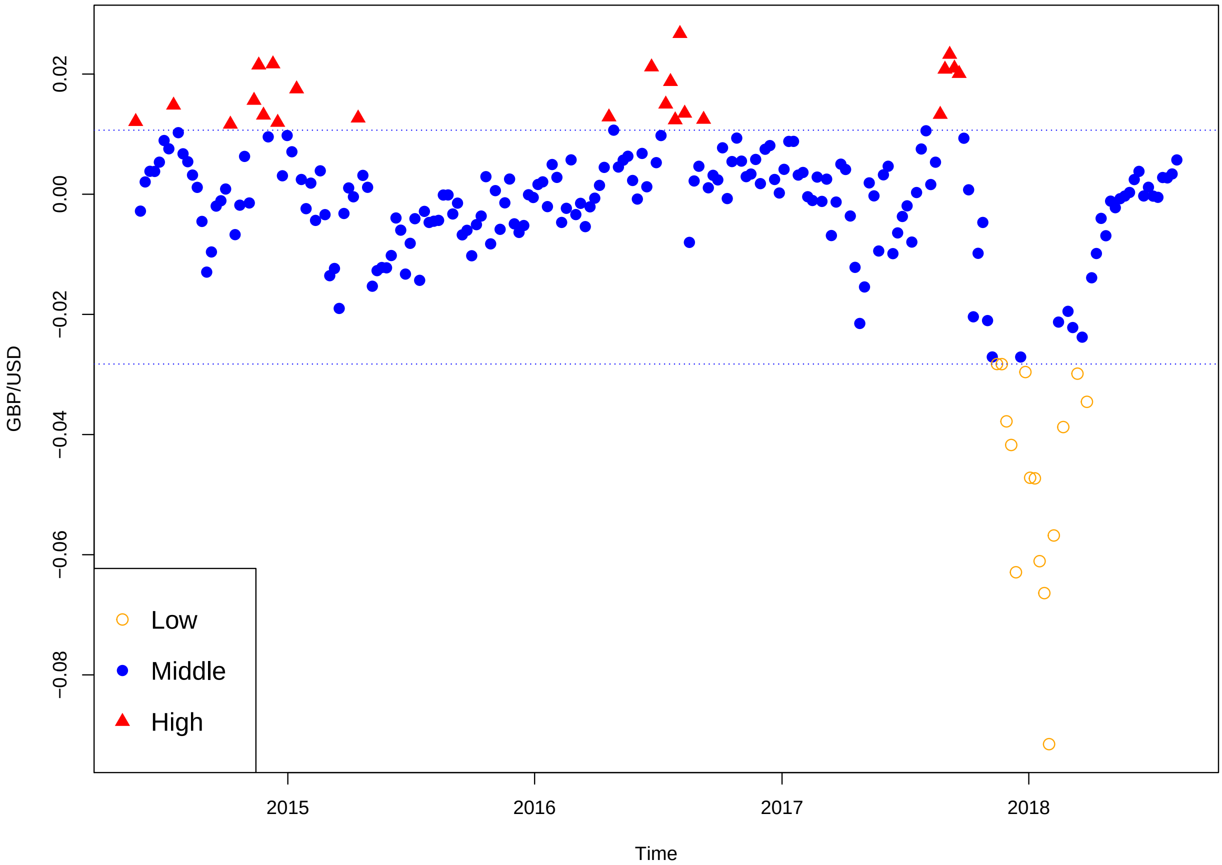

Figure 7: Error Correction Term and Threshold Values: GBP/USD

Note: Dotted horizontal lines represent the threshold values.

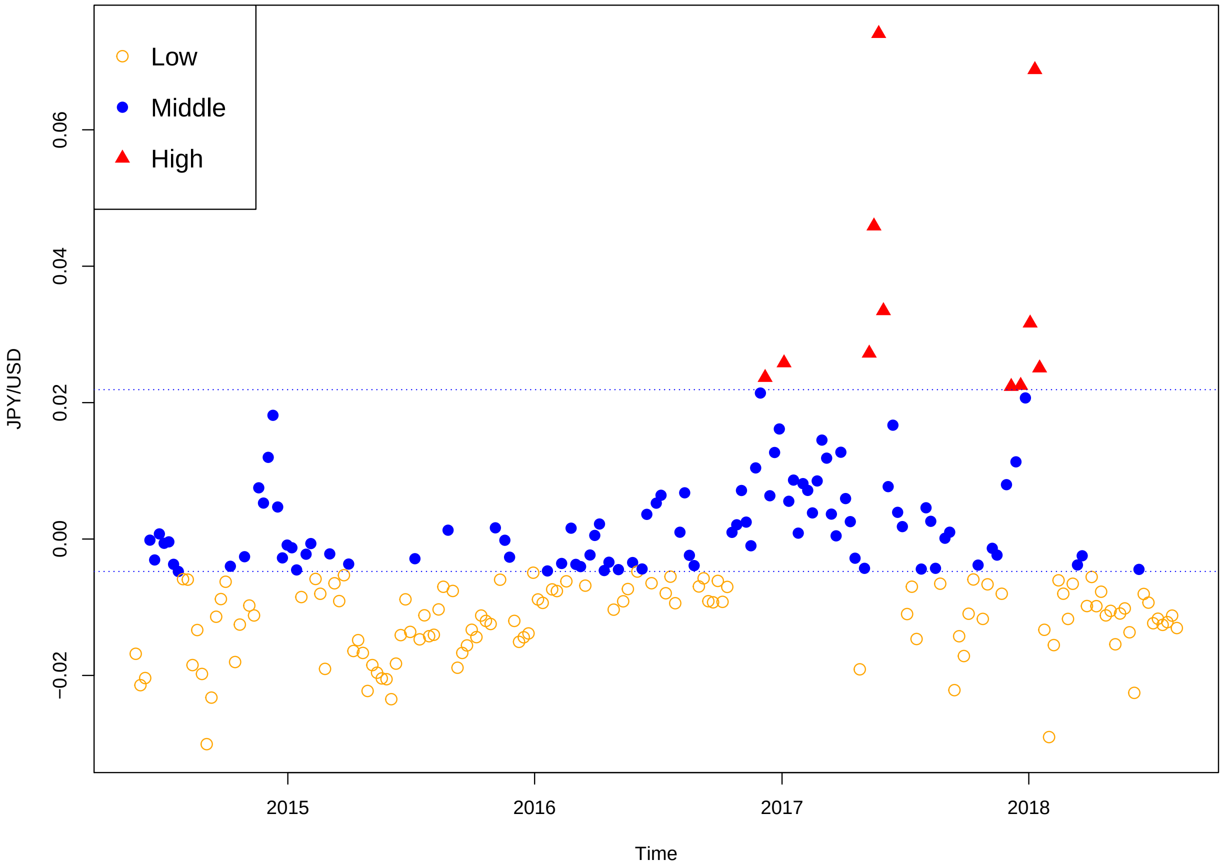

Figure 8: Error Correction Term and Threshold Values: JPY/USD

Note: Dotted horizontal lines represent the threshold values.

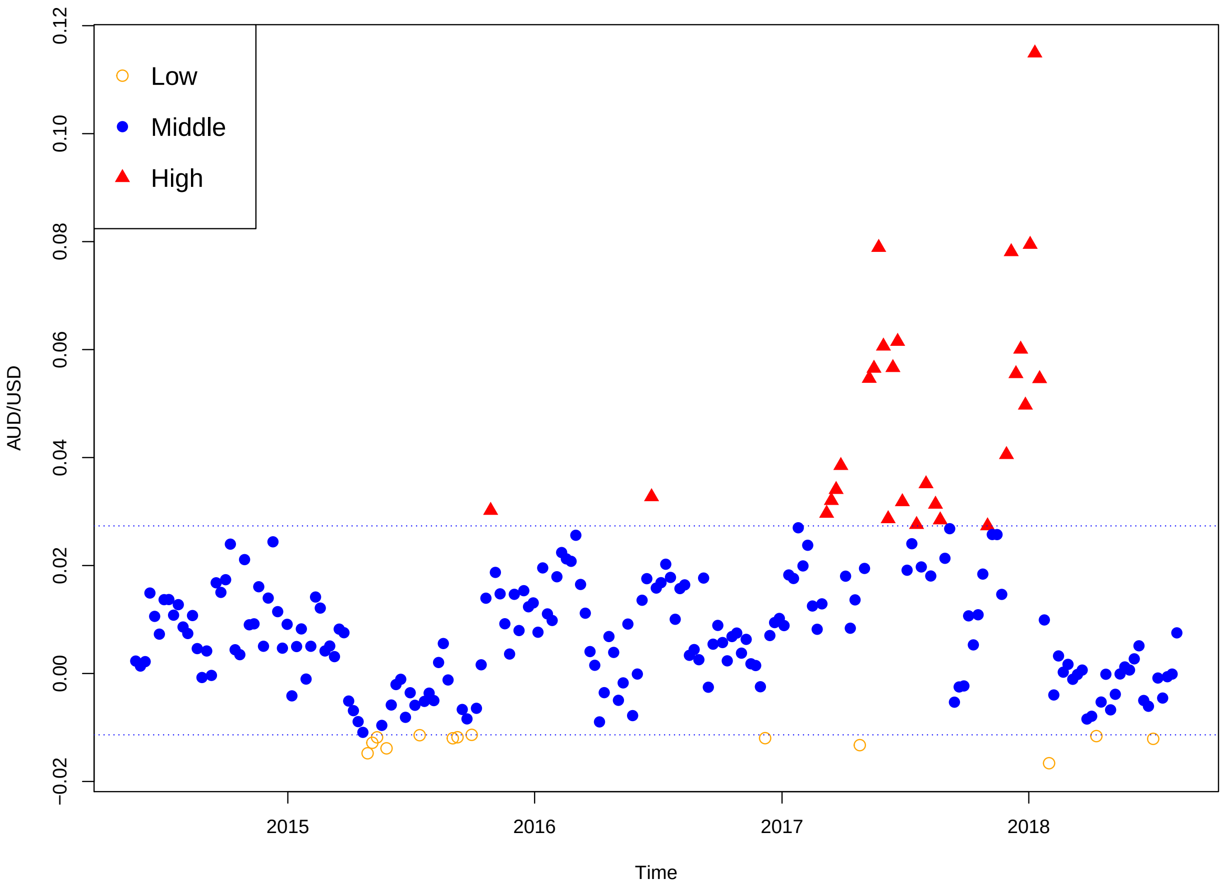

Figure 9: Error Correction Term and Threshold Values: AUD/USD

Note: Dotted horizontal lines represent the threshold values.