Forbearance in Prudential Regulation

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Abstract

We construct a model of a self-interested financial regulation agency who incurs private cost when a bank defaults. The agency decides whether to regulate the bank conditional on signal observation, which contains information about the risk of the bank. The agency may choose not to regulate a bank even if its condition looks bad enough, which we call forbearance. The forbearance occurs due to the conflict of interests among the regulator and the bank. In particular the regulator postpones the resolution of uncertainty when he expects the bank would not comply with the regulation.

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*Preliminary, please do not quote.

1 Introduction

The recent financial crises which have originated from the U.S. subprime mortgage market had a devastating effect on the global economy. While there are still debates on who are responsible, there is no doubt that the prudential regulation system did not perform. The present paper investigates the incentive problems in the prudential regulation system.

Financial markets are unique in that regulation is a norm rather than an exception. Due to the significance of the financial market for the proper functioning of the economy and the possibility of excessive risk taking on the part of financial firms, the financial markets are subject to regulation. Regulators monitor the risk assumed by financial firms and apply corrective actions in case the measures, which represent the riskiness of the financial firm, enter a dangerous zone.

After the global financial crises, there have been many attempts to design a better prudential regulation framework. However approaches which examine only the policy tools of current prudential regulatory system may be begging a more fundamental question, "Does the regulator have a proper incentive to regulate?" since despite the market had many regulatory tools available, the crises happened. This paper focuses on the incentive issues of the regulator.

Prudential regulation involves systematic collection and analysis of detailed information about the risk taking of financial firms. The regulator has to make costly efforts to collect and analyze relevant information about the target firm. Moreover the regulator has to apply certain rules to determine whether the target firm deserves a disciplinary action. The regulator approaches the task of regulation with a set of trigger rules for corrective action in case the target firm takes excessive risk. In the U.S., the FDIC Improvement Act of 1991 provides the procedure for the regulator's intervention in case of prudential concern. The Act sets out 5 categories of capital, for each of which a particular course of action is prescribed. For instance a bank, which is "undercapitalized," is required a capital restoration plan so that it is adequately capitalized upon its completion.

It is often argued that the regulator has an incentive to defer the application of the corrective action in the hope that the target firm's financial situation turns positive later by luck. The problem is called the *forbearance* of the regulator. The paper explains that forbearance may occur due to the incentive of the regulator, who incurs a private cost in the case the monitored bank is found in financial difficulty.

We construct a model of a self-interested financial regulation agency who incurs a privatel cost when a bank defaults. The agency decides whether to regulate the bank conditional upon a signal observation, which contains information about the risk of the bank. The agency may choose not to regulate a bank even if its condition looks bad enough, which we call forbearance.

In the model, the bank chooses an investment project, which will generate informative signal about the default risk of the bank before the true state is revealed at the end of the game. Different projects generate signals with different precisions. Since the optimal decision rule of the regulator depends on the precision of the signal, the bank chooses the project with the signal which will induce its preferred decision from the regulator. We explain why the regulator's intervention rule may be sub-optimal and we also show that the bank's project choice is made in such a way to induce a sub-optimal intervention of the regulator.

Section 2 constructs a model which will be employed to analyze the equilibrium behavior of the regulator and the bank. Section 3 explains how forbearance occurs at the equilibrium of the model, while Section 4 analyzes the socially optimal intervention rule for the regulator so that we can compare the optimal regulatory rule against the equilibrium regulatory rule. Section 5 concludes with a brief discussion of desirable regulatory framework.

2 Model

The model has 2 agents, a regulator and a bank. The regulator is self-interested in that he maximizes his own payoff from the regulation, which is different from the payoff of the economy or the bank. The payoff depends on the state of the bank, which is either *default* or *solvent.* The regulator gets payoffs $\underline{V}$ when the bank is in default and $\overline{V}$ when it is solvent, respectively. Similarly the total payoffs of all agents in the economy except for the regulator and the bank are denoted $\underline{W}$ when the bank is in default and $\overline{W}$ when it is solvent, respectively. Finally the bank gets payoffs 0 when the bank is in default and π when it is solvent, respectively. We define a few notations for future use and assume that $\Delta V = \overline{V} - \underline{V} > 0$, $\Delta W = \overline{W} - \underline{W} > 0$, and $\pi > 0$.

Let θ be a solvency index:[1] the bank is in default when the solvency index is below a cut-off

[1] As an example of the solvency index, consider the BIS capital adequacy ratio, which provides information about the solvency of the bank.

value, which we set equal to 0. We assume that θ is uniformly distributed on $[-1, 1]$.[2]

The bank makes an investment[3] which yields a constant profit π when the bank is solvent and 0 otherwise (due to the limited liability of the bank). Before the profit is realized, the project generates a noisy signal about the solvency of the bank: $s = \theta + \epsilon$ where ϵ is a noise, distributed uniformly on $[-\sigma, \sigma]$. The bank chooses among projects with different precisions, which we identify with σ for the convenience of discussion.[4] In particular, all invesment projects the bank may choose have the same profit π when the bank is solvent at the end of the game but the different investment projects generate signals with different values of σ.[5]

We assume that $\frac{1}{2} > \sigma > 0$, where the left inequality rules out pathological cases where the noise overwhelms the interaction of the bank and the regulator while the right inequality implies that it is impossible to intrinsically remove uncertainty in the bank's operation.

After the bank makes investment decision but before the true state is revealed, the regulator receives the noisy signal. The information the regulator receives includes the signal value together with the precision of the signal.[6] The regulator decides whether to intervene once the signal is observed.

Since the signal s is a random variable which is the sum of two uniformly distributed random variables, the probability density function of s is computed as:

$$f_\sigma(s) = \begin{cases} \frac{1}{4\sigma}(\sigma + 1 + s) & -1 - \sigma \leq s < -1 + \sigma \\ \frac{1}{2} & -1 + \sigma \leq s < 1 - \sigma \\ \frac{1}{4\sigma}(\sigma + 1 - s) & 1 - \sigma \leq s \leq 1 + \sigma \end{cases} \tag{1}$$

Upon observing s, the regulator decides whether to take a prompt corrective action (PCA). To take PCA, the regulator pays a cost, γ. If the regulator decides not to take PCA, no further action is taken by any agent until the true state is realized at the end. If the regulator takes PCA,

[2]We take the particular parameterization on the support of θ and the cut-off value of 0 to avoid unnecessary computational and notational complicatrion. They are not responsible for major results.

[3]This investment decision problem is not analyzed in the model; instead we take the invetment decision as given in the model until we extend the model to calculate the payoffs from investment projects with different noise property.

[4]The smaller σ is, the higher the precision is.

[5]It would be more realistic to assume that the profit of the bank varies with the solvency index θ. However this change makes the calculation far more difficult without adding much to the main results.

[6]We can think of this assumption as the description of a situation where the regulator observes the intermediate outcome from the bank's investment as well as the statistic profile of the investment. For instance the bank's portfoilio may be composed of various assets, whose signal's precision can be assessed from its identities.

he immediately resolves the uncertainty in the following way: when the regulator takes PCA, the bank either increases the capital ratio to the required level or is put under restructuring by the regulator where once the capital ratio is increased to the required level or the bank is restructured, the default probability is zero, that is, the bank is solvent with certainty.

When the regulator takes PCA, the bank is asked to increase the capital ratio to the required level. If the bank is asked to increase the captail ratio, the bank decides whether to comply. To comply with the request, the bank pays the compliance cost proportional to the size of improvement, which is equal to the difference between the cutoff level of θ and s: if the bank with the noisy signal of s is asked to improve the solvency index to 0, it pays the compliance cost, $\alpha \cdot \max\{0, -s\}$.

If the bank does not comply, the regulator restructures the bank. The restructuring incurs the regulator an additional cost, λ. During the restructuring, the bank stops regular business operations until the problem is resolved, while after the restructuring completes, the probability of default is zero. We assume that when the bank is restructured, the bank gets zero payoff as in the case of default.[7] The regulator's payoff after paying the PCA cost γ and the restructuring cost λ is assumed still bigger than his payoff from the bank's default so that the regulator prefers PCA and restructuring to default: $\overline{V} - \gamma - \lambda > \underline{V}$.

The noisy signal s is private information of the regulator. In particular, the regulator may decide not to take PCA even if the signal s indicates a high probability of default. Except for the low private payoff in the case of default, the regulator is not punished for not taking PCA at low signal value.

Summarizing the description, the game has the following order of moves:

1. The bank makes an investment choice.

2. The regulator, upon observing the project's signal together with its precision, decides whether to intervene.

3. The bank decides whether to comply if the regulatortakes PCA.

4. The regulator restuructures the bank if the bank does not comply.

5. The state whether the bank is solvent is realized.

[7]The incumbent shareholders of the bank, who we identify as the bank in the model, will get zero payoff if the bank ownership is transferred to a new owner after restructuring.

We assume that the structure of the game as well as the parameters of the game including the payoffs of the bank and the regulator are common knowledge among the players.

Next we consider the bank's decision problem. Since the investment choice of the bank is taken as given in the model, we consider only the bank's decision on whether to comply if asked by the regulator. We assume that the bank is passive in the sense that it does not increase the capital ratio unless requested by the regulator.[8]

The bank is requested to improve its capital ratio depending on the signal value. There are 3 cases, first no intervention by the regulator, second the regulator's intervention which the bank complies, and third the regulator's intervention which the bank does not comply.

When the regulator requests the bank to improve its capital ratio, the bank has to decide whether to comply with the regulator's request. Define c^B as the signal value at which the bank is indifferent between complying with the regulator's request to increase the capital ratio and not complying:

$$\pi - \alpha \cdot \max\{0, -c^B\} = 0 \tag{2}$$

The bank will comply with the regulator's request to increase the capital ratio only when the index s is bigger than c^B, since otherwise the cost is greater than the benefit from the compliance. It follows that the regulator can anticipate the bank to comply only when the signal value is greater than c^B.

Next we analyze the decision problem of the regulator. The expected payoff of the regulator who takes no further action after observing s is:

$$Pr\{\theta > 0|s\}\overline{V} + \Pr\{\theta \leq 0|s\}\underline{V} \tag{3}$$

where the probability of default is computed as follows:

$$\Pr\{\theta \leq 0|s\} = \begin{cases} 0, & \text{if } s \geq \sigma, \\ \frac{-s+\sigma}{2\sigma}, & \text{if } \sigma \geq s \geq -\sigma, \\ 1, & \text{if } s \leq -\sigma. \end{cases} \tag{4}$$

[8] We make this assumption since self-regulation by the bank lacks in credibility. Hence to make a credible improvement of the capital ratio, it requires the intervention of the regulator.

Taking account of the default probability, the expected payoff of the regulator from taking no action at the realization of the signal s changes as follows.

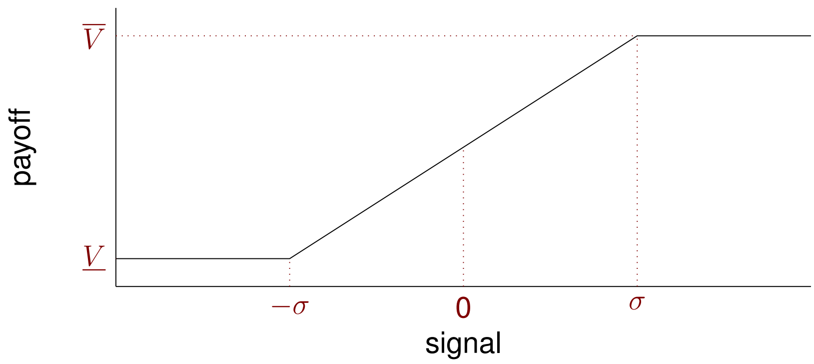
i) When $s \geq \sigma$, it is $\overline{V}$ since there is zero probability of default conditional on the signal.

ii) When $\sigma > s > -\sigma$, there is a positive probability of default if no action is taken and thus the expected payoff is $\frac{s+\sigma}{2\sigma}\overline{V} + \frac{-s+\sigma}{2\sigma}\underline{V}$.

iii) Finally when $-\sigma > s$ the expected payoff from no action is $\underline{V}$ since the bank is in default with probability 1.

The following picture describes how the payoff of the regulator changes as s changes when no action is taken.

Figure 1: **Payoff from No Action**



On the other hand the regulator has to pay costs if PCA is taken. If the bank complies with PCA, the regulator pays γ while if the bank does not comply, he pays $\gamma + \lambda$. It follows that the payoff of the regulator from taking PCA at the realization of the signal s changes as follows.

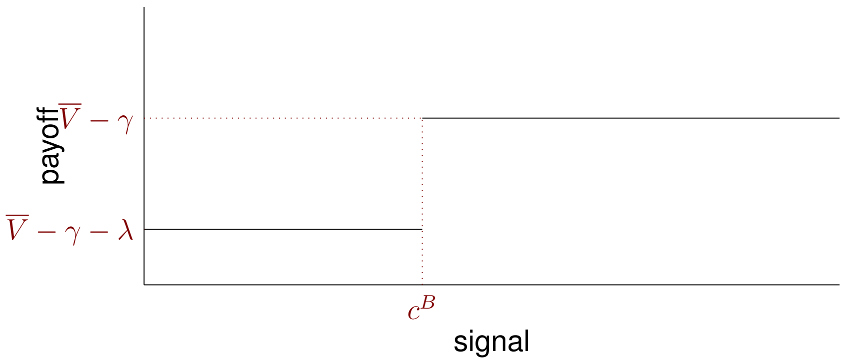
i) When $s \geq c^B$, the regulator gets $\overline{V} - \gamma$.

ii) When $s < c^B$, the regulator gets $\overline{V} - \gamma - \lambda$.

The following picture describes how the payoff of the regulator changes as s changes when PCA is taken.

The optimal behavior of the regulator can be obtained as follows: when the noisy signal is observed by the regulator, he compares the expected payoff from taking no action with that from taking PCA and chooses the one with higher payoff.

Figure 2: **Payoff from PCA**



3 Forbearance

In the whole game, the bank initially makes investment project choice which determines the precision of the noisy signal and subsequently the regulator decides whether to take PCA conditional on the realization of the signal. Here we first analyze the game after the bank makes the investment project choice since the bank should have a rational prediction of the game when making the project choice initially. Hence we take the precision of the signal, σ, as given.

Financial regulators are often blamed for exercising forbearance in the regulation: the regulator does not take regulatory action when a bank appears bearing too much risk, in the hope that the risk unravels in a favorable way.[9] This kind of behavior is frequently observed when an immediate action makes a bad outcome certain, while a delay in the action may allow the realization of a favorable outcome although the chance is small.

It is obviously rational that the regulator waits for an uncertain good outcome instead of fixing a certain bad outcome. However this choice becomes harmful if the delay makes the matter worse in the sense that the payoff from an immediate resolution of uncertainty is greater than the payoff from a delay. Moreover if the chance of a good outcome after delay is negligible, the cost of delay may be substantial.

We first analyze the equilibrium in this part of the game and then do comparative static analysis to understand the role of various parameters in the determination of the equilibrium.

[9] Dewatripont and Tirole (1994) define that forbearance "consists in lowering the capital adequacy requirements or not enforcing it."

In the present analysis we define forbearance in two forms: **non-monotonic forbearance** occurs if the regulator does not take PCA for a range of the signal values even if he takes PCA for signal values higher than the range and **extreme forbearance** occurs if the regulator takes PCA only when the signal value is extremely low.[10]

Non-monotonic forbearance appears particularly perverse since it is reasonable that if a bank with a certain capital ratio is put under PCA, another bank with a lower capital ratio must be put under PCA *a fortiori*. While a delay in regulatory action is usually implied by forbearance, it is hard to determine how lenient the regulator should be to exhibit forbearance. Non-monotonic forbearance is easy to distinguish since the regulator chooses apparently an inconsistent course of action.

We start by characterizing the equilibrium of the model and subsequently state the condition for forbearance.

Lemma 1 *In the equilirbium,*

1. **the bank** *complies if and only if* $s \geq c^B$ *and*
2. (a) *if* $s > \sigma$, *then* **the regulator** *takes* **no action**,
   (b) *if* $\sigma \geq s > c^B$, *then* **the regulator** *takes* **no action** *when* $\frac{s+\sigma}{2\sigma}\overline{V} + \frac{-s+\sigma}{2\sigma}\underline{V} > \overline{V} - \gamma$, *and* **PCA** *otherwise,*
   (c) *if* $c^B \geq s > -\sigma$, *then* **the regulator** *takes* **no action** *when* $\frac{s+\sigma}{2\sigma}\overline{V} + \frac{-s+\sigma}{2\sigma}\underline{V} > \overline{V} - \gamma - \lambda$, *and* **PCA** *otherwise,*
   (d) *if* $-\sigma \geq s$, **the regulator** *takes* **PCA**.

The Lemma states that the regulator takes PCA when early resolution of uncertainty yields a higher payoff than delayed resolution. While taking PCA is costly, the benefit of increasing the capital ratio which resolves uncertainty may outweigh the cost, in which case the regulator moves early. However taking PCA requires an additional restructuring cost if the bank refuses to comply. Since the bank refuses to comply when the cost of increasing the capital ratio exceeds its benefit, it is more likely that the regulator pays additional restructuring cost

[10]There is a correspondence among the two forms of forbearance and the characterization of forbearance due to Dewatripont and Tirole (1994) in the following way: the case of non-monotonic forbearance corresponds to not enforcing the captial adequacy requirements while that of extreme forbearance corresponds to lowering it.

as the signal deteriorates. Balancing the additional restructuring cost and the benefit, the regulator may defer the resolution of uncertainty and take no action even if the signal indicates a substantial probability of default.

The Lemma is easy to understand. When the signal is very high, there is no probability of default so that no intervention is optimal. Similarly when the signal is very low, the probability of default is 1 so that intervention is optimal. When the signal takes an intermediate value, intervention is optimal if and only if its cost is greater than its benefit. Since the cost of intervention changes depending on whether the bank complies, i.e., whether s is greater or smaller then c^B, the regulator compares the appropriate cost against the benefit which also changes with the signal value.

There are 3 different characterizations of the equilibrium depending on the pattern of intervention. An **equilibrium without forbearance** obtains if the regulator intervenes for s both greater than c^B *and* smaller than c^B, and an **equilibrium with extreme forbearance** obtains if the regulator does not intervene for s both greater than c^B *and* smaller than c^B. An **equilibrium with non-monotonic forbearance** obtains if the regulator intervenes for s greater than c^B but does not intervene for s smaller than c^B.

The names of the equilibria are intuitive; in the equilibrium without forbearance, the regulator intervenes without hesitation as long as there is enough probability of default while in the equilibrium with extreme forbearance, the regulator intervenes only when the default is very likely. In the equilibrium with non-monotonic forbearance, the regulator's choice changes non-monotonically around c^B since intervention occurs for s greater than c^B and no intervention occurs for s smaller than c^B although default is more likely as the signal deteriorates.

Proposition 1 *Define* $i^{NF} = [1 - \frac{2\gamma}{(\Delta V)}]\sigma$ *and* $i^F = [1 - \frac{2(\gamma+\lambda)}{(\Delta V)}]\sigma$.

1. *Equilibrium without forbearance: if* $\frac{c^B+\sigma}{2\sigma}\overline{V} + \frac{-c^B+\sigma}{2\sigma}\underline{V} < \overline{V} - \gamma - \lambda$, *the regulator intervenes for all* $s \leq i^{NF}$.

2. *Equilibrium with non-monotonic forbearance: if* $\overline{V} - \gamma > \frac{c^B+\sigma}{2\sigma}\overline{V} + \frac{-c^B+\sigma}{2\sigma}\underline{V} > \overline{V} - \gamma - \lambda$, *the regulator intervenes for* $c^B \leq s \leq i^{NF}$ *and* $s \leq i^F$

3. *Equilibrium with extreme forbearance: if* $\frac{c^B+\sigma}{2\sigma}\overline{V} + \frac{-c^B+\sigma}{2\sigma}\underline{V} > \overline{V} - \gamma$, *the regulator intervenes for all* $s \leq i^F$.

In the proposition, i^{NF} denotes the point at which the regulator starts intervention without forbearance and i^F denotes the one with forbearance. At $s = i^{NF}$, the regulator takes PCA with the knowledge that the bank will comply while at $s = i^F$, the regulator takes PCA with the knowledge that the bank will not comply. In the former case the intervention is preemptive so that it prevents further difficulty even if the regulator incurs the PCA cost. In contrast the regulator is forced to take PCA even if he knows that the bank will not comply in the latter case; in a sense it is an intervention, *too little, too late.*

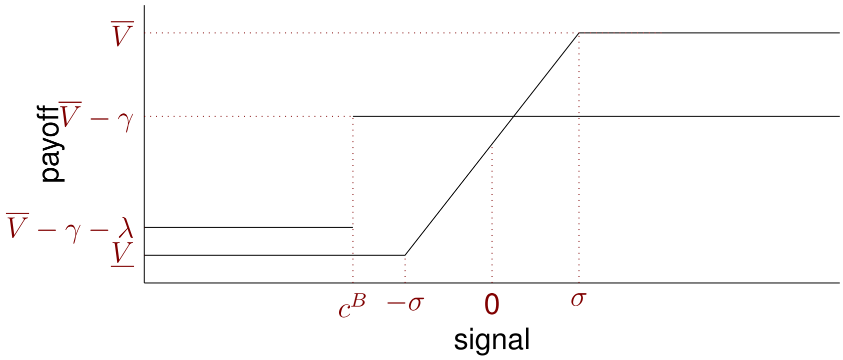
The 3 different equilibria have the following characterization. In the equilibrium without forbearance, the regulator starts intervention for a signal realzation at which the bank is wiling to comply and for any worse signals, the regulator always intervenes. If the signal realization is low enough, the bank starts not to comply when the regulator pays the restructuring cost additonally but still the regulator continues intervention. In the equilibrium with non-monotonic forbearance, the regulator starts intervention for a signal realization at which the bank is willing to comply. However the regualtor stops intervention when the bank stops complying with the request for capital ratio increase and resumes intervention when the signal realization is very bad. In the equilibrium with extreme forbearance, the regulator starts intervention only when the signal realization is very bad and the bank is not willing to comply. The intervention in this equilibrium is made only when it is unavoidable since the bank will almost certainly default if left alone. Hence we can say that the regulator hardly makes any preemptive move in maintaining the solvency of the bank.

It is obvious that due to the existence of the restructuring cost, an equilibrium with the intervention pattern opposite to non-monotonic equilibrium is not feasible. Indeed it is the additional restructuring cost that induces the regulator to hesitate to intervene although early intervention may prevent default.

The following figures help us understand the conditions of the proposition for different equilibria. Figure 3 describes the equilibrium without forbearance.

The equilibrium without forbearance obtains when the precision of the signal is high, i.e., σ is small. If the precision is high, the slanted section of the expected payoff from no action is steep so that at the signal realization c^B, the payoff is lower than the payoff from PCA and restructuring. It follows that the regulator chooses to take PCA at the threshold level preventing forbearance. This is the case where forbearance does not occur since the regulator takes PCA

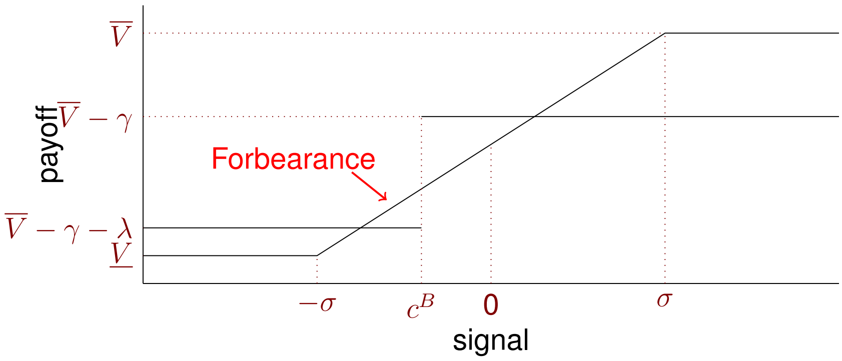
Figure 3: **Equilibrium without Forbearance**



continuously from the first point where the PCA is less costly than no action without taking account of the restructuring cost.

Figure 4 describes the equilibrium with non-monotonic forbearance.

Figure 4: **Equilibrium with Non-monotonic Forbearance**

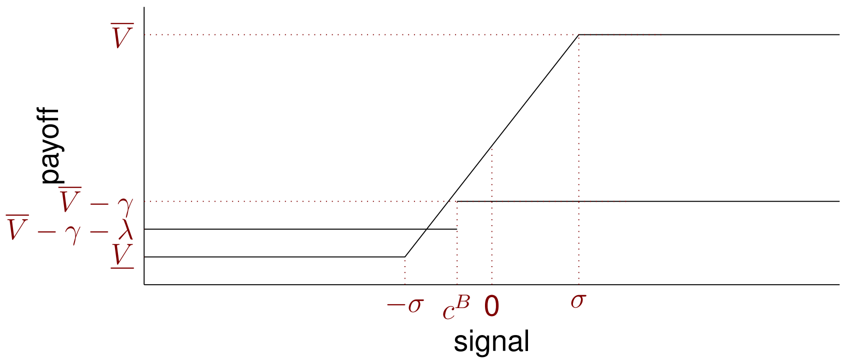


Forbearance occurs in the marked region in Figure 4. When the restructuring cost introduces a discontinuous downward jump in the payoff from PCA, forbearance occurs. To avoid the restructuring cost, the regulator delays immediate resolution of uncertainty through PCA. When the regulator takes PCA, he hopes that the bank will comply so that he can avoid paying default cost. However he knows that taking PCA will cost him restructuring effort as well, if the bank does not comply. Hence the regulator ceases taking PCA when the bank will refuse to comply since he can prevent paying the additional restructuring cost.

According to the proposition, forbearance occurs if the expected payoff from no action when the noisy signal is at the threshold level c^B is neither too high nor too low. It turns out that this condition restricts the precision of the noisy signal, σ, the cost of PCA, γ, and the restructuring cost, λ.

Figure 5 describes the equilibrium with extreme forbearance.

Figure 5: **Equlibrium with Extreme Forbearance**



In the equilibrium with extreme forbearance, PCA cost is so high that the regulator delays the early resolution of uncertainty even if the bank complies. This is an extreme case of forbearance since the regulator does not take PCA unless the solvency index is very low.

Which equilibrium characterization holds under a given economic condition depends on the parameter values. Next we anaylyze the condition on the parameter values for each equlibrium characterization so that we can determine which equilirbium is observed under a certain economic condition.

Proposition 2 *1. An equilibrium without forbearance occurs if* $[\frac{2(\gamma+\lambda)}{\Delta V}-1]\sigma \leq \frac{\pi}{\alpha}$.

2. An equilibrium with non-monotonic forbearance occurs if $[\frac{2(\gamma+\lambda)}{\Delta V}-1]\sigma > \frac{\pi}{\alpha} \geq [\frac{2\gamma}{\Delta V}-1]\sigma$.

3. An equilibrium with extreme forbearance occurs if $[\frac{2\gamma}{\Delta V}-1]\sigma \geq \frac{\pi}{\alpha}$.

The proof of the proposition follows from rearranging the conditions in Proposition 1 and replacing c^B with $-\frac{\pi}{\alpha}$.

A close inspection of the condition in Proposition 2 reveals interesting properties. From part 3 of the proposition, equilibrium with extreme forbearance cannot be observed if $\Delta V > 2\gamma$. Under this condition, the left hand side of the condition in part 3 is positive while the right hand side is negative so that the left hand side cannot be smaller than the right hand side. It follow that when the PCA cost is smaller than half of the regulator's payoff difference from default and solvency, the equilibrium with extreme forbearance will never be observed.

A similar reasoning implies that only the equilibrium without forbearance will be observed if $\Delta V > 2(\gamma + \lambda)$ from part 1 of the proposition. Since the restructuring cost λ is positive it is always the case that $\gamma + \lambda > \gamma$, that is, PCA alone is less constly than PCA and restructuring. Hence when $\Delta V > 2(\gamma + \lambda)$, then $\Delta V > 2\gamma$ so that when equilibrium with non-monotonic forbearance is not feasible, equilibrium with extreme forbearance is not feasible either.

The above properties imply that different equilibrium characterizations prevail depending on the relative size of the regulation costs, γ and λ, and the net payoff of solvency of the bank, ΔV. Forbearance equilibrium is more likely when PCA cost and restructuring cost are more substantial compared to the net payoff of solvency. For instance, the regulator hesitates to intervene when its cost is big. When the restructuring cost is big while PCA cost is not as big, the regulator is willing to intervene as long as the bank is prepared to comply with the capital ratio increase but stops intervention when the bank stops complying so that equilibrium with non-monotonic forbearance occurs. If PCA cost is already very big, the regulator is reluctant to intervene at all since PCA alone is very costly.

An interesting obervation is that if equilibrium with extreme forbearance is feasible, then other 2 equilibria are also feasible depending on the parameter values. Similarly the existence of equilibrium with non-monotonic forbearance implies the existence of equilibrium without forbearance although equilibrium with extreme forbearance may not exist. Finally equilibrium without forbearance always exists although the other 2 equilibrium may not exist. For future reference, we summarize the property in the following corollary without proof.

Corollary 1

1. *If equilibrium with extreme forbearance is feasible, then equilibrium with non-monotonic forbearance and equilibrum without forbearance are also feasible depending on the values of* σ, π, *and* α.

2. *If equilibrium with non-monotonic forbearance is feasible, then equilibrum without no forbearance is also feasible depending on the values of* σ, π, *and* α.

We can expect that both costs are moderate when the financial condition of an economy is good. Under this economic condition, the regulator is not likely suffering from the problem for forbearance. However when the economic condition excerbates as during the recent global financial crises, both costs get bigger. Restructuring cost is high since the capital needed for the restructuring may have to be funded from taxpayer's money which requires the approval of the parliament. PCA cost could be also high since the society may take the action of PCA as a signal of regulation failure. Moreoever during the crises, a hint of PCA on a bank may trigger a run on the bank so that restructuring is almost automatic if PCA is taken.

Next we analyze the change of the bank's compliance pattern when the payoff parameters of the bank change. Notice that the bank complies only when its cost is greater than its benefit. In particular the bank complies if and only if $s \geq c^B$ where $c^B = -\frac{\pi}{\alpha}$ which is smaller than 0. Moreover c^B gets smaller when π is bigger or α is smaller. The bank is more willing to comply when the benefit from the compliance, $\pi,$ is bigger or when the unit cost of compliance, $\alpha,$ is smaller.

From the conditions of Proposition 2, the forbearance problem of the regulator is less likely when either π is bigger, or α is smaller, or both. Again we can interpret this result for different economic conditions. If the economy is doing well, the profit of the bank may be big and also the financial cost to capital ratio increase is small so that forbearance problem is not sever. However as the economic condition deteriorates, the bank can expect a smaller profit even if it survives and the financial cost of capital ratio increase is prohibitively high since nobody is willing to lend money under uncertain economic condition.

Finally we analyze the effect of the size of precision on the equilibrium characterization. Remember that the precision is a control variable of the regulator although in the subgame after its choice, it is a parameter which is beyond the control of the regulator. In the analysis of the entire game, we have to find the regulator's optimal solution with respect to the size of precision, which should be determined in view of the equilibrium in the subgame after the precision choice. Hence the comparative static result here provides a ground work for the analysis of the entire game.

From the analysis above, some equilibrium characterizations are not feasible unless the regulator's payoff parameters satisfy certain conditions. For instance when the costs of interventions are negligible relative to the benefit of solvency, forbearance equilibrium ceases to be feasible. For the following analysis we assume that cost and payoff parameters of the regulator allow all 3 equilibrium characterizations: $\Delta V \leq 2\gamma$.

First observe that when $c^B = \frac{\pi}{\alpha} < -\sigma$, the bank is willing to comply for all cases where uncertainty remains about the true state conditional on the noisy signal. It follows that equilibrium without forbearance obtains if $\sigma < \frac{\pi}{\alpha}$: if the precision of the signal is high enough, then the regulator does not suffer from forbearance.

The conditions in Proposition 2 compare linear functions of σ, $[1 - \frac{2\gamma}{(\Delta V)}]\sigma$ and $[1 - \frac{2(\gamma+\lambda)}{(\Delta V)}]\sigma$ against the cutoff value of the bank's compliance, c^B. When σ is small, the condition for equilibrium without forbearance is satisfied; as σ grows bigger, the condition for equilibrium with non-monotonic forbearance is satisfied and finally when σ is big enough, the condition for equilibrium with extreme forbearance is satisfied. As expected, the regulator suffers from more severe forbearance problem as the signal gets less precise.

Next we analyze the effect of parameter change on the points at which the regulator starts intervention and the bank changes compliance decision.

Proposition 3 *1. i^{NF} and i^F are increasing in ΔV and decreasing in γ and λ*

2. i^{NF} and i^F are decreasing in σ if $[1-\frac{2\gamma}{\Delta V}] < 0$ while i^{NF} is increasing in σ if $[1-\frac{2\gamma}{\Delta V}] > 0$.

3. c^B is increasing in α and decreasing in π.

As expected, Part 1 indicates that the regulator intervenes at a higher signal when the benefit from the intervention, ΔV, is bigger or the its costs, γ and λ are smaller. This is natural since the regulator has more to gain from the intervention when the benefit from the intervention is bigger or the its costs are smaller. Also Part 3 shows that the bank's decision behavior is consistent with the intuition because as the benefit from the compliance grows bigger or its cost gets smaller, the bank is more willing to comply. Part 2 reveals an interesting aspect of the effect of the noise on the regulator's behavior. When the regulator starts intervention without forbearance in the negative range of the signal, an increase in the noise induces the regulator to intervene later. However the regulator starts intervention earlier for a bigger noise if the starting point of intervention without forbearance, i^F, is in the positive range. This is due to the fact that when there is a bigger noise, even a more positive signal allows a non-negligible probability of default, which induces the regulator to intervene earlier. This property gives the bank an incentive to choose an investment project with bigger noise so that the regulator may intervene ealier if the intervention will start even for a positive signal. A similar property cannot hold true for i^F since by definition, it is in the negative range smaller than c^B.

4 Socially Efficient Regulation

The economy's payoff from the performance of the bank is different from the regulator's payoff. In particular the social cost of a bank's failure is immense and the social benefit of keeping a bank solvent is also huge. In addition the social costs of PCA and restructuring may differ from those of the regulator. The social cost of PCA may not be big since PCA involves the bank to take a set of actions to avoid deterioration of the financial strength. While the social cost of restructuring may be big since the taxpayers' money needed for the restructuring may be substantial, it cannot be compared to the social cost of bank's failure and the social benefit of keeping the bank solvent through restructuring.

The present section analyzes socially optimal outcome. We ask when it is socially optimal to allow no forbearance although the regulator may choose either equilibrium with non-monotonic forbearance or equilibrium with extreme forbearance.

We compute the welfare effect of different choices for the whole economy including the bank and the regulator. The socially optimal choice is computed from the following scenario. Given a fixed precision level of signal, a social planner who internalizes payoffs to all agents in the economy observes the signa. He increase the capital ratio if it yields higher payoff for the economy including the bank and the regulator. In contrast to the game with the regulator and the bank, the social planner maximizes the sum of payoff for particitants. In particular the cost of intervention should be paid regardless of who pays it. Hence the intervention cost consists of two parts, the PCA cost and the capital ratio increase cost.

Since the comparison of payoffs from different action choice is straightfroward when there is no uncertainty conditional on the signal observation, we focus on the signal range, $s \in [-\sigma, \sigma]$.[11]

The social payoff is $(\overline{W} + \overline{V} + \pi)$ if the bank is solvent and $(\underline{W} + \underline{V} + 0)$ if default. It follows that the expected payoff from no action conditional on the observation of the signal s is computed as

$$\frac{s+\sigma}{2\sigma}(\overline{W} + \overline{V} + \pi) + \frac{-s+\sigma}{2\sigma}\underline{W} + \underline{V}.$$

If an intervention on the bank is made, PCA should be taken and the capital ratio is increased conditional on the signal observation if the cost from capital ratio improvement is smaller than

[11] When the signal is above σ or below $-\sigma$, the bank is either solvent with probability one or default with probability one.

the resutructuring cost while PCA and resturucturing takes place if the cost from capital ratio improvement is greater than the resutructuring cost so that the overall payoff is $\overline{W} + \pi - \gamma - \min\{\alpha \max\{0, -s\}, \lambda\}$. Comparing the payoffs, the optimal decision rule is to increase the capital ratio when $\frac{-s+\sigma}{2\sigma}((\overline{W} + \overline{V}) + \pi - (\underline{W} + \underline{V})) \leq \gamma + \min\{\alpha \max\{0, -s\}, \lambda\}$. Since the regulator starts intervention for signal value at which PCA is taken and the capital ratio is improved before the compliance cost exceeds the restructuring cost, we can ignore the effect of λ in the comparison.

Define i^{FB} as the cutoff point for socially optimal capital ratio increase. If $i^{FB} < 0$, the solvency index s is increased if the realized signal is smaller than the cutoff point, while it is set equal to 0 if an intrervention is made for $s > 0$. However even for the case when the solvency index is set to 0 from a positive signal value, PCA cost should be paid by the economy.

$$i^{FB} = \left[\frac{\Delta W + \Delta V + \pi}{2} - \gamma\right] / \left[\frac{\Delta W + \Delta V + \pi}{2\sigma} - \alpha\right]. \tag{5}$$

Remember that the cutoff point in equation (5) is computed under the assumption that it is negative. We can assume that the numerator of equation (5) is positive since the net benefit of the bank's solvency for the whole economy would be huge. Hence there is a well-defined cutoff point only when $\frac{\Delta W+\Delta V+\pi}{2\sigma} - \alpha < 0$ since the numerator is always positive. When the benefit of solvency of the bank dominates the unit cost of capital increase, i.e., $\frac{\Delta W+\Delta V+\pi}{2\sigma} > \alpha$, the socially optimal decision rule is to intervene for all $s \leq 0$. In fact, there is not forbearance in the socially optimal intervention rule since an intervention is made whenever capital ratio increase is needed.

From the decision rule for the current problem, we can conclude that non-monotonic forbearance cannot occur in the socially optimal solution. In the regulator-bank problem, the regulator pays the PCA cost and the bank pays the capital ratio increase cost. Hence the bank stops cooperating when the cost of compliance gets smaller than its benefit. Non-monotonic forbearance is a consequence of the conflicts among agents with respect to the payoff and cost of intervention.

In contrast, the benevolent social planner who solves the problem of social optimum has to pay PCA and restructuring cost and the capital ratio increase cost whenever intervention takes place. Since the intervention is made when its benefit is greater than its cost and its cost is monotone increasing as the signal deteriorates, it is not possible to have a decision rule similar to equilibrium with non-monotonic forbearance.

5 Conclusion

The paper explains why forbearance by the regulator may occur. The regulator wants to delay the resolution of uncertainty in the hope that a good luck may produce a good outcome, although not very likely. Knowing this incentive problem of the regulator, the bank deliberately chooses the investment project which induces the regulator's forbearance.

The analysis indicates that the precision of the signal which provides information about the risk of the bank plays a crucial role in determining the decision rule of the regulator. In particular, the regulator is not subject to the forbearance problem when the signal has a high precision. Hence the financial regulation system should find a way to improve the precision of the signal which the regulator uses in the intervention decision. For instance an introduction of information agency which can improve the precision of the signal to be used by the regulator appears desirable.

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