Reading a Central Banker's Mind: A Time-varying Coefficient Regression Approach[1]

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Abstract: We examine the role of the US Federal Reserve System's (the Fed's) preference in the understanding of inflation rate and unemployment rate evolution in the US. We show that the coefficients in a reduced-form regression between two variables, derived from the first-order condition of the Fed's optimization problem, have been unstable. Based on this evidence, we run a time-varying coefficient regression and find evidence that the Fed's preference parameters have moved, implying that its preference can be represented by the Cukierman–Gerlach model approximately before the era of Volcker's chairmanship of the Fed and by the strict inflation targeting model since Volcker's term as chairman.

Keywords: Monetary policy; time-varying parameter; inflation; unemployment.

JEL Classification: E31; E52; E61.

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1. Introduction

Since Phillips (1958) reported a negative relation between the unemployment rate and the growth rate of money wages, the relation between unemployment and inflation, known as the Phillips curve, has attracted attention from economists. As shown in Figure 1, however, the pronounced negative relation during the 1960s in the US has disappeared since the 1970s, a phenomenon that has posed a challenge to economists. The broken relation can also be observed from plots in Figure 2 that show inflation rate and unemployment rate movements together with the variance of the unemployment rate over time. The inflation rate rose rapidly during the 1970s, dropped drastically during the early 1980s, and then stabilized, while the unemployment rate shows long swings. Moreover, the variance of the unemployment rate does not seem to be constant but varies over time, as Ruge-Murcia (2003) emphasizes. Many studies have considered the behavior of the inflation rate and unemployment rate since the 1960s and examined the US central bank's role in the evolution of these two variables. Sargent (1999) explains the inflation rate's evolution with the US Federal Reserve System's (the Fed's) gradual learning of the private sector's behavior. Clarida, Gali, and Gertler (2000) provide evidence that Taylor-rule type monetary policy rule changed significantly before and after the appointment of Volcker as the Fed's chairman. They also argue that monetary policy has become more stabilizing since Volcker's era. Sims and Zha (2006) demonstrate that the time variations of innovation volatilities have played an influential role in the evolution of the US macroeconomy. Later, however, Bianchi (2013) shows that changes in volatilities and monetary policy are both important for understanding the dynamics of macroeconomic variables. Further, Best and Hur (2016) argue that time-variation in volatilities, changes in monetary policy, and a central banker's learning all matter when explaining US post-war data.

(Figures 1 and 2 here)

In addition to the abovementioned works, studies such as Kydland and Prescott

(1977), Barro and Gordon (1983), and Cukierman and Gerlach (2003) attempt to explain the behavior of the inflation rate and unemployment rate based on a central banker's preference and rational expectation, whereby equilibrium often ends up with the time-inconsistency problem. The empirical validity of such studies is also examined to determine whether the interpretation based on the time-inconsistency problem is consistent with the data of the inflation rate and unemployment rate. Ireland (1999) shows that the long-run cointegration relation between the two variables can be explained by the time-inconsistency problem in the Barro–Gordon model. Ruge-Murcia (2003) compares the Barro–Gordon model and a version of the Cukierman–Gerlach model and demonstrates that the asymmetric loss function, which is similar to that of Cukierman and Gerlach (2003), is more compatible with the data. Following Ireland (1999) and Ruge-Murcia (2003), we also investigate which type of central banker's preference is supported by the data on the inflation rate and unemployment rate. While doing so, we allow the Fed's preference parameters, which govern the degree of asymmetry for positive and negative unemployment deviations from the target in the preference, the relative weight between the inflation and unemployment components in the preference, and the unemployment rate target, to vary over time in order to explain the rise and fall of the inflation rate and the fluctuation of the unemployment rate. Then, we examine how the variation in these parameters results in different reduced-form relations between the inflation rate and unemployment rate.

Do the Fed's preference parameters vary over time? We have no direct evidence for this. Sargent (1999) admits the difficulty of explaining the rise and fall of the inflation rate using one type of time-inconsistency model with the rational expectation hypothesis. Through the expectation-augmented Phillips curve, Sargent (1999) also emphasizes the variation in the Fed's view of the private sector's behavior. Many empirical studies also provide evidence that the Fed's monetary policy rule has varied over time.[2] Among them, studies such as Dolado, Pedrero, and Ruge-Murcia (2004) and Surico (2007, 2008) estimate US monetary policy reaction functions based on non-quadratic loss functions and suggest that the Fed's preferences were different during the pre-Volcker and Volcker–Greenspan eras. In addition, Komlan (2013) estimates the Canadian monetary policy reaction function and asymmetric preference parameters for different subperiods and different regimes. Komlan (2013) then provides evidence that the Canadian monetary authority's preferences have changed since 1991. Although one could conjecture that variations in a central banker's view or the monetary policy rule could be related to, or caused by, shifts in the central banker's preference, Debortoli and Nunes (2014) show that changes in the parameters of a monetary policy rule do not necessarily correspond to changes in a monetary authority's preference. Hence, it is worth examining how far we can go to explain the behavior of the inflation rate and unemployment rate based only on variations of the Fed's preference. It is also worth comparing our results with those of prior studies to check whether changes in monetary policy are related to changes in policymakers' preferences. In doing so, however, our approach differs from previous studies in that we do not assume the number of breaks or known structural break dates *a priori*. Namely, we consider the possibility that the relation among the inflation and unemployment variables is not permanently fixed and that the coefficients in the reduced-form regression evolve gradually and smoothly.

In order to address this question, the current study is organized as follows. Section 2 presents the Fed's preference (or loss function), nesting the Barro–Gordon model, the Cukierman–Gerlach model, or strict inflation targeting as a special case.[3] A reduced-form

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[2] Limited examples of studies that report changes in central banks' monetary policy rules are Clarida, Gali, and Gertler (2000), Dolado, Pedrero, and Ruge-Murcia (2004), Surico (2007, 2008), Bae, Kim, and Kim (2012), Bianchi (2013), Komlan (2013), and Best and Hur (2016).

[3] The Fed's loss function examined in this study is developed by Ruge-Murcia (2003).

relation between the inflation rate, unemployment rate, and the volatility of the unemployment rate is derived from the Fed's first order condition, as in Ruge-Murcia (2003). Section 3 provides a brief description of the data used in this study. In Section 4, we provide results from the constant coefficient regression and argue that these results could provide a completely different interpretation. After presenting evidence that coefficients in the constant coefficient regression are unstable, we employ time-varying coefficient regression and show how the reduced-form coefficients have varied over time. We also relate the variation of these coefficients to the variation of the Fed's preference. Our results suggest that the Fed's preference can be described by the Cukierman and Gerlach (2003) model approximately before Volcker's era, and by strict inflation targeting since Volcker's era. Our results also imply that changes in the monetary policy rule since Volcker's era, reported in many studies, may be related to movements in the Fed's preferences. Concluding remarks are offered in Section 5.

2. An Economic Model with Time-varying Parameters

The preference of the Fed (or the loss function) is represented by the following function:

$$L(\pi_t, u_t; \pi_t^*, \kappa_t, \lambda_t, \gamma_t) = \frac{1}{2}(\pi_t - \pi_t^*)^2$$
$$+ \frac{\lambda_t}{\gamma_t^2}\left[exp\left(\gamma_t(u_t - \kappa_t E_{t-1}(u_t^n))\right) - \gamma_t(u_t - \kappa_t E_{t-1}(u_t^n)) - 1\right], \qquad (1)$$

where π_t denotes the inflation rate, u_t denotes the unemployment rate, u_t^n denotes the natural rate of unemployment, and π_t^* denotes the implicit target (or desirable) inflation rate set by the Fed at *t*-1. Ruge-Murcia (2003) uses equation (1) to test whether the Barro–Gordon quadratic preference or the Cukierman–Gerlach asymmetric preference is supported by data on the inflation rate and unemployment rate. κ_t, λ_t, and γ_t are the Fed's preference parameters, which are assumed to be given at *t*-1, in a central banker's mind and are known to the public through full understanding of the central banker's problem. κ_t is related to the implicit target of the unemployment rate. When the Fed's implicit target for the unemployment rate is lower than the expected natural unemployment rate (that is, $\kappa_t E_{t-1}(u_t^n) < E_{t-1}(u_t^n)$ or $0 < \kappa_t < 1$), then the Fed tends to be keen to generate inflation bias in models such as those of Kydland and Prescott (1977) and Barro and Gordon (1983). γ_t governs the degree of asymmetry for positive and negative unemployment deviations from the target in the preference. When $\gamma_t > 0$, a central banker puts more weight on the unemployment rate during a recession than during a boom with regard to the loss function. As $\gamma_t \to 0$, the unemployment component in the preference becomes a quadratic form.[4] When $\gamma_t \to 0$ with $0 < \kappa_t < 1$, the loss function in equation (1) collapses to the quadratic loss function in the Barro–Gordon model which is tested by Ireland (1999). Finally, $\lambda_t \geq 0$ determines the relative weight between the inflation and unemployment components in the preference. A smaller value for λ_t implies greater importance for the inflation component in the loss function, and $\lambda_t = 0$ means strict inflation targeting. Hence, the representation of the Fed's preference in equation (1) nests the Barro–Gordon model ($\gamma_t \to 0$ and $\lambda_t > 0$ with $0 < \kappa_t < 1$), the Cukierman–Gerlach type model ($\gamma_t > 0$ and $\lambda_t > 0$ with $\kappa_t = 1$), or strict inflation targeting ($\lambda_t = 0$ and $\gamma_t \neq 0$) as a special case.[5]

In addition to a central banker's preference, other parts in the model are standard in the literature. We assume the following expectation-augmented Phillips curve:

$$u_t = u_t^n - \alpha\big(\pi_t - E_{t-1}(\pi_t)\big) + v_t, \tag{2}$$

where α is assumed to be positive and v_t denotes a supply-side disturbance. $E_{t-1}(\pi_t)$ denotes the expectation of the inflation rate conditional on all available data at time *t*-1. We assume that the public has the same information set as the Fed and that the public and the

4 This can be verified by applying L'Hôpital's rule twice.

5 Svensson (1999) names the loss function with the inflation component only as "strict inflation targeting."

central banker form rational expectations.

The natural rate of unemployment (u_t^n) evolves a stationary process.[6] Studies such as Ireland (1999) assume a nonstationary process for the natural rate of unemployment. In the appendix, we also consider this possibility that the unemployment rate follows a nonstationary process. Although parameters in the expectation-augmented Phillips curve and the process of u_t^n could vary over time because of technological developments, demographic changes, or other changes in the economic environment, we assume that the parameters are constant in order to see how much we can explain the behavior of the inflation rate and unemployment rate based only on variations in a central banker's preference parameters.

Finally, we assume that the Fed can affect the inflation rate as follows:

$$\pi_t = i_t + \eta_t, \tag{3}$$

where i_t denotes a central banker's policy instrument, and η_t denotes a control error. The specification in equation (3) states that although the Fed can affect the inflation rate, its control is not perfect, as in Ruge-Murcia (2003).

A central banker sets a policy instrument to minimize the loss function in equation (1) subject to the expectation-augmented Phillips curve, equation (3), and the private agents' anticipation formed at *t*-1 regarding the central banker's action at *t*. The first-order condition can be written as

$$E_{t-1}(\pi_t) = \pi_t^* + \frac{\alpha\lambda_t}{\gamma_t} E_{t-1}\left(\exp\left(\gamma_t\left(u_t - \kappa_t E_{t-1}(u_t^n)\right)\right) - 1\right). \tag{4}$$

As explained in Ruge-Murcia (2003), the assumptions of the rational expectation hypothesis and the normal distribution for the unemployment rate mean that equation (4) can be rewritten as

[6] Ruge-Murcia (2003) assumes ARIMA (2,0,2) or ARIMA (1,1,2) for u_t^n.

$$\pi_t = \pi_t^* + \frac{\alpha\lambda_t}{\gamma_t}(\exp(\gamma_t(1-\kappa_t)E_{t-1}(u_t^n) + \frac{\gamma_t^2\sigma_{u,t}^2}{2}) - 1) + \eta_t, \tag{5}$$

where $\sigma_{u,t}^2$ is the conditional variance of u_t. Then, the first-order Taylor approximation and the replacement of $E_{t-1}(u_t^n)$ with $E_{t-1}(u_t)$ provide the following reduced-form relation among π_t, $E_{t-1}(u_t)$, and $\sigma_{u,t}^2$:

$$\pi_t = a_t + b_t E_{t-1}(u_t) + c_t \sigma_{u,t}^2 + e_t, \tag{6}$$

where e_t is a disturbance in the reduced form. Although we cannot identify the Fed's preference parameters (κ_t, λ_t, and γ_t) separately from the reduced form regression in equation (6), preference parameters have relations with b_t and c_t depending on the central banker's preference. Since $b_t = \alpha\lambda_t(1-\kappa_t)$ and $c_t = \frac{\alpha\lambda_t\gamma_t}{2}$, we have three possible interpretations for estimation results from the reduced form regression.

When the Fed's preference can be represented by the Barro–Gordon model ($\gamma_t \to 0$ and $\lambda_t > 0$ with $0 < \kappa_t < 1$), then b_t will be positive and $c_t = 0$. If the Fed's preference can be represented by the Cukierman–Gerlach model ($\gamma_t > 0$ and $\lambda_t > 0$ with $\kappa_t = 1$), then $b_t = 0$ and c_t will be positive.[7] Finally, if strict inflation targeting describes the Fed's preference ($\lambda_t = 0$), then $b_t = 0$ and $c_t = 0$. We investigate which implication is supported by data.

In this exercise, we assume that the Fed's preference parameters vary over time. Although we have no direct evidence for variations in the preference parameters, studies such as Clarida, Gali, and Gertler (2000) and Bae, Kim, and Kim (2012) report that the Fed's monetary policy function has shifted over time. For example, Clarida, Gali, and Gertler (2000) demonstrate that the estimated Fed's monetary policy rule implies that the Fed has placed

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[7] When $\kappa_t = 1$, one cannot rule out the possibility that $\gamma_t < 0$ and $\lambda_t < 0$, which also results in $b_t = 0$ and a positive c_t. Hence, we do not claim that the condition we mentioned is necessary and sufficient to distinguish the Cukierman–Gerlach model from the Barro–Gordon model. Thus, the abovementioned condition should be interpreted as suggestive information rather than a necessary condition.

more weight on the inflation component in the loss function since 1979. Bae, Kim, and Kim (2012) argue that the evolution of the Fed's monetary policy rule has coincided approximately with each chairperson's term at the Fed. Bianchi (2013) and Best and Hur (2016) contend that a change in the monetary policy rule is one of the important factors in understanding US macroeconomic dynamics. Since Debortoli and Nunes (2014) demonstrate that changes in the parameters of the monetary policy rule are not necessarily related to changes in the Fed's preference, we attempt to examine whether the findings described here are related to changes in the Fed's preference parameters by comparing our results with those findings. In this regard, the current study is expected to supplement the literature.

3. Data

Quarterly US data on inflation and unemployment are used for this study's analysis. The percentage change in the GDP implicit price deflator and the quarterly average of the monthly civilian unemployment rate are taken for the construction of the inflation rate and the unemployment rate respectively. These data are obtained from the website of the Federal Reserve Bank of St. Louis.[8] The sample starts from the first quarter of 1960 and ends in the fourth quarter of 2007. We exclude the data since 2008 to avoid the sudden and drastic change in the economic environment resulting from the global financial crisis in that year.

4. Empirical Analysis

The results of various unit root tests are reported in Table 1. Depending on the test method, the results are quite sensitive. For example, when the augmented Dickey–Fuller (ADF) test is conducted, the unit root null hypothesis is rejected for the unemployment rate but is not rejected for the inflation rate. However, the test results are reversed when the Phillips–Perron

[8] The web address is https://fred.stlouisfed.org/.

test is employed. Under the Phillips–Perron test, the unit root null hypothesis is not rejected for the unemployment rate but is rejected for the inflation rate. Faced with these conflicting results, we conduct the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test. Under the KPSS test, the null hypothesis of stationarity is not rejected for any variables. Hence, we assume that both the unemployment rate and inflation rate are stationary, although the results depend on the test method and sample period. We also examine the relation between the unemployment rate and inflation rate when both are cointegrated, as in Ireland (1999) (see the appendix).

(Table 1 here)

4.1. Constant Coefficient Regression

Table 2 shows the results of constant coefficient regression for equation (6) when the Fed's preference parameters, κ_t, λ_t, and γ_t, are assumed to be unchanged. The first row is the result when $E_{t-1}(u_t)$ is replaced with $u_t + \varsigma_t$ under the rational expectation hypothesis (ς_t denotes forecast error). Further, $\sigma^2_{u,t}$ is the realized variance of the unemployment rate through the use of monthly actual unemployment rates during each quarter. If ς_t (forecast error) is uncorrelated with e_t (disturbance in the reduced form) under the rational expectation hypothesis, the coefficients in the reduced form regression in equation (6) can be consistently estimated. As shown in the first row, b (the coefficient of $E_{t-1}(u_t)$) is insignificant and c (the coefficient of the volatility of u_t) is significantly positive. These results support the Cukierman–Gerlach-type asymmetric model. When $E_{t-1}(u_t)$ and $\sigma^2_{u,t}$ are constructed from the ARIMA (2,0,2) with GARCH (1,1) assumption for the unemployment rate, as in Ruge-Murcia (2003), b is insignificant while c is significantly positive. This result is consistent with Ruge-Murcia (2003), although our sample period is longer. These results together suggest the Cukierman–Gerlach-type model for the Fed's preference when constant coefficient regressions are run.

(Table 2 here)

If a version of the Cukierman–Gerlach model is a good description of the Fed's preference for the entire sample period, however, it is a little difficult to justify the change in the Fed's stance in lowering and stabilizing inflation since Volcker's era. The US inflation trend actually began to drop in the early 1980s in spite of the high level and considerable volatility of the unemployment rate. A possible interpretation of the movements of the inflation rate since the 1980s, based on the results in Table 2, would be that the marginal benefit of unexpected inflation has become smaller as the volatility of the unemployment rate has decreased exogenously. Hence, the inflation has been stabilized because the lower volatility of the unemployment rate has led to a lower inflation bias rather than the Fed's initiation to reduce the inflation rate. This interpretation, based on Table 2, sounds similar to studies such as Sims and Zha (2006) in the sense that influential role of time-varying volatilities rather than changes in the Fed's stance is needed to understand US macroeconomic dynamics.

However, the results from the constant coefficient regressions could be misleading because variations of the monetary policy rule and/or changes in the Phillips curve relation that are reported in many studies may imply variations of the Fed's preference. This conjecture suggests that time-varying coefficients in the reduced form regression in equation (6) should be run. In order to check this possibility, we run rolling regressions for equation (6) with the window size equal to 40 quarters. The estimated coefficients and their 95% confidence intervals from the rolling regressions are plotted in Figure 3. Although our rolling regressions are arbitrary in terms of the window size, Figure 3 shows that the regression coefficients, b_t and c_t, are far from being constant. Instead, b_t is sometimes significantly positive and at other times significantly negative, although it fluctuates around zero. c_t is significantly positive between the late 1960s and early 1980s, becomes insignificant between the early 1980s and 1995, and then becomes significantly negative from 1995. These plots imply that the results from the constant coefficient regression are sensitive to sample periods, possibly due to changes in the Fed's preference.

(Figure 3 here)

We also conduct structural break tests for the coefficients in the constant coefficient regression to determine whether these coefficients are unstable during the sample period. The Bai and Perron (1998) test is conducted for the constant coefficient regression. The results are presented in Table 3. As shown in Table 3, the number of breaks and estimated break points are quite sensitive to test statistics (e.g. whether the F-statistic, UDmax statistic, WDmax statistic, Schwarz criterion, or LWZ criterion is employed in the test), but all the results indicate strongly that the coefficients in the constant coefficient regression are unstable. The estimated break points can be found in the 1960s, 1970s, 1980s, 1990s, and 2000s, which may imply gradual changes of the coefficients in the regression.

(Table 3 here)

4.2.Time-varying Coefficient Regression

Considering the evidence presented in Figure 3 and Table 3, we run the time-varying coefficient regression for equation (6). Assume that the coefficients vary smoothly so that $b_t = b\left(\frac{t}{T}\right)$ and $c_t = c\left(\frac{t}{T}\right)$, where $b(\cdot)$ and $c(\cdot)$ are smooth functions defined on [0, 1] and T is the sample size. Under the assumption that $b(\cdot)$ and $c(\cdot)$ are sufficiently smooth functions that can be approximated with a series of polynomials and/or trigonometric functions, the reduced form regression in equation (6) can be written as follows:

$$
\begin{aligned}
\pi_t &= a_t + b_t E_{t-1}(u_t) + c_t \sigma^2_{u,t} + e_t \\
&= a\left(\tfrac{t}{T}\right) + b\left(\tfrac{t}{T}\right) E_{t-1}(u_t) + c\left(\tfrac{t}{T}\right) \sigma^2_{u,t} + e_t \\
&= a + \left[\textstyle\sum_{i=1}^{n} \theta_i^b\, \varphi_i^b\left(\tfrac{t}{T}\right)\right] E_{t-1}(u_t) + \left[\textstyle\sum_{i=1}^{n} \theta_i^c\, \varphi_i^c\left(\tfrac{t}{T}\right)\right] \sigma^2_{u,t} + e_{nt} \\
&= a + \chi_{nt}^{b\prime} a_n^b + \chi_{nt}^{c\prime} a_n^c + e_{nt},
\end{aligned}
\tag{7}
$$

where $\chi_{nt}^{b\prime} = \left[\varphi_1^b\left(\frac{t}{T}\right), \dots, \varphi_n^b\left(\frac{t}{T}\right)\right]' E_{t-1}(u_t)$, $\chi_{nt}^{c\prime} = \left[\varphi_1^c\left(\frac{t}{T}\right), \dots, \varphi_n^c\left(\frac{t}{T}\right)\right]' \sigma^2_{u,t}$, $a_n^b = [\theta_1^b, \dots, \theta_n^b]'$, $a_n^c = [\theta_1^c, \dots, \theta_n^c]'$, $b_n\left(\frac{t}{T}\right) = \sum_{i=1}^{n} \theta_i^b\, \varphi_i^b\left(\frac{t}{T}\right)$, $c_n\left(\frac{t}{T}\right) = \sum_{i=1}^{n} \theta_i^c\, \varphi_i^c\left(\frac{t}{T}\right)$, and $e_{nt} = e_t + \left[a\left(\frac{t}{T}\right) - a\right] + \left[b\left(\frac{t}{T}\right) - b_n\left(\frac{t}{T}\right)\right] E_{t-1}(u_t) + \left[c\left(\frac{t}{T}\right) - c_n\left(\frac{t}{T}\right)\right] \sigma^2_{u,t}$.

Once $\chi_{nt}^{b\prime}$ and $\chi_{nt}^{c\prime}$ are constructed, a_n^b and a_n^c can be estimated by the least squares approach. In this regard, Andrews (1991a) demonstrates desirable asymptotic results for the estimates of a_n^b and a_n^c. It is straightforward to recover $b_n\left(\frac{t}{T}\right)$ and $c_n\left(\frac{t}{T}\right)$ with the estimates of a_n^b and a_n^c. Even if π_t and u_t are nonstationary, Park and Hahn (1999) show that the time-varying cointegration coefficients can be estimated in a similar way.

Since we cannot expand $b\left(\frac{t}{T}\right)$ and $c\left(\frac{t}{T}\right)$ with an infinite number of terms, it is important to decide n (the number of series functions) in the empirical analysis to obtain a good approximation for $b\left(\frac{t}{T}\right)$ and $c\left(\frac{t}{T}\right)$. Regarding this issue, the *h*-block cross-validation (CV) and the modified *h*-block CV criteria, as suggested by Burman, Chow, and Nolan (1994) and Racine (1997), are utilized as selection criteria for n.[9] For a given block size (*h*), the *h*-block CV criterion can be expressed as:

$$
CV = T^{-1} \textstyle\sum_{t=h}^{T-h} (\pi_t - \hat{a}(t,h) - \chi_{nt}^{b\prime} \hat{a}_n^b(t,h) - \chi_{nt}^{c\prime} \hat{a}_n^c(t,h))^2, \tag{8}
$$

where $\hat{a}(t,h)$, $\hat{a}_n^b(t,h)$, and $\hat{a}_n^c(t,h)$ are estimators of the coefficients in equation (7)

[9] The block size, *h*, is set as the integer nearest to $\frac{T}{6}$ in accordance with the suggestion of Burman, Chow, and Nolan (1994).

obtained by removing the t-th observation, and the h observations preceding and following the t-th observation in the dependent and independent variables in the regression. The modified h-block CV criterion, motivated by cases where $\frac{n}{T}$ is not negligible, can be written as follows:

$$MCV = T^{-1}\sum_{t=h}^{T-h}(\pi_t - \hat{a}(t,h) - \chi_{nt}^{b'}\hat{a}_n^b(t,h) - \chi_{nt}^{c'}\hat{a}_n^c(t,h))^2$$

$$+T^{-2}\sum_{t=h}^{T-h}\sum_{i=1}^{T}(\pi_i - \hat{a}(t,h) - \chi_{ni}^{b'}\hat{a}_n^b(t,h) - \chi_{ni}^{c'}\hat{a}_n^c(t,h))^2$$

$$+T^{-1}\sum_{i=1}^{T}(\pi_i - \hat{a} - \chi_{ni}^{b'}\hat{a}_n^b - \chi_{ni}^{c'}\hat{a}_n^c)^2 \quad (9)$$

The n that minimizes CV or MCV is selected.

As shown in Table 4, we test various forms of series functions. Among them, both CV criteria are minimized when 1, m, m^2, $\cos(m)$, $\sin(m)$, $\cos(2m)$, $\sin(2m)$, $\cos(3m)$, and $\sin(3m)$ are used. Hence, we choose this Fourier flexible form (FFF) to approximate $b\left(\frac{t}{T}\right)$ and $c\left(\frac{t}{T}\right)$ in the empirical analysis.

(Table 4 here)

The estimates of b_t and c_t that are based on the time-varying coefficient regression with the selected n are plotted in Figure 4. We replace $E_{t-1}(u_t)$ with $u_t + \varsigma_t$ under the rational expectation assumption and the realized variance of u_t is used for $\sigma_{u,t}^2$ as in the first row of Table 2. Interesting points emerge from Figure 4. First, the estimate of b_t is never significant during the entire sample period. This finding differs from the estimates of b_t from the rolling regression but seems more sensible because all models we consider imply non-negative b_t; however, b_t is occasionally significantly negative in the rolling regression. Second, c_t is significantly positive between the late 1960s and early 1980s, and becomes insignificant from the early 1980s except for a brief period in the early 2000s. Unlike b_t, the evolution of c_t is somewhat similar to that depicted in the rolling regressions. Third, but most importantly, the evolution of b_t and c_t provide hints about the Fed's preference parameters. The insignificant b_t during the entire sample period suggests that it is difficult to explain movements of the inflation rate and unemployment rate using the Barro–Gordon model for the Fed's preference. Instead, the prediction from the Cukierman–Gerlach type model is consistent with the insignificant b_t and significantly positive c_t between the late 1960s and early 1980s. Since both b_t and c_t are insignificant from the early 1980s, the Cukierman–Gerlach-type model has a problem explaining these movements of b_t and c_t. Insignificant b_t and c_t are possible with $\lambda_t = 0$, implying that the Fed is thinking of strict inflation targeting. Except for the period when c_t briefly becomes negative during the early 2000s,[10] the time-varying coefficient regression suggests that the Fed's preference can be described by the Cukierman–Gerlach-type model approximately before Volcker's era, and by strict inflation targeting since Volcker's era.

(Figure 4 here)

Figure 5 presents time-varying coefficients estimated with an alternative method of constructing $E_{t-1}(u_t)$ and $\sigma^2_{u,t}$ in equation (6). With regard to Figure 5, $E_{t-1}(u_t)$ and $\sigma^2_{u,t}$ are constructed from the ARIMA (2,0,2) model with GARCH (1,1) for u_t as in Ruge-Murcia (2003). The results are similar to those in Figure 4. While b_t is insignificant during the entire sample period, c_t is significantly positive between the late 1960s and early 1970s, insignificant until 2006, and significantly positive again at the end of the sample. These movements of b_t and c_t are consistent with the Cukierman–Gerlach-type model during the period between the late 1960s and early 1970s. The movements are also mostly consistent with the strict inflation-targeting model since then.

(Figure 5 here)

[10] Negative c_t is possible when $\gamma_t < 0$, which implies that the Fed intends to generate a deflation bias, not an inflation bias.

Figure 6 compares the fitted values of the inflation rate from the constant coefficient regression and the time-varying coefficient regression. As shown in Figure 6, the time-varying coefficient regression presents a much tighter relation between the inflation rate and unemployment rate.

(Figure 6 here)

5. Discussion and Conclusion

This study's results provide supplementary evidence for the literature that examines the role of the Fed in understanding the dynamics of the inflation rate and unemployment rate in the US. According to the results from the constant coefficient regression, the Cukierman–Gerlach model can be selected to represent the Fed's preference for the entire sample period. Considering this result, the fall of the inflation rate since the early 1980s may not have been initiated by the Fed. Instead, an exogenous fall in the volatility of the unemployment rate around the mid-1980s caused the Fed to perceive the lower marginal benefit of unexpected inflation, which resulted in the stabilization of the inflation rate since then. This interpretation is similar to that of Sims and Zha (2006) and those of subsequent studies that emphasize the role of the volatility of innovations rather than changes in the monetary policy rule in order to explain the evolution of the inflation rate and unemployment rate.

However, econometric evidence in this study implies that coefficients in the reduced-form regression are not constant, which indicates that the parameters of the Fed's preference have moved during the sample period. Considering this possibility, we run a time-varying coefficient regression. The results from the regression suggest that the relation between the inflation rate and unemployment rate can be explained by the Cukierman–Gerlach model approximately before Volcker's era, and by the strict inflation-targeting model since Volcker's term. Although the Fed did not adopt inflation targeting until 2012,[11] it is well-known that the philosophy of the Volcker–Greenspan era at the Fed is very similar to inflation targeting (see Bernanke and Mishkin (1997)). Moreover, the direction and timing of the movement in the Fed's preference parameters implied by the time-varying regression are approximately consistent with changes in the monetary policy rule, indicating that the monetary policy rule has been more stabilizing since Volcker's term than before (see Clarida, Gali, and Gertler (2000)). Hence, the evidence from the time-varying regression suggests that changes in the Fed's preference correspond to changes in the monetary policy rule in the US data before the 2008 global financial crisis.

[11] See Reuters (2012) at http://www.reuters.com/article/us-usa-fed-inflation-target-idUSTRE80O25C20120126.

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Appendix. Nonstationary Inflation Rate and Unemployment Rate

As shown in Table 1, the results of the unit root tests are quite sensitive. Hence, in this appendix, we assume that both the inflation rate and unemployment rate are nonstationary and examine whether the two variables have a time-varying cointegration relation. For this purpose, we conduct model specification tests proposed by Park and Hahn (1999) and Bierens and Martins (2010) to see whether the two variables have a time-varying cointegration relation rather than a constant cointegration relation. As shown in Appendix Table 1, the data are favorable toward the time-varying cointegration relation between the inflation rate and unemployment rate rather than the constant cointegration relation, regardless of test statistics. Based on this result, we plot the time-varying coefficients when unemployment follows the ARIMA (1,1,2) process. The results in Appendix Figure 1 are quite similar to those in Figures 4 and 5. While b_t is insignificant during the entire sample period, c_t is significantly positive between the late 1960s and early 1970s, and is then mostly insignificant up to 2007.

Appendix Table 1. Model Specification Tests for the Time-varying Cointegration

	τ_1	τ_2	Bierens and Martin test
$E_{t-1}(u_t) = u_t$ and $\sigma^2_{u,t}$ is the realized variance of u_t	164.7453	3.7975	23.8531 (0.0006)
$E_{t-1}(u_t)$ and $\sigma^2_{u,t}$ are from the ARIMA (1,1,2) model with GARCH (1,1) for u_t	164.8065	3.7565	18.8007 (0.0045)

Notes: Regarding τ_1, τ_2, and the Bierens and Martin test, the null hypothesis is cointegration with constant coefficients, while the alternative hypothesis is time-varying cointegration. The number of Chebyhev polynomials in the Bierens and Martin test is set to be three but the result is not sensitive to this choice. The numbers in parentheses in the Bierens and Martin test column are p-values. The results in the third row of Appendix Table 1 are based on the assumption that $E_{t-1}(u_t)$ is formed from the ARIMA (1,1,2) specification for the unemployment rate. The results are not sensitive to the choice of specification for the unemployment rate process. The 5% critical values for τ_1, and τ_2 reported in Shin (1994) and Park and Hahn (1999) are 11.071 and 0.895 respectively.

Appendix Figure 1.

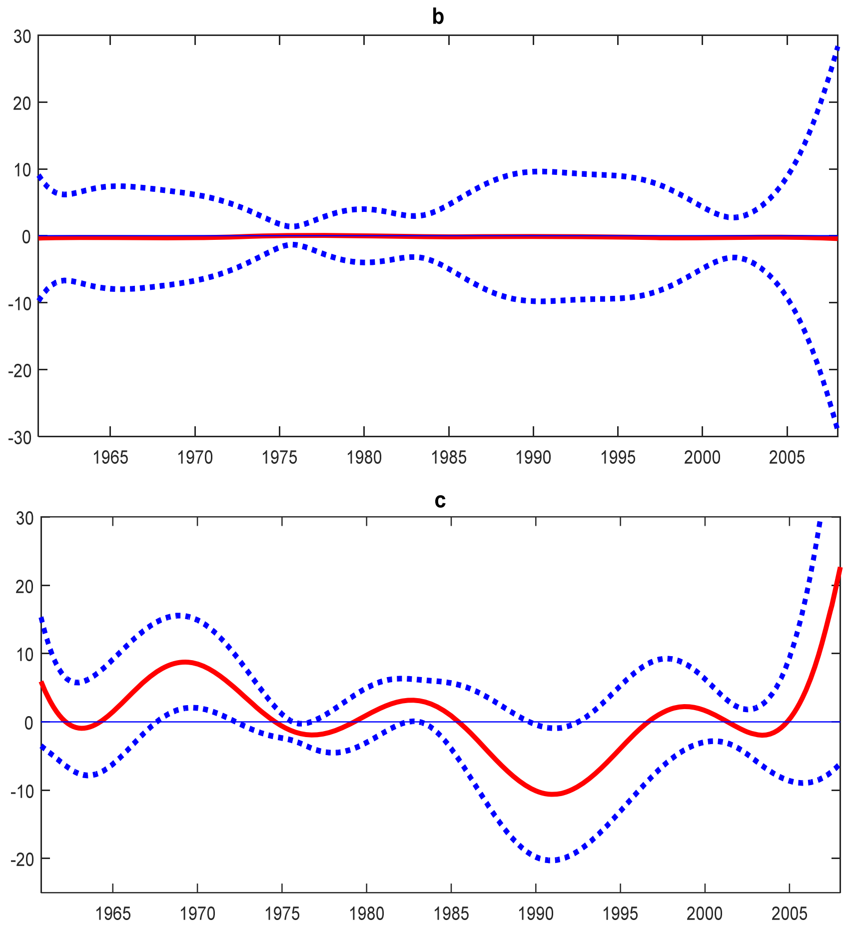


Table 1. Unit Root Test Results

	Unemployment rate	Inflation rate
Augmented Dickey–Fuller	-3.1821**	-2.2882
Phillips and Perron	-2.3576	-2.8779**
Kwiatkowski–Phillips–Schmidt–Shin	0.1477	0.2156

Notes: When a unit root test is conducted, an intercept is included in a test equation. A lag length is selected by the Akaike information criterion (AIC) for the augmented Dickey–Fuller test. Andrews (1991b)' bandwidth selection is used for the Phillips–Perron test and the Kwiatkowski–Phillips–Schmidt–Shin test. *, **, and *** denote that the null hypothesis is rejected at the 10%, 5%, and 1% levels respectively.

Table 2. Constant Coefficient Regression Results

	a	b	c
$E_{t-1}(u_t) = u_t$ and $\sigma_{u,t}^2$ is the realized variance of u_t	0.0039 (0.0029)	0.0725 (0.0508)	4.7141*** (1.2829)
$E_{t-1}(u_t)$ and $\sigma_{u,t}^2$ are from the ARIMA (2,0,2) model with GARCH (1,1) for u_t	0.7621*** (0.2732)	-0.0067 (0.0510)	3.0747*** (1.0557)

Notes: This table shows the regression results for $\pi_t = a + bE_{t-1}(u_t) + c\sigma_{u,t}^2 + e_t$. The sample period is 1960: I—2007: IV. The numbers in parentheses are Newey–West standard errors.

Table 3. Multiple Breakpoint Tests (1960: I—2007: IV)

Methods	Selection	The number of breaks	Break dates
1 to M globally determined breaks	Sequential F-statistic determined breaks	5	1967Q2, 1974Q2, 1983Q4, 1991Q4, 2000Q1
	Significant F-statistic largest breaks	5	1967Q2, 1974Q2, 1983Q4, 1991Q4, 2000Q1
	UDmax determined breaks	2	1971Q1, 1982Q4
	WDmax determined breaks	5	1967Q2, 1974Q2, 1983Q4, 1991Q4, 2000Q1
L + 1 vs. L globally determined breaks	Sequential F-statistic determined breaks	2	1971Q1, 1982Q4
	Significant F-statistic largest breaks	4	1967Q2, 1974Q2, 1983Q4, 1991Q4
Global information criteria for 0 to M globally determined breaks	Schwarz criterion selected breaks	4	1967Q2, 1974Q2, 1983Q4, 1991Q4
	LWZ criterion selected breaks	3	1967Q2, 1974Q2, 1983Q1

Notes: Breakpoints are determined by Bai–Perron tests of globally determined breaks. The test statistics are attained by employing HAC covariance. The number of breaks and estimated break dates are determined according to each methodology and reported in the third column. The obtained results are from the case when the trimming percentage of the sample is set to be 15%, the maximum number of breaks is 5, and the significance level is 0.05.

Table 4. *h*-block and Modified *h*-block Cross-Validation Criteria

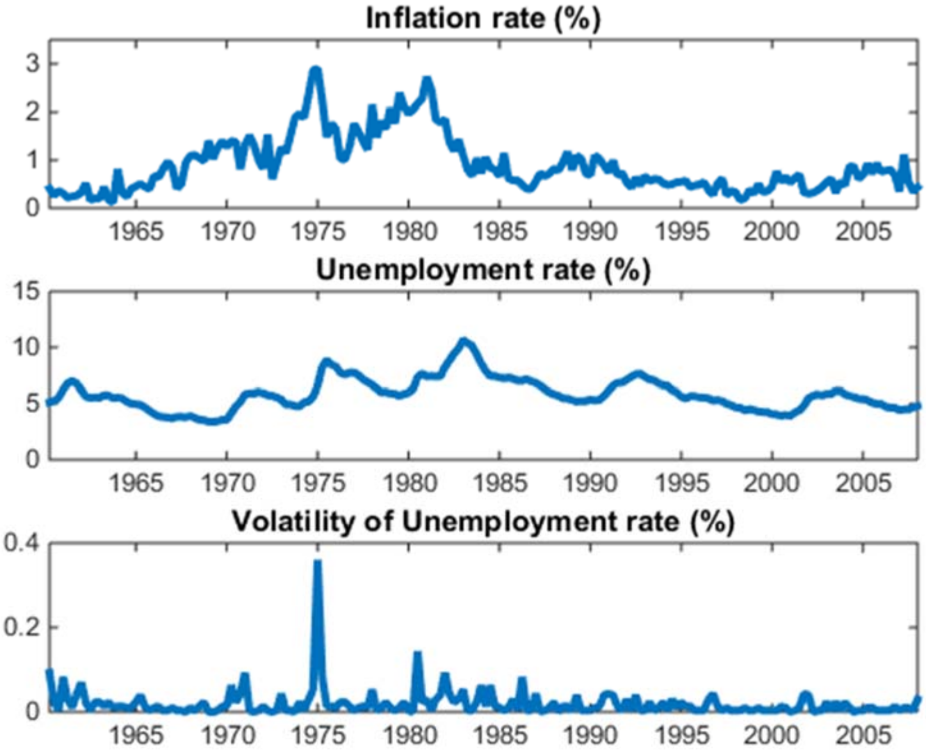
	h-block CV	Modified h-block CV
$1, m, m^2$	2970.7	5855.8
$1, m, m^2, m^3$	3184.9	6489.5
$1, m, m^2, m^3, m^4$	3303.8	6926.2
$1, m, m^2, \ldots, m^5$	3376.0	7251.9
$1, m, m^2, \cos(m), \sin(m)$	1316.7	4288.7
$1, m, m^2, \cos(m), \sin(m)$ $\cos(2m), \sin(2m)$	683.8	3649.0
$1, m, m^2, \cos(m), \sin(m), \ldots$ $, \cos(3m), \sin(3m)$	663.8	3633.1
$1, m, m^2, \cos(m), \sin(m), \ldots$ $, \cos(4m), \sin(4m)$	969.2	3984.7

Notes: h-block cross-validation and modified h-block cross-validation are data-dependent criteria for the selection of the optimal number of series functions. Statistics for h-block cross-validation and modified h-block cross-validation are computed from equations (8) and (9)~~,~~ respectively. The n that minimizes the above CV criteria is selected. See Burman, Chow, and Nolan (1994), and Racine (1997) for further discussion.

Figure 1. Inflation and Unemployment

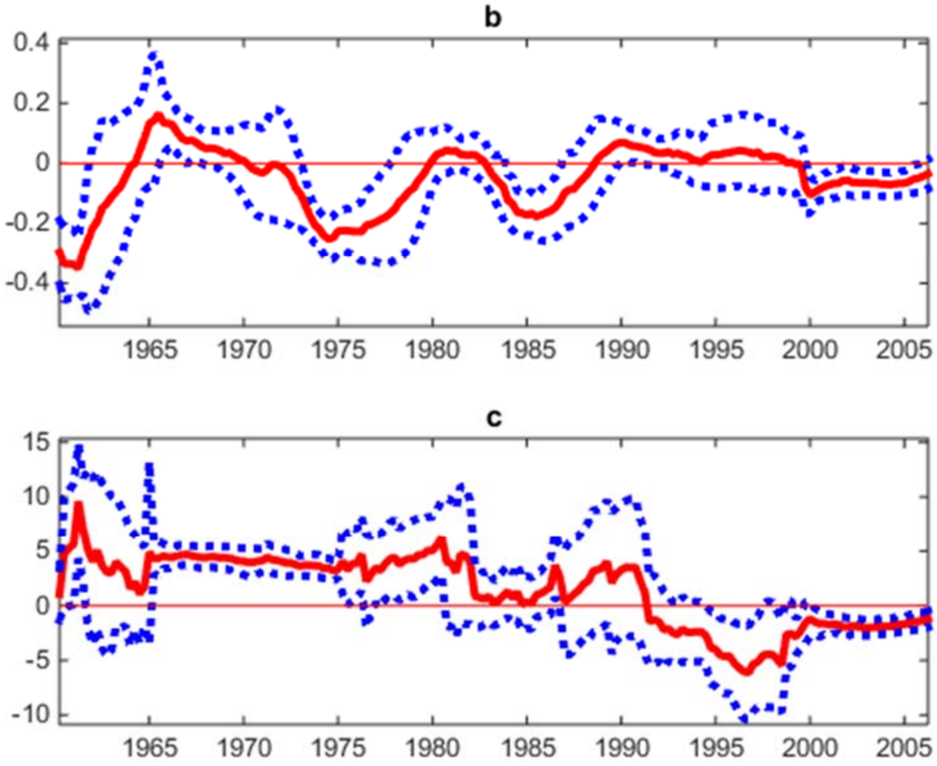
Note: The panels in this figure show the plots of unemployment rate and inflation rate during the 1960s and during the period between 1960 and 2007 respectively.

Figure 2. Movements of the Inflation Rate, Unemployment Rate, and Volatility of the Unemployment Rate



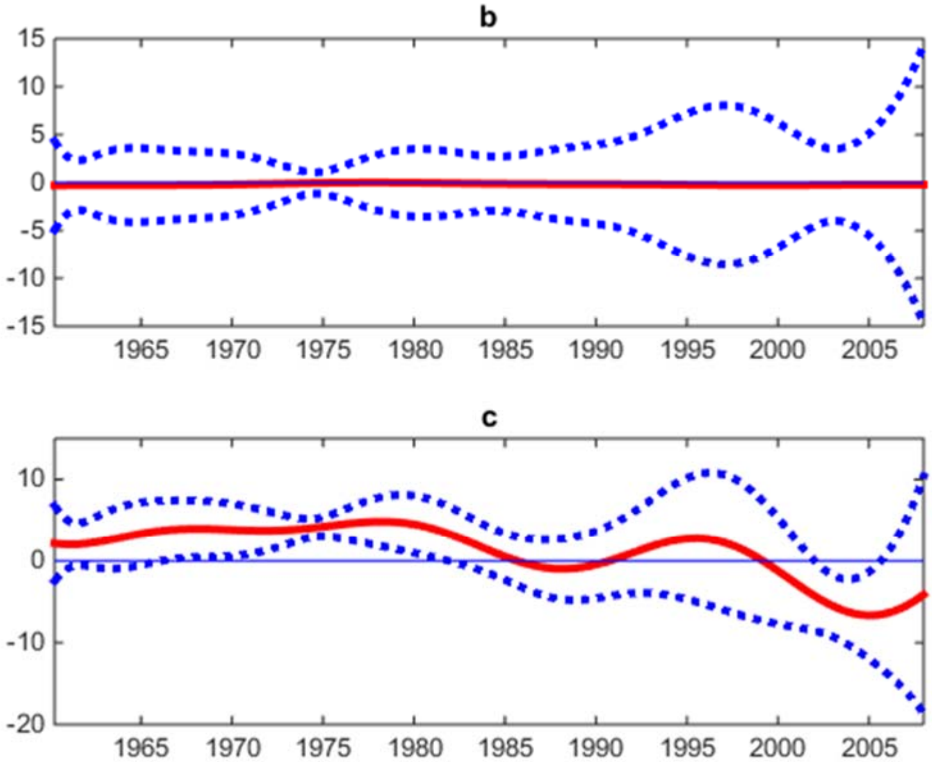
Notes: The panels in this figure show the plots of the inflation rate, unemployment rate, and volatility of the unemployment rate during the period between 1960 and 2007 respectively. The volatility of the unemployment rate is measured as the realized variance of the unemployment rate through the use of monthly actual unemployment rates during each quarter.

Figure 3. Regression Coefficients from the Rolling Regressions



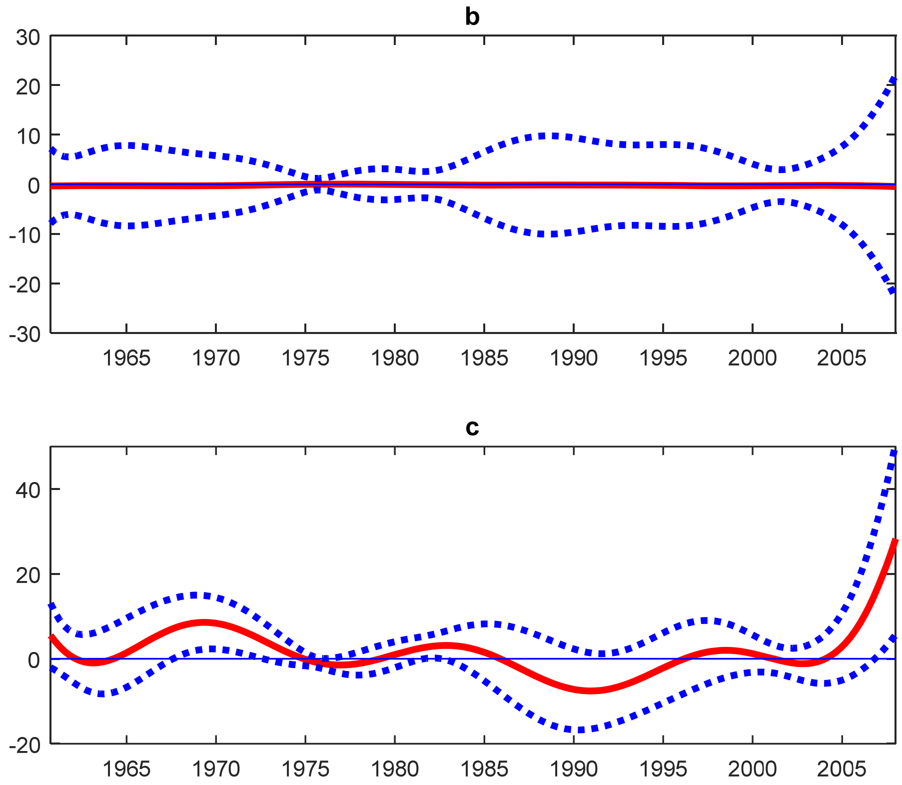
Note: The regression in equation (6) is regressed with a rolling window size of 40 quarters (a 10-year horizon).

Figure 4. Results from the Time-varying Coefficient Regression 1



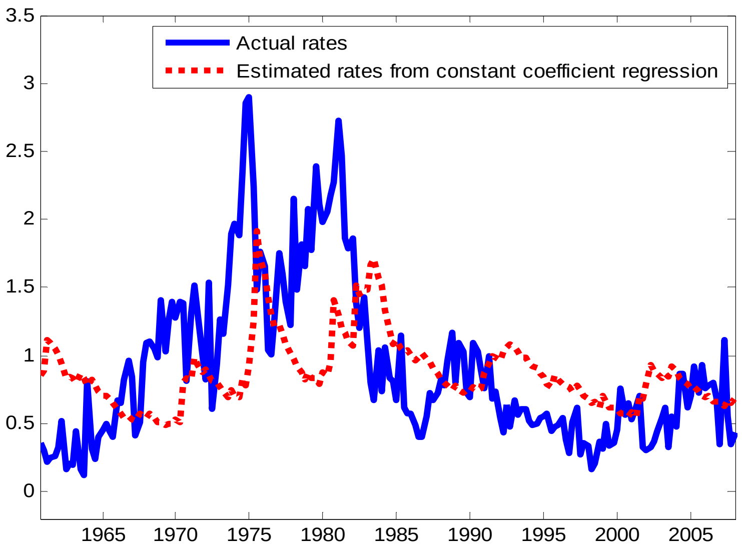
Notes: $\pi_t = a_t + b_t E_{t-1}(u_t) + c_t \sigma_{u,t}^2 + e_t$ is run. $E_{t-1}(u_t)$ is replaced with $u_t + \varsigma_t$ under the rational expectation assumption, and the realized variance of u_t is used for $\sigma_{u,t}^2$.

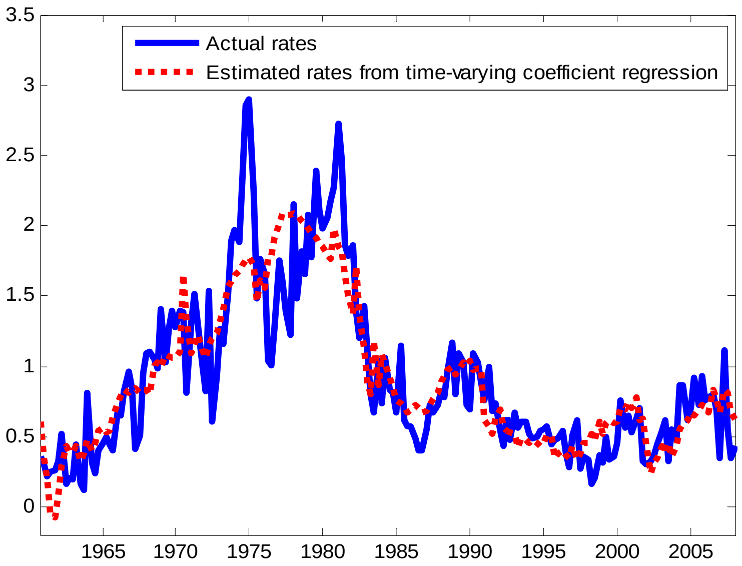
Figure 5. Results from Time-varying Coefficient Regression 2



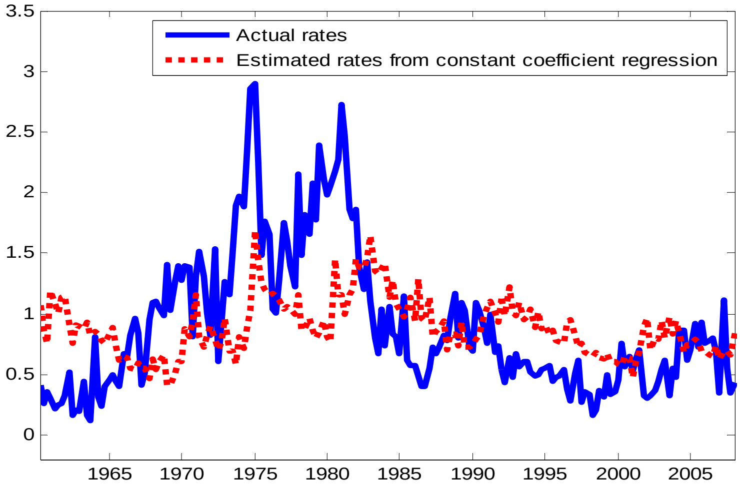
Notes: $\pi_t = a_t + b_t E_{t-1}(u_t) + c_t \sigma^2_{u,t} + e_t$ is run. $E_{t-1}(u_t)$ and $\sigma^2_{u,t}$ are constructed from the ARIMA (2,0,2) model with GARCH (1,1) for u_t, as in Ruge-Murcia (2003).

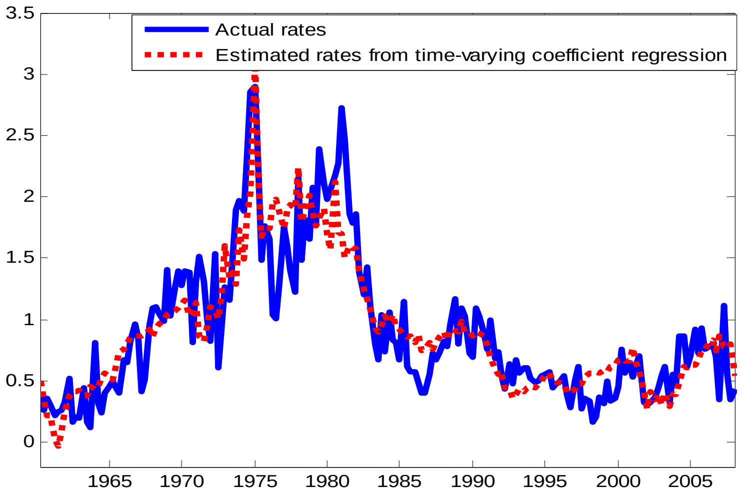
Figure 6. Estimated Inflation Rates from Constant Coefficient Regression and Time-varying Coefficient Regression





Notes: Plots in this figure compare the actual inflation rate with the fitted value of the inflation rate from the constant coefficient regression and time-varying coefficient regression respectively. $E_{t-1}(u_t)$ and $\sigma^2_{u,t}$ are constructed from the ARIMA (2,0,2) model with GARCH (1,1) for u_t.





Notes: Plots in this figure compare the actual inflation rate with the fitted value of the inflation rate from the constant coefficient regression and time-varying coefficient regression respectively. $E_{t-1}(u_t)$ is replaced with $u_t + \varsigma_t$ under the rational expectation assumption, and the realized variance of u_t is used for $\sigma^2_{u,t}$.