

# Long Term Financial Projection of Public Pension in Japan

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## Abstract

Providing better information on the financial status of social security pension is important for planning and maintaining the long-term financial viability or sustainability of those systems.

In this study, pension projections are implemented with different assumptions from those used in 2009 Actuarial Valuation. Under stochastic mortality and economic model, the future demographic structure and economic variables are forecasted and the financial status of EPI pension is projected until 2099.

For the first, of various stochastic mortality models the one which is best-suited for Japanese mortality trends are selected and then, population projections are implemented. Secondly, the economic assumptions for the future wage growth and inflation rate are forecasted introducing the Bayesian VAR model to examine how the changes in economic variables affect the financial status of pension finance. Finally combining the forecasts estimated from above two models it is possible to project the future pension finance under entire stochastic models.

When using the newly forecasted population the financial status of the EPI pension might be hugely exposed to longevity risk, showing the level of reserve drops drastically over time and exhausted at 2063. And the effect of the future economic assumptions on EPI pension system is positive but when combined both assumptions the level of reserve was projected still lower than that of 2009 Actuarial Valuation, which is 40 years earlier.

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# 1. Introduction

Planning and maintaining the long-term financial viability or sustainability of the pension scheme is one of the most important issues in many countries. One useful way of reaching that objective is to provide better information on the financial developments and status of those systems. Many countries have reported on these aspects in pay-as-you-go(PAYG) pension scheme and have provided actuarial reports to examine how the pension finances interact with demographic and economic environments over long-term periods. In the United States, for example, Board of Trustees (BOT) Federal Old-Age and Survivors Insurance (OASI) and Disability Insurance (DI) Trust Funds have provided an annual actuarial report and in Sweden the Swedish Pension System Annual Report has been published. For the case of Japan the government should regularly revise the Actuarial Valuation Report at least once in a five year in accordance with the stipulated law.

The main purpose of implementing these reports is to improve their transparency, credibility and solvency of pension system by projecting financial status of public pension. In line with these trends each of country is trying to develop the demographic, economic and actuarial methodologies which are needed for more accurate projection. In facts, financial projections for social security systems depend on demographic, economic and system-specific factors. Each of the underlying factors are listed such as mortality and fertility for demographic projection and participation rates, inflation and wage growth rates for economic forecasts. Also in the case of Japan statutory contribution rates and macroeconomic slide would be a system-specific factor, which affects the pension finances.

Therefore, appropriate forecasts for the future populations and economic variables are key components for pension projection. The most challengeable one for having appropriate projections for these factors is to forecaste extremely long-term periods, over 100 years in case of the Actuarial Valuation of Japan. For this reason financial stability inevitably entails a risk so that the projected results would vary over time. For this reason the BOT provides the projections not only based on deterministic assumptions also stochastic results since 2002 for the better understanding.

The aim of this study is to project the financial status of social security pension system of Japan using different models for demographic and economic variables from those used in the 2009 Actuarial Valuation published by Ministry of Health, Labor and Welfare (MHLW). For this analysis, stochastic mortality models for projecting future demographic structure and a Bayesian Vector autoregression(BVAR) model for forecasting future economic variables are employed. Under these two models, which allow the variables affecting the pension finances to vary stochastically over time, the pension projection is carried out by 2099. The three cases are set and compared with the results of base case from the 2009 Actuarial Valuation.

Firstly, of various stochastic mortality models the one which is best-suited for Japanese mortality trends are selected and then, population projections are implemented using

estimated mortality rates. Assuming the fertility rates are the same as the projection used in 2009 Actuarial Valuation, the longevity risks can be measured in terms of the Japan's public pension finance. For the second, the economic assumptions for the future wage growth and inflation rate are forecasted introducing the Bayesian VAR model. It is allowed to examine how the changes in economic variables affect the financial status of pension finance. Finally combining the forecasts estimated from above two models it is possible to project the future pension finance under entire stochastic models.

The remainder of this study is organized as follows. Chapter 2 stochastic mortality models are introduced, which is used for forecasting the number of contributors and beneficiaries and in chapter 3 economic variables such as wage growth rates, inflation and long-term interest rates are estimated by Bayesian VAR model. In chapter 4 pension projections are carried out using actuarial model and the results will be suggested and then, chapter 5 concludes this study.

## **2. Stochastic Mortality Models for the Japanese Population**

### **2.1 Outline**

Obtaining appropriate population forecasts is one of the important components for projecting the financial status of social security pension. Many countries have created a pay-as-you-go pension scheme that is very sensitive to demographic changes for its sustainability. In particular, Japan has experienced more rapid population aging for the last few decades and expected to stay out longer for the future. Decreases in a number of contributors caused by low fertility rates and increases in beneficiaries from mortality improvement may affect negative effects on the balance of pensions and government debts.

Though it is key component for pension projection and public policy it does not seem to fully captured the uncertainty over the long-term period. For example, most widely used Lee-Carter model for deriving life tables and population projection, which is also adopted by the National Institute of Population and Social Security Research (NIPSSR), has founded many limitations so far. For example the age effect parameter is constant over time and could not reflect the cohort effect.

For this reason, it is needed to examine various models that could capture the uncertainty in future mortality rates. There have been many studies in this field since the Lee-Carter (1992) study on a stochastic mortality model was published. Most recognized models can be categorized into the Lee-Carter and its generalized family of Lee-Cater models and Cairns-Black-Dowd (CBD) and its generalized family models. As one extension

of Lee-Carter model (1992) Brouhns et al. (2003) noted that potential inflexibility with respect to age effect in Lee-Carter (1992) failed to capture more recent trends in mortality experience. They tried to incorporate additive age effect term into the original Lee-Carter (1992) model to capture the important age differential effects. In the similar motivation Cairns, Blake, Dowd (2006) developed a two-factor stochastic model focusing on higher ages, 60 to 89. This model is designed to reflect the mortality rate trends at higher age are much more than at lower ages. As a result, these two multi-factor models have significantly upgraded the original Lee-Carter (1992), but still failed to tackle the cohort effect (Cairns et al., 2008a). Renshaw and Haberman (2006) proposed the first stochastic mortality model, one of the generalized Lee-Carter model, to include a cohort effect. Significant improvements have been founded compared to the Lee-Carter model, reducing the dependence of standardized residuals caused by cohort. But CMI (2007) pointed out that by changing the range of years or ages the parameter estimates could be quite different, that is, suffering from a lack of robustness. Cairns et al.(2007) introduced three generalized models and all of them incorporate a cohort effect term as extension of their original version.

Various stochastic mortality models may result in different performance according to countries' mortality experience for the past several decades. In this chapter a model that is best suited for Japanese mortality data will be selected based on the criteria suggested by Cairns et al. (2007, 2008b)

## **2.2 Stochastic Mortality Model and Selection Criterion**

Cairns et al. (2007, 2008b) have compared eight stochastic mortality models developed since the early 1990s mainly focusing on quantitative criteria. The eight models are labeled such as M1, M2, etc. and listed in [Table 2-1]. They developed consistent notations and conventions to ensure comparability between models. Thus, in all the models the conventions represent age-related, period-related and cohort-related effects, respectively.

M2 is the generalized version of Lee and Carter (1992) model, M1, including a cohort effect and M3 is a special case of model M2, which is called Age-Cohort-Period model. M6 to M8 proposed by Cairns et al. (2007) are the models which generalize the original version of M5. The M6 is CBD model including a cohort effect and M7 adds a quadratic term into the age effect to capture the possibility of some curvature in the US data. M8 can be regarded as the CBD version of model M2. With these various models they also suggested several qualitative and quantitative measures to evaluate performance of each model with the use of England & Wales male data.

[Table 2-1 Stochastic mortality models]

| Model | Formula  | Publisher                 |
|-------|--|---------------------------|
| M1    | $\log m(t, x) = \beta_x^{(1)} + \beta_x^{(2)} \kappa_t^{(2)}$  | Lee & Carter (1992)       |
| M2    | $\log m(t, x) = \beta_x^{(1)} + \beta_x^{(2)} \kappa_t^{(2)} + \beta_x^{(3)} \gamma_{t-x}^{(3)}$   | Renshaw & Haberman (2006) |
| M3    | $\log m(t, x) = \beta_x^{(1)} + \kappa_t^{(2)} + \gamma_{t-x}^{(3)}$   | Currie (2006)             |
| M4    | $\log m(t, x) = \sum_{i,j} \theta_{ij} \kappa_t^{(2)} B_{ij}^{ay}$   | Currie et al. (2004)      |
| M5    | $\text{logit } q(t, x) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x})$   | Cairns et al.(2006)       |
| M6    | $\text{logit } q(t, x) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) + \gamma_{t-x}^{(3)}$  | Cairns et al.(2007)       |
| M7    | $\text{logit } q(t, x) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) + \kappa_t^{(3)}((x - \bar{x})^2 - \hat{\sigma}_x^2) + \gamma_{t-x}^{(4)}$ | Cairns et al.(2007)       |
| M8    | $\text{logit } q(t, x) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) + \gamma_{t-x}^{(3)}(x_c - x)$   | Cairns et al.(2007)       |

A number of criteria are proposed by Cairns et al. (2007, 2008a) and implemented various methodologies for comparing between these models. [Table 2-2] is cited from the Cairns et al. (2007) study , which shows whether each model satisfies each of the stated criteria. Also they add two more important additional criteria when fitting the data; consistency with historical data and parameter estimates robustness.

Of these, except for relatively obvious criteria, consistency and parsimony can be tested by methods such as Bayse Information Criterion (BIC), Likelihood Ratio(LR) and standardized residuals(SR). And by changing the range of age and period data used, one can implement robustness test.

[Table 2-2 Model selection criteria]

| Model                               | M1 | M2 | M3 | M4 | M5 | M6 | M7 | M8 |
|-------------------------------------|----|----|----|----|----|----|----|----|
| Ease of implementation              | Y  | ?  | Y  | ?  | Y  | Y  | Y  | ?  |
| Parsimony                           | Y  | ?  | ?  | Y  | Y  | ?  | ?  | ?  |
| Transparency                        | Y  | Y  | Y  | ?  | Y  | Y  | Y  | Y  |
| Ability to generate sample paths    | Y  | Y  | Y  | N  | Y  | Y  | Y  | Y  |
| Ability to generate percentiles     | Y  | Y  | Y  | Y  | Y  | Y  | Y  | Y  |
| Allowance for parameter uncertainty | Y  | Y  | Y  | Y  | Y  | Y  | Y  | Y  |
| Incorporation of cohort effects     | N  | Y  | Y  | Y  | N  | Y  | Y  | Y  |
| Non-trivial correlation structure   | N  | N? | N? | N  | Y  | Y  | Y  | Y  |

They suggested that incorporating cohort effects are more desired if one believes that cohort effects are important and needed. In addition to consistency of model, Dowd et al. (2011) used back-testing for evaluating the forecasting performance.

First of all, M4 is excluded in this study because it has no ability to generate sample paths which are needed to forecasting the future mortality rates. M7 inspired to reflect the specific characteristics of US mortality data is also excluded from this analysis.

All the models are estimated by Maximum Likelihood Estimation with a log-likelihood function defined as follows.

$$L(\phi; D, E) = \sum_{t, x} D(t, x) \log[E(t, x) m(t, x; \phi)] - E(t, x) m(t, x; \phi) - \log[D(t, x)!]$$

where, D: number of death, E: Exposure to Risk, t: year, x: age,  $\phi$ : set of parameter

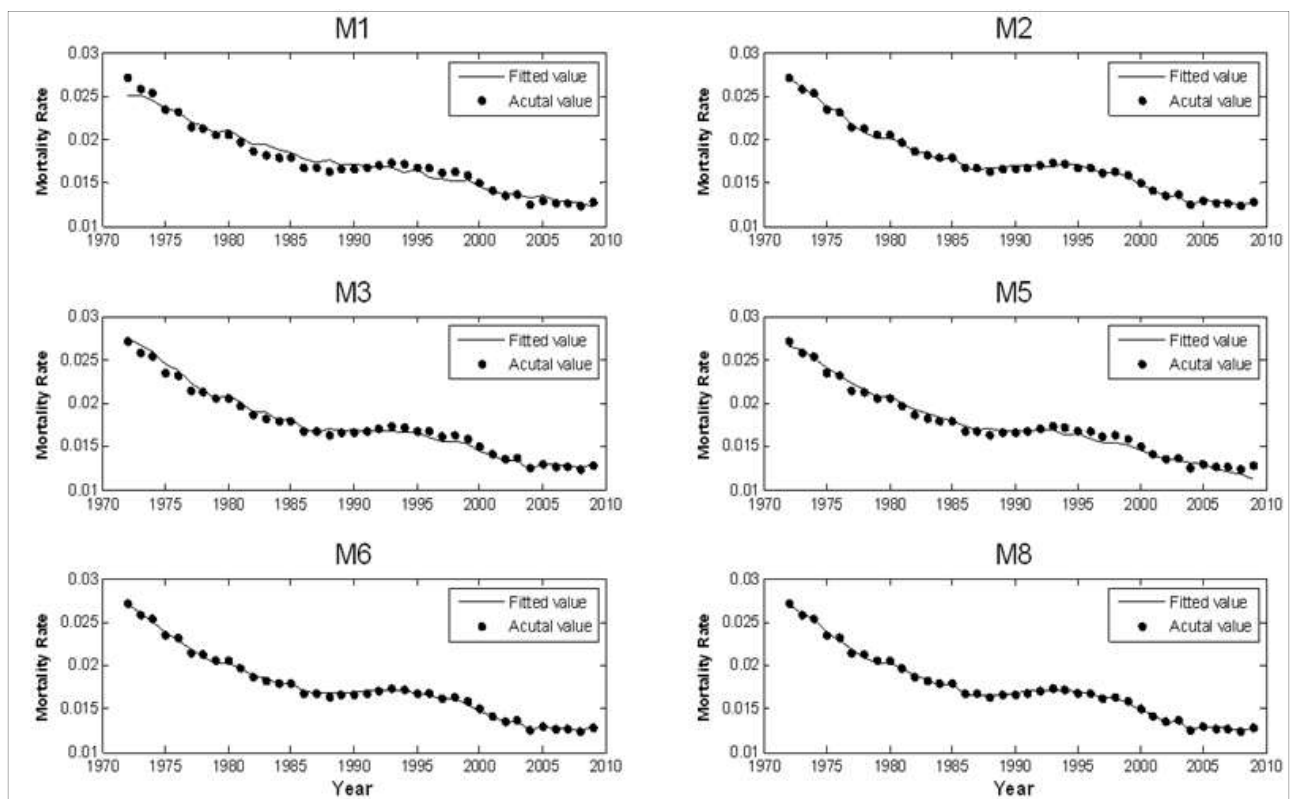
## 2.3 Results

With the quantitative comparisons of six selected models Bayse Information Criterion (BIC) and robustness test are implemented. For the first, the actual and fitted values are plotted in [Figure 2-1]. Only for the male aged 65 suggest because of restriction on spaces. Visually all the models fit well the past mortality trends except for M1 in case of Japanese male. BIC are given in [Table 2-3] and the models are ranked in higher BIC order.

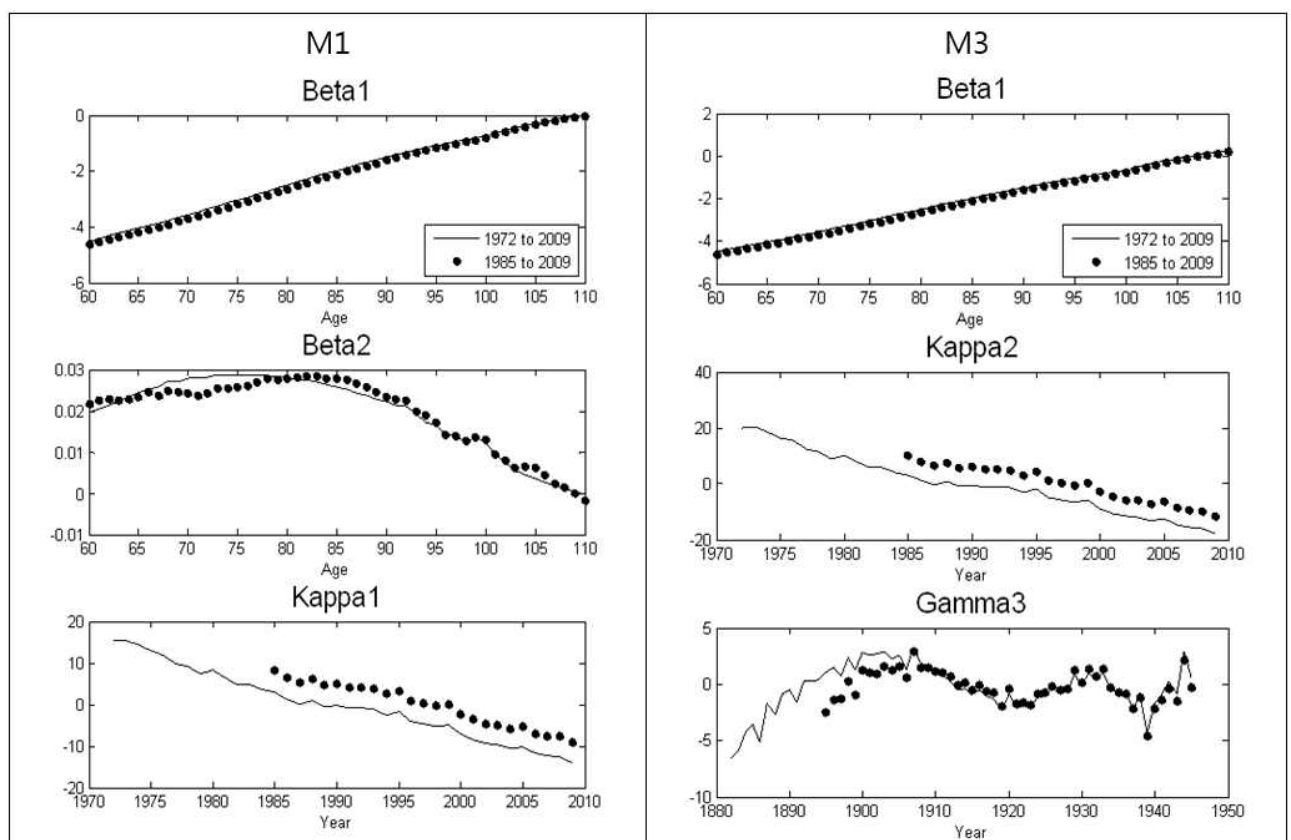
[Table 2-3 Result of BIC]

| Model | Maximum log-likelihood | Effective number of parameters | BIC      | Rank |
|-------|------------------------|--------------------------------|----------|------|
| M1    | -13984.3               | 138                            | -14498.1 | 5    |
| M2    | -9519.8                | 275                            | -10543.7 | 1    |
| M3    | -12108.5               | 174                            | -12756.4 | 4    |
| M5    | -19062.2               | 76                             | -19345.2 | 6    |
| M6    | -12140.0               | 162                            | -12743.2 | 3    |
| M8    | -11185.7               | 163                            | -11792.6 | 2    |

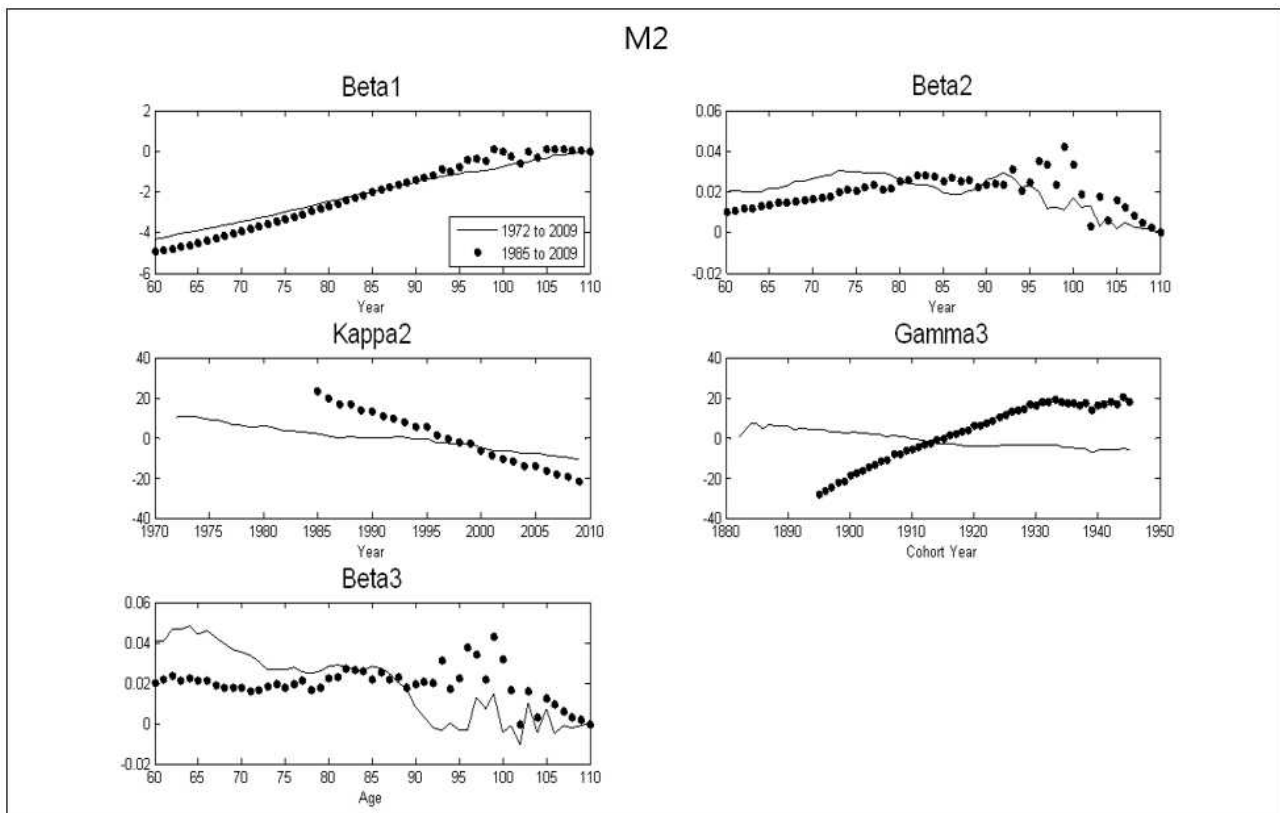
[Figure 2-1] Actual and fitted values for Japan male aged 65



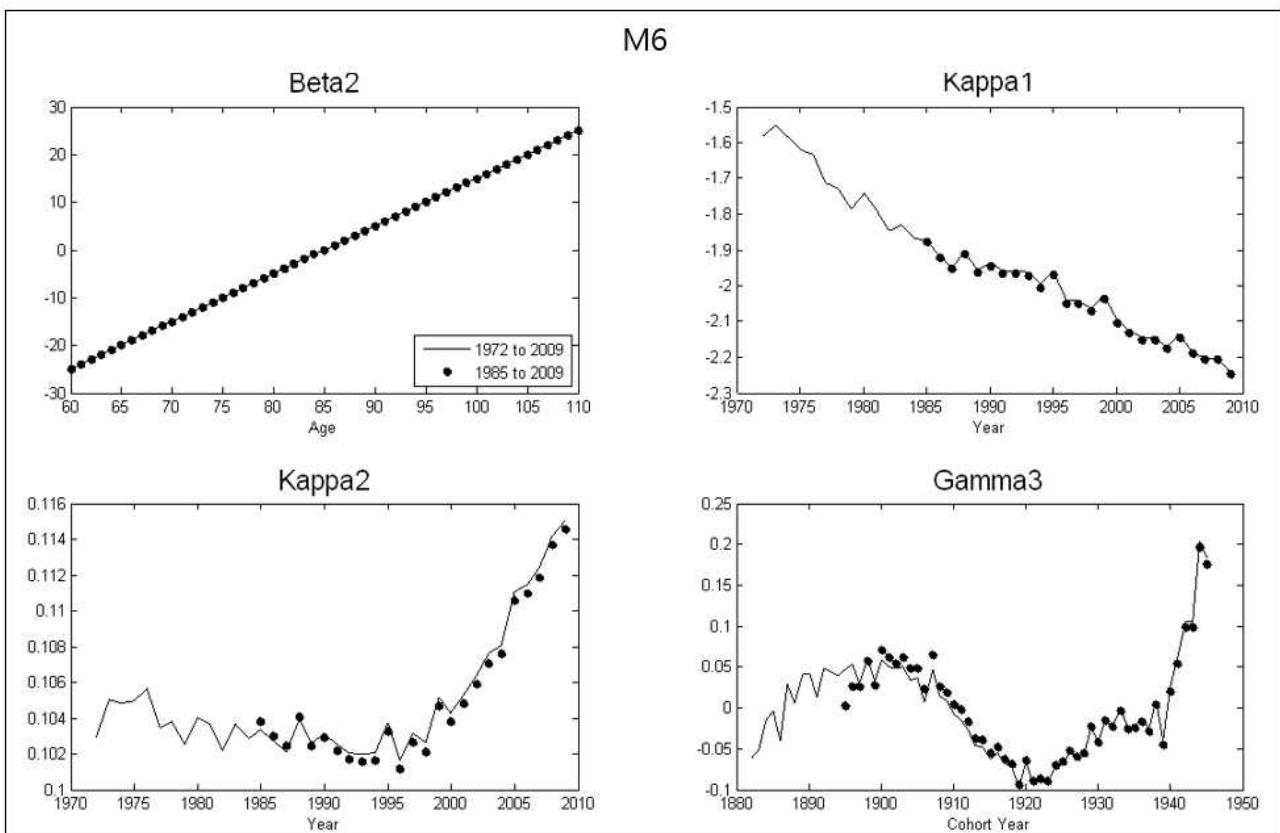
[Figure 2-2] Result of robustness test: M1, M3



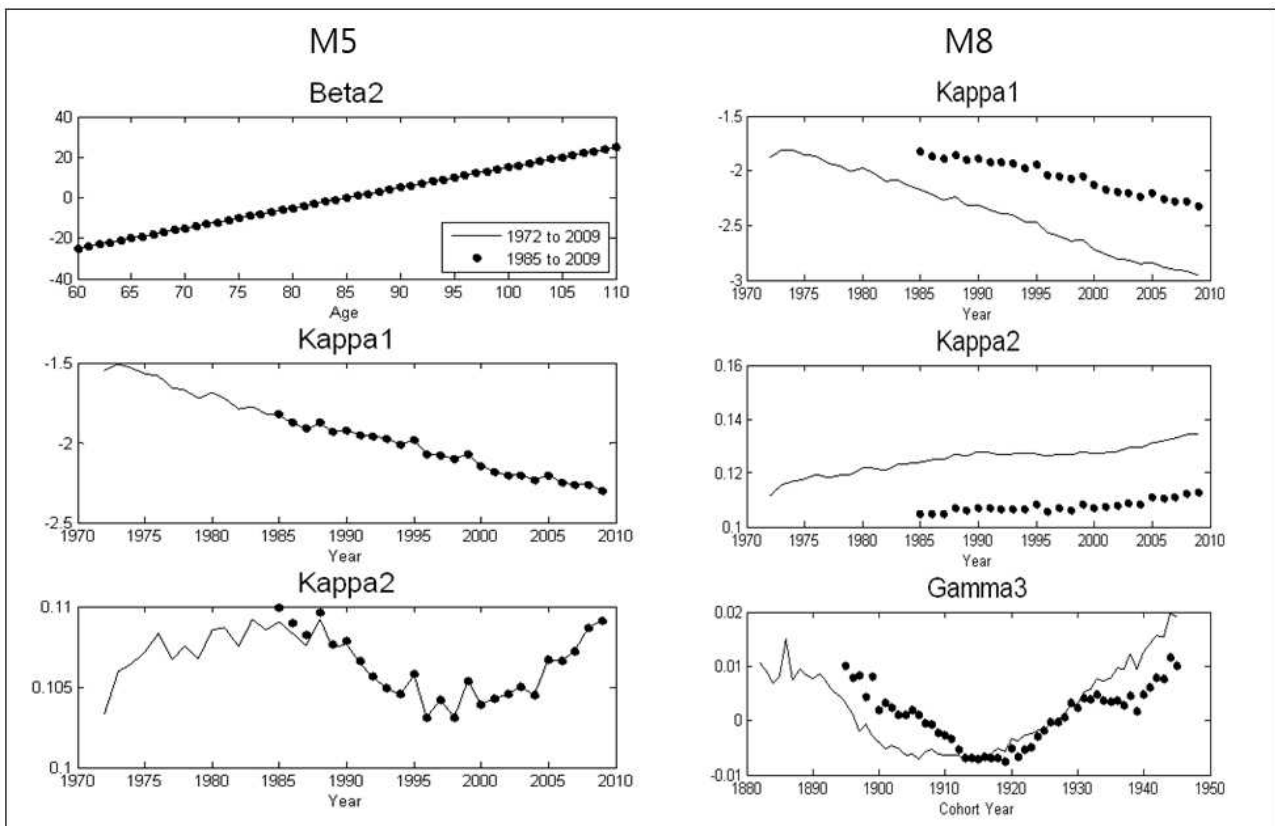
[Figure 2-3] Result of robustness test: M2



[Figure 2-4] Result of robustness test: M6



[Figure 2-5] Result of robustness test: M8



Robustness test is implemented by changing the range of period and estimate new parameters based on this data. Robust models should not be dramatically changed its parameters though the range of analyzed data is changed. With the data from 1985 to 2009 parameters of each model are re-estimated and compared with ones derived from entire periods from 1972 to 2009. The results are illustrated [Figure 2-2]~[Figure 2-6]. Solid lines means derived parameters from data from 1972 to 2009 and dots are those from data from 1985 to 2009. One notable result is that M2 has significantly changed when the data used is reduced. From this result, it may conclude M2 is not appropriate in terms of robustness criterion though it is ranked highly according to BIC. This is not a new and the Renshaw and Haberman (2006) cohort model turns out to suffer from a lack of robustness. Cairns et al. (2007, 2008) found the similar result when they changed the range of year with the England & Wales and US male data. For this model of lack robustness, the likelihood function possibly might have more than one maximum (Cairns et al., 2008). For another models, M1 shows higher period effect than original one, implying this model has also a lack of robustness. As a result M5 and M6 seemed to be better performance compared to other models showing the parameters are almost same from two data series.

Based on these results no single model stands out as being the best in all aspects. But

there are some notable and useful information to select the model for Japanese mortality data. First, in case of Japan there are evidences for existing a cohort effect. The inclusion of a cohort effect provided a statistically and significantly better fits. In this sense models capturing the year-of-birth effects are preferred, thus, excluding M1 and M5. Secondly, M2 is not a reliable model because of lack of a robustness although including cohort effect term. On the basis of these findings M6 is best suited model for Japanese showing relatively high goodness of fit and robustness as well as considering a cohort effect. In conclusion, M6 is employed and used for forecasting the future mortality and population from next.

Next step is to forecast the future mortality rates with the fitted parameters under M6. In model M6 three parameters change over time,  $K_t^{(1)}, K_t^{(2)}, Y_{t-x}^{(3)}$  which reflect time-period and cohort effects, respectably. These parameters of two time-period effect will be simulated for future mortality rates by random walk process. A Cairns et al. (2006)'s study proposed model labeled M5 introduced 2-dimensional random walk with drift to make forecasts of the future distribution of  $K_t^{(1)}, K_t^{(2)}$ . For the dynamic process on cohort effect,  $Y_{t-x}^{(3)}$ , it does not seem to  $Y_{t-x}^{(3)}$  drives by simple random-walk process. Thus, with dynamics of  $Y_{t-x}^{(3)}$  is derived with ARIMA (2,1,0).

$$K(t+1) = K(t) + \mu + CZ(t+1)$$

$$\text{where, } K(t) = [K_t^{(1)}, K_t^{(2)}],$$

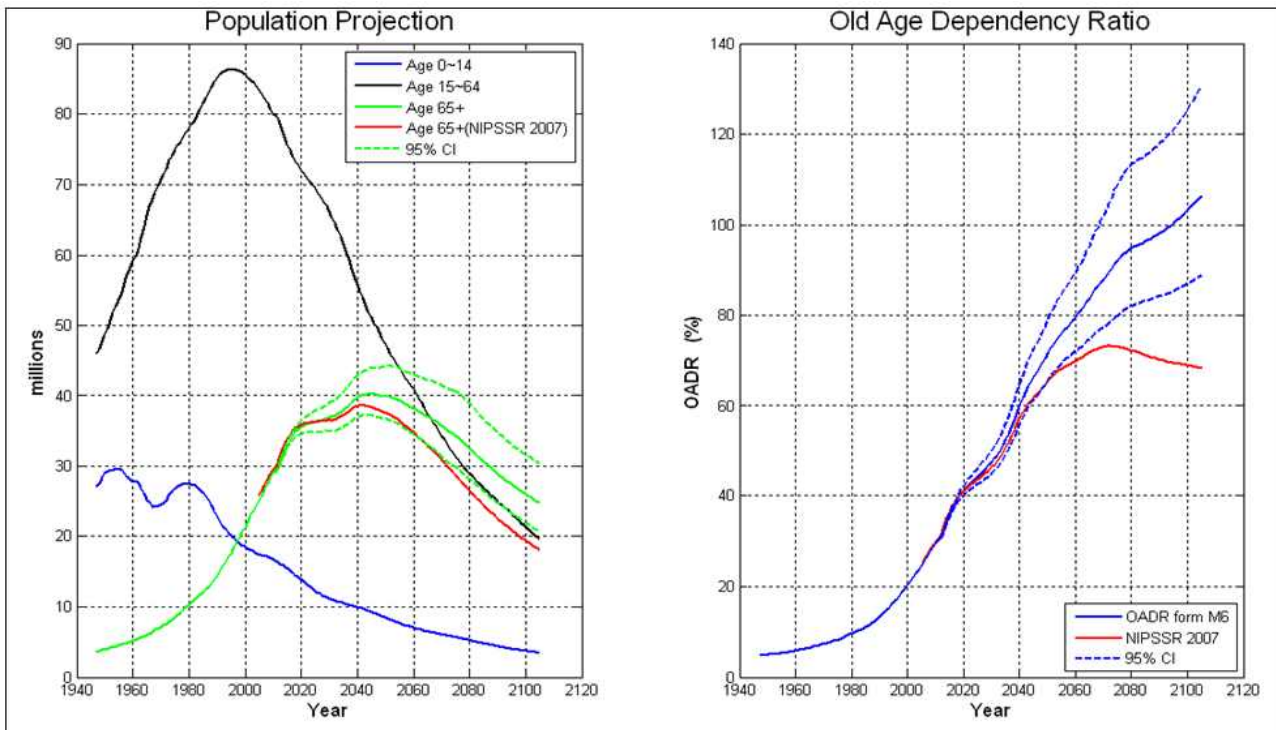
$$\mu = [\mu_1, \mu_2](\text{drift}),$$

C: constant 2 by 2 upper triangular matrix,

Z(t): dimensional standard normal random variable

Finally, the population projection is implemented under forecasted future mortality rates until 2099. The projections are suggested [Figure 2-6]. By age groups the young and working people, aged from 0 to 64, have declined continuously over time, while old population aged above 65 have increased and peak around 2040s'. The ratio of the number of working people to old age population, Old Age Discrepancy Ratio (OADR), has increased rapidly, which might threaten the sustainability of pay-as-you-go pension system. In chapter 6 an analysis how impact the demographic changes on financial status is performed.

[Figure 2-6] Population Projection and OADR



### 3. Bayesian VAR model for Economic Variables

#### 3.1 Outline

In this chapter macro-economic model is constructed for the future economic environment of Japan. As described early, a financial projection for social security system is also depend on economic variables such inflation rate, wage growth rate, and interest rate, as well as fertility and mortality rate for population projection.

For example, in Japan case, pension benefits are re-evaluated in accordance with inflation and wage growth rates every year. Also, wage growth rate is used to revalue contributory salaries, thus affects the total income of contributions. The rate of investment return is also necessary such as long-term interest rates and returns on stock, etc.

To project the benefit expenditures, the contribution incomes, and the level of reserves for future years assumptions on macro economic variables should be imposed by a certain theories or models. In practices, for example 2009 Actuarial Valuation in Japan, they have been adopted the Cobb-Douglas production function in neo-classical economics to estimate the real GDP growth rate and nominal wage growth rate for the long-term economic assumptions. One advantage in this model is that the future real GDP growth rate can be estimated to be consistent with the decreasing population in Japan. But it has a limitation in that assuming the annual increase rate of the CPI is exogenously which

might strongly correlated to these variables. A study of Kato (2006) analyzed the impacts of the macro-economic variables to financial status of pension introducing a vector autoregressive (VAR) and vector error correction (VEC) model to reflect the correlation these three macro variables. But the result of this showed inflation rate has negative value in the long-term equilibrium, which it is unlikely to occur in reality. Also, the VAR model has been employed to project the unemployment rate, inflation rate and real interest rate by OCACT for a stochastic model of the long-range financial status of the OASDI program. Another analysis estimating these economic variables was employed mean reversion process (Kitamura et al., 2004 and Ueda, 2011). They considered the process of inflation rate and wage growth rate tend to drift towards its long-term mean assumed the same level as those used in Actuarial Valuation in 2009.

### 3.2 VAR and Bayesian VAR Model

A univariate auto-regression is a single-equation in which the current value of a variable is explained by its own lagged values. After Sims(1980)'s study providing vector autoregression (VAR) it is allowed to capture dynamics in multiple time series.

In VAR model with n variables each dependent variable is a liner function of its own past values, plus the current and past values of other n-1 variables and a serially uncorrelated error term.

VAR(p) model can be written as:

$$\vec{Y}_t = \vec{\alpha}_0 + A_1 \vec{Y}_{t-1} + A_2 \vec{Y}_{t-2} + \dots + A_p \vec{Y}_{t-p} + \vec{\varepsilon}_t$$

$$\text{where, } \vec{Y}_t = [y_{1,t}, y_{2,t}, \dots, y_{N,t}] , A_i = \begin{bmatrix} \alpha_{11,i} & \dots & \alpha_{1N,i} \\ \dots & \dots & \dots \\ \alpha_{N1,i} & \dots & \alpha_{NN,i} \end{bmatrix} , \vec{\varepsilon}_t = [\varepsilon_{1,t}, \varepsilon_{2,t}, \dots, \varepsilon_{N,t}] , \forall t, i$$

But VAR has a great number of parameters to estimate which might be not parsimonious models thus, over-parameterization problems may arise. To overcome this drawback of VAR Bayesian methods have been popular. Bayesian VAR used a variety of priors that can be posed a certain restrictions on the parameters to estimate. This approach is so-called "shrinkage" that can be of great benefit in reducing over-parameterization problems.

VAR(p) model in equation (3) can be also written in matrix form as follows.

$$Y = XA + E$$

The likelihood function can be derived from the sampling density,  $p(y|\alpha, \Sigma)$ .

$$\alpha|\Sigma, y \sim N(\hat{\alpha}, \Sigma \otimes (X'X)^{-1})$$

and

$$\Sigma^{-1}|y \sim W(S^{-1}, T - K - M - 1)$$

where,  $\hat{A} = (X'X)^{-1}X'Y$  is the OLS Estimator of A,  $\hat{\alpha} = \text{vec}(\hat{A})$  and,

$$S = (Y - X\hat{A})'(Y - X\hat{A})$$

There are 4 Priors used in BVAR model for forecasting:

- Diffuse(Jeffrey) Prior

$$p(A, \Sigma) \propto |\Sigma|^{-(n+1)/2}$$

- Natural Conjugate Prior

$$A|\Sigma \sim N(A, \Sigma \otimes V)$$

$$\Sigma^{-1} \sim W(v, S^{-1})$$

where,  $W(\cdot)$ : Wishart Distribution,  $v$ : Population Variance Matrix,  $S = V^{-1/2}$

- Independent Wishart Distribution Prior

$$p(A, \Sigma^{-1}) = p(A)p(\Sigma^{-1})$$

- Minnesota Prior

$$S(i, j, p) = \lambda \times p^{-1} \times f(i, j) \times \frac{s_j}{s_i}$$

Where ,  $S(i, j, p)$ : Element of Standard Deviation Matrix of p Lag.

$i$ : subset of Dependant Variable,  $j$ : subset of Independent Variable

$\lambda$ : tightness of  $S(i, j, p)$

$f(i, j)$ : Ratio of Standard Deviation between  $i$  and  $j$

$s_i$ : Standard Error of Regression Equation

Of these four priors, natural conjugate priors is used in this analysis. In natural conjugate priors, the prior likelihood and posterior comes from the same family of distributions.

### 3.3 Results

The real GDP growth rate, inflation rate, nominal wage growth and long-term interest rate are simulated using VAR and BVAR. It is able to capture correlation among the economic variables and generate the sample paths needed to project the financial status of public pension. Firstly, unit-root tests are implemented for each four variables using Dickey-Fuller test. [Table 3-1] shows data description and p-values from unit-root test. The p-values of all variables indicate rejection of the unit-root null hypothesis.

[Table 3-1] Data description and unit root test

| Variable             | Description                                    | p-value |
|----------------------|--|---------|
| Real GDP growth rate | Change from the previous year                  | 0.0018  |
| Inflation rate       | Change of CPI from the previous year(2010base) | 0.0483  |
| wage growth rate     | Change of previous year: nominal term          | 0.0254  |
| Real interest rate   | Japanese Government Bond                       | 0.0114  |

And to determine the appropriate lag of the VAR model the AIC is calculated by each VAR(p),  $p=0, 1, 2, 3, 4, 5$ . Based on AIC criteria VAR(4) is selected which has the lowest AIC.

As the number of data used is 41 and parameters to be estimated in VAR(4) model with exogenous dummy variables are 18, thus over-parameterization problems might arise. For this reason the same data is also estimated by using the Bayesian VAR model.

[Table 3-2] Result of AIC criterion for VAR(p)

| Lag | Maximum log-likelihood | Effective number of parameters | AIC    |
|-----|------------------------|--------------------------------|--------|
| 0   | -339.224               | 18                             | 714.44 |
| 1   | -240.939               | 34                             | 549.87 |
| 2   | -210.632               | 50                             | 521.26 |
| 3   | -149.667               | 66                             | 431.33 |
| 4   | -121.942               | 82                             | 407.88 |
| 5   | -108.715               | 98                             | 413.43 |

Mean square forecast errors (MSFE) are provided to evaluate forecast performance these two models. For this the out-of-sample data runs from 2002 to 2011 with VAR and B-VAR model and forecasted with parameters from 1972 to 2001. The results from two models are similar, implying no over-parameterization problem in VAR model. But according to the

MSFE presented [Table 3-3] it can be seen the B-VAR has better predictive performance, especially interest rate, thus BVAR(4) model is employed from the next.

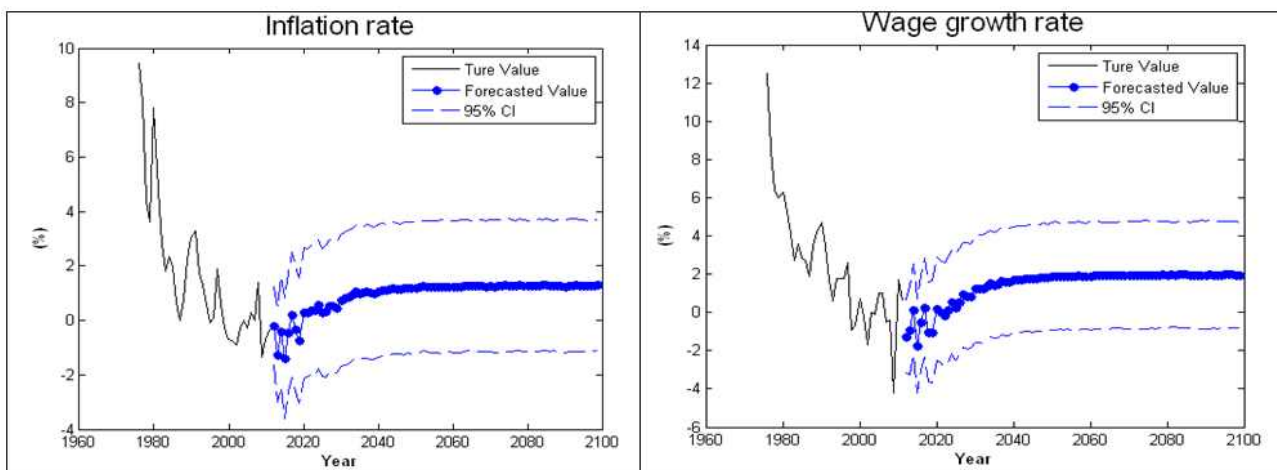
[Table 3-3] Result of MSFE

| Variables        | VAR   | BVAR  |
|------------------|-------|-------|
| GDP growth rate  | 5.537 | 5.481 |
| Inflation rate   | 0.741 | 0.703 |
| Wage growth rate | 2.459 | 2.395 |
| Interest rate    | 0.774 | 0.583 |

The forecasted mean values of inflation rate and wage growth rate are used as input variables for project pension. Using the BVAR model including exogenous dummy variable forecasts carry out from 2012 to 2099. Dummy variable is assigned as,  $X_t = 1$  if  $t=1972\sim1984$  and  $X_t = 0$  if  $t=1985\sim2011$ .

In fact, to project financial status of social security pension long-term forecasts are necessary. The economic growth of the Japan has experienced slowdown but it is not appropriate assumptions that this situation will keep going for the long-term period over 100 years. Thus based on assumptions that the Japanese economic recovery for the future. This assumption reflected the model by give one into the dummy variables. The results are suggested in [Figure 3-1]

[Figure 3-1] Result of forecasting: inflation rate and wage growth rate

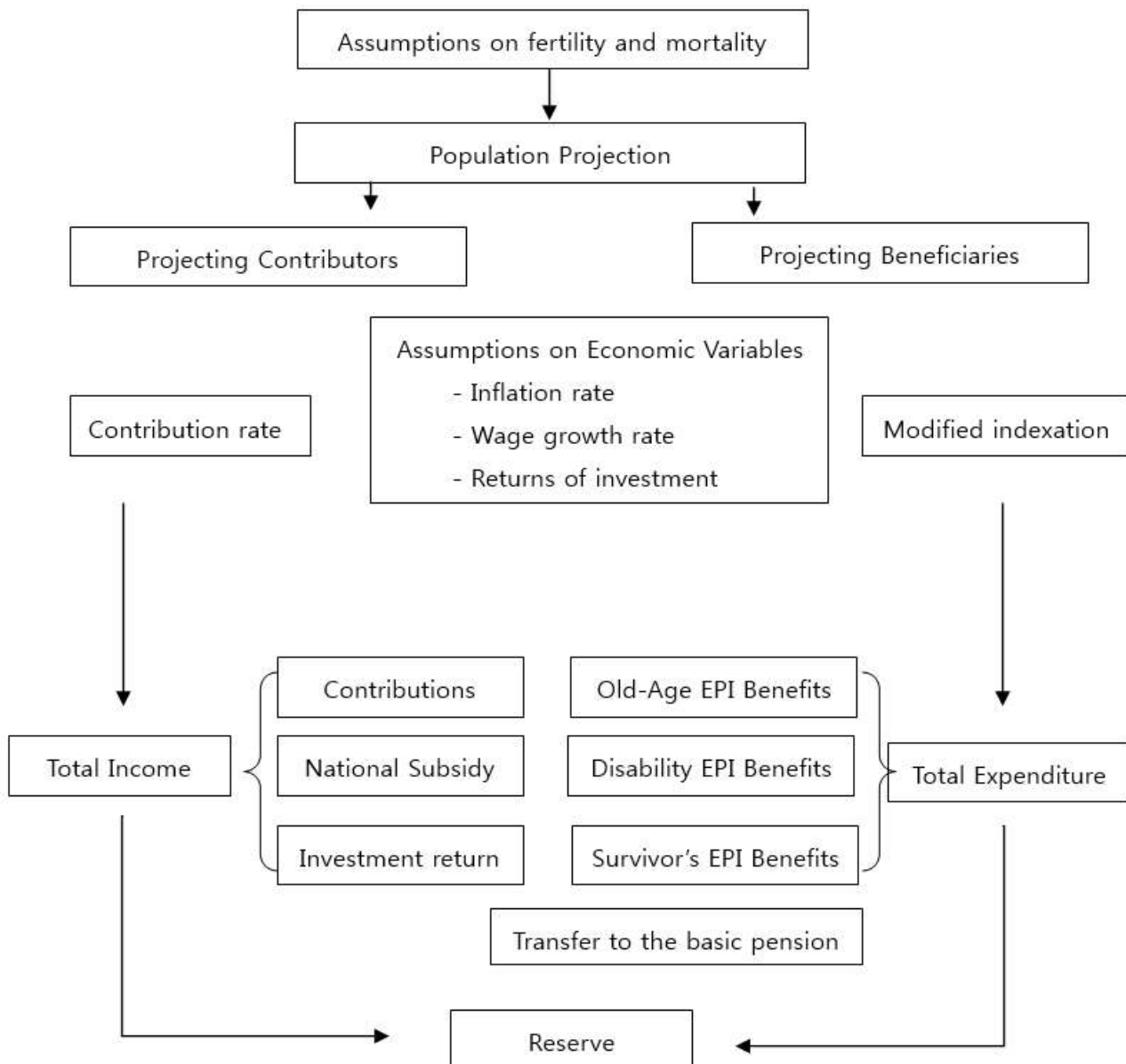


## 4. Actuarial Model for the Pension Projection

### 4.1 Outline

So far three main factors- population, inflation rate and wage growth rate- that affect the financial status of social security pension were forecasted using stochastic models. In this chapter pension projection is carried out focusing on the Employees' Pension Insurance (EPI). For a financial projection of EPI, an actuarial model is employed with the same one used in 2009 Actuarial Valuation. The process of EPI pension projection are presented [Figure 4-1].

[Figure 4-1] process of EPI pension projection



## 4.2 Actuarial Model

On the website the Ministry of Health, Labor and Welfare(MHLW) has provided the base data and estimating results. And the mathematical equation for deriving the balance is suggested in 2009 Actuarial Valuation Report. Except for the specifically noted ones, the projection is carried out with the same data and equations. Here, briefly summarize the each step and used data.

### ■ Projecting Covered people

Based on forecasted population structure the covered people is calculated as following equation.

Number of hired people=

Projected population (aged 16~69) X Participant rate X Employment rate

Number of covered people=

(Full-time hired people X Ratio of Covered people to Full-time hired people  
+ Part-time-job people X Ratio of Covered people to Part-time-job people)

### ■ Projecting beneficiary

Beneficiaries are divided into two types; newly awarded one who has firstly eligibility to pension benefits and already awarded beneficiary. The pension benefits of former are revaluated by real wage growth and inflation rate and the latter is adjusted by only inflation rate.

The number of beneficiaries is determined when covered people reach at ages 65 and then dynamics by the rate of losing the pension right. In this analysis, an assumption on the rate of losing the pension right is different from the 2009 Actuarial Valuation. They provide the rate of losing the pension right but these values are assumed to be fixed over projecting periods thus, it is difficult to reflect the effects of improvement in mortality. In this analysis the forecasted mortality rates is used instead of a rate of losing the pension right.

### ■ Contributions

Income of Contributions =

Contribution rate X amount of standard remuneration X Covered people

From this equation the amount of standard remuneration is revalued by nominal wage growth rate so that it is influenced the future forecasts of wage growth rates. And the contribution rate is determined in 2004 Pension Reform, which is scheduled to increase 0.354% every year starting from 2005 and fixed at the level of 18.30 % after 2017.

#### ■ National subsidy

As noted chapter II, the rate of subsidy was raised to one-half of its benefits by 2009. National subsidy can be calculated multiplying one-half to the amount of transfer to basic pension.

#### ■ Investment return

Rate of investment return does not change and uses the same assumption from 2009 Actuarial Valuation. But to examine the effects of the changes in investment return, sensitive analysis is implemented.

#### ■ Mechanism of applying slide: Modified indexation

Firstly, estimated nominal wage growth rate and inflation rate is used for slides. The former is a slide for revaluating the pension benefits of newly awarded beneficiaries and the latter is a slide used for adjusting the pension benefit of after awarded ones. But under circumstance that contribution rates is scheduled to increase statutorily some restraints are posed in applying slides, not to burden on working generations too much. For example, when the real wage growth rate has negative value and inflation rate has positive value (Condition 2 as below), inflation rate is greater than nominal wage growth, slides to both newly awarded and after awarded beneficiaries will be zero. The following is the rules of these mechanism.

Condition 1.  $P(t) > 0$  and  $W(t) > 0$ , and  $P(t) > Z(t)$

Condition 2.  $P(t) \geq 0$  and  $W(t) < 0$

Condition 3.  $P(t) < 0$  and  $W(t) < 0$ , and  $P(t) > Z(t)$

$$\text{Slide for newly awarded } a(t) = \begin{cases} W(t) & \text{if Condition 1} \\ 0 & \text{if Condition 2} \\ P(t) & \text{Otherwise} \end{cases}$$

$$\text{Slide for after awarded } b(t) = \begin{cases} P(t) & \text{if Condition 3} \\ 0 & \text{if Condition 2} \\ W(t) & \text{Otherwise} \end{cases}$$

where,  $P(t)$ : inflation rate at year  $t$  and  $W(t)$ : nominal wage growth rate at year  $t$   
In the second modifier is applied as follows.

$$W^{**} = \begin{cases} W^* & \text{if } W < 1 \\ 1 & \text{if } W > 1 \text{ and macro slide } X W < 1 \\ W \times \text{modifier} & \text{Otherwise} \end{cases}$$

$$P^{**} = \begin{cases} P^* & \text{if } P < 1 \\ 1 & \text{if } P > 1 \text{ and macro slide } X P < 1 \\ P \times \text{modifier} & \text{Otherwise} \end{cases}$$

where,  $W^*$  : slide before modified and  $W^{**}$  : wage slide after modified  
 $P^*$  : slide before modified and  $P^{**}$  : price slide after modified

In short, the modifier does not be applied when the wage and inflation are negative and have a lower limitation at zero even though it is adjusted. The automatic balancing mechanism is introduced either until the pension finance attains equilibrium or the replacement ratio becomes below 50%. In 2009 Actuarial Valuation projected the adjustment periods start from 2012 to 2019 for EPI and to 2038 for Basic pension. The same periods are adopted in this analysis.

## ■ Total Expenditure

Old-Age EPI benefits =

Avg. pensionable remuneration X Accrue rate X Avg. months enrolled X  
Beneficiaries

Avg pensionable remuneration

= (total revalued monthly pay + total revalued bonuses)/months enrolled

As seen above old-age pension benefits are affected by wage growth rate for revaluating

pensionable remuneration and inflation rate for adjusting the benefits every year. After projecting the old-age EPI benefits, Disability and Survivor's EPI Benefits is calculated by the ratio of past total benefits to expenditure of old-age EPI. Because the ratio is stable for the past 20 years around 81.4% averaged, it is adopted for simplicity. Transfers to the basic pension are estimated using the ratio of beneficiaries of EPI to beneficiaries of National Pension.

### 4.3 Result

To examine the effects of changes in demographic and economic assumptions on EPI system the projections under base case of 2009 Actuarial Valuation are compared with other three cases. In the base case assumptions and data used in 2009 Actuarial Valuation are employed, which are based on the intermediate fertility, mortality rates and economic assumptions.

Three different cases are analyzed. In the first case, demographic projection is changed but not economic variables. As seen chapter III, the new projection shows more rapid aging than that of base case, thus it is expected to negative effects on the financial status of the EPI. The improvements in mortality rates simulated from M6 model, longevity risk can be estimated using the same economic assumptions of base case. In the second case, changes in economic variables are considered with the same assumptions on population structure in the base case. Short- and long-term economic assumptions are replaced by the forecasts estimated from the B-VAR in chapter IV. Finally, in the third case it is allowed both demographic and economic assumptions to be replaced. [Table 4-1] presents the summary of each case.

[Table 4-1] Summary of analysis\*

|                  | base          | case1       | case2         | case3        |
|------------------|---------------|-------------|---------------|--------------|
| Population       | NIPSSR (2007) | Chapter III | NIPSSR (2007) | Chapter III  |
| Inflation rate   | 1.00%         | 1.00%       | BVAR (1.29%)  | BVAR (1.29%) |
| Wage growth rate | 2.50%         | 2.50%       | BVAR (2.01%)  | BVAR (2.01%) |

\* All the value of inflation rate and wage growth rate represents the long-term assumptions.

[Figure 4-2] shows the dynamics of number of contributors and beneficiaries of EPI from base case and case1. The difference in total contributors is small, but the gap of beneficiaries is significantly widened over time. These results are caused by the mortality rates estimated from difference models using the same total fertility rates; in case of base

those are from the NIPSSR while in case1 they are estimated by generalized CBD model in chapter III. These significant mortality improvements inevitably give bad effects on the level of reserves of EPI, which can be called longevity risk. As seen in [Figure 4-3] it is clear that the increases in beneficiaries cause the level of reserve to be exhausted more rapidly than base case indicated in 2063 years. It is notable that even in the case of less improvements in mortality the reserve is expected to be zero in 2075, which is 30-years earlier than that of 2009 Actuarial Valuation, 2105.

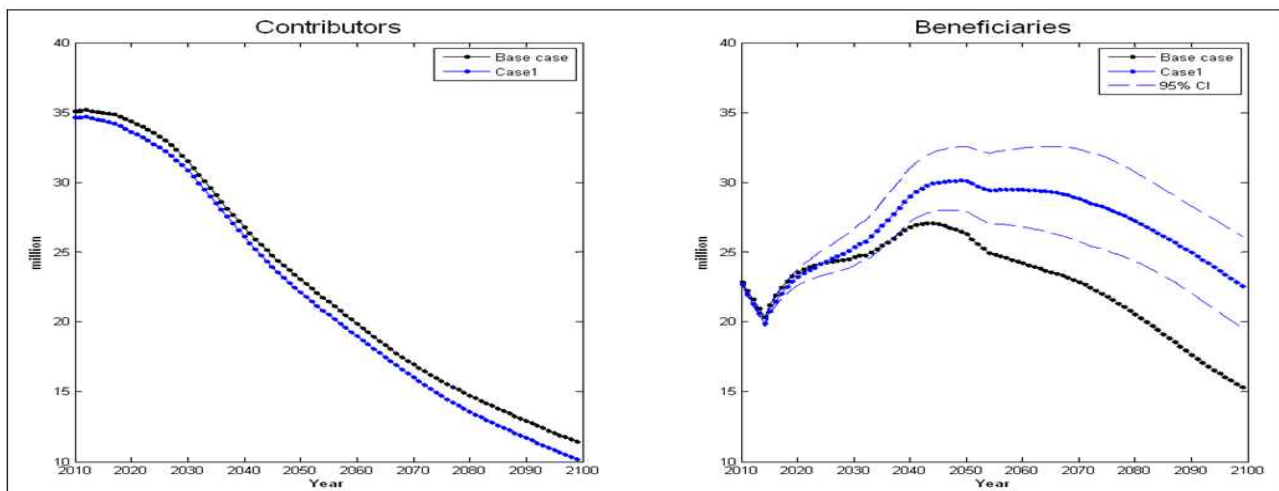
[Figure 4-6] presents the level of reserve for all cases. In Case2 the lowered wage growth rate than that of base case results in not only decreasing contribution income, but also benefits expenditure. In fact, the contributory salaries are cumulatively re-evaluated every year so that contributions are very sensitive to the long-term level of wage growth rate, while benefits after awarded at ages 65 are adjusted by the inflation rate. The large difference between base case and case1 in contributions income and benefits expenditure is caused by its totally different assumptions on economic variables. [Figure 4-4] represents assumptions of each case on inflation rate and wage growth rate. Even the benefits are supposed to be re-evaluated by inflation rate firstly awarded benefits have been affected wage growth rate cumulatively until they reach at pensionable age. But two economic variables give an effect on contributions and benefits at the same time, it is hard to distinguish how much it affects in this analysis. As a result, with the newly estimated assumptions, it has positive effects on the level of EPI reserve in term of the financial sustainability although downsizing the scale of reserve.

In case3 it shows the level of reserve with entirely new assumptions on demographic and economic variables. Combining the these two assumptions generated by stochastic mortality model and B-VAR it is possible to project the financial status of the EPI stochastically. The positive effects of economic assumptions on the reserve in case2 are disappeared when combined with newly projected population. This result indicates that even the economic environments move positively for the future it might be eroded by the population aging. The result shows the time of depletion is extended only 2 years compared to the case1 while shortened 28-years earlier than base case.

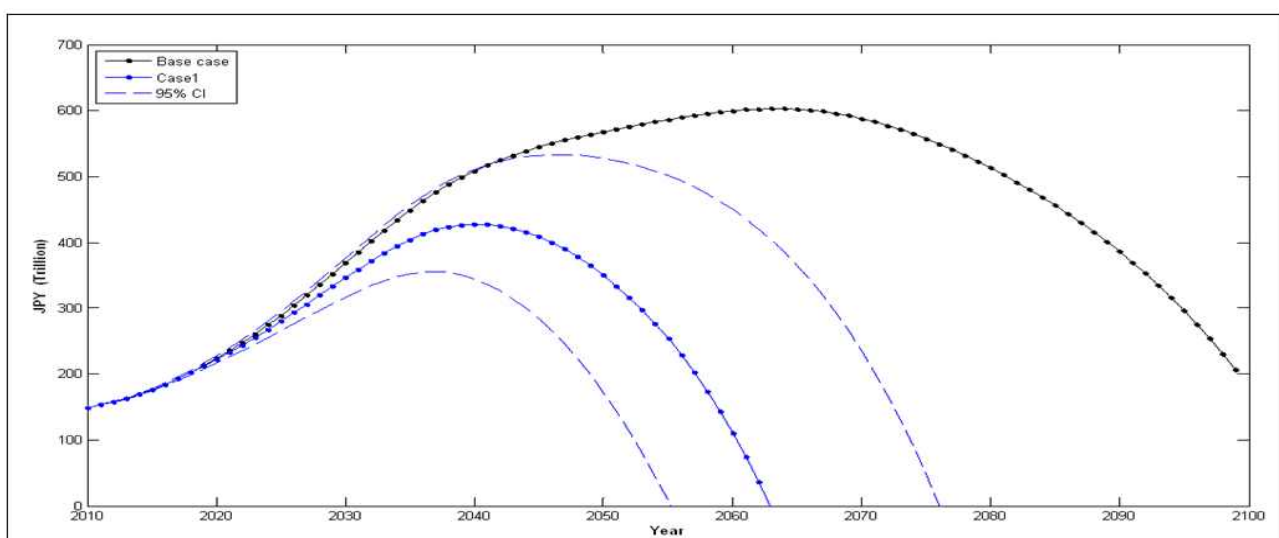
The future EPI projection is also hugely influenced by the rate of investment return. In this study the same investment return on reserve that is used in 2009 Actuary Valuation are employed for the better comparison on the gaps from different demographic and economic changes. But it is worthy to examine how the rate investment return affects the transition of the level of reserve. In fact, it is often criticized that the long-term investment return set in 2009 Actuarial Valuation is too high by historical and present Japanese

economic situation. Thus, with three different scenario, that is, minus 0.5%, 1% and 2% from the original ones(4.1%), sensitive analysis is implemented in all cases. Based on the results of [Figure 4-6] estimated with 4.1% of investment rate, this long-term rate of return is replaced with 3.6%, 3.1% and 2.1% in each case. It is natural that the effects of decreased rate of investment return on financial status of EPI are negative in the all cases. But one notable result is that the reserve of base case are hugely affected by the changes in assumptions on rate of investment return. As plotted in [Figure 4-7], the reserve in base case is seemed to be exhausted out at 2082, 2073 and 2063 from each scenario, which is more sensitive than other cases. Considering the scenario that is set the 3.6%, 3.1% and 2.6% of investment return is not unreasonable values, it is needed to re-examine for the long-term viability and sustainability of pension system.

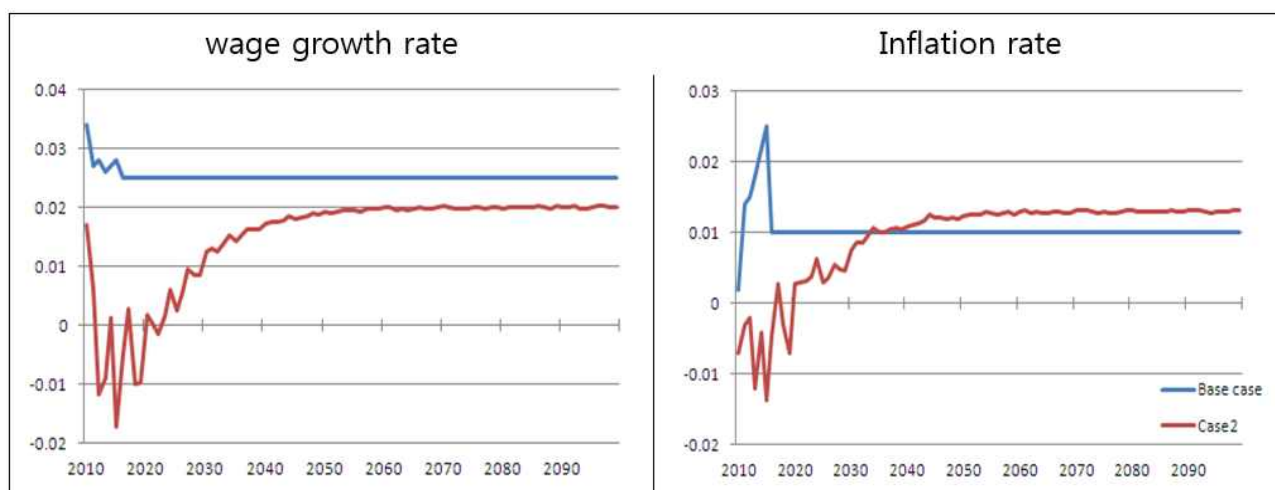
[Figure 4-2] Contributors and Beneficiaries : Base case VS Case1



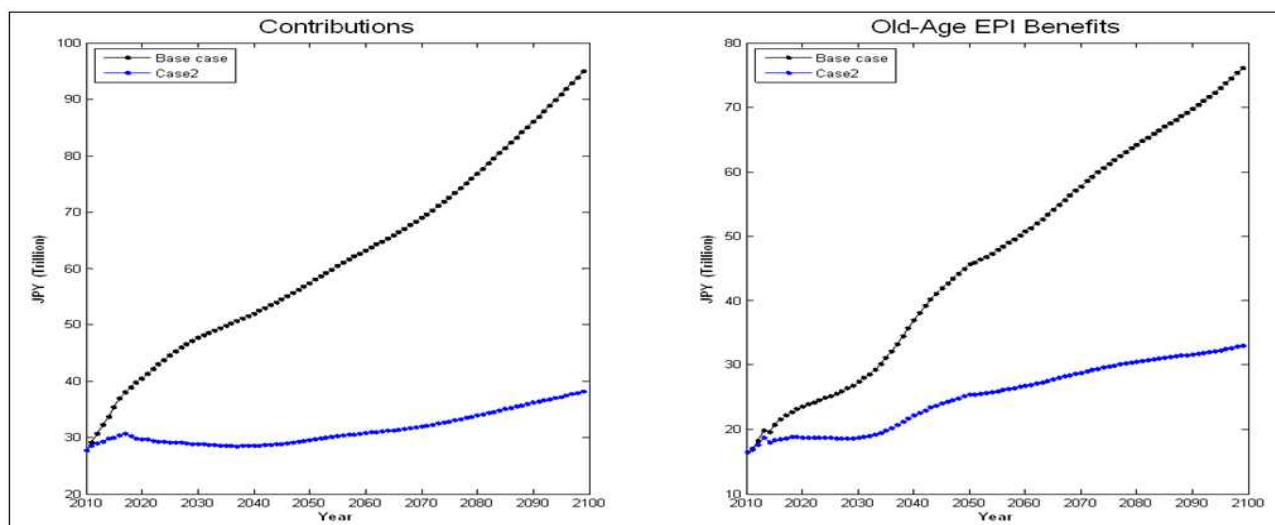
[Figure 4-3] Projected level of reserve: base case and case1



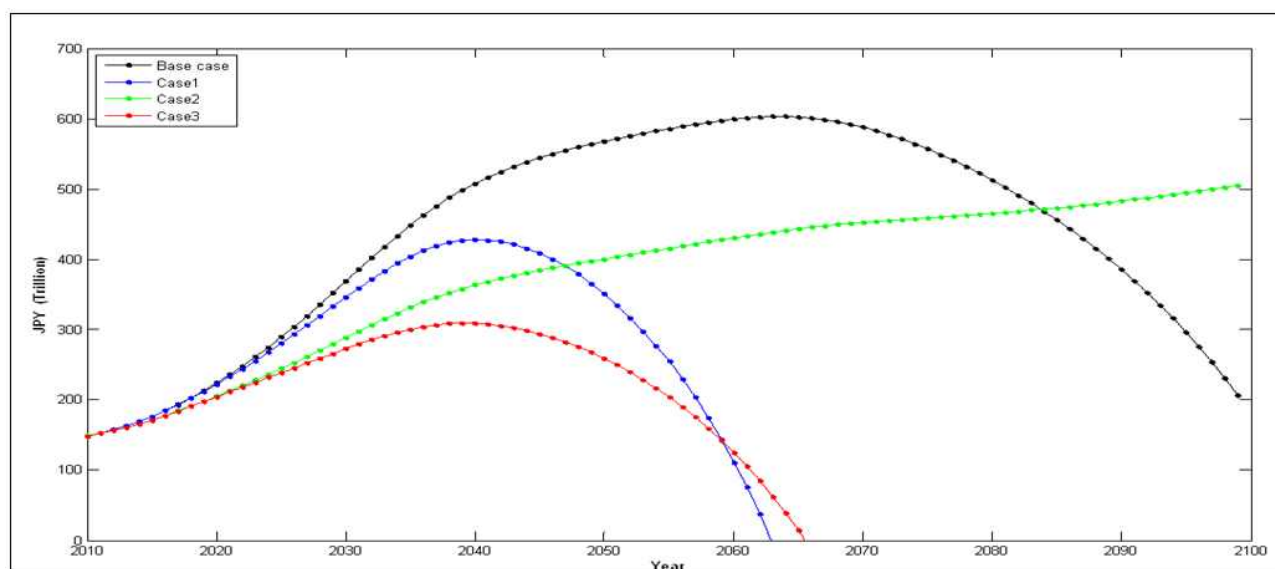
[Figure 4-4] Economic assumptions



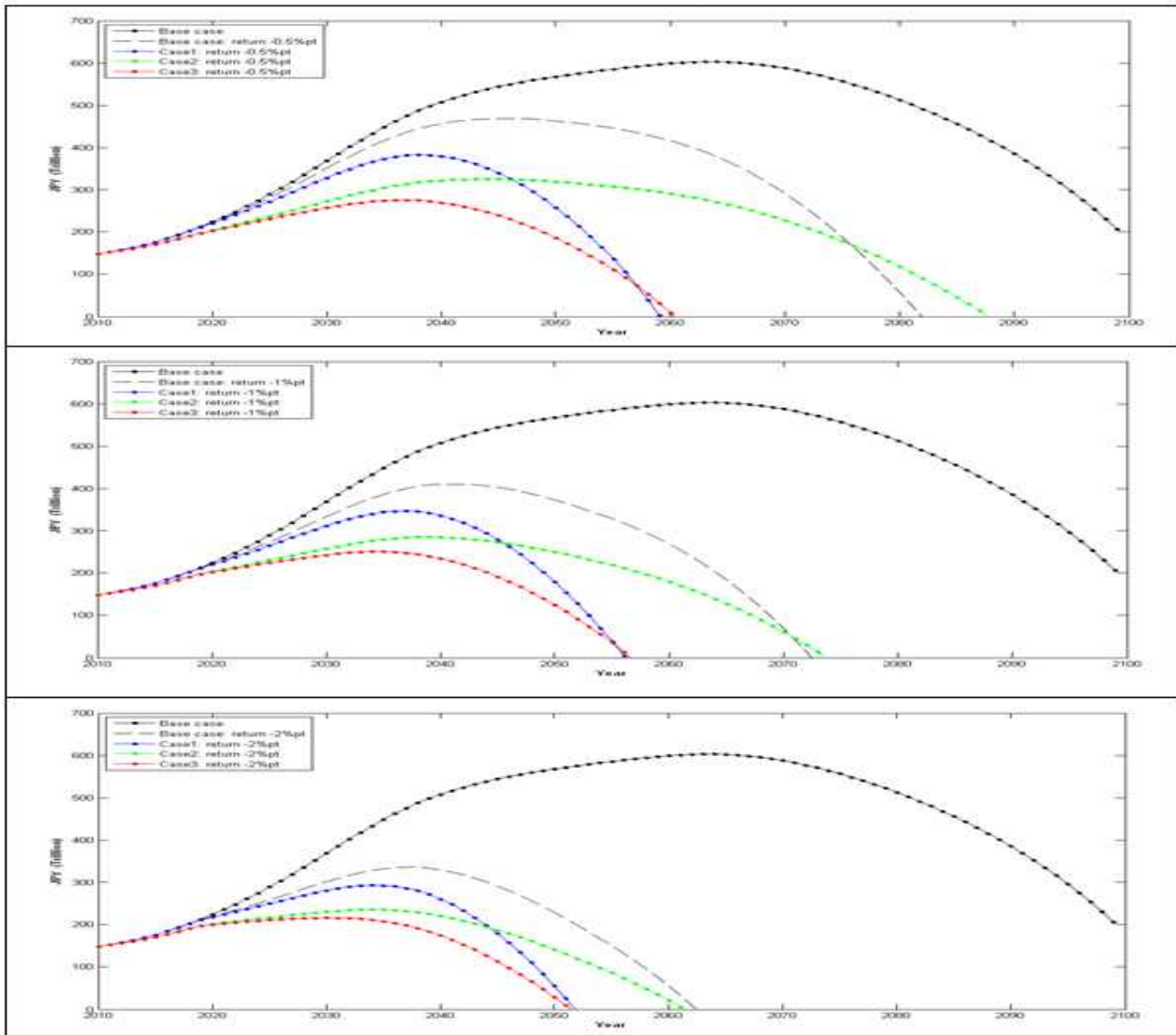
[Figure 4-5] Contributions and Benefits : Base case VS Case2



[Figure 4-6] Projected level of reserve: all cases



[Figure 4-7] Results of sensitive analysis



## 5. Conclusion

In this study pension projections were implemented with different assumptions from those used in 2009 Actuarial Valuation. Under stochastic mortality and economic model the future demographic structure and economic variables was forecasted and the financial status of EPI pension was projected until 2099. The main results are as follows.

Firstly, when using the newly forecasted population the financial status of the EPI pension might be hugely exposed to longevity risk. Considering only the improvements in mortality rate, the level of reserve drops drastically over time and exhausted at 2063, which is 43 years earlier than that of 2009 Actuarial Valuation.

Secondly, the future economic environments seemed to have advantageous effects on pension finances. But the benefits to be paid are affected by wage growth rate and inflation at the same time it is hard to distinguish the effects from the two economic variables .

Finally, with the assumptions combining these two assumptions it is founded the level of reserve still lower than that of 2009 Actuarial Valuation, which is 25 years earlier.

There are some limitations in this analysis. A stochastic model for future fertility model did not independently considered in this analysis. It is likely that the fertility rates are mainly influenced by policies and public tendency. At the same reason stochastic mortality models can not consider other factors that affect deaths only depend on historical time series data.

The pension projection is highly affected by the long-term forecasts on wage growth, inflation rate and return on reserve because the contributions, benefits and reserve are re-evaluated or re-invested cumulatively over extremely long periods. Therefore it is important to note that the results presented here should be interpreted with cautions on these limitations.

It is very difficult to project the pension finance accurately based on forecasts of the future population and macro economic variables over a long-term period. Many counties have been trying to develop methodologies used for pension projection so as to improve their credibility and substantiality of social security pension system. This study contributes as an effort in line with those trends.