Measuring Sovereign Risk Contagion in the Eurozone*

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Abstract

This paper proposes new measures of contagion effects during the recent Eurozone sovereign debt crisis. The new measures, called as the contagion value-at-risk and the contagion expected shortfall, are based on popular risk exposure measures and therefore can provide useful information from the practical perspective of actual investors. For that purpose, this paper also sets up a new model which can disentangle contagion from interdependence. We find that contagion effects dynamically fluctuate, sometimes greatly deviating from the mean level. In addition, the economic values of contagion effects prove to be quite large even for the stable countries.

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Keywords: Sovereign Risk, Contagion, Value-at-risk, Expected Shortfall.

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1 Introduction

For several years, the Eurozone sovereign debt crisis has exerted negative and significant impacts, not only on the Eurozone economy, but also on other economies; these impacts are ongoing. Given its importance, many studies have been conducted on the Eurozone crisis from multi-dimensional viewpoints. In this paper, we aim to provide useful information about contagions associated with the recent Eurozone crisis from the perspective of actual investors who have exposures on the Eurozone sovereign debts. In particular, we try to measure the extent of investors' risk exposure due to contagions from a crisis country. This perspective is rare in Eurozone crisis literature; hence, this information will be useful from a risk management view.

The literature on the Eurozone sovereign yield (spreads) is recently growing. Gibson et al. (2012) use a cointegrated VAR to identify the long-run determinants of the Greek sovereign debt spreads. Arghyrou and Kontonikas (2011) analyze the effects of Greek crisis on other countries with a linear regression model. Caporale and Girardi (2011) use a Global VAR model to analyze the dynamic effects of fiscal imbalances on the borrowing costs of other countries in the euro area. Von Hagen et al. (2011) examine the determinants of European sovereign yield spreads via panel analysis with time fixed-effects. Caceres et al. (2010) investigate the effects of both global risk aversion and country-specific risks on euro area sovereign spreads. Particularly, they construct the spillover index based on the probability of distress of a country conditional on other countries becoming distressed. Gerlach et al. (2010) employ a seemingly unrelated regression model to study the determinants of sovereign bond spreads in the euro area. Attinasi et al. (2009) use a dynamic panel approach to explain the determinants of widening sovereign bond yield spreads vis-a-vis Germany in selected euro area countries. Sgherri and Zoli (2009) document the effects of a single time-varying common factor associated with international risk appetite on sovereign debt yields in the Eurozone during the global financial crisis period. These studies belong to the large literature on the empirical determinants of sovereign yield spreads. Beyond the euro area, this line of research includes Edwards (1986), Eichengreen and Mody (1998), Min (1998), Beck (2001), Ferrucci (2003), and Hilsher and Nosbusch (2010), among others. In sum, these studies largely focus on whether contagion exists or not, as well as on the measurements of the mean contagion effects, or its determinants; thus, they lack the risk management viewpoint from investors. Clearly, since the contagion effects would be time-varying, it would be more informative to provide time-varying effects rather than only the mean effects for the purpose of risk management.

The contagion issue also has been extensively studied for international transmission through stock or foreign exchange markets. Several studies, including Boyer et al. (1999), Loretan and English (2000), Forbes and Rigobon (2002), Rigobon (2003), and Corsetti et al. (2005), among others, define contagion as the increase in correlation between two variables during a crisis period. Latent factor models are widely employed to distinguish between interdependence and contagion, as demonstrated in Corsetti et al. (2005), Dungey and Martin (2001), Forbes and Rigobon (2002), Bekaert et al. (2005), and Dungey et al. (2007). Other methods are also utilized for the contagion analysis: for example, the VAR approach of Favero and Giavazzi (2002), the probability model of Eichengreen et al. (1995, 1996) and the co-exceedance approach of Bae et al. (2003). Dornbusch et al. (2000), Pericoli and Sbracia (2003), and Dungey et al. (2005) provide an overview of the literature. Noteworthily, the dynamics of stock returns or foreign exchange rates among a group of countries differ from those of sovereign yields with a common currency. For instance, sovereign yields typically exhibit close co-movements before crisis while they show divergent movements during a crisis period. Therefore, contagion may be detected not by the increase but by the decrease in correlation for sovereign yields. This difference in the dynamics may necessitate a development of a different methodology.

To provide practically useful information, we propose a new model which can disentangle contagion from interdependence and use it to propose new contagion measures based on popular risk measures, such as the Value-at-Risk (VaR) and the Expected Shortfall (ES), both of which allow clear economic interpretations. As a measure of the risk of loss on a portfolio of financial assets, the VaR is defined as a threshold value such that the mark-to-market loss on the portfolio over a given time horizon exceeds this value at a given probability. As an alternative to the VaR, the ES is the expected return on a portfolio in the worst cases at a given probability. Our approach is novel in that it links contagion measure with the two most popular risk management measures. We provide the measurements of the contagion effects for ten selected Eurozone countries, which consist of five crisis countries and five stable countries. We find that contagion effects dynamically fluctuate, sometimes greatly deviating from the mean level. In addition, the economic values of contagion effects prove to be quite large even for the stable countries.

The rest of this paper is organized as follows. The next section briefly describes the Eurozone sovereign-debt crisis developments as well as the characteristics of sovereign yield data. Section 3 provides the model and the measures of contagion effects. Section 4 first reports the estimation results of the ECB policy rate effects on sovereign yields and then offers empirical results related to the measurements of sovereign risk contagion. Section 5 concludes. The appendix provides a chronicle of the Eurozone sovereign-debt crisis.

2 Eurozone Sovereign-debt Crisis Developments

The Eurozone sovereign debt crisis is not only a fiscal but also a financial crisis, which has never occurred after the inception of the Euro (January 1, 1999); yet, it is still ongoing. From late 2009, fears of sovereign debt insolvency have developed among investors; causes of the debt crisis vary by country. In several countries, governments have greatly increased their debt level due to bailouts of weak banking systems and/or expansionary fiscal responses to slowing economies, both of which have arisen from a property bubble and its burst. In Greece, unsustainable public wage and pension commitments engendered a sovereign debt increase. The structure of the Eurozone has been blamed as a fundamental reason. The Eurozone is a monetary union with one currency, not a fiscal union with different taxes and fiscal expenditures; therefore, it lacks an appropriate stabilizing force.

Concerns intensified in early 2010 and thereafter.[1] While sovereign debt has risen substantially only in a few countries, it has become a perceived problem of the Eurozone as a whole; the speculation of a possible breakup has continuously arisen, which may be an important channel of contagion to other countries in the Eurozone. To restore confidence in the Eurozone, Eurozone leaders have made various measures and efforts to help countries

[1] Refer to the Appendix for the detailed chronicle of the Eurozone sovereign-debt crisis developments.

in a debt crisis. By contributing to or promising a huge amount of bailout packages, debt burdens have been transferred to other countries in the Eurozone, which may be another contagion channel.

For an empirical analysis, we select ten countries (Austria, Belgium, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, and Spain) among 17 Eurozone countries. Cyprus, Estonia, Malta, Slovakia, Slovenia are excluded because they recently joined the Eurozone. Finland and Luxembourg are also excluded due to their small economic size. Because of the popularity as a benchmark indicator for sovereign risk, we select 10-year sovereign debt yields. To balance between tranquil and turbulent periods, we choose a sample period of 7 years from October 1, 2005 to September 30, 2012. Figure 1 demonstrates the 10-year benchmark sovereign debt yields during the whole sample period.

To facilitate the analysis, we exogenously determine the tranquil and turbulent periods. During the tranquil period without significant sovereign risk, the sovereign debt of the member countries would exhibit virtually the same yields, whereas the yields would be differentiated during the turbulent period with significant sovereign risk. To determine the turbulent period, we consult the maximum difference among the ten sovereign debt yields, illustrated in figure 2, and set the turbulent period as the period from October 1, 2008 to September 30, 2012; therefore, in total there is a tranquil period of 3 years and a turbulent period of 4 years.

Table 1 reports the summary statistics of the benchmark sovereign debt yields for the ten countries for both sub-periods. During the tranquil period, all sovereign debt yields show similar mean levels and low volatilities, whereas they exhibit divergent patterns during the turbulent period: Greek sovereign debt yields show the highest mean yield and the most volatile movements, Portuguese sovereign debt yields claim the next place, followed by Irish sovereign debt yields, then by Spanish and Italian sovereign debt yields, respectively. The other five sovereign debt yields remain stable in their movements with only a little increase in the volatilities. This feature can be seen more clearly from the correlations of yields, as illustrated in table 2. During the tranquil period, all sovereign debt yields show almost perfect correlations, which, however, does not hold during the turbulent period. The stable five sovereign debt yields show relatively high correlations, but to a lesser extent. Moreover, they are largely negatively correlated with the five crisis countries. The yields of the five crisis countries show positive correlations, but to a lesser extent, compared to the tranquil period.

With this preliminary result, we try to set up a new model to match the empirical characteristics of sovereign yields during both periods in the next section.

3 Model

3.1 Setup

We focus on the benchmark 10-year sovereign debt yields and denote this by $r_{i,t}$, which is country i's sovereign debt yield at time t. These long-term sovereign debt yields are affected by various factors, among which the policy rate set by the central bank is the most influential one. The ECB policy rate at time t is denoted as $r_{f,t}$.

We apply Cox, Ingersoll, and Ross (CIR) (1985) model to eliminate the effects of changes in the ECB policy rate on individual sovereign debt yields. The CIR model specifies the dynamics of the risk-free rate as follows:

$$dr_{f,t} = \kappa\left(\theta - r_{f,t}\right)dt + \sigma\sqrt{r_{f,t}}dW_t, \tag{1}$$

where $\{W_t\}_{t\geq 0}$ indicates the Wiener process. Under this model, the risk-free yield-to-maturity (YTM) of a pure discount bond with a maturity of $t^*\left(> t\right)$, denoted as r_{t,t^*}, becomes

$$\begin{aligned}
r_{t,t^*} &= \frac{r_{f,t}B\left(t,t^*\right) - \log A\left(t,t^*\right)}{t^* - t}, && (2)\\
A\left(t,t^*\right) &= \left[\frac{2\gamma\exp\left(\left[\left(\kappa+\lambda+\gamma\right)\left(t^*-t\right)\right]/2\right)}{\left(\kappa+\lambda+\gamma\right)\left(\exp\left[\gamma\left(t^*-t\right)\right]-1\right)+2\gamma}\right]^{2\kappa\theta/\sigma^2}, && (3)\\
B\left(t,t^*\right) &= \frac{2\left(\exp\left[\gamma\left(t^*-t\right)\right]-1\right)}{\left(\kappa+\lambda+\gamma\right)\left(\exp\left[\gamma\left(t^*-t\right)\right]-1\right)+2\gamma}, && (4)\\
\gamma &= \left[\left(\kappa+\lambda\right)^2+2\sigma^2\right]^{1/2}. && (5)
\end{aligned}$$

The risk-free YTM of pure discount bonds allow us to readily calculate the YTM of coupon-bearing bonds, which we denote as R_{t,t^*}. Then, we calculate the change in long-term sovereign debt yields after adjusting the effects of the ECB policy rate on individual sovereign debt yields. In particular, the change in long-term sovereign debt yields of country i at time t, $x_{i,t}$ is defined as:

$$x_{i,t} \equiv (r_{i,t} - R_{t,t^*}) - (r_{i,t-1} - R_{t-1,t^*-1}) . \tag{6}$$

Next, we divide the whole period into two sub-periods: pre-crisis period $\mathcal{T}_1 \equiv \{1, ..., T_1\}$ and crisis period $\mathcal{T}_2 \equiv \{T_1 + 1, ..., T\}$. Consistent with the literature on international economic contagions, we assume that only interdependence exists during $\mathcal{T}_1$, whereas both interdependence and contagion exist during $\mathcal{T}_2$.

During $\mathcal{T}_1$, only the interdependence-related shock $u_{i,t}$ determines the dynamics of $x_{i,t}$ as follows:

$$x_{i,t} = \mu_i + u_{i,t}, \tag{7}$$

$$E(u_{i,t}) = 0, \tag{8}$$

$$E\left(u_{i,t}^2\right) = \sigma_i^2, \tag{9}$$

$$E(u_{i,t} u_{j,t}) = \rho_{ij} \sigma_i \sigma_j . \tag{10}$$

Empirical findings characterize high correlations and low volatilities during pre-crisis periods, implying that ρ_{ij}s are close to one and σ_i^2s are low.

On the other hand, during $\mathcal{T}_2$, the interdependence-related shock $u_{i,t}$ and the contagion-related shock $w_{i,t}$ jointly determine the dynamics of $x_{i,t}$ as follows:

$$x_{i,t} = \mu_i + u_{i,t} + w_{i,t}, \tag{11}$$

$$\equiv \mu_i + \varepsilon_{i,t}. \tag{12}$$

For the purpose of identification, we assume that the dynamics of the interdependence-related shock $u_{i,t}$ remain the same during $\mathcal{T}_2$ and are independent of $w_{i,t}$; i.e.,

$$E(u_t w_t') = \mathbf{0}, \tag{13}$$

where u_t and w_t are the corresponding vectors of $u_{i,t}$s and $w_{i,t}$s, respectively. Instead of specifying the dynamics of the contagion-related shock $w_{i,t}$, we specify the dynamics of the combined shock $\varepsilon_{i,t}$, which follows the multivariate diagonal-vech GARCH(1,1) model, that is,

$$\begin{aligned} \varepsilon_t &\equiv \mathbf{H}_t^{1/2} z_t \sim MVN\left[\mathbf{0}, \mathbf{H}_t\right], && (14) \\ h_{ij,t} &= c_{ij} + a_{ij}\varepsilon_{i,t-1}\varepsilon_{j,t-1} + b_{ij}h_{ij,t-1}, && (15) \end{aligned}$$

where $h_{ij,t}$ is the (i,j)th element of $\mathbf{H}_t$. Notably, the contagion-related shock $w_{i,t}$ also follows multivariate diagonal-vech GARCH(1,1) model

$$\begin{aligned} w_t &\equiv \widetilde{\mathbf{H}}_t^{1/2} z_t \sim MVN\left[\mathbf{0}, \widetilde{\mathbf{H}}_t\right], && (16) \\ \widetilde{h}_{ij,t} &= \widetilde{c}_{ij} + \widetilde{a}_{ij}\varepsilon_{i,t-1}\varepsilon_{j,t-1} + \widetilde{b}_{ij}\widetilde{h}_{ij,t-1}. && (17) \end{aligned}$$

Moreover, the dynamics of the contagion-related shock $w_{i,t}$ can be recovered from the dynamics of the combined shock $\varepsilon_{i,t}$ as follows:

$$\begin{aligned} \widetilde{c}_{ij} &= c_{ij} - (1 - b_{ij})\,\rho_{ij}\sigma_i\sigma_j, && (18) \\ \widetilde{a}_{ij} &= a_{ij}, && (19) \\ \widetilde{b}_{ij} &= b_{ij}, && (20) \end{aligned}$$

which are deduced from the following relation:

$$h_{ij,t} = \widetilde{h}_{ij,t} + \rho_{ij}\sigma_i\sigma_j. \quad (21)$$

We infer that the correlation during $\mathcal{T}_2$ is time-varying and less than the correlation during $\mathcal{T}_1$.[2] This result can help the model to match the empirical characteristics of low correlations and high volatilities during crisis periods.

For contagion analysis, the usual factor models introduce common and idiosyncratic factors; the models also investigate whether an idiosyncratic factor of a country affects other countries or not by estimating the effects with fixed coefficients. The factor dynamics may be different between tranquil period and turbulent period, which poses difficult problems regarding identification and estimation (refer to Dungey et al. (2005) for detailed discussion). In contrast, by distinguishing factors into an interdependence-related one and a contagion-related one, rather than common and idiosyncratic factors, we can greatly facilitate a contagion analysis since we can separately estimate the dynamics of the interdependence-related factor during the tranquil period and then the dynamics of the contagion-related factor during the turbulent period. Moreover, the contagion effects are captured by the correlations between the contagion-related factors, and by modeling them as time-varying, we can flexibly estimate the contagion effects.

3.2 Contagion measurement

We will propose measures for contagion effects from the perspective of investors who invest in sovereign debt instruments. In particular, we intend to gauge as to how riskier investors' exposure to sovereign debt instruments becomes due to contagion. This risk-management

[2] This point can be shown as follows:

$$\begin{aligned}
\frac{h_{ij,t}}{\sqrt{h_{i,t}}\sqrt{h_{j,t}}} &= \frac{\rho_{ij}\sigma_i\sigma_j + \widetilde{h}_{ij,t}}{\sqrt{\sigma_i^2+\widetilde{h}_{i,t}}\sqrt{\sigma_j^2+\widetilde{h}_{j,t}}} \\
&= \frac{\rho_{ij}\sigma_i\sigma_j}{\sqrt{\sigma_i^2+\widetilde{h}_{i,t}}\sqrt{\sigma_j^2+\widetilde{h}_{j,t}}} + \frac{\widetilde{h}_{ij,t}}{\sqrt{\sigma_i^2+\widetilde{h}_{i,t}}\sqrt{\sigma_j^2+\widetilde{h}_{j,t}}} \\
&= \rho_{ij}\frac{1}{\sqrt{1+\frac{\widetilde{h}_{i,t}}{\sigma_j^2}}\sqrt{1+\frac{\widetilde{h}_{j,t}}{\sigma_i^2}}} + \widetilde{\rho}_{ij,t}\frac{1}{\sqrt{1+\frac{\sigma_i^2}{\widetilde{h}_{j,t}}}\sqrt{1+\frac{\sigma_j^2}{\widetilde{h}_{i,t}}}} \\
&< \rho_{ij} \quad \text{if } \rho_{ij} > \widetilde{\rho}_{ij,t},
\end{aligned}$$

where $\widetilde{\rho}_{ij,t} \equiv \widetilde{h}_{ij,t}/\sqrt{\widetilde{h}_{i,t}\widetilde{h}_{j,t}}$.

viewpoint may be relevant for actual investors; however, it is scarce in the literature on sovereign risk contagion nonetheless. As previously explained, we will utilize popular risk management measures, the VaR, and the ES but modify them to suit the sovereign risk contagion measurement.

First, we construct the contagion-only yield $y_{i,t,t+\tau}$ by accumulating the contagion-related shocks from time $t+1$ until $t+\tau$ for a chosen horizon of τ, that is,

$$y_{i,t,t+\tau} \equiv w_{i,t+1} + ... + w_{i,t+\tau}. \tag{22}$$

This contagion-only yield $y_{i,t,t+\tau}$ allows for a risk-management analysis at the point in time of $t+\tau$. Alternatively, the maximum contagion-only yield, $\overline{y}_{i,t,t+\tau} \equiv \max\{y_{i,t,t+1}, ..., y_{i,t,t+\tau}\}$ permits risk analysis about maximum risk until $t+\tau$.

Given a confidence level of $1-\alpha$, the usual VaR for $\overline{y}_{i,t,t+\tau}$, denoted as $\overline{VaR}_{i,t}^{\alpha}$, is defined as

$$\Pr\left[\overline{y}_{i,t,t+\tau} > \overline{VaR}_{i,t}^{\alpha}\right] = \alpha. \tag{23}$$

The significance level α is typically set low as 1%, 5%, or 10%. Similarly, we define $VaR_{i,t}^{\alpha}$ for $y_{i,t,t+\tau}$. Clearly, $\overline{VaR}_{i,t}^{\alpha}$ $\left(VaR_{i,t}^{\alpha}\right)$ is a legitimate risk measure for sovereign debt investors because they incur losses as the bond yield becomes higher. Moreover, changes in bond yields have approximately linear and negative relationship with the bond portfolio return through bond duration.

Next, let us define the $\overline{VaR}_{i|j,t}^{\alpha}$ as the conditional VaR of country i at a given confidence level of $1-\alpha$ conditional on the event that a rare and negative shock hits country j.[3] Specifically, we assume the rare and negative shock as the event that $\overline{y}_{j,t,t+\tau}$ belongs to its high tail of size α, i.e.,

$$\Pr\left[\overline{y}_{i,t,t+\tau} > \overline{VaR}_{i|j,t}^{\alpha} | \overline{y}_{j,t,t+\tau} > \overline{VaR}_{j,t}^{\alpha}\right] = \alpha. \tag{24}$$

Similarly, we may define $VaR_{i|j,t}^{\alpha}$ for $y_{i,t,t+\tau}$.

Alternatively, we may define the rare and negative shock as the event that $\overline{y}_{j,t,t+\tau}$ $(y_{j,t,t+\tau})$

[3] Conventionally, conditional VaR refers to the expected shortfall.

exceeds a predetermined absolute level (say, 1%p). This definition may have an advantage of clearer interpretation; however, the same absolute level would imply a high threshold during the tranquil period but a low one during the turbulent period. Based on this consideration, and to be consistent with the conventional choice of a relative risk threshold, we choose our relative conditioning approach.

Now, we propose our VaR-based sovereign risk contagion measure, $\Delta\overline{VaR}^{\alpha}_{i|j,t}$, (which we name as *contagion VaR*) as follows:

$$\Delta\overline{VaR}^{\alpha}_{i|j,t} \equiv \overline{VaR}^{\alpha}_{i|j,t} - \overline{VaR}^{\alpha}_{i,t}. \tag{25}$$

Clearly, this measure becomes zero when country i is not affected by a rare and negative shock to country j, in which case, both the conditional and the unconditional probabilities become the same, and therefore, $\overline{VaR}^{\alpha}_{i|j,t} = \overline{VaR}^{\alpha}_{i,t}$. The more country i gets affected by the rare and negative shock to country j, the greater the $\overline{VaR}^{\alpha}_{i|j,t}$; thus, the greater the $\Delta\overline{VaR}^{\alpha}_{i|j,t}$. This measure provides investors with a clear and meaningful information as to how riskier their exposure to country i becomes, conditional on a rare and negative event to country j. Evidently, the contagion VaR is directional in that the contagion effect from country j to i differs from the effect from country i to j. The measure $\Delta VaR^{\alpha}_{i|j,t}$ is similarly defined for $y_{i,t,t+\tau}$.[4]

We construct similar measures based on the ES. The usual ES for $\overline{y}_{i,t,t+\tau}$, denoted as $\overline{ES}^{\alpha}_{i,t}$, is defined for a given significance level α as

$$\overline{ES}^{\alpha}_{i,t} = \mathbb{E}_t\left[\overline{y}_{i,t,t+\tau}\middle|\,\overline{y}_{i,t,t+\tau} > \overline{VaR}^{\alpha}_{i,t}\right], \tag{26}$$

where $\mathbb{E}_t[\cdot]$ indicates the expectation operator based on information available up to time t. Then, we similarly define the conditional ES, $\overline{ES}^{\alpha}_{i|j,t}$, by

$$\overline{ES}^{\alpha}_{i|j,t} = \mathbb{E}_t\left[\overline{y}_{i,t,t+\tau}\middle|\,\overline{y}_{i,t,t+\tau} > \overline{VaR}^{\alpha}_{i,t}, \overline{y}_{j,t,t+\tau} > \overline{VaR}^{\alpha}_{j,t}\right] \tag{27}$$

[4] Adrian and Brunnermeier (2011) similarly utilize the VaR in order to measure the systemic risk level of the financial system in a country by using individual equity returns of financial institutions. Differing from our approach, they do not explicitly model the dynamics of equity returns but employ quantile regression method to provide numerical results.

Finally, the ES-based sovereign risk contagion measure, $\Delta\overline{ES}^{\alpha}_{i|j,t}$, (which we name as *contagion ES*) is defined by

$$\Delta\overline{ES}^{\alpha}_{i|j,t} \equiv \overline{ES}^{\alpha}_{i|j,t} - \overline{ES}^{\alpha}_{i,t}. \tag{28}$$

We similarly introduce $ES^{\alpha}_{i,t}$, $ES^{\alpha}_{i|j,t}$, and $\Delta ES^{\alpha}_{i|j,t}$ for $y_{i,t,t+\tau}$.

4 Empirical Results

In this section, we first show the estimation results regarding the effects of changes in the ECB policy rate on individual sovereign debt yields. After eliminating the policy rate effects, we then calculate and provide the sovereign risk contagion effects according to the proposed measures.

4.1 Estimation of Policy Rate Effects

For the purpose of eliminating the effects of changes in the ECB policy rate on individual sovereign debt yields, we employ the CIR model to generate model-implied long-term yields. Since this model-implied long-term yield reflects only the dynamics of the policy rate, we can eliminate the effects of policy rate changes by subtracting the model-implied yields from the market yields.

We choose German sovereign debt as the benchmark bond and exclude the crisis period from the estimation period, being concerned over the possibility that the benchmark bond price may have been affected by contagion during the crisis period. Specifically, we select the estimation period from the beginning of the Euro (January 1, 1999) to June 30, 2007 (before the inception of the global financial crisis).

Even though the CIR model provides the analytic probability density function (pdf) for long-term zero rates, typical long-term sovereign debts are coupon-bearing bonds, for which an analytic formula for the pdf is not available. Because of this consideration, we estimate the model parameters by non-linear minimization of the sum of the squared difference between the model-implied and the actual yields. Instead of using the actual (and time-varying) coupon rates, we simply assume the benchmark bond as a par bond. This simplification affects the result minimally.

Figure 3 contrasts market yields with the model-implied yields for German 10-year government bonds. The model-implied yields show jumps at the times of policy rate change; yet, they are non-linearly related with the ECB policy rates. During the estimation period, the actual market yields fluctuate around the model-implied yields whereas the market yields are consistently below the model-implied yields during the out-of-estimation period. This overestimation may not pose a significant problem because we try to model not the yield levels but the yield changes. Indeed, the model-implied yields look to well reflect the effects of policy rate changes during the out-of-estimation period.

4.2 Measurements of Sovereign Risk Contagion

We can easily estimate the dynamics (7)-(10) during $\mathcal{T}_1$. However, it may be problematic to estimate multivariate GARCH models during $\mathcal{T}_2$ when we consider many countries in our system, which is the case. To circumvent this estimation problem, we first separately estimate the GARCH(1,1) model for each country during $\mathcal{T}_2$. Then, we apply Ledoit et al.'s (2003) algorithm, which can be regarded as an approximate multivariate GARCH model and also guarantees positive-semidefinite covariance matrix.

Once we obtain the estimates for the multivariate GARCH model during $\mathcal{T}_2$, we employ simulation methods in order to calculate our sovereign risk contagion measures. We choose a horizon of one month (21 business days). We simulate 50,000 hypothetical paths for the calculation of the measures.

Figure 4 contrasts market 10-year sovereign debt yields with the VaR ($\overline{VaR}^{\alpha}_{i,t}$ in the text) with a significance level α of 5% for each of the ten countries during $\mathcal{T}_2$. Among the ten countries, the first five countries (Greece, Ireland, Portugal, Spain, and Italy) show significant changes in the VaR as well as high VaR levels. Moreover, the VaRs seem to appropriately reflect the 'riskiness' by showing the tendency to co-move with market yield fluctuations. On the other hand, for the remaining five countries, the VaRs also show changes, but to a lesser extent.

Table 3 presents the mean riskiness of sovereign debts gauged by the proposed measures during $\mathcal{T}_2$. Greek sovereign debts could be regarded as the riskiest: investors were faced with the risk of rising yields up to 394 bps (341 bps) in terms of VaR of the maximum yields until one month (yields in one month) at the 5% significance level, conditional on which the expected yield change was 505 bps (456 bps). Portuguese and Irish sovereign debts followed, which were then followed by Italian and Spanish sovereign debts. Consistent with intuition, the VaR of the maximum yields until the horizon, $\overline{VaR}_{i,t}^{\alpha}$, is greater than $VaR_{i,t}^{\alpha}$, which is the VaR of the yields at the end of the horizon. That relation also holds for the ES-based measures, $\overline{ES}_{i,t}^{\alpha}$ and $ES_{i,t}^{\alpha}$. Furthermore, the ES-based measures are greater then the corresponding VaR-based measures.

Table 4 reports the mean contagion effects from the selected five affecting countries to the ten Eurozone countries during $\mathcal{T}_2$. The contagion effects are gauged by the four proposed measures. The corresponding p-values are also reported for the null hypothesis of no contagion-effect against the alternative hypothesis of positive contagion-effect. In terms of the contagion VaR $\left(\Delta\overline{VaR}_{i|j,t}^{\alpha}\right)$, as shown in panel A, Italy is the most influential country by increasing the VaR by 27 bps, on average, for the other nine countries. The second most influential country is Ireland (21 bps), followed by Spain (18 bps) and Portugal (11 bps), respectively. Greece is the least influential country (10 bps) among the selected five countries, even though Greece is the most severely-distressed country, implying that the impact of the sovereign debt crisis in Greece has spread to others to a limited extent. Clearly, the relatively small size of the Greece economy in the Eurozone partly contributes in limiting the contagion. Another reason may be found in the timing of the crisis. Greece is the first crisis country in the Eurozone; the crisis occurred at a relatively healthy condition in the Eurozone economy, and other healthy countries might be affected little, significantly contributing to contain the risk contagion.

Among the selected five affecting countries, which country is the most influential one for each individual country? Greece has been affected by Italy, Ireland, and Spain; however, the extents are statistically insignificant. Ireland has been mostly affected by Italy (58 bps) with a statistical significance. Portugal has been affected by Spain (17 bps) and Italy (13 bps) with a statistical significance. Spain has been affected mostly by Italy (22 bps), and Italy by Ireland (37 bps) and Spain (26 bps), with a statistical significance. The remaining five countries (Germany, France, Netherlands, Belgium, and Austria) have been mostly and significantly affected by Italy, with extents ranging from 16 to 25 bps, followed by Ireland with a statistical significance (except for Germany). Panel B of table 4 shows the mean contagion effects measured by the contagion VaR $\left(\Delta VaR_{i|j}^{\alpha}\right)$, which is the VaR-based measure of the yields in one month. The contagion VaR of $\Delta VaR_{i|j}^{\alpha}$ shows qualitatively the same results with the contagion VaR of $\Delta\overline{VaR}_{i|j}^{\alpha}$; however, the former indicates slightly greater extents in most cases.

On the other hand, the ES-based measures provide different patterns. Recalling the well-known fact that the VaR is more sensitive to the shape of the loss distribution in the tail of the distribution than the ES, this difference between the two measures is attributable to the tail shape. Panels C and D of table 4 show the mean contagion effects gauged by the contagion ES, $\Delta\overline{ES}_{i|j}^{\alpha}$ and $\Delta ES_{i|j}^{\alpha}$, respectively. Different from the VaR-based measures, Italy is not the most-influential country with ES-based measures, but is one of the four countries among the selected five affecting countries (excepting Greece) who have exerted similar contagion effects to the other nine countries; moreover, Greece has claimed the smallest influence. Instead, the contagion effects are more clearly differentiated among the five affecting countries; i.e., the contagion ES $\left(\Delta\overline{ES}_{i|j}^{\alpha}\right)$ ranges between 362 to 365 bps for Greece and between 66 to 72 bps for Spain. Further, the mean contagion effects seem largely commensurate with the corresponding ES itself. Consistent with the contagion VaR, the contagion ES of $\Delta ES_{i|j}^{\alpha}$ indicates greater contagion effects than the $\Delta\overline{ES}_{i|j}^{\alpha}$.

Clearly, contagion effects tend to dynamically fluctuate, which cannot be captured by the measurements of the mean contagion effects, as shown in table 4. To demonstrate time-varying contagion effects, figure 5 illustrates the contagion VaR of $\Delta\overline{VaR}_{i|j,t}^{\alpha}$ at each point in time for each of the ten countries against each of the five affecting countries. This figure is useful for the comparison among the affected countries.[5]

Panel A of figure 5 shows that the contagion effects from Greece have shown relatively stable movements for Germany, France, Netherlands, Belgium, and Austria; yet, it has

[5] By rearranging figure 5, we may provide information from the other point of view, which is helpful for comparing the five affecting countries. To save space, we omit this figure; however, it is available upon request.

shown more time-varying patterns for Ireland, Portugal, Spain, and Italy. To characterize the time-varying patterns of the contagion effects, table 5 shows the volatilities and the correlations of the contagion VaR $\left(\Delta\overline{VaR}_{i|j,t}^{\alpha}\right)$ for each of the five affecting countries at a 5% significance level. As demonstrated in panel A of table 5, the contagion effects (from Greece) on Ireland have the greatest volatility and are relatively uncorrelated with the effects on other countries. On the other hand, the effects (from Greece) on the five countries (Germany, France, Netherlands, Belgium, and Austria) show similar volatilities and are closely correlated to each other.

In sum, the contagion effects on the stable five countries show similar volatilities and are also closely correlated with each other for all cases. The contagion effects on Greece prove to be the greatest in most cases. The effects on the stable five countries largely show decoupled movements (even negative relations in some cases) from the five affecting countries with only a few exceptions. When Greece affects countries, the effects on Portugal and Spain are moderately correlated with those of the stable five countries. When Portugal affects countries, the effects on Spain and Italy are also closely correlated with those of the stable five countries. Among the five affecting countries, the effects on Spain and Italy are closely correlated one another in all cases.

To provide more clear economic interpretations, table 6 shows the economic value of the contagion effects. The economic values, gauged by the duration of the benchmark bond times the bond yield changes caused by contagions, indicate the extent of the increase in investors' risk exposures due to contagion effects in terms of the yield from their investment in Eurozone sovereign debts. For example, from the perspective of German sovereign debt investors, the mean riskiness measured by the conditional VaR $\left(\overline{VaR}_{i|j}\right)$ at the 5% significance level was 4.93% losses within a one month horizon. A rare and significantly negative shock to Italy would have engendered an additional investment risk of 1.39% losses on average. The second-most influential country to Germany was Ireland (1.28%) and then Spain (0.98%) consecutively. On average, the contagion effects have heightened the investment risk by 0.98%, which is equivalent to 20% of the mean riskiness. With the contagion VaR of $VaR_{i|j}^{0.05}$, the contagion effects have risen to 1.12% on average, which is equivalent to 25.7% of the mean riskiness. With ES-based measures, the contagion effects have risen even further; the contagion effects on Germany have risen to 3.81% (63% of the mean riskiness) and 4.71% (84.6% of the mean riskiness) measured by contagion ES, $\overline{ES}_{i|j}^{0.05}$ and $ES_{i|j}^{0.05}$, respectively.

5 Conclusion

In this paper, we aim to provide useful information regarding contagions, which have occurred during the recent Eurozone crisis. This information comes from the practical perspective of actual investors who have been exposed to the Eurozone sovereign debts. In particular, we measure the extent of the increase in investors' risk exposure due to contagions from a crisis country. This perspective is rare in Eurozone crisis literature; hence, this will be useful from a risk management view. For that purpose, we set up a new model which can disentangle the contagion from interdependence; it can also propose new contagion measures, called as the contagion VaR and the contagion ES. We find that contagion effects tend to dynamically fluctuate and sometimes greatly deviate from the mean level. We also find that the economic values of contagion effects prove to be quite large even for the stable countries. The economic values of contagion effects are larger with the contagion ES than with the contagion VaR.

This risk management perspective provides practical investors with useful information in the context of the recent Eurozone crisis. We believe that measurements of contagion effects from the risk management perspective would also be valuable for international transmissions through stock markets and foreign exchange markets. Our proposed methods might also be applicable for contagions through stock or foreign exchange markets.

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Appendix: Chronicle of Eurozone Sovereign-debt Crisis

Greece

- 2010
  - 23 April: Greece officially requested an initial loan of €45 billion from the EU and IMF.
  - 27 April: S&P downgraded Greece's sovereign debt rating to BB+.
  - 2 May: Greece announced the latest, fourth, raft of austerity measures.
  - 3 May: The ECB announced that it would accept Greek Government bonds as collateral irrespective of their rating.
  - 4 May: First day of strikes against austerity measures.
  - 5 May: General nationwide strike and demonstrations in two major cities in Greece turned violent.
  - 8 May: Leaders of the Eurozone countries resolved in Brussels to take drastic action in order to protect the euro from further market turmoil after approving a $100 billion bailout plan for Greece.
  - 20 May: Fourth strike in Greece against wage cuts.
  - 24 May: Greek government announced a deficit reduction by 41.5% for the first four months.
  - 5 July: The central Bank of Greece announced a reduction of central government cash deficit by 41.8%, for the first half-year of 2010.
  - 7 September: Finance Ministers of the EU countries approved the second of the bailout installments for Greece (€6.5 billion).
  - 11 September: IMF approved the second installment of their bailout package for Greece (€2.57 billion).
- 2011
  - 23 May: Greece unveiled a series of privatization, part of a goal to raise €50 billion by 2015 in order to pay down its debt mountain.
  - 4 June: Greece hit by further protests in central Athens, as PM Papandreou agreed to make "significant" cuts in public sector employment.

- 15 June: The Greek public turned violent after the failure of European leaders to resolve their disagreements over the Greek debt crisis.
- 21 July: The Council of the European Union reached an agreement on measures aimed at solving the Greek debt crisis.
- 22 September: Greeks reacted with anger and disbelief at a new wave of austerity cuts enacted to keep the country in the Eurozone.
- 24 September: IMF urged EU leaders to act decisively on Greece to stem the debt crisis.
- 1 November: Greek PM Papandreou announced a referendum on the new Eurozone debt deal which shocked European markets and had thrown the future of the euro back into disarray.
- 3 November: Prime Minister Papandreou withdrew from the promised Greek referendum on the bailout package amid heavy pressure from Germany and France.
- 7 December: The new interim national union government led by Lucas Papademos submits its plans for the 2012 budget, promising to cut its deficit from 9% of GDP 2011 to 5.4% in 2012, mostly due to the write-off of debt held by banks.
- 8 December: Fitch cut Greece's rating to BBB+ from A-, with a negative outlook.
- 14 December: Greek PM Papandreou outlined the first round of policies in order to cut the deficit and regain investor trust.
- 22 December: Moody's cut Greece's rating to A2 from A1.

- 2012

  - 21 February: The Eurogroup finalizes the second bailout package with the private holders of Greek governmental bonds accepting a slightly bigger haircut of 53.5%. EU Member States agree to an additional retroactive lowering of the bailout interest rates and pass on all central bank profits related to Greece until 2020. Altogether, this should bring down Greece's debt to 120.5% by 2020.
  - 6 May: In Greek legislative election, no party gains an overall majority, which worsened the market falls.
  - 13 May: Greece's President tried to form a coalition government.

Ireland

- 2010

- 13 November: The potential for loss in the value of government bonds triggered the selling of Irish debt. The 10-year Irish government bond premium surged to a record of 652 basis points premium against the German bond.
- 16 November: Ireland started talks with the EU over a bailout.
- 21 November: Ireland accepted an EU-IMF multi-billion euro package in order to help alleviate its debt burden.
- 22 November: Following the withdrawal of the Irish Green Party from the governing coalition, a new election was called.

- 2011
  - April: Moodys' downgraded the banks' debt to junk status.
  - July: European leaders agreed to cut the interest rate that Ireland was paying on its EU/IMF bailout loan from around 6% to between 3.5% and 4% and to double the loan time to 15 years.
  - 14 September: European Commission announced it would cut the interest rate on its €22.5 billion loan coming from the European Financial Stability Mechanism, down to 2.59%.

Portugal

- 2011
  - 3 May: Portugal has agreed to a three-year €78 billion IMF-EU bailout package in a bid to stabilize its public finances.
  - 6 July: Moodys' cut Portugal's credit rating to junk status.

Spain

- 2010
  - Early May: Spain announced new austerity measures designed to reduce the country's budget deficit.
  - 29 May: Fitch downgraded the Government of Spain bonds one notch from AAA to AA+

- 2011

- 7 October: Fitch cut Spain's rating to AA- from AA+.
- 13 October: S&P cut Spain's long-term credit rating by one notch from AA to AA- with a negative outlook.

- 2012
  - 13 January: Standard & Poor's downgraded Spain's rating
  - 9 June: Eurogroup granted Spain a credit line of up to €100 billion.
  - September: ECB took pressure from Spain when it announced its "unlimited bond-buying plan."

Italy

- 2011
  - 21 September: S&P has downgraded seven Italian banks after they've dropped Italy's sovereign rating two days ago.
  - 7 October: Fitch cut Italy's credit rating by one notch to A+ from AA-.
  - 13 November: Silvio Berlusconi resigns as Prime Minister of Italy as a result of the country's debt crisis.
- 2012
  - 13 January: Standard & Poor's downgraded Italy's rating.

Table 1: Summary statistics. This table reports the summary statistics of 10-year sovereign debt yields for ten Eurozone countries during both sub-periods.

	Tranquil: 10/1/2005 - 9/30/2008				Turbulent: 10/1/2008 - 9/30/2012			
	Mean	St. dev.	Skewness	Kurtosis	Mean	St. dev.	Skewness	Kurtosis
GRE	4.339	0.417	-0.130	2.834	13.630	9.288	0.944	2.750
IRE	4.117	0.436	-0.046	2.382	6.744	2.104	0.986	3.295
POR	4.217	0.413	-0.231	2.475	7.413	3.393	0.579	1.876
SPA	4.096	0.405	-0.250	2.480	4.822	0.889	0.660	2.471
ITA	4.311	0.410	-0.213	2.796	4.782	0.823	0.977	3.244
GER	3.991	0.340	-0.289	2.687	2.697	0.693	-0.525	2.099
FRA	4.070	0.380	-0.273	2.482	3.265	0.469	-0.581	2.918
NET	4.063	0.384	-0.277	2.517	3.057	0.682	-0.356	2.038
BEL	4.125	0.409	-0.226	2.414	3.781	0.505	-0.325	3.672
AUT	4.089	0.388	-0.351	2.568	3.355	0.619	-0.453	2.623

Table 2: Correlations of yields. This table shows the correlations of 10-year sovereign debt yields for ten Eurozone countries during both sub-periods.

	GRE	IRE	POR	SPA	ITA	GER	FRA	NET	BEL
	A. October 1, 2005 - September 30, 2008								
IRE	0.99								
POR	0.99	0.99							
SPA	0.99	0.99	1.00						
ITA	1.00	0.99	0.99	0.99					
GER	0.94	0.93	0.95	0.96	0.95				
FRA	0.98	0.98	0.99	0.99	0.98	0.98			
NET	0.98	0.98	0.99	0.99	0.98	0.98	1.00		
BEL	0.99	0.99	1.00	1.00	0.99	0.96	0.99	0.99	
AUT	0.98	0.98	0.99	0.99	0.98	0.95	0.98	0.98	0.99
	B. October 1, 2008 - September 30, 2012								
IRE	0.48								
POR	0.93	0.68							
SPA	0.78	0.61	0.80						
ITA	0.85	0.49	0.84	0.82					
GER	-0.83	-0.23	-0.75	-0.70	-0.65				
FRA	-0.59	-0.11	-0.49	-0.52	-0.32	0.89			
NET	-0.82	-0.24	-0.72	-0.68	-0.58	0.98	0.92		
BEL	-0.08	0.37	0.06	-0.08	0.21	0.49	0.76	0.55	
AUT	-0.67	-0.21	-0.59	-0.62	-0.41	0.91	0.97	0.95	0.70

Table 3: Mean riskiness of sovereign debts. This table shows the mean riskiness of sovereign debts gauged by measures proposed in the text during October 1, 2008 to September 30, 2012.

	$\overline{VaR}_i^{0.05}$	$VaR_i^{0.05}$	$\overline{ES}_i^{0.05}$	$ES_i^{0.05}$
GRE	3.937	3.412	5.048	4.559
IRE	1.321	1.134	1.761	1.573
POR	1.527	1.339	1.907	1.741
SPA	0.836	0.731	1.055	0.960
ITA	0.844	0.722	1.138	1.012
GER	0.564	0.499	0.692	0.637
FRA	0.589	0.523	0.719	0.663
NET	0.582	0.516	0.709	0.654
BEL	0.616	0.540	0.777	0.707
AUT	0.587	0.519	0.724	0.665

Table 4: Mean contagion effects. This table shows the mean contagion effects from the selected five affecting countries to the ten Eurozone countries during October 1, 2008 to September 30, 2012. The contagion effects are gauged by the measures proposed in the text. The corresponding p-values are also reported for the null hypothesis of no contagion-effect against the alternative hypothesis of positive contagion-effect.

	Affecting country									
	Mean Contagion Effects					p-value				
	GRE	IRE	POR	SPA	ITA	GRE	IRE	POR	SPA	ITA
	A. Contagion VaR $\Delta\overline{VaR}_{i\|j}^{0.05}$									
GRE		0.336	0.104	0.275	0.446		0.226	0.264	0.185	0.196
IRE	0.177		0.224	0.239	0.584	0.116		0.103	0.082	0.022
POR	0.069	0.187		0.166	0.134	0.131	0.099		0.005	0.010
SPA	0.098	0.125	0.114		0.222	0.037	0.038	0.016		0.009
ITA	0.156	0.374	0.107	0.260		0.063	0.016	0.030	0.031	
GER	0.059	0.148	0.084	0.114	0.162	0.131	0.105	0.047	0.014	0.053
FRA	0.079	0.180	0.108	0.158	0.242	0.096	0.033	0.042	0.001	0.001
NET	0.086	0.165	0.100	0.150	0.208	0.129	0.034	0.040	0.003	0.010
BEL	0.087	0.206	0.094	0.126	0.245	0.057	0.001	0.027	0.002	0.000
AUT	0.056	0.159	0.076	0.097	0.177	0.086	0.008	0.035	0.002	0.000
Mean	0.096	0.209	0.112	0.176	0.269					
	B. Contagion VaR $\Delta VaR_{i\|j}^{0.05}$									
GRE		0.347	0.118	0.291	0.459		0.225	0.244	0.134	0.186
IRE	0.174		0.217	0.240	0.597	0.088		0.081	0.048	0.015
POR	0.081	0.191		0.187	0.161	0.134	0.079		0.003	0.004
SPA	0.106	0.133	0.127		0.234	0.037	0.017	0.014		0.002
ITA	0.156	0.378	0.120	0.260		0.041	0.009	0.022	0.009	
GER	0.069	0.168	0.095	0.131	0.186	0.112	0.089	0.037	0.006	0.032
FRA	0.092	0.205	0.123	0.179	0.271	0.086	0.021	0.034	0.001	0.000
NET	0.099	0.187	0.115	0.171	0.237	0.114	0.022	0.033	0.001	0.004
BEL	0.097	0.219	0.106	0.145	0.262	0.049	0.000	0.026	0.001	0.000
AUT	0.064	0.175	0.085	0.111	0.196	0.082	0.003	0.031	0.001	0.000
Mean	0.104	0.222	0.123	0.190	0.289					

Table 4 continued.

	Affecting country									
	Mean Contagion Effects					p-value				
	GRE	IRE	POR	SPA	ITA	GRE	IRE	POR	SPA	ITA
	C. Contagion ES $\Delta\overline{ES}_{i\mid j}^{0.05}$									
GRE		3.652	3.636	3.617	3.631		0.257	0.257	0.261	0.262
IRE	1.260		1.237	1.224	1.088	0.026		0.029	0.030	0.045
POR	1.336	1.321		1.286	1.306	0.095	0.091		0.104	0.098
SPA	0.721	0.724	0.701		0.664	0.035	0.027	0.041		0.036
ITA	0.796	0.714	0.783	0.719		0.039	0.049	0.044	0.048	
GER	0.465	0.425	0.448	0.429	0.407	0.000	0.000	0.000	0.000	0.001
FRA	0.468	0.419	0.447	0.412	0.361	0.000	0.000	0.000	0.000	0.000
NET	0.457	0.420	0.443	0.411	0.377	0.000	0.000	0.000	0.000	0.000
BEL	0.519	0.477	0.505	0.480	0.430	0.001	0.001	0.001	0.002	0.005
AUT	0.492	0.449	0.477	0.462	0.427	0.000	0.000	0.000	0.000	0.000
Mean	0.724	0.956	0.964	1.004	0.965					
	D. Contagion ES $\Delta ES_{i\mid j}^{0.05}$									
GRE		4.501	4.450	4.422	4.463		0.257	0.258	0.261	0.264
IRE	1.498		1.468	1.435	1.214	0.023		0.026	0.026	0.047
POR	1.670	1.665		1.588	1.615	0.099	0.092		0.110	0.103
SPA	0.884	0.891	0.850		0.798	0.037	0.027	0.045		0.042
ITA	0.929	0.788	0.899	0.816		0.035	0.046	0.040	0.046	
GER	0.587	0.525	0.559	0.526	0.489	0.000	0.000	0.000	0.000	0.003
FRA	0.589	0.505	0.554	0.499	0.418	0.000	0.000	0.000	0.000	0.001
NET	0.576	0.509	0.550	0.498	0.443	0.000	0.000	0.000	0.000	0.001
BEL	0.636	0.562	0.612	0.569	0.494	0.001	0.001	0.001	0.003	0.007
AUT	0.620	0.544	0.596	0.566	0.509	0.000	0.000	0.000	0.000	0.000
Mean	0.888	1.166	1.171	1.213	1.160					

Table 5: Volatilities and correlations of the contagion VaR $\Delta\overline{VaR}^{0.05}_{i|j,t}$. This table shows the standard deviations (S.d.) and the correlations of the contagion VaR $\Delta\overline{VaR}^{0.05}_{i|j,t}$ for each of the five affecting countries.

	A. Affecting country: Greece								
	IRE	POR	SPA	ITA	GER	FRA	NET	BEL	AUT
S.d.	0.149	0.062	0.054	0.102	0.053	0.061	0.076	0.055	0.041
POR	0.109								
SPA	0.305	0.549							
ITA	0.404	0.227	0.624						
GER	-0.001	0.627	0.634	0.332					
FRA	0.045	0.689	0.698	0.404	0.925				
NET	0.046	0.633	0.664	0.388	0.949	0.941			
BEL	0.202	0.587	0.707	0.548	0.851	0.883	0.877		
AUT	0.054	0.619	0.666	0.394	0.889	0.905	0.894	0.861	
	B. Affecting country: Ireland								
	GRE	POR	SPA	ITA	GER	FRA	NET	BEL	AUT
S.d.	0.446	0.146	0.070	0.175	0.118	0.098	0.090	0.069	0.066
POR	0.435								
SPA	0.246	0.380							
ITA	0.219	0.258	0.741						
GER	-0.247	-0.405	-0.057	0.156					
FRA	-0.234	-0.395	0.039	0.260	0.905				
NET	-0.247	-0.431	-0.001	0.197	0.941	0.952			
BEL	-0.074	-0.163	0.291	0.463	0.670	0.759	0.724		
AUT	-0.192	-0.369	0.051	0.251	0.862	0.902	0.896	0.803	
	C. Affecting country: Portugal								
	GRE	IRE	SPA	ITA	GER	FRA	NET	BEL	AUT
S.d.	0.165	0.177	0.053	0.057	0.050	0.062	0.057	0.049	0.042
IRE	-0.026								
SPA	0.173	0.091							
ITA	0.186	0.076	0.776						
GER	0.187	-0.168	0.757	0.726					
FRA	0.197	-0.090	0.817	0.791	0.939				
NET	0.214	-0.098	0.826	0.791	0.926	0.963			
BEL	0.235	-0.054	0.805	0.779	0.877	0.907	0.926		
AUT	0.204	-0.105	0.773	0.754	0.908	0.922	0.918	0.898	

Table 5 continued.

	D. Affecting country: Spain								
	GRE	IRE	POR	ITA	GER	FRA	NET	BEL	AUT
S.d.	0.307	0.172	0.065	0.139	0.052	0.053	0.055	0.043	0.034
IRE	0.151								
POR	0.040	0.370							
ITA	0.165	0.269	-0.026						
GER	-0.277	-0.087	0.217	-0.280					
FRA	-0.217	-0.106	0.266	-0.224	0.905				
NET	-0.239	-0.046	0.299	-0.298	0.928	0.940			
BEL	-0.164	-0.008	0.395	-0.264	0.769	0.830	0.845		
AUT	-0.186	-0.058	0.317	-0.268	0.801	0.855	0.855	0.872	
	E. Affecting country: Italy								
	GRE	IRE	POR	SPA	GER	FRA	NET	BEL	AUT
S.d.	0.519	0.291	0.058	0.094	0.100	0.077	0.089	0.059	0.051
IRE	0.088								
POR	0.011	0.325							
SPA	0.241	0.208	-0.054						
GER	-0.522	0.016	0.085	-0.400					
FRA	-0.357	-0.172	-0.033	-0.369	0.843				
NET	-0.499	0.027	0.112	-0.433	0.949	0.863			
BEL	-0.196	0.075	0.084	-0.093	0.564	0.614	0.537		
AUT	-0.422	-0.095	0.103	-0.383	0.822	0.802	0.807	0.718	

Table 6: Economic value of contagion effects. This table shows the economic value of the contagion effects from the selected five affecting countries to the ten Eurozone countries during October 1, 2008 to September 30, 2012. The economic values are gauged by the duration of the benchmark bond times the bond yield changes caused by contagions.

	Affecting countries					Mean	Mean	Ratio(%)
	GRE	IRE	POR	SPA	ITA	riskiness	contagion	b/a
	A. Contagion VaR $\Delta\overline{VaR}_{i\|j}^{0.05}$					$(\overline{VaR}_{i\|j}^{0.05}, a)$	effects(b)	
GRE		1.654	0.661	1.461	2.199	17.336	1.494	8.6
IRE	1.263		1.592	1.702	4.187	9.317	2.186	23.5
POR	0.523	1.274		1.191	0.964	9.980	0.988	9.9
SPA	0.775	0.977	0.904		1.737	6.498	1.098	16.9
ITA	1.226	2.933	0.852	2.034		6.577	1.761	26.8
GER	0.506	1.281	0.720	0.982	1.390	4.925	0.976	19.8
FRA	0.662	1.522	0.907	1.335	2.047	4.991	1.295	25.9
NET	0.718	1.401	0.848	1.270	1.766	4.980	1.201	24.1
BEL	0.715	1.701	0.776	1.036	2.022	5.082	1.250	24.6
AUT	0.468	1.336	0.634	0.812	1.491	4.954	0.948	19.1
Mean	0.762	1.564	0.877	1.314	1.978			19.9
	B. Contagion VaR $\Delta VaR_{i\|j}^{0.05}$					$(VaR_{i\|j}^{0.05}, a)$	(b)	(b/a)
GRE		1.713	0.759	1.573	2.296	15.032	1.585	10.5
IRE	1.247		1.552	1.715	4.279	8.002	2.198	27.5
POR	0.615	1.304		1.345	1.148	8.757	1.103	12.6
SPA	0.846	1.038	1.010		1.833	5.682	1.182	20.8
ITA	1.234	2.967	0.956	2.036		5.631	1.798	31.9
GER	0.593	1.458	0.820	1.129	1.604	4.359	1.121	25.7
FRA	0.769	1.730	1.032	1.507	2.292	4.428	1.466	33.1
NET	0.829	1.597	0.972	1.450	2.015	4.420	1.372	31.0
BEL	0.796	1.805	0.874	1.196	2.164	4.456	1.367	30.7
AUT	0.529	1.467	0.711	0.935	1.650	4.377	1.059	24.2
Mean	0.829	1.675	0.965	1.432	2.142			24.8

Table 6 continued.

	Affecting countries					Mean	Mean	Ratio(%)
	GRE	IRE	POR	SPA	ITA	riskiness	contagion	b/a
	C. Contagion ES $\Delta\overline{ES}_{i\|j}^{0.05}$					$(\overline{ES}_{i\|j}^{0.05}, a)$	effects(b)	
GRE		15.953	15.842	15.709	15.785	22.206	15.822	71.3
IRE	8.853		8.680	8.590	7.595	12.404	8.429	68.0
POR	8.673	8.590		8.315	8.469	12.455	8.511	68.3
SPA	5.588	5.622	5.428		5.148	8.201	5.447	66.4
ITA	6.177	5.538	6.074	5.573		8.859	5.841	65.9
GER	4.067	3.724	3.919	3.756	3.568	6.043	3.807	63.0
FRA	3.972	3.554	3.789	3.497	3.066	6.090	3.576	58.7
NET	3.925	3.604	3.800	3.529	3.242	6.069	3.620	59.7
BEL	4.279	3.929	4.168	3.959	3.540	6.408	3.975	62.0
AUT	4.153	3.789	4.032	3.901	3.607	6.109	3.896	63.8
Mean	5.521	6.034	6.192	6.314	6.002			64.7
	D. Contagion ES $\Delta ES_{i\|j}^{0.05}$					$(ES_{i\|j}^{0.05}, a)$	(b)	(b/a)
GRE		19.649	19.338	19.135	19.354	20.065	19.369	96.5
IRE	10.529		10.309	10.082	8.472	11.087	9.848	88.8
POR	10.822	10.820		10.247	10.453	11.375	10.585	93.1
SPA	6.849	6.916	6.582		6.182	7.458	6.632	88.9
ITA	7.215	6.115	6.977	6.332		7.879	6.660	84.5
GER	5.137	4.593	4.893	4.608	4.293	5.561	4.705	84.6
FRA	4.997	4.282	4.700	4.236	3.547	5.617	4.352	77.5
NET	4.951	4.370	4.720	4.278	3.815	5.600	4.427	79.0
BEL	5.243	4.634	5.050	4.692	4.068	5.835	4.737	81.2
AUT	5.241	4.598	5.033	4.783	4.300	5.611	4.791	85.4
Mean	6.776	7.331	7.511	7.599	7.165			86.0

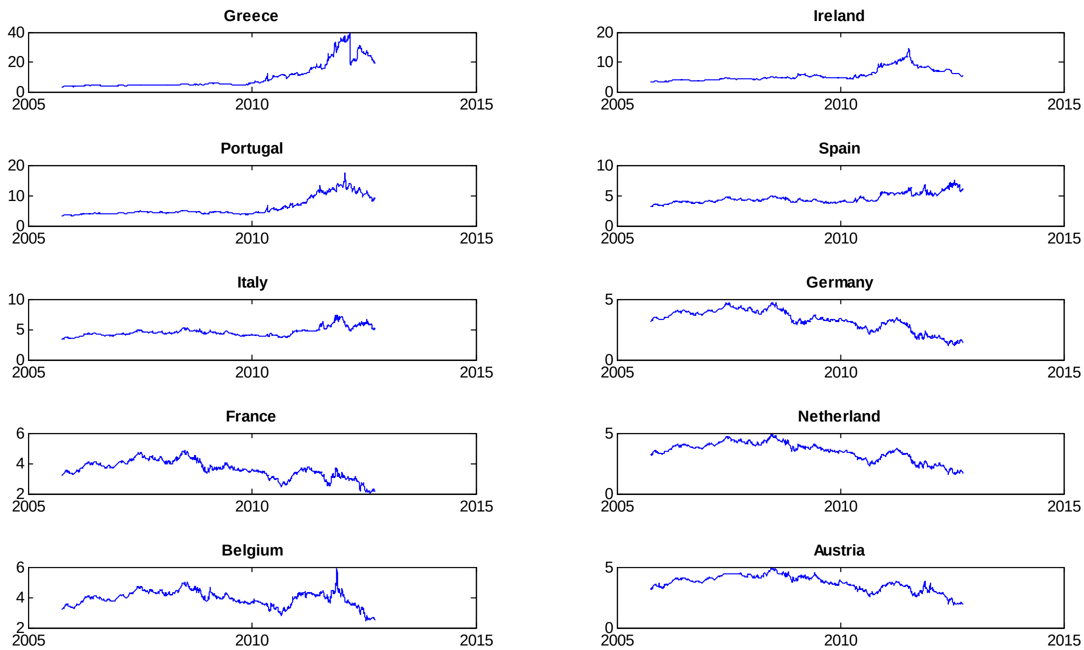

Figure 1: 10-year benchmark sovereign risk debt yields. This figure shows the 10-year benchmark sovereign risk debt yields for each of the ten countries during the sample period from October 1, 2005 to September 30, 2012.

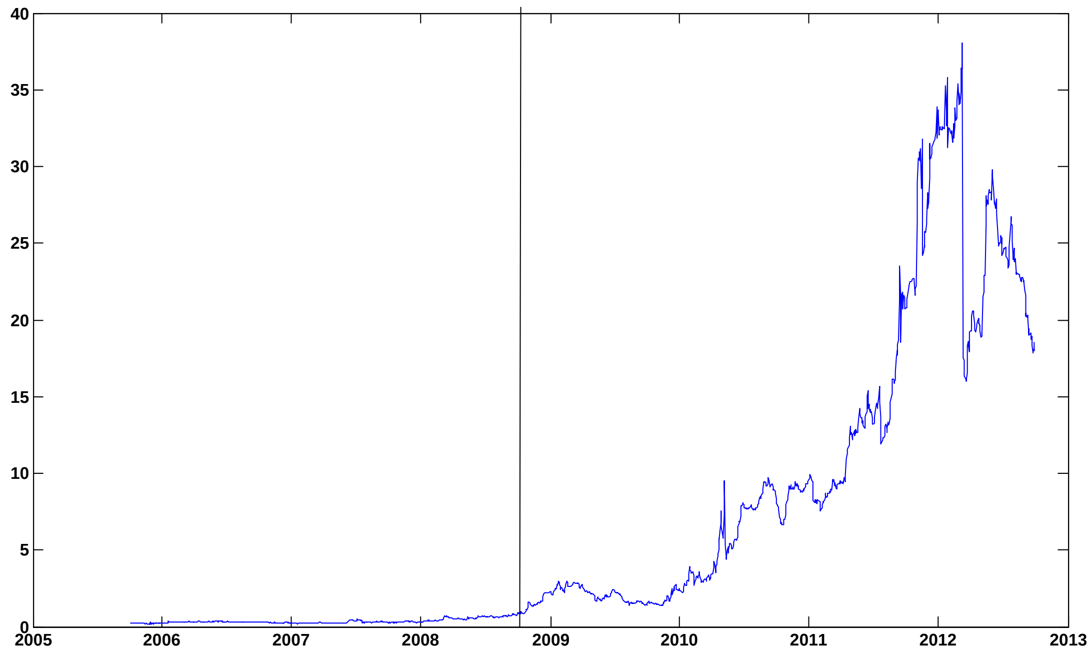

Figure 2: Maximum difference among the ten sovereign debt yields. This figure shows the maximum difference among the ten sovereign debt yields for the purpose of tranquil/turbulent period determination, as indicated by the vertical line.

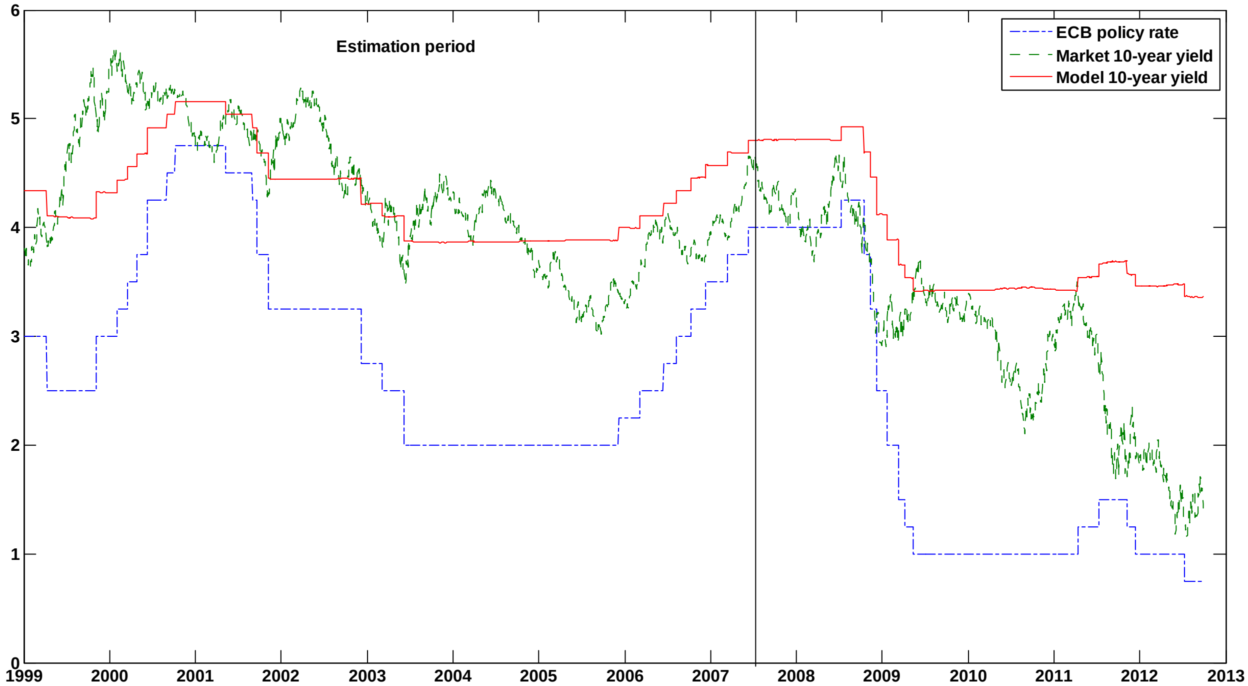

Figure 3: Market versus model-implied German 10-year yields. This figure contrasts market yields with model-implied yields for German 10-year government bonds, where the CIR model is utilized to produce the model-implied yields. The CIR model is first estimated with data from January 1, 1999 to June 30, 2007; the estimation results are used to generate the model-implied yields for the whole period until September 30, 2012.

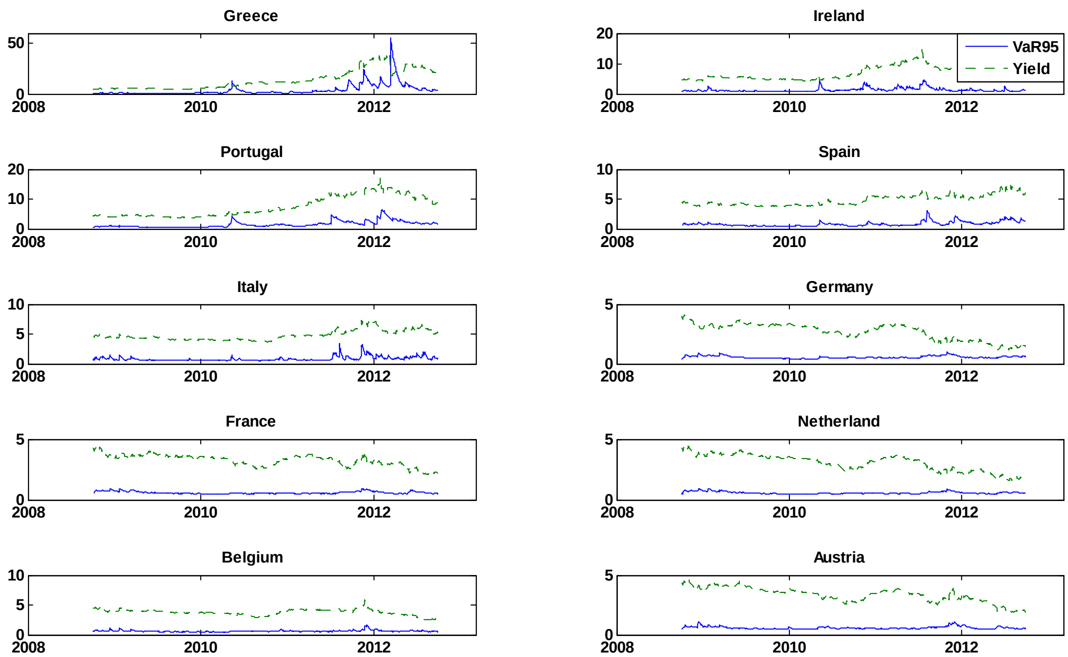

Figure 4: Market 10-year sovereign debt yields versus the VaR. This figure contrasts market 10-year sovereign debt yields (denoted as 'Yield') with the VaR (denoted as 'VaR95') which indicates $\overline{VaR}_{i,t}^{\alpha}$ in the text with a significance level α of 5%.

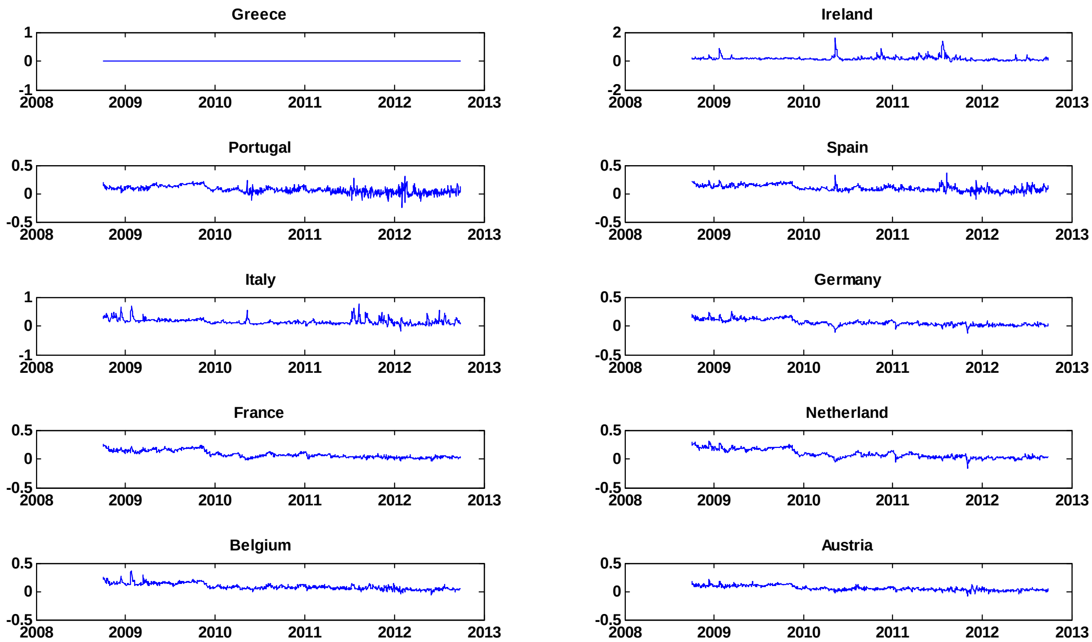

Figure 5-A: Contagion VaR $\Delta \overline{VaR}^{\alpha}_{i|j,t}$ conditional on a rare and negative shock to Greece. This figure shows the contagion VaR $\Delta \overline{VaR}^{\alpha}_{i|j,t}$ in the text when country j is Greece for each country with a significance level α of 5%.

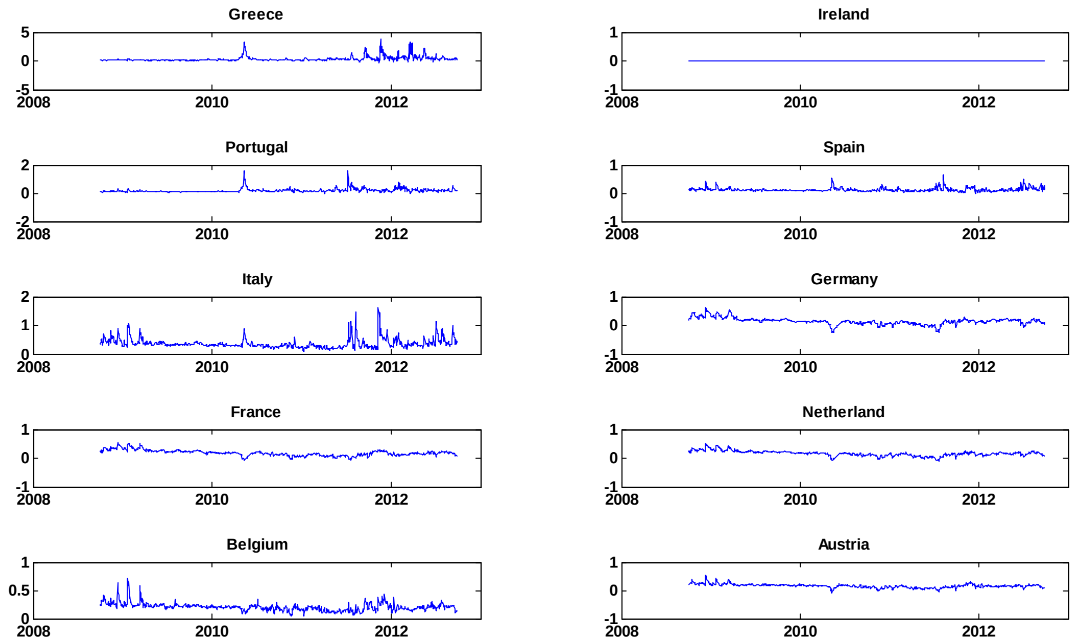

Figure 5-B: Contagion VaR $\Delta\overline{VaR}^{\alpha}_{i|j,t}$ conditional on a rare and negative shock to Ireland. This figure shows the contagion VaR $\Delta\overline{VaR}^{\alpha}_{i|j,t}$ in the text when country j is Greece for each country with a significance level α of 5%.

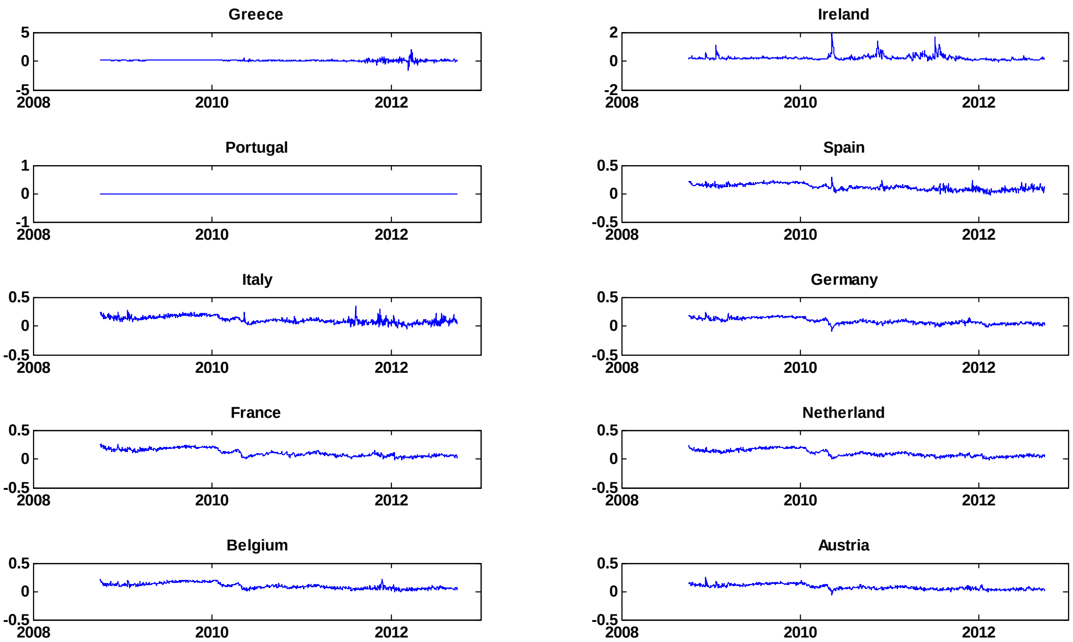

Figure 5-C: Contagion VaR $\Delta\overline{VaR}_{i|j,t}^{\alpha}$ conditional on a rare and negative shock to Portugal. This figure shows the contagion VaR $\Delta\overline{VaR}_{i|j,t}^{\alpha}$ in the text when country j is Greece for each country with a significance level α of 5%.

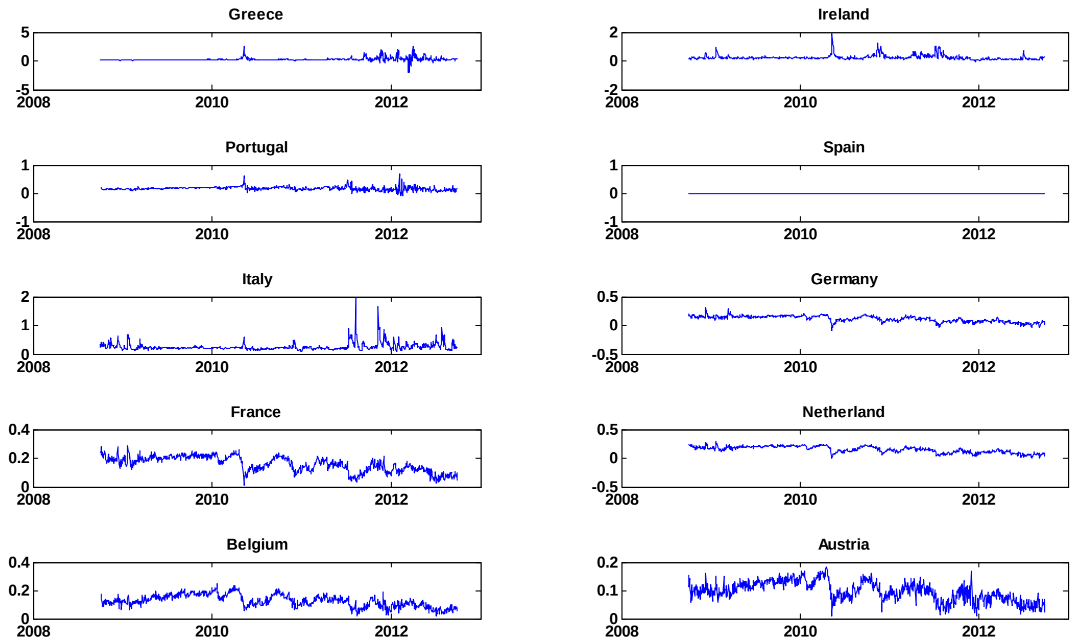

Figure 5-D: Contagion VaR $\Delta\overline{VaR}_{i|j,t}^{\alpha}$ conditional on a rare and negative shock to Spain. This figure shows the contagion VaR $\Delta\overline{VaR}_{i|j,t}^{\alpha}$ in the text when country j is Greece for each country with a significance level α of 5%.

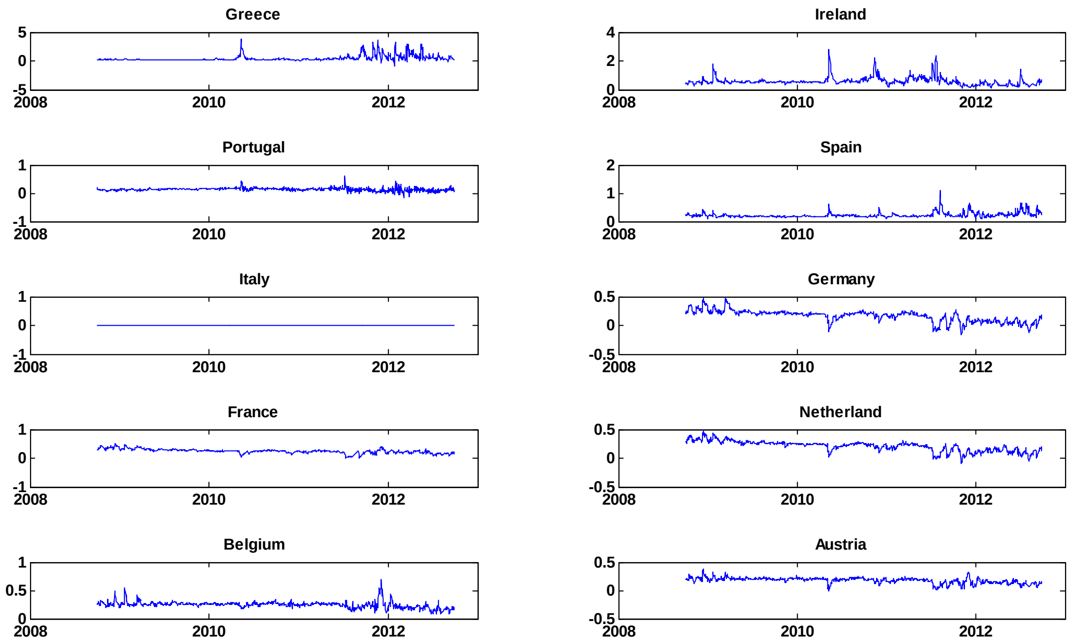

Figure 5-E: Contagion VaR $\Delta \overline{VaR}^{\alpha}_{i|j,t}$ conditional on a rare and negative shock to Italy. This figure shows the contagion VaR $\Delta \overline{VaR}^{\alpha}_{i|j,t}$ in the text when country j is Greece for each country with a significance level α of 5%.