Option-implied Tail Risks and Their Information Contents

Sangwon Suh[*], Eungyu Yoo[†], Sun-Joong Yoon[‡]

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Abstract

In this paper, we use the notion of tail risk swaps whose payoff depends upon a prescribed tail risk event to derive various tail risk premia and tail risk indicators. The proposed tail risk indicators are model-free and implied from equity index option prices. We find time-varying tail risk premia which are not induced simply by the underlying return risk and return risk premium but represent an additional source of risk. The proposed tail risk indicators not only gauge the "investor fear" but also possess predictive powers for future returns and future tail risks.

Keywords: Tail risk, Variance risk, Tail risk premium, Variance risk premium, VIX.

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[*]Corresponding author. School of Economics, Chung-Ang University; Email: ssuh@cau.ac.kr.
[†]KAIST; Email: crux26@kaist.ac.kr.
[‡]Dongguk University; Email: sunyoon@dongguk.edu.

1 Introduction

Tail risk refers to the risk that an asset or portfolio return belongs to one side of the return distribution tails. Theories of rational behavior argue that investors place greater weights on downside risk than upside uncertainty in their utility functions. Such examples include the lower-partial moment framework (Bawa and Lindenberg, 1977), the loss aversion in their prospect theory (Kahneman and Tversky, 1979), and the disappointment aversion (Gul, 1991; Routledge and Zin, 2010). Therefore, the asymmetric treatment of downside risk versus upside uncertainty by investors is not only theoretically grounded but also has led to the development of new concepts in asset pricing and risk management, like the value-at-risk (VaR) and the expected shortfall (ES).

A recently emerging literature documents evidence that left tail risk factors play an important role in explaining return dynamics or variations. For example, Andersen, Fusari, and Todorov (2015) find that the left tail risk is priced but cannot be spanned by market volatility. In a later paper, Andersen, Fusari, and Todorov (2017) also report that the negative jump tail risk is not spanned by market volatility and helps predict future equity returns. Lu and Murray (2018) find that the risk of future bear market state is priced in the cross-section of stock returns. Farago and Tèdongap (2018) suggest three disappointment-related factors and show that the factors are priced besides the market return and market volatility.[1]

It has been well documented that volatility (or variance) risk is not only important in understanding asset return dynamics or cross-sectional variations but also useful in portfolio investments or financial risk management. The development of the VIX index has significantly contributed to the importance of volatility or variance risk from both academics and practitioners. The VIX index has been widely used since the inception in 1993 and is a model-free annualized volatility measure implied from stock market index option prices.[2]

[1] In less related works, Bakshi, Kapadia, and Madan (2003) find that risk-neutral skewness implied from individual option prices is related with the cross sectional variations of stock returns. Xing, Zhang, and Zhao (2010) document evidence that the shape of the volatility smirk can significantly predict future equity return.

[2] For example, Gonzalez-Perez (2015) offers a survey on various usages of the VIX index in the financial literature.

For tail risks, however, VIX-like measures are not introduced yet. The success of the VIX index motivates us to develop similar measures for tail risks. We expect that such measures would greatly contribute to a more extensive usage of tail risks in various directions. In this paper, we develop VIX-like model-free tail risk measures implied from stock index option prices.[3] To this end, we use the notion of a VaR swap, which is an over-the-counter contract that pays one if a prescribed tail risk (i.e., an event that a realized return rate is less than a prescribed return rate for a given risk level) is realized and zero otherwise. Since VaR swaps cost zero at entry, the prescribed return rate represents the risk-neutral expected value of the realized return. We show that the prescribed return rate can be obtained from option prices in a model-free way. We propose to use the difference between the ex post realized VaR swap payoff and the fixed VaR swap rate to quantify the VaR risk premium. We also propose the prescribed return rate to quantify the left tail risk. In addition to the VaR, we also consider similar swaps based on the ES. To elicit information from an asymmetry between downside risks and upside uncertainty, we also devise alternative swaps using upside uncertainties corresponding to downside risks measured by the VaR and the ES.

We find that both of the average VaR and ES risk premiums are strongly negative, which implies that investors price tail risks and are willing to accept a significantly negative average return to hedge tail risks. Next, we run asset pricing regressions of the tail risk premia on conventional common risk factors and find that the tail risk premia are not induced simply by the underlying return risk and return risk premium but represent an additional source of risk. We also investigate whether the tail risk premia are time-varying and find that while they are time-varying and correlated with the tail risk indicator level, downside tail risk premia exhibit a more conspicuous time-varying feature than upside premia.

As the VaR- and the ES-based downside tail risk measures closely co-move with the VIX index, we investigate whether the downside tail risk measures possess additional information content. To this end, we devise alternative tail risk indicators by subtracting the VIX-implied VaR or ES under a normal distribution assumption from the model-free VaR or ES.

[3] There exists an approach using econometric modelling to estimate time-varying jumps in the tail of asset return. Refer to, for example, Bollerslev and Todorov (2014). However, this approach differs from ours in that it does not offer model-free estimates.

In addition, we also devise alternative tail risk indicators as the difference between downside risk and upside uncertainty based on the VaR or the ES to exploit the asymmetry. Although the VaR- or the ES-based left tail risk indicators behave similarly with the VIX index, we find that the other alternative tail risk indicators behave differently not only from the VIX but also each other.

It is well documented that the VIX index rises at a faster rate when the stock market falls than when it rises. Due to this "leverage effect", the VIX index is often referred to as an "investor fear gauge". We find that all of the tail risk indicators also rise at a faster rate during a market downturn and thus gauge the "investor fear".

We empirically evaluate the predictive power of tail risk indicators for future stock returns. We find that increased current tail risks tend to predict future portfolio return declines. Although this negative relation between current tail risks and future returns is largely statistically significant, it is more conspicuous in small-sized stocks than big-sized stocks, and the predictive power varies across tail risk indicators.

We also empirically assess the forecasting power of tail risk indicators for future tail events. We show that increased current tail risks tend to predict a higher probability of future tail risk events. Although the forecasting power varies across tail risk indicators, the forecasting powers are statistically significant even after controlling for the effect of the VIX. In addition, we examine whether tail risk indicators possess any predictive power for future ex ante tail risks and find that some tail risk indicators possess significant additional predictive power.

In related works, Bakshi and Kapadia (2003) use delta-hedged option portfolios and find that the volatility risk premium is negative. Carr and Wu (2008) propose a non-parametric method to quantify the variance risk premium from option prices. They find that the average variance risk premiums are negative and that the majority of the variance risk premiums is generated by an independent variance risk factor. This paper is closely related to Carr and Wu (2008). Consistent with the variance risk, we find that tail risk premiums are negative and time-varying and call for an independent risk factor.

Whaley (2009) documents evidence for the leverage effect, and Bandi and Renò (2012) extends the analysis to the time-varying leverage effect. We find similar leverage effect for tail risk indicators.

Banerjee, Doran, and Peterson (2007) show that implied volatility can predict future portfolio returns. Bollerslev, Todorov, and Xu (2015) find that the variance risk premium helps predict future market returns. Jiang and Tian (2005) provide evidence that implied volatility is an efficient forecast for future realized volatility. Similar with the evidences for the information content of the implied volatility, the tail risk indicators also possess information content which is useful for forecasting not only future returns but also future realized or ex ante tail risks.

This paper proceeds as follows: Section 2 formally introduces several tail risk swap contracts, derives swap rates and premia, and also introduces several tail risk indicators implied from the tail risk swap contracts. Section 3 explains the data to be used in the analysis and the methodology to obtain the tail risk indicators from option prices. In Section 4, several empirical analyses are conducted for comparing non-parametric and semi-parametric estimates of tail risk indicators, characterizing tail risk premia and indicators, and examining predictive power of tail risk indicators for future stock return, future tail events, and future ex ante tail risks. Section 5 concludes.

2 Tail risk swap rates, premia and indicators

In this section, we introduce several swap contracts which exchange uncertain future tail risks with current fixed costs. We then derive swap rates and premia that are associated with the tail swap contracts. We also introduce several tail risk indicators implied from the tail swap contracts.

2.1 Value-at-Risk swap

We use S_t to denote the time-t spot price of an asset, and $R_{t,T}$ its continuously compound return from time t to T, that is, $R_{t,T} \equiv \log S_T - \log S_t$. The negative of the lower (downside)

$\alpha\%$ percentile of $R_{t,T}$, denoted by $D_{t,T}(\alpha)$, is defined as

$$\Pr\left[R_{t,T} \leq -D_{t,T}(\alpha)\right] = \alpha\%. \tag{1}$$

We consider a value-at-risk (VaR) swap. At maturity, the payoff to the long side of the swap is one if the realized percentile over the life of the contract is less than a prescribed significance level α-percentile and zero otherwise; the profit from the VaR swap contract is

$$L\left[I\left(R_{t,T} \leq -D_{t,T}(\alpha)\right) - VaR_{t,T}^{SW}(\alpha)\right], \tag{2}$$

where $I(\cdot)$ denotes an indicator function, $VaR_{t,T}^{SW}(\alpha)$ denotes the fixed VaR swap rate that is determined at time t and paid at time T. Here, a notional amount of the swap contract L is normalized to be one without loss of generality. The value-at-risk (VaR) swap has zero net market value at entry. No arbitrage dictates that the VaR swap rate equals the risk-neutral expected value of the indicator

$$VaR_{t,T}^{SW}(\alpha) = \mathbb{E}_t^{\mathbb{Q}}\left[I\left(R_{t,T} \leq -D_{t,T}(\alpha)\right)\right], \tag{3}$$

where $\mathbb{E}_t^{\mathbb{Q}}[\cdot]$ denotes the time-t conditional expectation operator under some risk-neutral measure $\mathbb{Q}$. As the VaR swap rate $VaR_{t,T}^{SW}(\alpha)$ depends on the threshold level $D_{t,T}(\alpha)$, we fix the VaR swap rate $VaR_{t,T}^{SW}(\alpha)$ to be α and determine the VaR swap threshold level $D_{t,T}^{SW}(\alpha)$, that is,

$$\frac{\alpha}{100} = \mathbb{E}_t^{\mathbb{Q}}\left[I\left(R_{t,T} \leq -D_{t,T}^{SW}(\alpha)\right)\right]. \tag{4}$$

We show that the VaR swap threshold level $D_{t,T}^{SW}(\alpha)$ can be calculated from option prices of the same horizon T.

Proposition 1 *Under no arbitrage, the time-t risk-neutral expected value of the VaR swap threshold at $\alpha\%$ significance level of an asset over horizon [t, T] defined in Equation (4) can be calculated from European out-of-the-money option prices across strikes $K > 0$ and at the same maturity T*

$$D_{t,T}^{SW}(\alpha) = \log P'^{-1}\left(e^{-r(T-t)}\frac{\alpha}{100}\right) - \log S_t, \tag{5}$$

where $P'^{-1}(\cdot)$ denotes the inverse function of the first derivative of the European put option price function $P(K)$ of strike price K, and r denotes the risk-free rate over horizon [t, T].

The proof is provided in the Appendix. While the Breeden-Litzenberger (1978) formula derives the risk-neutral density function of future asset return from option prices, the density function is obtained from the second derivative of option price function with respect to strike prices. The second derivative, however, typically provides oscillating and thus unreliable densities. Noteworthily, our result, however, can avoid the usage of the second derivative and is engaged with the first derivative, which greatly helps to reduce errors in the estimation.

2.2 Alternative tail risk swaps

2.2.1 Upside swap

If we denote the upper (upside) $\alpha\%$ percentile of $R_{t,T}$ by $U_{t,T}(\alpha)$ as follows:

$$\Pr\left[R_{t,T} \geq U_{t,T}(\alpha)\right] = \alpha\%, \tag{6}$$

Similar with the VaR swap, we also consider an upside (UP) swap. At maturity, the payoff to the long side of the swap is one if the realized upper percentile over the life of the contract is less than a prescribed significance level α-percentile and zero otherwise; the profit from the UP swap contract is

$$I\left(R_{t,T} \geq U_{t,T}(\alpha)\right) - UP^{SW}_{t,T}(\alpha), \tag{7}$$

where $UP^{SW}_{t,T}(\alpha)$ denotes the fixed UP swap rate that is determined at time t and paid at time T. From the no-arbitrage condition, the UP swap rate equals the risk-neutral expected value of the indicator

$$UP^{SW}_{t,T}(\alpha) = \mathbb{E}^{\mathbb{Q}}_t\left[I\left(R_{t,T} \geq U_{t,T}(\alpha)\right)\right]. \tag{8}$$

We fix the UP swap rate $UP^{SW}_{t,T}(\alpha)$ to be α and determine the UP swap threshold level $U^{SW}_{t,T}(\alpha)$, that is,

$$\frac{\alpha}{100} = \mathbb{E}^{\mathbb{Q}}_t\left[I\left(R_{t,T} \geq U^{SW}_{t,T}(\alpha)\right)\right]. \tag{9}$$

we can show that the UP swap threshold can be calculated from option prices of the same horizon T.

Corollary 2 *Under no arbitrage, the time-t risk-neutral expected value of the UP at $\alpha\%$ significance level of an asset over horizon [t, T] defined in Equation (6) can be calculated from European out-of-the-money option prices across strikes $K > 0$ and at the same maturity T*

$$U_{t,T}^{SW}(\alpha) = \log C'^{-1}\left(e^{-r(T-t)}\frac{\alpha}{100}\right) - \log S_t, \tag{10}$$

where $C'^{-1}(\cdot)$ denotes the inverse function of the first derivative of the European call option price function $C(K)$ of strike price K, and r denotes the risk-free rate over horizon [t, T].

The proof is provided in the Appendix.

2.2.2 Expected-shortfall swap

The expected shortfall at $\alpha\%$ significance level is defined as

$$ED_{t,T}(\alpha) = E\left[-R_{t,T} \left| R_{t,T} \leq -D_{t,T}(\alpha)\right.\right] = \frac{-100}{\alpha}\int_{-\infty}^{-D_{t,T}(\alpha)} R \cdot f_R(R)\, dR, \tag{11}$$

where $f_R(R)$ denotes the pdf of the log return over $[t, T]$.

Now, we consider an expected-shortfall (ES) swap. At maturity, the payoff to the long side of the swap is

$$-R_{t,T} \cdot I\left(R_{t,T} \leq -D_{t,T}(\alpha)\right) - ES_{t,T}^{SW}(\alpha), \tag{12}$$

where $ES_{t,T}^{SW}(\alpha)$ denotes the fixed ES swap rate that is determined at time t and paid at time T. We fix the ES swap threshold $D_{t,T}(\alpha)$ as the same with the VaR swap threshold level $D_{t,T}^{SW}(\alpha)$. No arbitrage dictates that the ES swap rate equals the following risk-neutral expected value:

$$ES_{t,T}^{SW}(\alpha) = -\mathbb{E}_t^{\mathbb{Q}}\left[R_{t,T} \cdot I\left(R_{t,T} \leq -D_{t,T}^{SW}(\alpha)\right)\right], \tag{13}$$

We show that the ES swap rate can be calculated from option prices of the same horizon T.

Corollary 3 *Under no arbitrage, the time-t risk-neutral expected value of the ES at $\alpha\%$ significance level of an asset over horizon [t, T] defined in Equation (11) can be calculated from European out-of-the-money option prices across strikes $K > 0$ and at the same maturity T*

$$ES_{t,T}^{SW}(\alpha) = D_{t,T}^{SW}(\alpha) + \frac{100}{\alpha}\left[\frac{P\left(S_t e^{-D_{t,T}^{SW}(\alpha)}\right)}{S_t e^{-D_{t,T}^{SW}(\alpha)}} + \int_0^{S_t e^{-D_{t,T}^{SW}(\alpha)}} \frac{P(K)}{K^2} dK\right], \tag{14}$$

where $D_{t,T}^{SW}(\alpha)$ denotes the VaR swap threshold level at $\alpha\%$ percentile over $[t, T]$, and $P(K)$ a European put option price function of strike price K, and r denotes the risk-free rate over horizon [t, T].

The proof is provided in the Appendix.

2.2.3 Expected upside swap

Similarly, the expected upside at $\alpha\%$ significance level is defined as

$$EU_{t,T}(\alpha) = E\left[R_{t,T} \left| R_{t,T} \geq U_{t,T}(\alpha)\right.\right] = \frac{100}{\alpha}\int_{U_{t,T}(\alpha)}^{\infty} R \cdot f_R(R)\, dR. \tag{15}$$

Now, we consider an expected upside (EUP) swap. At maturity, the payoff to the long side of the swap is

$$R_{t,T} \cdot I\left(R_{t,T} \geq U_{t,T}(\alpha)\right) - EUP_{t,T}^{SW}(\alpha), \tag{16}$$

where $EUP_{t,T}^{SW}(\alpha)$ denotes the fixed EUP swap rate that is determined at time t and paid at time T. We fix the EUP swap threshold $U_{t,T}(\alpha)$ as the same with the UP swap threshold level $U_{t,T}^{SW}(\alpha)$. The no-arbitrage condition implies that the EUP swap rate equals the following risk-neutral expected value:

$$EUP_{t,T}^{SW}(\alpha) = \mathbb{E}_t^{\mathbb{Q}}\left[R_{t,T} \cdot I\left(R_{t,T} \geq U_{t,T}^{SW}(\alpha)\right)\right], \tag{17}$$

We can show that the EUP swap rate can be calculated from option prices of the same horizon T.

Corollary 4 *Under no arbitrage, the time-t risk-neutral expected value of the EUP at α% significance level of an asset over horizon [t, T] defined in Equation (15) can be calculated from European out-of-the-money option prices across strikes $K > 0$ and at the same maturity T*

$$EUP_{t,T}^{SW}(\alpha) = U_{t,T}^{SW}(\alpha) + \frac{100}{\alpha}\left[\frac{C\left(S_t e^{U_{t,T}^{SW}(\alpha)}\right)}{S_t e^{U_{t,T}^{SW}(\alpha)}} + \int_{S_t e^{U_{t,T}^{SW}(\alpha)}}^{\infty} \frac{C(K)}{K^2} dK\right], \tag{18}$$

where $U_{t,T}^{SW}(\alpha)$ denotes the UP swap threshold level at α% percentile over $[t, T]$, and $C(K)$ a European call option price function of strike price K, and r denotes the risk-free rate over horizon [t, T].

The proof is provided in the Appendix.

2.3 Tail risk premia

It is straightforward to derive tail risk premia from the above tail swap contracts as follows:

- VaR risk premium (VaRrp): The VaRrp is defined by

  $$VaR_{t,T}^{RP}(\alpha) = \mathbb{E}_t^{\mathbb{P}}\left[I\left(R_{t,T} \leq -D_{t,T}^{SW}(\alpha)\right)\right] - \alpha, \tag{19}$$

  where $\mathbb{E}_t^{\mathbb{P}}\left[I\left(R_{t,T} \leq -D_{t,T}^{SW}(\alpha)\right)\right]$ indicates the time-series conditional mean of the realized payoff to the long side of the VaR swap contract. The average VaR risk premium is directly estimated by the sample average of the difference between the realized payoff and the VaR swap rate:

  $$RVaR_{t,T}^{RP}(\alpha) = I\left(R_{t,T} \leq -D_{t,T}^{SW}(\alpha)\right) - \alpha. \tag{20}$$

- UP risk premium (UPrp): The UPrp is similarly defined as

  $$UP_{t,T}^{RP}(\alpha) = \mathbb{E}_t^{\mathbb{P}}\left[I\left(R_{t,T} \geq U_{t,T}^{SW}(\alpha)\right)\right] - \alpha. \tag{21}$$

- ES risk premium (ESrp): The ESrp is defined as

$$ES_{t,T}^{RP}(\alpha) = \mathbb{E}_t^{\mathbb{P}}\left[-R_{t,T} \cdot I\left(R_{t,T} \leq -D_{t,T}^{SW}(\alpha)\right)\right] - \alpha \cdot ES_{t,T}^{SW}(\alpha). \tag{22}$$

- EUP risk premium: The EUPrp is defined as

$$EUP_{t,T}^{RP}(\alpha) = \mathbb{E}_t^{\mathbb{P}}\left[R_{t,T} \cdot I\left(R_{t,T} \geq U_{t,T}^{SW}(\alpha)\right)\right] - \alpha \cdot EUP_{t,T}^{SW}(\alpha). \tag{23}$$

2.4 Tail risk indicators

As we fix the tail event probability, time variations of the threshold that determines the tail risk event would quantity tail risks. We will as tail risk indicators use the thresholds associated with VaR, UP, ES, and EUP swaps as follows:

- VaR-based tail risk indicator (VaRtr): $D_{t,T}^{SW}(\alpha)$.
- UP-based tail risk indicator (UPtr): $U_{t,T}^{SW}(\alpha)$.
- ES-based tail risk indicator (EStr): $ES_{t,T}^{SW}(\alpha)$.
- EUP-based tail risk indicator (EUPtr): $EUP_{t,T}^{SW}(\alpha)$.

In addition, the following two indicators represent how much downside risks are greater than upside uncertainty:

- Down-minus-up tail risk indicator (DMU): $D_{t,T}^{SW}(\alpha) - U_{t,T}^{SW}(\alpha)$.
- Expected down-minus-up tail risk indicator (EDMU): $ES_{t,T}^{SW}(\alpha) - EUP_{t,T}^{SW}(\alpha)$.

Lastly, we assume the VIX as the standard deviation of a normal distribution (with mean zero) and derive VIX-implied tail risk indicators: VIXVaRtr, VIXUPtr, VIXEStr and VIXEUPtr. Using these VIX-implied tail risk indicators, we introduce the following relative tail risk indicators:

- VaR-based tail risk difference indicator (VaRD): VaRtr $-$ VIXVaRtr.
- UP-based tail risk difference indicator (UPD): UPtr $-$ VIXUPtr.
- ES-based tail risk difference indicator (ESD): EStr $-$ VIXEStr.
- EUP-based tail risk difference indicator (EUPD): EUPtr $-$ VIXEUPtr.

A fatter tail represents a greater risk which is hardly captured by the VIX indicator. The above four relative tail risk indicators convey information about non-normal features of return distribution in the tail.

3 Data and methodology

We use OptionMetrics to obtain daily options data on the U.S. S&P500 index calls and puts from January 1996 to December 2015. To be consistent with the VIX, we choose a 30-day horizon for the synthetic tail swap contracts. At each trading date, we choose the two consecutive maturities which include the 30-day horizon within the range. We first calculate tail risk indicators at each maturity and then linearly interpolate them to obtain the corresponding 30-day horizon indicators.

At each maturity, we apply a filtering algorithm into option price data to obtain reasonable tail risk estimates. First, we delete option prices greater than $0.05 at the greatest strike price for calls or at the lowest strike price for puts, which leads to the last out-of-the-money (OTM) option prices of $0.05. Second, we allow at most two consecutive option prices of $0.05 at the end of option series by cutting off the extreme strike prices. Third, we repetitively eliminate option price data which violate monotonicity of option price and/or its first derivative. For comparison with the non-parametric method, we also apply Figlewski's (2008) semi-paramentric method to obtain tail risk indicators.[4]

[4]The Internet Appendix provides the Matlab codes to implement our method and Figlewski's method (2008).

4 Empirical analyses

In this section, we focus on downside tail risks and perform several empirical analyses. We make comparison between our non-parametric estimates and Figlewski's semi-paramentric estimates of the tail risk indicators. We also characterize tail risk premia and the behavior of tail risk indicators. Lastly, we examine whether tail risk indicators possess any additional predictive power for future returns and future tail risks in the presence of the VIX index.

4.1 Comparison between non-parametric and semi-parametric tail risk estimates

For comparison, we demonstrate the time trends of the VaRtr (Figure 1) and the EStr (Figure 2) obtained by our non-parametric method and the Figlewski's semi-paramentric method. Both tail risk estimates correspond to the tail event probability of 5% ($\alpha = 0.05$). Although both the non-parametric and the semi-paramentric methods largely yield similar estimates for both indicators, the semi-paramentric method provides estimates that seem to be not only unusually low levels in several days but also plagued with erratic fluctuations. Based on this observation, we choose the non-parametric method as the benchmark method.

4.2 Tail risk premia

Do investors price tail risks? If investors price tail risks, the sample averages of the realized payoffs to the tail swap contracts will differ from the corresponding swap rates. Figure 3 shows the time trend of the realized VaRrp (Figure 3) and the realized ESrp (Figure 4) along with the realized variance risk premium (VIXrp). As both tail risk swap contracts provide positive payoffs only when tail risk events occur, the realized tail risk premia exhibit patterns which are quite different from the realized variance risk premium. Table 1 shows summary statistics of various tail risk indicators. Both sample averages of the VaRrp and the ESrp are significantly negative. For the same reason, the sample proportion of positive tail risk premia is 2.2% which is less than the ex-ante tail risk probability of 5%. Clearly, investors are willing to accept a significantly negative average return to hedge tail risks.

Can common risk factors explain the tail risk premia? To investigate whether the tail risk premia are induced simply by the underlying return risk and return risk premium or represent an additional source of risk, we run asset pricing regressions of the tail risk premia on conventional common risk factors such as market portfolio's excess return, Fama-French (1993) size and boot-to-market ratio, Carhart's (1997) momentum (MOM), Fama and French's (2015) operating profitability and investment factors. In addition, we also consider the variance risk premium. Tables 2 and 3 report the regression results of the VaR and the ES tail risk premia on various combinations of common risk factors, respectively. For both tail risk premia, the risk-adjusted alphas (i.e., constant term in the regression) are significantly negative and are not much smaller than the average tail risk premia reported in Table 1. This result implies that additional risk factors are called for explaining the tail risk premia. The beta estimates of the market excess return are strongly negative in all cases. This result is related to the fact that a tail risk event occurs only when the index return sharply declines. While the Fama and French operating profitability factor is also significantly and negatively associated with the tail risk premia, other common risk factors are insignificant. Interestingly, although the beta estimates of the variance risk premium are significantly positive, the risk-adjusted alphas are not smaller even after incorporating the variance risk premium. In sum, not only the conventional common risk factors but also the variance risk factor fail to fully account for the negative tail risk premia on the stock index.

Are tail risk premia constant or time-varying? We run regressions based on the expectation hypothesis. Specifically, for the VaR tail risk swap, we run the following quantile regression:

$$Q_{-R_{t,T}|D^{SW}_{t,T}}(\alpha) = a + bD^{SW}_{t,T}(\alpha) + \epsilon_t, \tag{24}$$

where $Q_{-R_{t,T}|D^{SW}_{t,T}}(\alpha)$ is the α-th quantile function of the negative of the stock index returns conditional on the VaR tail risk indicator level. Under the null hypothesis of zero VaR tail risk premium, Equation (19) posits that we have $a = \alpha$, $b = 1$. In particular, the VaR risk premium would be time-varying and correlated with the VaR tail risk indicator level if b significantly deviates from one. Similarly, for the UP tail risk swap, we run a quantile regression as follows:

$$Q_{R_{t,T}|U_{t,T}^{SW}}(1-\alpha) = a + bU_{t,T}^{SW}(\alpha) + \epsilon_t. \tag{25}$$

On the other hand, for the ES tail risk swap, we run the following linear regression:

$$-R_{t,T} \cdot I\left(R_{t,T} \leq -D_{t,T}^{SW}(\alpha)\right) = a + bD_{t,T}^{SW}(\alpha) + \epsilon_t. \tag{26}$$

Under the null hypothesis of zero ES risk premium and the expectation hypothesis equation (22) , we would have $a = 0$, $b = 1$. Still, the slope estimate less than one implies time-varying risk premium. Similarly, for the EUP tail risk swap, we run a linear regression as follows:

$$R_{t,T} \cdot I\left(R_{t,T} \geq U_{t,T}^{SW}(\alpha)\right) = a + bU_{t,T}^{SW}(\alpha) + \epsilon_t. \tag{27}$$

Table 4 reports the estimation results of the four expectation hypothesis regressions (24) to (27) . Interestingly, there exists an asymmetric time variation between downside and upside tail risk premiums. Specifically, the slopes of the VaRrp and the ESrp are significantly less than one (0.685 and 0.443, respectively), whereas the slopes of the UPrp and the EUPrp are slightly less than one (0.941 and 0.827, respectively). It implies that downside tail risk premia exhibit a more conspicuous time-varying feature than upside risk premia.

4.3 Behavior of tail risk indicators

Table 1 reports summary statistics of six tail risk indicators, that is, the VaRtr, the EStr, the VaRD, the ESD, the DMU, and the EDMU, and their time trends are demonstrated along with the VIX index in Figures 5 to 10, respectively. Table 5 shows correlations of the tail risk indicators.

Do the tail risk indicators behave differently from the VIX index? Figures 5 and 6 show that both the VaRtr and the EStr closely co-move with the VIX index. Indeed, correlation coefficients between the VIX index and the VaRtr and the EStr are 0.965 and 0.981, respectively. However, if we measure relative tail risks by subtracting the VIX-implied tail risks under a normality assumption, the relative tail risk indicators show behaviors which are quite different from that of the VIX index. Figures 7 and 8 show that the VaRD and the ESD behave differently not only from the VIX index but also each other. Correlation coefficients between the VIX index and the VaRD and the ESD are reduced to 0.281 and 0.751, respectively, and the VaRD and the ESD are also weakly correlated (with correlation coefficient of 0.327). On the other hand, information from asymmetric changes in upside and downside risks are relatively closely related with the VIX index, compared to the VaRD or the ESD indicator. Figures 9 and 10 illustrate the DMU and the EDMU along with the VIX index. Correlation coefficients between the VIX and the DMU and the EDMU are 0.765 and 0.837, respectively, which are lower than those of the VaRtr and the EStr but higher than those of the VaRD and the ESD. Both are also strongly correlated each other (with correlation coefficient of 0.808). In sum, whereas the VaRtr and the EStr behave similarly with the VIX index, the VaRD and the ESD behave differently not only from the VIX index but also each other. The DMU and the EDMU lie in between the two groups of tail indicators.

Do the tail risk indicators gauge "investor fear"? It is well documented that when investor fear heightens, increased hedging demand for index puts affects the VIX index level. Specifically, the VIX index rises at a faster rate when the stock market falls than when it rises. This relation is called as "leverage effect", and thus the VIX index is an "investor fear gauge". To investigate whether the tail risk indicators also respond more sensitively to a stock market decline than its rise, we run the following linear regressions:

$$Y_t = \beta_0 + \beta_1 R_t + \beta_2 R_t^- + \varepsilon_t, \tag{28}$$

where R_t denotes daily S&P500 index return, and R_t^- takes only negative return or zero otherwise, i.e., $R_t^- \equiv \min\{R_t, 0\}$.[5] We regress daily changes of VaRtr ($\Delta VaRtr_t$) and EStr ($\Delta EStr_t$), and daily VaRD, ESD, DMU, and EDMU using (28). Table 6 shows the regression results not only for the tail risk indicators but also for the VIX index. Consistent with the leverage effect, the coefficient β_2 is significantly negative for daily changes of VIX (ΔVIX_t), implying that the VIX index rises at a faster rate during a market down turn. Similar results are also found for all of the tail risk indicators (although the coefficient β_2 for daily changes of the EStr is statistically insignificant). It suggests that the tail risk indicators also gauge

[5] This regression equation is similar to that used in Whaley (2009).

the “investor fear”.

4.4 Return predictability

Do tail risk indicators possess any predictive power of future stock returns? To empirically evaluate the predictive power, we run the following linear predictive regressions:

$$R_{t+1,t+h} = \beta_0 + \beta_1 X_t + \beta_2 \Delta X_t + \varepsilon_{t+1}, \tag{29}$$

where $R_{t+1,t+h}$ denotes a $h-$horizon future stock return, X_t a time-t predictor variable, ΔX_t daily change in X. Using the predictive regression model (29), we investigate whether the VIX, the VaRtr, and the EStr possess any predictive power on future stock index returns. Table 7 (Panel A) reports the regression results for various horizons: 1-, 2-, 3-, 4-week, and 60-day. While the VIX and the VIX innovations are positively associated with future S&P 500 index returns, the positive relation is statistically significant only for the VIX innovation.[6] Similar results are obtained for the EStr and the VaRtr although the predictive power of the VaRtr is statistically insignificant.

To examine whether the tail risk indicators possess additional information to forecast future stock index returns, we employ the following linear predictive regressions:

$$R_{t+1,t+h} = \beta_0 + \beta_1 VIX_t + \beta_2 \Delta VIX_t + \beta_3 TRI_t + \varepsilon_{t+1}, \tag{30}$$

where TRI_t denotes a time-t tail risk indicator. Table 7 (Panel B) shows the estimation results of the regression (30) for each of the four tail risk indicators. Although current tail risk indicators are positively associated with future stock index returns in the presence of the VIX, their predictive ability is statistically insignificant.

As economic agents dislike tail risks, they require additional risk premium for holding high tail-risk assets. Therefore, we expect that a high level of a tail risk indicator corresponds to a high equity risk premium, or high equity return if tail risks are not realized. However, when

[6] This result is largely consistent with prior studies. Refer to, for example, Banerjee, Doran, and Peterson (2007).

crash occurs, the realized future return will be negative. As the size of the tail risk indicators reflect the strength of this expectation, a higher tail risk indicator would correspond to a positive and higher future equity return in case of no-crash but a negative and lower future return when crash occurs. To empirically evaluate this hypothesis, we use the following linear explanatory regressions:

$$R_{t+1,t+h} = \beta_0 + \beta_1 VIX_t + \beta_2 \Delta VIX_t + \beta_3 TRI_t \times Dum^{\perp}_{t+1,t+h} + \beta_4 TRI_t \times Dum_{t+1,t+h} + \varepsilon_{t+1}, \tag{31}$$

where $Dum_{t+1,t+h}$ denotes a dummy variable which takes one when crash occurs during h-horizon and zero otherwise, and $Dum^{\perp}_{t+1,t+h}$ a dummy variable orthogonal to $Dum_{t+1,t+h}$ (i.e., $Dum^{\perp}_{t+1,t+h} = 1 - Dum_{t+1,t+h}$).[7] The coefficients β_3 and β_4 in the regression (31) indicate the relationship between a current tail risk indicator and future returns when crash does not occur and when crash occurs, respectively. Table 7 (Panel C) shows the estimated β_3 and β_4 of the regression (31) for each of the four tail risk indicators. In most cases, the hypothesis is statistically confirmed. Current tail risk indicators are significantly and positively associated with future stock index returns when crash does not occur but significantly and negatively associated with future stock index returns when crash occurs.[8]

To check the robustness of the conditional relationship between current tail risk indicators and future equity returns, we additionally analyze three sets of six (i.e., 18 in total) portfolio returns: 2×3 size-by-book-to-market, size-by-operating-profitability, and size-by-investment.[9] Table 8 reports the explanatory regression results of 30-day portfolio returns.[10] Main findings are as follows: First, current tail risk indicators are positively associated with future portfolio returns during normal periods but negatively associated during crash periods in most cases. Second, the tail risk indicators differ with the statistical significance of the conditional relationship. The VaRD has a statistical insignificance of the conditional relationship during crash periods while the conditional relationship for the EDMU is insignif-

[7] The crash is defined as an event when a return is less than the tenth percentile of the sample distribution of returns.

[8] This result is largely consistent with Vilkov and Xiao (2012).

[9] The data for portfolio returns and risk factors are retrieved at Kenneth French's data library.

[10] For 60-day horizon, we also obtain similar results which are available upon request.

icant during normal periods. Third, the conditional relationship during normal periods is significantly observed only for big-sized stocks. We guess that this fact is attributable to relatively stable returns of big stocks.

4.5 Risk predictability

Do tail risk indicators possess any forecasting power for future tail events? To empirically assess the forecasting power, we perform the following logistic predictive regressions:

$$\ln \frac{p_{t+1,t+h}}{1-p_{t+1,t+h}} = \beta_0 + \beta_1 TRI_t + \varepsilon_{t+1}, \tag{32}$$

$$\ln \frac{p_{t+1,t+h}}{1-p_{t+1,t+h}} = \beta_0 + \beta_1 TRI_t^{\perp} + \beta_2 VIX_t + \varepsilon_{t+1}, \tag{33}$$

$$p_{t+1,t+h} \equiv \Pr\left[R_{t+1,t+h} \leq \underline{R}\right] = \Pr\left[Dum_{t+1,t+h} = 1\right], \tag{34}$$

where $Dum_{t+1,t+h}$ denotes a dummy variable indicating a tail event which occurs when a portfolio return is less than a threshold level $\underline{R}$.

Similar to the above return predictability regressions, we consider the same four tail risk indicators, the same sets of portfolio returns, and 30-day horizon. We choose the 10th percentile of historical portfolio returns as the threshold portfolio return level. Table 9 shows the logistic predictive regression results of 30-day portfolio returns. The results are summarized as follows: First, the results from the regression (32) show that increased current tail risks tend to predict a higher probability of future tail risk events. Second, the regression (33) shows that an inclusion of the VIX as a predictor variable not only lowers the forecasting power of the tail risk indicators but also even reverses the predictive direction in some cases. For example, the forecasting power of the VaRD becomes insignificant after the VIX is included. The other three indicators show that their predictive direction becomes negative in several cases. This result is understandable as each of the four tail risk indicators is positively correlated with the VIX. Third, although the VaRD loses its forecasting power in the presence of the VIX, the other three indicators still possess their forecasting powers. The ESD, the EDMU, and the DMU are statistically significant in 13, 13, and 7 cases (out of 18 cases) even after controlling the forecasting power of the VIX.

Do tail risk indicators possess any predictive power of future ex ante tail risks? As the VIX index is used as an underlying price upon which various derivatives are written, it might be useful for investments to accurately predict future VIX index. In a similar vein, it might be also useful to accurately predict future tail risk indicators, although tail risk indicators are not used in practice yet. To examine the predictive power, we run the following linear predictive regressions:

$$Y_{t+h} = \beta_0 + \beta_1 TRI_t^{\perp} + \beta_2 VIX_t + \beta_3 Y_t + \varepsilon_{t+1}, \quad (35)$$

$$Y_{t+h} = \beta_0 + \beta_1 TRI_t^{\perp} + \beta_2 VIX_t + \varepsilon_{t+1}, \quad (36)$$

where Y_{t+h} denotes an $h-$horizon future ex ante tail risk indicator. We include as a predictor variable each of the same four tail risk indicators (TRIs) orthogonal to the VIX and investigate whether it has any additional predictive power. In addition to the two tail risk indicators (VaRtr and EStr) to predict, we also consider the VIX index for comparison and for its usefulness in the VIX-related derivatives trading. Four prediction horizons are considered: 1, 2, 3, and 4 weeks. We will apply the model (36) into the prediction of a future VIX index and the model (35) otherwise.

Table 10 shows the results of the linear predictive regressions. For the prediction of the two ex ante tail risk indicators, both the ESD and the EDMU posses significant additional predictive power for all horizons. Specifically, an increase in both indicators tend to predict an increase in future ex ante tail risks. The VaRD possesses the predictive power for the VaRtr over all horizons and for the EStr only over 1- and 2-week horizons. The DMU has the predictive power only for the EStr over 1-, 2- and 3-week horizon. However, for the prediction of the VIX index, the four tail risk indicators possess insignificant predictive powers in many cases.

5 Conclusion

In this paper, we use the notion of a value-at-risk swap whose payoff depends upon a prescribed tail risk event to derive left tail risk premium and tail risk indicators. In addition to the VaR, we also consider similar swaps based on the expected-shortfall. Based on an asymmetry between downside risks and upside uncertainty, we also devise alternative swaps using upside uncertainties. We use those tail risk swaps to propose several tail risk indicators which are model-free and implied from equity index option prices.

We find that investors price tail risks and are willing to accept a significantly negative average return to hedge tail risks. Moreover, the tail risk premia are not induced simply by the underlying return risk and return risk premium but represent an additional source of risk. We also find that tail risk premia, specifically downside tail risk premia rather than upside premia, are time-varying.

We show that all of the tail risk indicators also rise at a faster rate during a market downturn and thus gauge the "investor fear". We also investigate information contents conveyed by tail risk indicators and find that they possess predictive powers not only for future returns but also for future realized or ex ante tail risks.

The development of the VIX index has significantly contributed to the importance of volatility or variance risk in research and in practice as well. Similar with the success of the VIX index, we expect that the proposed option-implied model-free tail risk indicators would contribute to a more extensive usage of tail risks in various directions.

Appendix

A1. Proof of Proposition 1.

We begin with the Breeden-Litzenberger (1978) formula:

$$f^{\mathbb{Q}}(K) = e^{r(T-t)} P''(K), \tag{37}$$

where $f^{\mathbb{Q}}(\cdot)$ indicates a risk-neutral pdf of S_T under some risk-neutral measure $\mathbb{Q}$, $P(K)$ is a European put option prices with strike price K (and the current underlying asset price S_t), and $P''(K)$ denotes its second derivative. Under the risk-neutral measure $\mathbb{Q}$, simple algebraic manipulation yields

$$\begin{aligned}
\Pr{}^{\mathbb{Q}}\left[R_{t,T} \leq -D_{t,T}^{SW}(\alpha)\right] &= \Pr{}^{\mathbb{Q}}\left[S_T \leq S_t \cdot e^{-D_{t,T}^{SW}(\alpha)}\right] \\
&= \int_0^{S_t \cdot e^{-D_{t,T}^{SW}(\alpha)}} f^{\mathbb{Q}}(K)\, dK \\
&= e^{r(T-t)} \int_0^{S_t \cdot e^{-D_{t,T}^{SW}(\alpha)}} P''(K)\, dK \\
&= e^{r(T-t)} P'\left(S_t \cdot e^{-D_{t,T}^{SW}(\alpha)}\right) - e^{r\tau} P'(0) \\
&= e^{r(T-t)} P'\left(S_t \cdot e^{-D_{t,T}^{SW}(\alpha)}\right),
\end{aligned}$$

which leads to

$$-D_{t,T}^{SW}(\alpha) = \log P'^{-1}\left(e^{-r(T-t)} \frac{\alpha}{100}\right) - \log S_t. \tag{38}$$

A2. Proof of Corollary 2.

We begin with the Breeden-Litzenberger formula:

$$f^{\mathbb{Q}}(K) = e^{r(T-t)} C''(K), \tag{39}$$

where $f^{\mathbb{Q}}(\cdot)$ indicates a risk-neutral pdf of S_T, $C(K)$ is a European call option prices with strike price K (and the current underlying asset price S_t), and $C''(K)$ denotes its second derivative. Under some risk-neutral measure $\mathbb{Q}$, simple algebraic manipulation yields

$$\begin{aligned}
\Pr{}^{\mathbb{Q}}\left[R_{t,T} \geq U_{t,T}^{SW}(\alpha)\right] &= \Pr{}^{\mathbb{Q}}\left[S_T \geq S_t \cdot e^{U_{t,T}^{SW}(\alpha)}\right] \\
&= \int_{S_t \cdot e^{U_{t,T}^{SW}(\alpha)}}^{\infty} f^{\mathbb{Q}}(K)\, dK \\
&= e^{r(T-t)} \int_{S_t \cdot e^{U_{t,T}^{SW}(\alpha)}}^{\infty} C''(K)\, dK \\
&= e^{r(T-t)} C'(\infty) - e^{r(T-t)} C'\left(S_t \cdot e^{U_{t,T}^{SW}(\alpha)}\right) \\
&= -e^{r(T-t)} C'\left(S_t \cdot e U_{t,T}^{SW}(\alpha)\right),
\end{aligned}$$

which leads to

$$U_{t,T}^{SW}(\alpha) = \log C'^{-1}\left(-e^{-r(T-t)} \frac{\alpha}{100}\right) - \log S_t. \tag{40}$$

A3. Proof of Corollary 3.

Note that from the change of variables,

$$\begin{aligned}
R &= \log S_T - \log S_t, \\
dR &= \frac{1}{S_T} dS_T, \\
\Pr{}^{\mathbb{Q}}[R < x] &= \Pr{}^{\mathbb{Q}}[S_T < S_t e^x], \\
F_R^{\mathbb{Q}}(x) &= F^{\mathbb{Q}}(S_t e^x), \\
f_R^{\mathbb{Q}}(x) &= f^{\mathbb{Q}}(S_T) S_T,
\end{aligned}$$

we obtain

$$
\begin{aligned}
\int_{-\infty}^{-D_{t,T}^{SW}(\alpha)} R\cdot f_R^{\mathbb{Q}}(R)\,dR &= \int_0^{S_t e D_{t,T}^{SW}(\alpha)} \log K\cdot f^{\mathbb{Q}}(K)\,dK - \log S_t \int_0^{S_t e^{-D_{t,T}^{SW}(\alpha)}} f^{\mathbb{Q}}(K)\,dK \\
&= e^{r(T-t)}\int_0^{S_t e^{-D_{t,T}^{SW}(\alpha)}} \log K\cdot P''(K)\,dK - \frac{\alpha}{100}\log S_t \\
&= e^{r(T-t)}\cdot \log K\cdot P'(K)\big|_0^{S_t e^{-D_{t,T}^{SW}(\alpha)}} \\
&\quad -\int_0^{S_t e^{-D_{t,T}^{SW}(\alpha)}} K^{-1}P'(K)\,dK - \frac{\alpha}{100}\log S_t \\
&= -\frac{\alpha}{100}D_{t,T}^{SW}(\alpha) - \int_0^{S_t e^{-D_{t,T}^{SW}(\alpha)}} K^{-1}P'(K)\,dK \\
&= -\frac{\alpha}{100}D_{t,T}^{SW}(\alpha) - S_t^{-1}e^{D_{t,T}^{SW}(\alpha)}P\left(S_t e^{-D_{t,T}^{SW}(\alpha)}\right) \\
&\quad -\int_0^{S_t e^{-D_{t,T}^{SW}(\alpha)}} K^{-2}P(K)\,dK,
\end{aligned}
$$

which leads to

$$
ES_{t,T}^{SW}(\alpha) = D_{t,T}^{SW}(\alpha) + \frac{100}{\alpha}\left[\frac{P\left(S_t e^{-D_{t,T}^{SW}(\alpha)}\right)}{S_t e^{-D_{t,T}^{SW}(\alpha)}} + \int_0^{S_t e^{-D_{t,T}^{SW}(\alpha)}} \frac{P(K)}{K^2}dK\right].
$$

A4. Proof of Corollary 4.

From the change of variables and simple algebraic manipulation, we obtain

$$
\begin{aligned}
\int_{U_{t,T}^{SW}(\alpha)}^{\infty} R \cdot f_R^{\mathbb{Q}}(R)\, dR &= \int_{S_t e^{U_{t,T}^{SW}(\alpha)}}^{\infty} \log K \cdot f^{\mathbb{Q}}(K)\, dK - \log S_t \int_{S_t e U_{t,T}^{SW}(\alpha)}^{\infty} f^{\mathbb{Q}}(K)\, dK \\
&= e^{r(T-t)} \int_{S_t e^{U_{t,T}^{SW}(\alpha)}}^{\infty} \log K \cdot C''(K)\, dK - \frac{\alpha}{100} \log S_t \\
&= e^{r(T-t)} \cdot \log K \cdot C'(K) \Big|_{S_t e^{U_{t,T}^{SW}(\alpha)}}^{\infty} \\
&\quad - \int_{S_t e^{U_{t,T}^{SW}(\alpha)}}^{\infty} K^{-1} C'(K)\, dK - \frac{\alpha}{100} \log S_t \\
&= \frac{\alpha}{100} U_{t,T}^{SW}(\alpha) - \int_{S_t e^{U_{t,T}^{SW}(\alpha)}}^{\infty} K^{-1} C'(K)\, dK \\
&= \frac{\alpha}{100} U_{t,T}^{SW}(\alpha) + \frac{C\left(S_t e^{U_{t,T}^{SW}(\alpha)}\right)}{S_t e^{U_{t,T}^{SW}(\alpha)}} \\
&\quad + \int_{S_t e^{U_{t,T}^{SW}(\alpha)}}^{\infty} K^{-2} C(K)\, dK,
\end{aligned}
$$

which leads to

$$
EUP_{t,T}^{SW}(\alpha) = U_{t,T}^{SW}(\alpha) + \frac{100}{\alpha} \left[\frac{C\left(S_t e^{U_{t,T}^{SW}(\alpha)}\right)}{S_t e^{U_{t,T}^{SW}(\alpha)}} + \int_{S_t e^{U_{t,T}^{SW}(\alpha)}}^{\infty} \frac{C(K)}{K^2} dK \right].
$$

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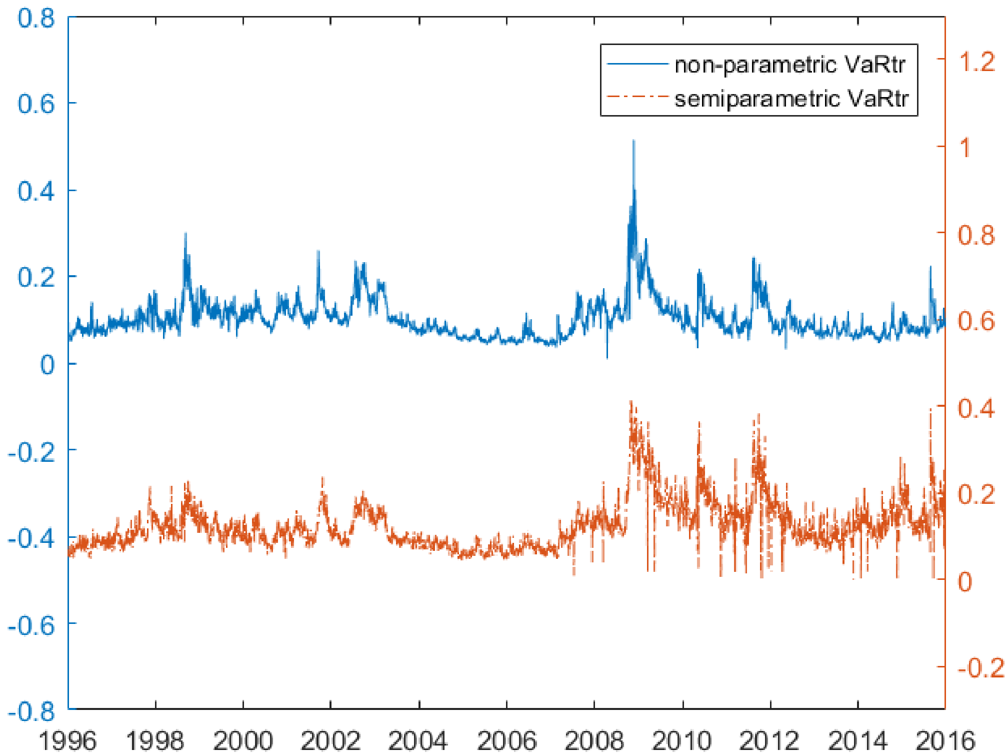

Figure 1. Time trend of the VaR-based tail risk indicators: non-parametric vs. semi-parametric method.

This figure shows the time trend of the VaR-based tail risk indicator (VaRtr) obtained by the non-parametric method (above line and left y-axis) and the Figlewski's semi-paramentric method (below line and right y-axis). The VaRtr corresponds to the tail event probability of 5% ($\alpha = 0.05$).

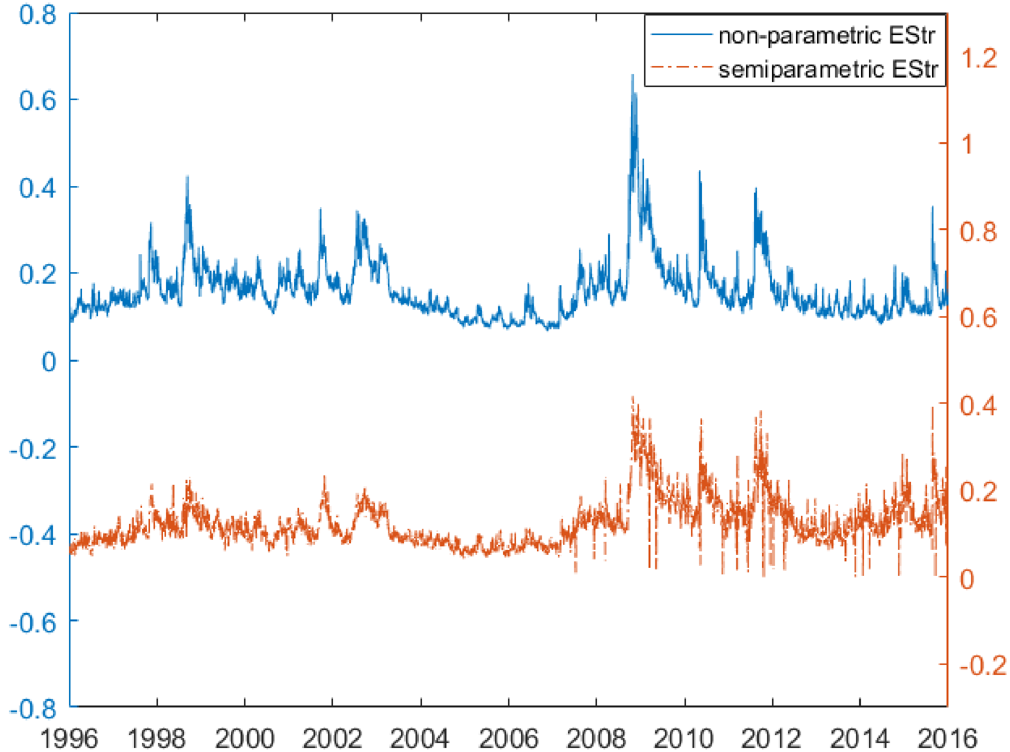

Figure 2. Time trend of the ES-based tail risk indicators: non-parametric vs. semi-parametric method.

This figure shows the time trend of the ES-based tail risk indicator (EStr) obtained by the non-parametric method (above line and left y-axis) and the Figlewski's semi-paramentric method (below line and right y-axis). The EStr corresponds to the tail event probability of 5% ($\alpha = 0.05$).

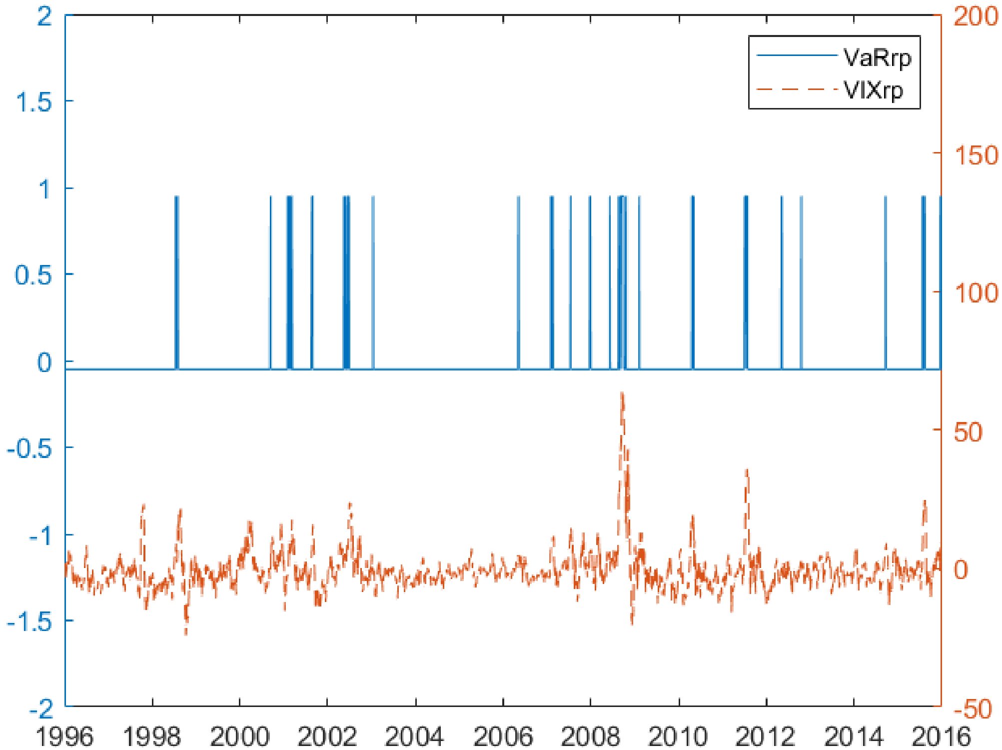

Figure 3. Time trend of the realized VaR risk premium.

This figure shows the time trend of the realized VaR risk premium (VaRrp, above line and left y-axis) and also the realized VIX risk premium (VIXrp, below line and right y-axis). The VaRrp corresponds to the tail event probability of 5% ($\alpha = 0.05$).

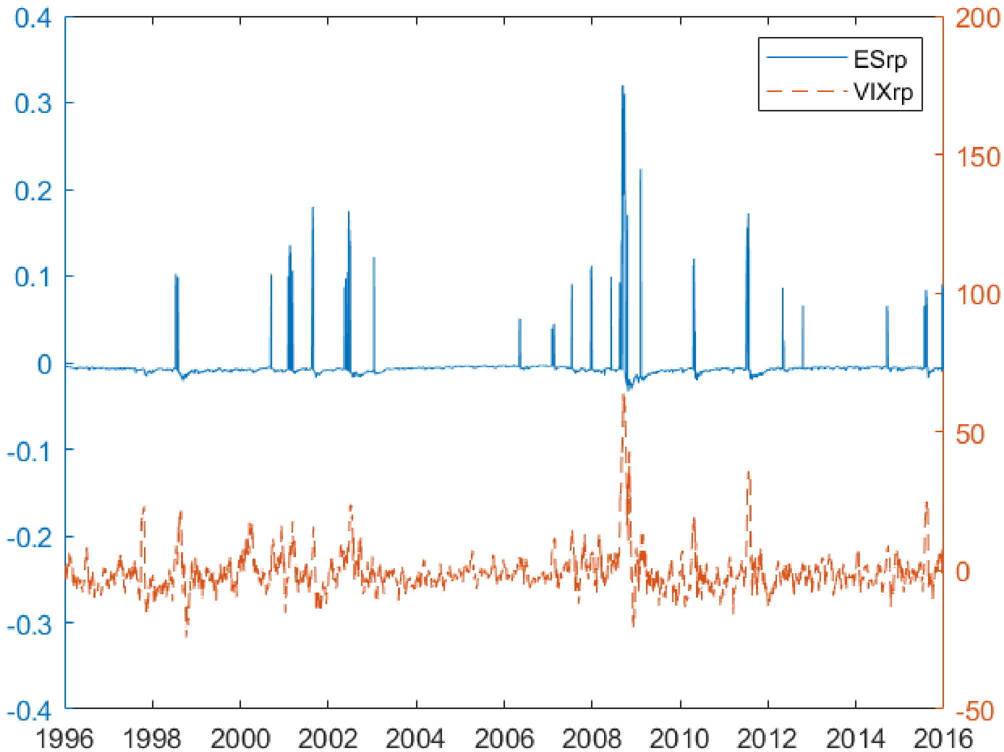

Figure 4. Time trend of the realized ES risk premium.

This figure shows the time trend of the realized ES risk premium (ESrp, above line and left y-axis) and also the realized VIX risk premium (VIXrp, below line and right y-axis). The ESrp corresponds to the tail event probability of 5% ($\alpha = 0.05$).

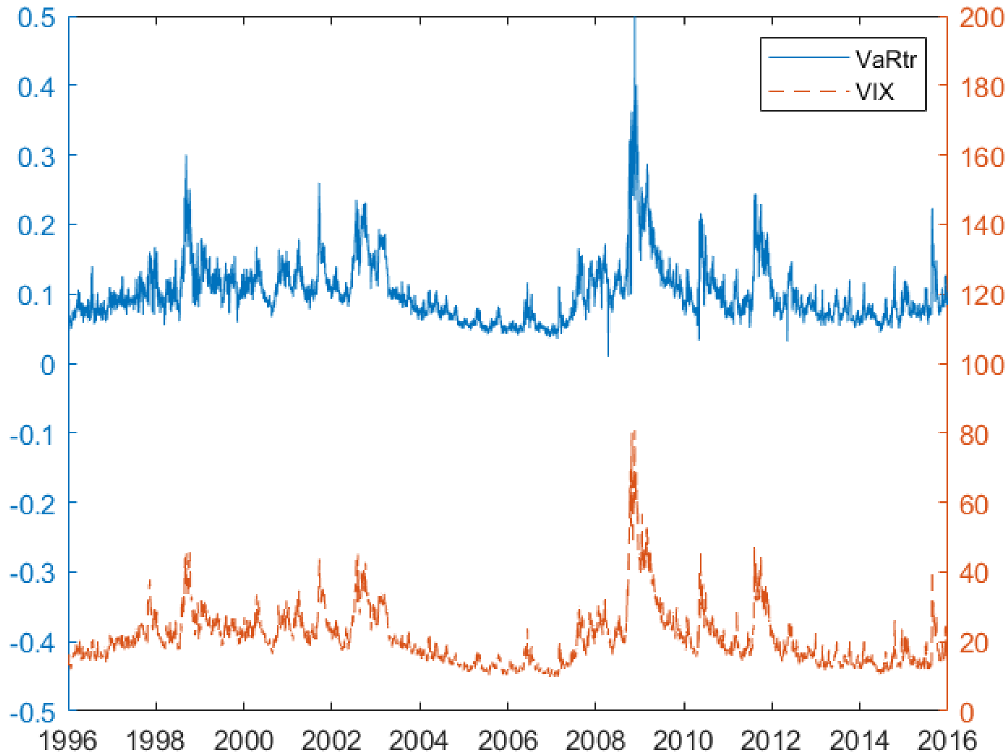

Figure 5. Time trend of the VaR-based tail risk indicator.

This figure shows the time trend of the VaR-based tail risk indicator (VaRtr, above line and left y-axis) and also the VIX index (VIX, below line and right y-axis). The VaRtr corresponds to the tail event probability of 5% ($\alpha = 0.05$).

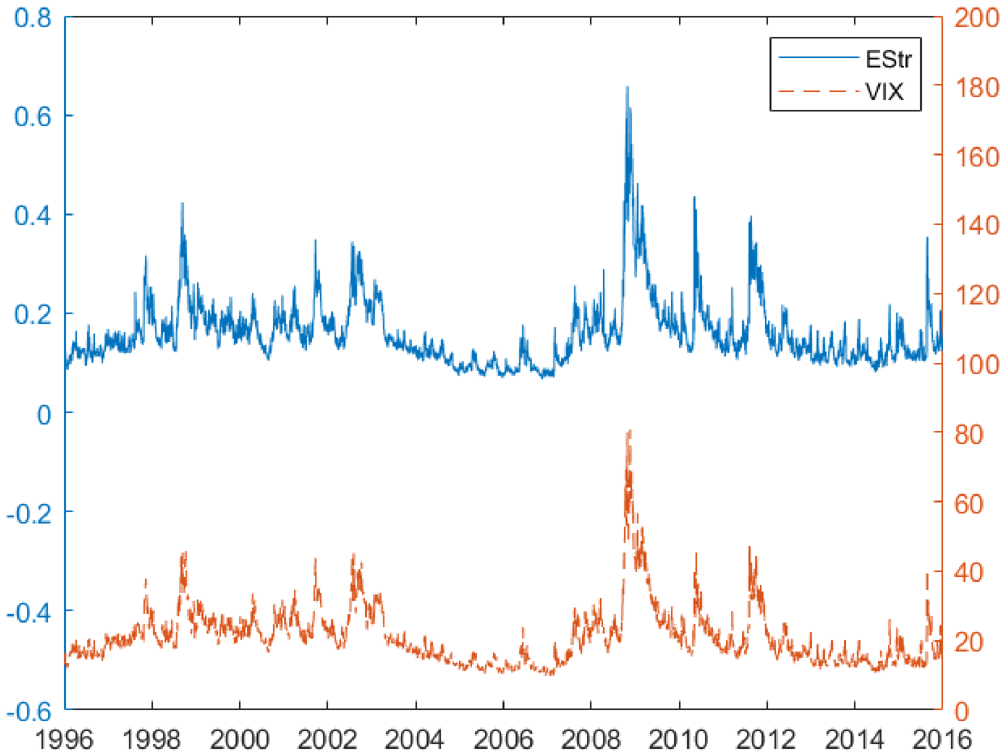

Figure 6. Time trend of the ES-based tail risk indicator.

This figure shows the time trend of the ES-based tail risk indicator (EStr, above line and left y-axis) and also the VIX index (VIX, below line and right y-axis). The EStr corresponds to the tail event probability of 5% ($\alpha = 0.05$).

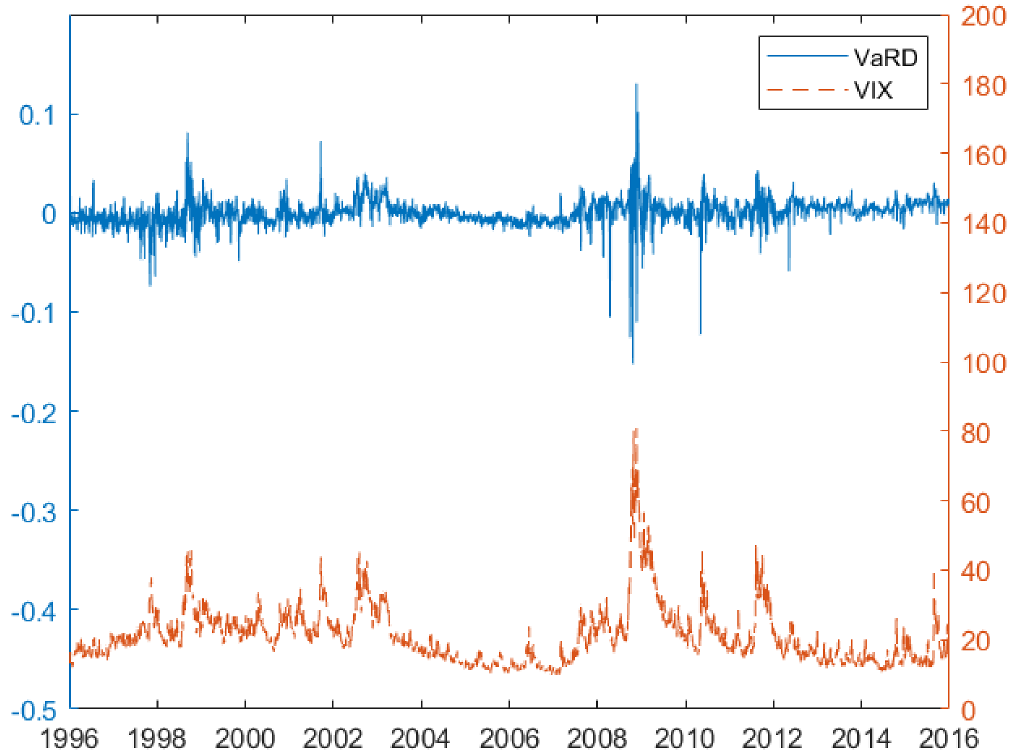

Figure 7. Time trend of the VaR-based tail risk difference indicator.

This figure shows the time trend of the VaR-based tail risk difference indicator (VaRD, above line and left y-axis) and also the VIX index (VIX, below line and right y-axis). The VaRD is the difference between the VaRtr and the VIX-implied VaR level under normality assumption and corresponds to the tail event probability of 5% ($\alpha = 0.05$).

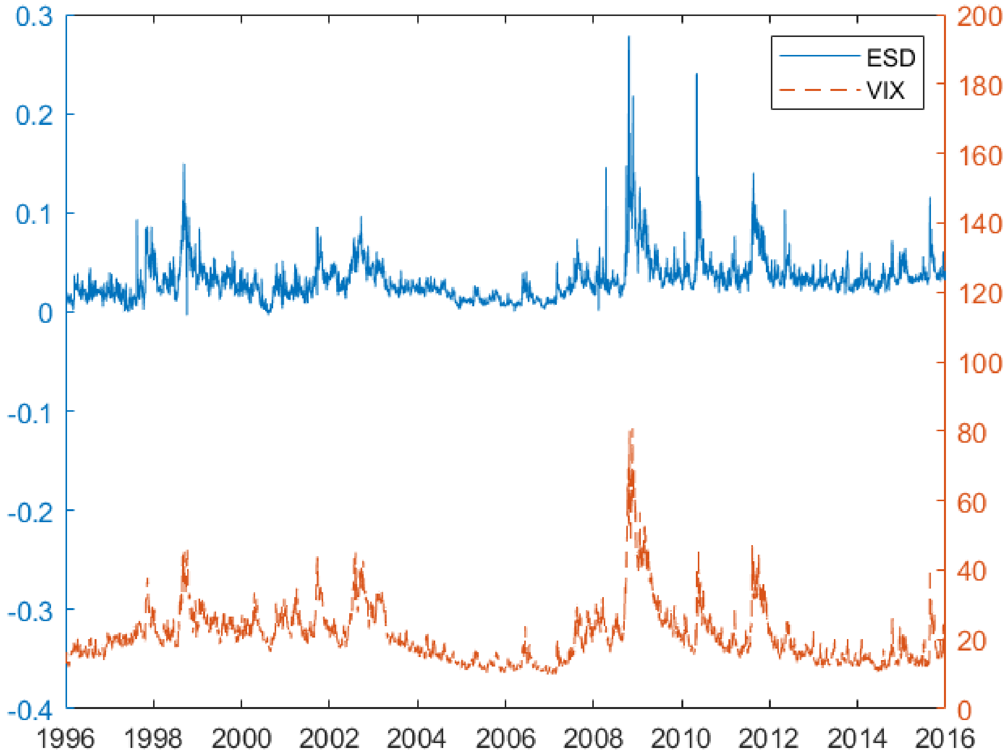

Figure 8. Time trend of the ES-based tail risk difference indicator.

This figure shows the time trend of the ES-based tail risk difference indicator (ESD, above line and left y-axis) and also the VIX index (VIX, below line and right y-axis). The ESD is the difference between the EStr and the VIX-implied ES level under normality assumption and corresponds to the tail event probability of 5% ($\alpha = 0.05$).

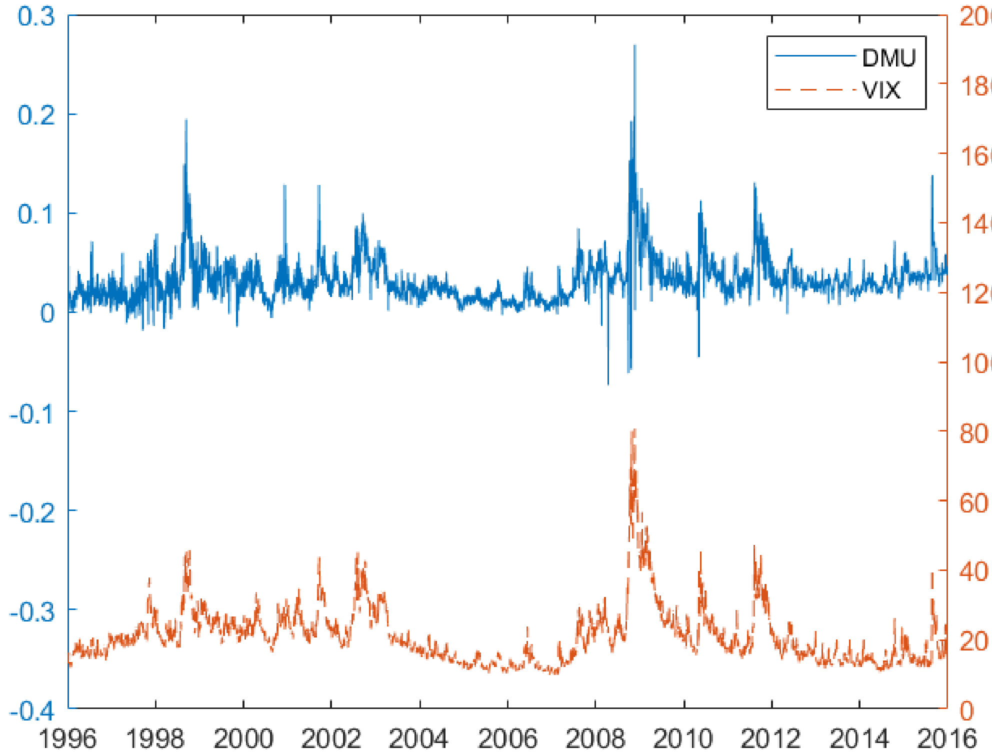

Figure 9. Time trend of the down-minus-up tail risk indicator.

This figure shows the time trend of the down-minus-up tail risk indicator (DMU, above line and left y-axis) and also the VIX index (VIX, below line and right y-axis). The DMU is the difference between the VaRtr and the UPtr and corresponds to the tail event probability of 5% ($\alpha = 0.05$).

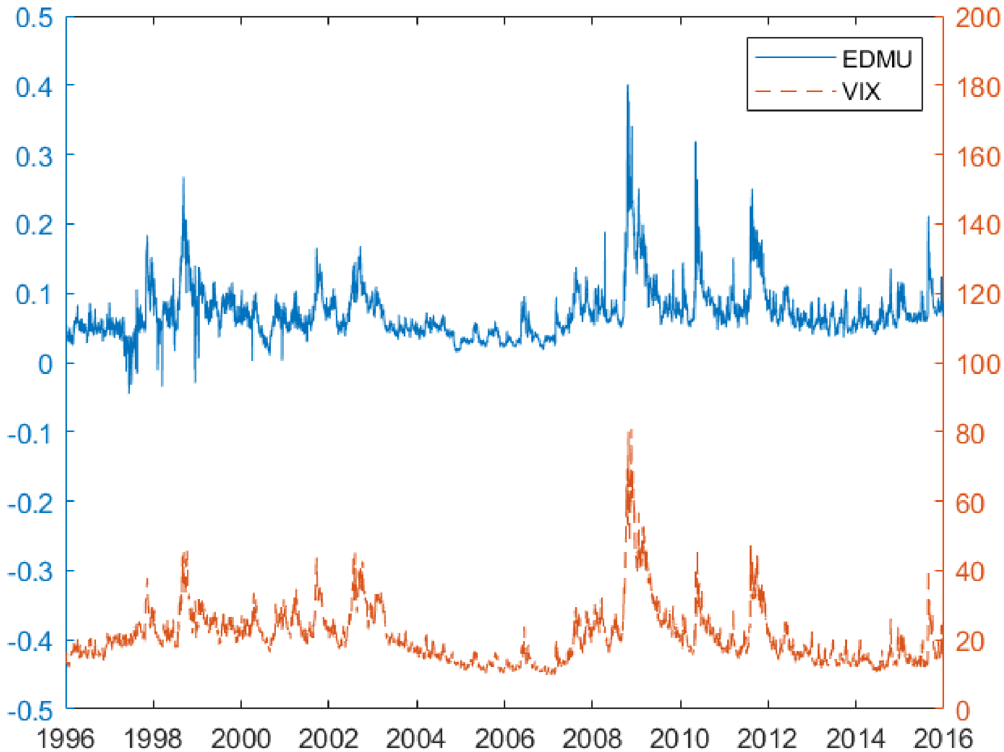

Figure 10. Time trend of the expected down-minus-up tail risk indicator.

This figure shows the time trend of the expected down-minus-up tail risk indicator (EDMU, above line and left y-axis) and also the VIX index (VIX, below line and right y-axis). The EDMU is the difference between the EStr and the EUPtr and corresponds to the tail event probability of 5% ($\alpha = 0.05$).

Table 1. Summary statistics of various tail risk indicators.

This table shows summary statistics of various tail risk indicators, including the sample mean, standard deviation, skewness, kurtosis, daily autocorrelation, and t-value. This table also reports percentiles and the proportion of negative values (Pr(<0)).

Indicator	Mean	S.d.	Skewness	Kurtosis	AC(1)	t-val
VIXtr	20.902	8.163	1.93	9.56	0.98	172.23
VaRtr	0.102	0.044	2.01	10.49	0.94	156.29
EStr	0.159	0.065	2.11	10.57	0.97	164.55
VaRD	0.000	0.012	-0.47	21.29	0.50	1.52
ESD	0.032	0.020	2.49	17.17	0.88	107.40
DMU	0.032	0.021	2.06	13.16	0.79	100.34
EDMU	0.071	0.037	2.06	10.97	0.93	128.78
VIXrp	-0.529	7.395	2.88	19.34	0.96	-4.81
VaRrp	-0.028	0.147	6.50	43.27	0.60	-12.77
ESrp	-0.005	0.022	8.56	89.05	0.76	-14.90
			Percentile			Pr(<0)
	0.05	0.1	0.5	0.9	0.95	
VIXtr	11.79	12.67	19.48	30.57	35.955	0.000
VaRtr	0.052	0.058	0.093	0.153	0.182	0.000
EStr	0.084	0.096	0.144	0.233	0.284	0.000
VaRD	-0.015	-0.012	0.000	0.013	0.019	0.511
ESD	0.008	0.011	0.029	0.053	0.069	0.002
DMU	0.007	0.011	0.028	0.056	0.070	0.009
EDMU	0.028	0.035	0.063	0.111	0.144	0.003
VIXrp	-8.892	-7.038	-1.577	6.174	10.811	0.645
VaRrp	-0.050	-0.050	-0.050	-0.050	-0.050	0.978
ESrp	-0.014	-0.012	-0.007	-0.005	-0.004	0.978

Table 2. Asset pricing regressions of the VaR risk premium.

This table shows the results of linear time-series regressions of the VaR risk premium (VaRrp) on various asset pricing factors, including market portfolio's excess return (MKT), Fama-French's (1993) size (SMB) and boot-to-market ratio (HML), Carhart's (1998) momentum (MOM), Fama and French's (2015) operating profitability (RMW) and investment (CMA) factors. In addition, the variance risk premium (VIXrp) is also considered. The t-values are obtained according to Newey and West (1987). R^2 is the unadjusted R-squared.

	Model 1		Model 2		Model 3		Model 4	
	coef	t-val	coef	t-val	coef	t-val	coef	t-val
const	-0.021	-3.737	-0.021	-3.663	-0.019	-2.976	-0.017	-2.390
MKT	-1.192	-4.437	-1.186	-4.469	-1.275	-4.480	-1.385	-4.281
SMB			-0.045	-0.347	-0.015	-0.116	-0.277	-1.570
HML			-0.021	-0.119	-0.120	-0.578	0.332	1.552
MOM					-0.206	-1.648		
RMW							-0.670	-2.595
CMA							-0.373	-1.150
R^2	0.164		0.164		0.168		0.175	
	Model 5		Model 6		Model 7		Model 8	
	coef	t-val	coef	t-val	coef	t-val	coef	t-val
const	-0.021	-4.415	-0.021	-4.489	-0.020	-3.879	-0.018	-3.252
MKT	-0.770	-4.590	-0.776	-4.610	-0.834	-4.485	-0.942	-4.502
SMB			0.064	0.500	0.077	0.605	-0.145	-1.017
HML			0.023	0.139	-0.030	-0.152	0.269	1.394
MOM					-0.107	-0.957		
RMW							-0.572	-2.695
CMA							-0.182	-0.660
VIXrp	0.005	3.692	0.005	3.774	0.005	3.679	0.005	3.918
R^2	0.205		0.205		0.207		0.213	

Table 3. Asset pricing regressions of the ES risk premium.

This table shows the results of linear time-series regressions of the ES risk premium (ESrp) on various asset pricing factors, including market portfolio's excess return (MKT), Fama-French's (1993) size (SMB) and boot-to-market ratio (HML), Carhart's (1998) momentum (MOM), Fama and French's (2015) operating profitability (RMW) and investment (CMA) factors. In addition, the variance risk premium (VIXrp) is also considered. The t-values are obtained according to Newey and West (1987). R^2 is the unadjusted R-squared.

	Model 1		Model 2		Model 3		Model 4	
	coef	t-val	coef	t-val	coef	t-val	coef	t-val
const	-0.004	-3.416	-0.004	-3.273	-0.004	-2.984	-0.003	-1.875
MKT	-0.196	-2.968	-0.196	-2.997	-0.200	-2.932	-0.238	-2.839
SMB			-0.004	-0.204	-0.003	-0.138	-0.052	-1.287
HML			-0.004	-0.118	-0.008	-0.242	0.072	1.865
MOM					-0.010	-0.554		
RMW							-0.138	-1.905
CMA							-0.083	-1.434
R2	0.177		0.178		0.179		0.212	
	Model 5		Model 6		Model 7		Model 8	
	coef	t-val	coef	t-val	coef	t-val	coef	t-val
const	-0.004	-4.330	-0.004	-4.327	-0.004	-4.402	-0.003	-2.732
MKT	-0.116	-4.007	-0.117	-3.986	-0.112	-3.594	-0.154	-3.447
SMB			0.017	0.851	0.016	0.799	-0.027	-0.921
HML			0.005	0.185	0.010	0.350	0.060	1.710
MOM					0.010	0.634		
RMW							-0.119	-1.992
CMA							-0.047	-0.968
VIXrp	0.001	2.584	0.001	2.637	0.001	2.639	0.001	2.720
R2	0.235		0.237		0.237		0.262	

Table 4. Expectation hypothesis regressions of tail risk premia.

This table shows the estimation results of the expectation hypothesis regressions of tail risk premia. "Model" refers to the corresponding regression equations in the text. Standard errors (s.e.) of the estimates are obtained according to Newey and West (1987).

Risk premium	Model	Regression	Coef.	estimate	s.e.
VaRrp	(24)	Quantile	a	-0.0090	0.0038
			b	0.6848	0.0332
UPrp	(25)	Quantile	a	-0.0007	0.0022
			b	0.9406	0.0286
ES	(26)	Linear	a	0.0620	0.0247
			b	0.4427	0.1624
EUP	(27)	Linear	a	0.0092	0.0061
			b	0.8273	0.0782

Table 5. Correlations of various tail risk indicators.

This table shows correlations of various tail risk indicators, tail risk premia and the VIX index.

	VIX	VaRtr	EStr	VaRD	ESD	DMU	EDMU	VIXrp	VaRrp	ESrp
VIX	1.000									
VaRtr	0.965	1.000								
EStr	0.981	0.951	1.000							
VaRD	0.281	0.519	0.302	1.000						
ESD	0.751	0.743	0.862	0.327	1.000					
DMU	0.765	0.876	0.799	0.734	0.752	1.000				
EDMU	0.837	0.820	0.914	0.310	0.947	0.808	1.000			
VIXrp	0.090	0.088	0.082	0.027	0.046	0.083	0.058	1.000		
VaRrp	0.010	-0.009	0.017	-0.058	0.033	-0.036	0.015	0.399	1.000	
ESrp	-0.091	-0.103	-0.093	-0.074	-0.079	-0.120	-0.098	0.462	0.891	1.000

Table 6. Regressions of tail risk indicators on stock market returns

This table shows the results of the regressions (Eq. (28)) of tail risk indicators on daily stock market return (R_t) and the negative stock market return variable (R_t^-) which is equal to $\min\{R_t, 0\}$. The t-values of the estimates are obtained according to Newey and West (1987).

Y_t		Explanatory variable			R^2
		Const.	R_t	R_t^-	
ΔVIX_t	coef	-0.079	0.385	-4.231	0.003
	t-val	-2.114	0.342	-2.081	
$\Delta VaRtr_t$	coef	-0.001	0.011	-0.031	0.001
	t-val	-2.341	1.496	-2.352	
$\Delta EStr_t$	coef	-0.001	0.000	-0.029	0.002
	t-val	-1.581	-0.010	-1.618	
$VaRD_t$	coef	-0.002	0.037	-0.126	0.027
	t-val	-3.691	3.334	-4.969	
ESD_t	coef	0.026	0.131	-0.343	0.067
	t-val	17.489	3.434	-5.087	
DMU_t	coef	0.025	0.160	-0.423	0.090
	t-val	18.519	4.293	-6.349	
$EDMU_t$	coef	0.059	0.282	-0.703	0.082
	t-val	20.149	3.826	-5.542	

Table 7. Linear predictive regression of stock market index returns

This table shows the results of the linear predictive regressions of the S&P 500 index returns on the VIX and tail risk indicators. Panel A reports the results of the regression (29) . Panel B shows only the results of coefficients corresponding to tail risk indicators of the regression equation (30), and Panel C shows only the results of coefficients corresponding to tail risk indicators of the regression equation (31) . The t-values of the estimates are obtained according to Newey and West (1987) and provided in the parenthesis. Bold numbers in the t-values indicate statistical significance at 5% level.

	Horizon				
	1w	2w	3w	4w	60D
	A. Regression (29)				
const	-0.001	0.000	0.000	0.001	-0.002
	(-0.337)	(0.019)	(0.025)	(0.075)	(-0.171)
VIX	0.000	0.000	0.000	0.000	0.001
	(0.696)	(0.508)	(0.528)	(0.397)	(0.671)
ΔVIX	0.001	0.001	0.001	0.001	0.000
	(**3.926**)	(**3.241**)	(**2.782**)	(**1.983**)	(0.825)
R^2	0.009	0.003	0.003	0.002	0.004
const	-0.001	0.000	-0.001	-0.001	-0.004
	(-0.729)	(-0.127)	(-0.171)	(-0.163)	(-0.338)
VaRtr	0.024	0.027	0.041	0.052	0.126
	(1.136)	(0.714)	(0.813)	(0.718)	(0.872)
ΔVaRtr	0.030	0.036	-0.014	0.015	-0.023
	(1.144)	(1.347)	(-0.427)	(0.445)	(-0.332)
R^2	0.003	0.002	0.002	0.003	0.007
const	-0.001	0.000	0.000	0.001	-0.005
	(-0.336)	(-0.008)	(0.037)	(0.088)	(-0.372)
EStr	0.012	0.015	0.020	0.023	0.086
	(0.717)	(0.584)	(0.563)	(0.428)	(0.879)
ΔEStr	0.100	0.084	0.086	0.059	0.007
	(**3.885**)	(**2.982**)	(**2.293**)	(1.318)	(0.112)
R^2	0.006	0.003	0.003	0.002	0.007

Table 7. Continued.

	Horizon				
	1w	2w	3w	4w	60D
	B. Regression (30)				
VaRD	0.062	0.078	0.110	0.239	0.372
	(0.939)	(1.010)	(1.059)	(1.743)	(1.580)
ESD	0.005	0.039	0.028	0.016	0.472
	(0.111)	(0.431)	(0.241)	(0.110)	(1.565)
DMU	0.020	0.016	0.019	0.101	0.393
	(0.445)	(0.261)	(0.212)	(0.823)	(1.561)
EDMU	-0.022	-0.020	-0.049	-0.045	0.241
	(-0.688)	(-0.321)	(-0.577)	(-0.415)	(1.017)
	C. Regression (31)				
VaRD×Dum$^{\perp}$	0.176	0.221	0.321	0.350	0.444
	(**2.604**)	(**2.656**)	(**2.468**)	(**2.130**)	(1.705)
VaRD×Dum	-0.409	-0.725	-0.872	-0.395	0.040
	(-1.219)	(-1.904)	(-1.756)	(-0.633)	(0.060)
ESD×Dum$^{\perp}$	0.209	0.256	0.314	0.309	0.865
	(**3.593**)	(**3.216**)	(**2.336**)	(1.886)	(**3.184**)
ESD×Dum	-0.923	-1.311	-1.543	-1.945	-2.342
	(**-11.049**)	(**-9.168**)	(**-6.504**)	(**-6.344**)	(**-4.543**)
DMU×Dum$^{\perp}$	0.227	0.278	0.311	0.402	0.663
	(**3.875**)	(**3.815**)	(**2.986**)	(**2.501**)	(**3.208**)
DMU×Dum	-0.858	-1.212	-1.530	-1.749	-2.678
	(**-10.239**)	(**-11.919**)	(**-8.247**)	(**-5.717**)	(**-5.992**)
EDMU×Dum$^{\perp}$	0.083	0.107	0.110	0.079	0.358
	(**2.877**)	(**2.415**)	(1.527	(0.871)	(**2.203**)
EDMU×Dum	-0.478	-0.660	-0.806	-1.033	-1.279
	(**-14.827**)	(**-12.281**)	(**-8.049**)	(**-7.773**)	(**-5.483**)

Table 8. Linear predictive regression of 30 days portfolios returns

This table shows the results of the linear predictive regressions (31) of 30 days portfolio returns on tail risk indicators. Three sets of six portfolio returns are predicted: 2×3 Size-by-BM, Size-by-INV, and Size-by-OP. Only the coefficient estimates of the tail risk indicator $\widehat{\beta}_3$, $\widehat{\beta}_4$ and their t-values are reported. Each of four tail risk indicators (TRIs) is used as a predictor variable. The t-values of the estimates are obtained according to Newey and West (1987). Bold numbers in the t-values indicate statistical significance at 5% level.

TRI			TRI×Dum$^{\perp}$				TRI×Dum			
			$\widehat{\beta}_3$		t-value		$\widehat{\beta}_4$		t-value	
		Size →	Small	Big	Small	Big	Small	Big	Small	Big
VaRD	BM	Low	0.319	0.288	1.295	1.847	-0.500	-0.502	-0.684	-0.919
		Middle	0.278	0.365	1.239	**2.031**	-0.657	-0.768	-1.091	-1.251
		High	0.241	0.448	0.811	1.595	-0.697	-0.844	-1.147	-1.227
	INV	Low	0.182	0.365	0.650	**2.176**	-0.675	-0.847	-1.024	-1.377
		Middle	0.294	0.299	1.273	1.826	-0.656	-0.699	-1.189	-1.289
		High	0.332	0.349	1.273	1.895	-0.454	-0.497	-0.758	-0.886
	OP	Low	0.319	0.453	1.160	1.993	-0.593	-0.476	-0.887	-0.754
		Middle	0.296	0.405	1.246	**2.320**	-0.571	-0.718	-1.056	-1.180
		High	0.230	0.231	0.964	1.520	-0.548	-0.525	-1.004	-1.040
ESD	BM	Low	0.254	0.501	0.923	**2.852**	-2.911	-1.663	**-6.405**	**-5.375**
		Middle	0.214	0.295	1.114	1.691	-2.148	-1.906	**-7.494**	**-6.519**
		High	0.142	0.267	0.675	1.286	-2.389	-2.334	**-8.083**	**-7.116**
	INV	Low	0.168	0.386	0.695	**2.552**	-2.570	-1.649	**-7.145**	**-6.299**
		Middle	0.202	0.379	1.108	**2.458**	-2.064	-1.625	**-7.737**	**-6.392**
		High	0.365	0.555	1.440	**2.606**	-2.494	-1.994	**-6.497**	**-5.892**
	OP	Low	0.308	0.529	1.142	**2.324**	-2.614	-2.195	**-6.225**	**-6.104**
		Middle	0.241	0.428	1.288	**2.476**	-2.042	-1.805	**-7.545**	**-6.274**
		High	0.196	0.419	0.928	**2.628**	-2.275	-1.564	**-7.448**	**-5.689**

Table 8. Continued.

			Model 1				Model 2			
			$\widehat{\beta}_1$		t-value		$\widehat{\beta}_1$		t-value	
		Size →	Small	Big	Small	Big	Small	Big	Small	Big
DMU	BM	Low	0.383	0.402	1.525	**2.669**	-2.575	-1.772	**-6.032**	**-6.276**
		Middle	0.208	0.297	1.022	1.758	-2.106	-1.857	**-6.684**	**-6.807**
		High	0.132	0.234	0.569	1.083	-2.286	-2.303	**-6.638**	**-6.414**
	INV	Low	0.129	0.301	0.529	**2.030**	-2.541	-1.718	**-6.463**	**-6.443**
		Middle	0.204	0.338	1.030	**2.235**	-1.988	-1.625	**-6.789**	**-6.601**
		High	0.333	0.479	1.341	**2.585**	-2.476	-2.032	**-6.317**	**-6.432**
	OP	Low	0.275	0.433	1.091	**2.004**	-2.604	-2.278	**-6.145**	**-6.157**
		Middle	0.228	0.339	1.123	**2.014**	-1.996	-1.873	**-6.815**	**-6.476**
		High	0.169	0.309	0.748	**2.069**	-2.241	-1.685	**-6.824**	**-6.170**
EDMU	BM	Low	-0.063	0.166	-0.404	1.631	-1.673	-0.951	**-8.140**	**-7.080**
		Middle	-0.077	0.020	-0.686	0.212	-1.275	-1.088	**-9.461**	**-8.869**
		High	-0.155	-0.049	-1.291	-0.398	-1.420	-1.338	**-9.729**	**-8.234**
	INV	Low	-0.132	0.088	-0.967	0.994	-1.528	-0.936	**-8.992**	**-7.996**
		Middle	-0.071	0.118	-0.664	1.348	-1.210	-0.895	**-9.393**	**-8.361**
		High	-0.022	0.144	-0.151	1.171	-1.488	-1.171	**-8.378**	**-7.675**
	OP	Low	-0.076	0.109	-0.487	0.837	-1.585	-1.283	**-7.979**	**-7.262**
		Middle	-0.043	0.085	-0.397	0.873	-1.193	-1.044	**-9.077**	**-8.219**
		High	-0.104	0.144	-0.855	1.551	-1.350	-0.867	**-9.448**	**-7.341**

Table 9. Logistic predictive regression of tail events

This table shows the results of the logistic predictive regressions of 30 days portfolio tail risks on tail risk indicators. The logistic predictive regression "Model 1" and "Model 2" are defined by (32) and (33), respectively. A tail risk event occurs when portfolio return is less than the 10th percentile of historical portfolio returns. Tail risk events of three sets of six portfolio returns are predicted: 2×3 Size-by-BM, Size-by-INV, and Size-by-OP. Only the coefficient estimate of the tail risk indicator $\widehat{\beta}_1$ and its t-value are reported. Each of four tail risk indicators (TRIs) is used as a predictor variable. Bold numbers in t-value indicate statistical significance at 5% level.

			Model 1				Model 2			
			$\widehat{\beta}_1$		t-value		$\widehat{\beta}_1$		t-value	
		Size →	Small	Big	Small	Big	Small	Big	Small	Big
VaRD	BM	Low	7.496	4.054	1.882	1.015	-2.517	-4.328	-0.743	-1.278
		Middle	10.860	12.304	**2.744**	**3.119**	-1.114	-0.767	-0.328	-0.226
		High	16.746	13.767	**4.282**	**3.501**	0.288	-2.393	0.084	-0.705
	INV	Low	10.537	10.199	**2.660**	**2.573**	-2.338	-0.111	-0.693	-0.032
		Middle	12.794	10.499	**3.247**	**2.651**	-1.161	-1.515	-0.343	-0.448
		High	6.125	6.551	1.535	1.642	-4.921	-3.882	-1.465	-1.153
	OP	Low	8.422	5.895	**2.117**	1.477	-3.221	-5.387	-0.957	-1.604
		Middle	11.994	9.071	**3.038**	**2.283**	-2.096	-3.262	-0.621	-0.970
		High	7.800	5.233	1.959	1.310	-3.328	-4.280	-0.989	-1.270
ESD	BM	Low	9.127	8.066	**4.367**	**3.808**	-16.023	-15.658	**-4.255**	**-4.146**
		Middle	20.079	18.562	**9.905**	**9.178**	10.295	3.374	**3.156**	1.035
		High	26.761	24.823	**12.925**	**12.083**	14.975	8.143	**4.448**	**2.497**
	INV	Low	19.436	15.469	**9.598**	**7.656**	4.448	4.648	1.370	1.423
		Middle	21.863	17.270	**10.742**	**8.549**	8.666	2.457	**2.665**	0.749
		High	14.325	10.109	**7.078**	**4.885**	-7.080	-16.631	**-2.011**	**-4.422**
	OP	Low	14.271	13.038	**7.051**	**6.419**	-6.917	-12.660	**-1.967**	**-3.464**
		Middle	23.157	14.882	**11.336**	**7.361**	10.517	-7.558	**3.211**	**-2.145**
		High	19.424	13.212	**9.593**	**6.508**	8.611	-4.044	**2.653**	-1.166

Table 9. Continued.

			Model 1				Model 2			
			$\widehat{\beta}_1$		t-value		$\widehat{\beta}_1$		t-value	
		Size →	Small	Big	Small	Big	Small	Big	Small	Big
DMU	BM	Low	12.060	9.736	**6.234**	**4.932**	-4.859	-7.933	-1.570	**-2.537**
		Middle	18.006	18.520	**9.471**	**9.739**	6.584	4.959	**2.072**	1.580
		High	24.349	22.298	**12.593**	**11.630**	11.467	4.133	**3.554**	1.330
	INV	Low	18.515	14.628	**9.736**	**7.658**	3.784	3.572	1.213	1.129
		Middle	20.650	16.714	**10.822**	**8.788**	7.623	2.474	**2.404**	0.795
		High	15.130	12.284	**7.933**	**6.359**	-2.806	-7.133	-0.912	**-2.300**
	OP	Low	15.400	14.054	**8.080**	**7.343**	-1.835	-6.844	-0.596	**-2.211**
		Middle	20.726	15.791	**10.860**	**8.292**	6.372	-2.864	**2.025**	-0.932
		High	17.514	12.810	**9.212**	**6.653**	5.222	-3.519	1.655	-1.139
EDMU	BM	Low	6.829	5.823	**6.128**	**5.128**	-7.639	-8.946	**-3.359**	**-3.900**
		Middle	11.956	11.741	**11.103**	**10.907**	9.569	5.861	**4.307**	**2.675**
		High	15.747	14.675	**14.312**	**13.455**	13.820	7.656	**6.075**	**3.482**
	INV	Low	12.104	9.461	**11.238**	**8.743**	6.283	4.923	**2.867**	**2.232**
		Middle	13.379	10.772	**12.366**	**10.004**	9.757	4.078	**4.395**	1.862
		High	9.777	7.226	**9.051**	**6.525**	-1.110	-8.840	-0.503	**-3.890**
	OP	Low	9.547	8.800	**8.828**	**8.094**	-1.808	-6.431	-0.817	**-2.869**
		Middle	13.876	9.995	**12.791**	**9.262**	10.480	-1.930	**4.703**	-0.875
		High	11.901	8.455	**11.053**	**7.752**	9.338	-1.437	**4.207**	-0.646

Table 10. Linear predictive regressions of ex ante tail risks

This table reports the results of the linear predictive regressions of two ex ante tail risk indicators (VaRtr and EStr) and the VIX index. The linear predictive regression model (35) is specified for the two ex ante tail risk indicators, and the model (36) is specified for the VIX. Four prediction horizons are considered: 1, 2, 3, and 4 weeks. Only the coefficient estimate of the tail risk indicator $\widehat{\beta}_1$ and its t-value are reported. Each of four tail risk indicators (TRIs) is used as a predictor variable. The t-values of the estimates are obtained according to Newey and West (1987). Bold numbers in the t-values indicate statistical significance at 5% level.

Ex ante	Horizon	$\widehat{\beta}_1$				t-value			
tail risk		VaRD	ESD	DMU	EDMU	VaRD	ESD	DMU	EDMU
VaRtr	1w	0.649	0.165	-0.066	0.074	**5.436**	**5.574**	-1.199	**4.415**
	2w	0.757	0.156	-0.061	0.074	**4.068**	**3.576**	-1.009	**3.025**
	3w	0.760	0.141	-0.004	0.067	**3.375**	**3.576**	-0.083	**2.730**
	4w	0.767	0.129	0.018	0.065	**2.890**	**3.109**	0.234	**2.192**
EStr	1w	0.152	0.858	0.199	0.180	**3.563**	**4.241**	**4.202**	**3.701**
	2w	0.134	1.043	0.175	0.202	**2.675**	**3.377**	**3.695**	**3.725**
	3w	0.101	1.157	0.143	0.219	1.806	**2.899**	**2.901**	**3.327**
	4w	0.026	1.171	0.099	0.218	0.417	**2.472**	1.746	**2.570**
VIX	1w	4.903	-1.610	2.172	-2.110	1.083	-0.570	0.580	-1.290
	2w	-0.849	-4.327	-5.147	-4.029	-0.213	-0.911	-1.725	-1.670
	3w	-5.964	-7.715	-9.742	-5.412	-1.197	-1.706	**-2.760**	-1.846
	4w	-13.653	-7.805	-14.874	-5.944	**-2.181**	-1.328	**-2.873**	-1.517