Determinacy in the New Keynesian Model with Market Segmentation*

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Abstract

This paper introduces the asset market segmentation into an otherwise standard New Keynesian model, and shows that the Taylor principle continues to be essential for macroeconomic stability. In particular, central banks should change nominal interest rates more than one-for-one in response to sustained increases in the inflation rate to guarantee a unique equilibrium. This finding reverses the results of previous studies that argued the Taylor principle, under asset market segmentation, no longer provide an important criterion for the stability properties of interest rate rules. (*JEL C62, E12, E21, E31, E43*)

Keywords: New Keynesian, market segmentation, liquidity effect, consumption smoothing, determinacy, Taylor principle.

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1 Introduction

There are two interesting empirical facts in the macroeconomic field. First, prices are sticky and second, asset markets are segmented. These two empirical facts have been separately developed to provide powerful frameworks for a monetary economic model in the New Keynesian model and the market segmentation model, respectively, over two decades. The exogenous market segmentation model has two features that the standard New Keynesian model does not capture.[1] First, traders can smooth their consumption but non-traders cannot smooth their consumption.[2] Second, the market segmentation model can generate the liquidity effect.[3] The market segmentation model, however, is yet to explain the dynamics of inflation and interest rates on the output, which the standard New Keynesian model can illustrate, because the latter assumes an exchange economy.[4] Some economists have tried to incorporate the market segmentation model into the standard New Keynesian model since the early 2000s because the individual disadvantages of the two models can thus be overcome.

Gali, Lopez-Salido and Valles (2004), Nakajima (2006), Gali, Lopez-Salido and Valles (2007), and Lama and Medina (2007) have tried to introduce the market segmentation model into the New Keynesian model to analyze the monetary or fiscal policies imple-

[1] The exogenous market segmentation means that there are two types of households, traders, who can participate in the asset market, and non-traders, who cannot participate in the asset market. Traders and non-traders cannot switch sides. In here, we assume that the market segmentation means the exogenous market segmentation.

[2] According to Mulligan and i Martin (2000), as of 1989, 59 percent of the U.S. population did not hold interest-bearing assets. Mankiw (2000) argues that there are three reasons why two canonical models, the Barro-Ramsey model and the overlapping generations Diamond-Samuelson model, are not appropriate for policy analysis. First, some agent follows the permanent-income hypothesis and the others follow the simple rule-of-thumb of consuming their current income. Second, many households have near-zero net worth. Third, much wealth is concentrated in the hands of a few people and bequests are an important factor in great wealth accumulation.

[3] Edmond and Weill (forthcoming) provide intuition on how the segmented market model can generate the liquidity effect in the short run and the Fisher effect in the long run.

[4] Landon-Lane and Occhino (2008) try to introduce the production economy into the exogenous market segmentation model, but they do not assume sticky prices.

mented at that time.[5] We find an interesting assertion from Gali et al. (2004) and Nakajima (2006). In Gali et al. (2004), Ricardians, or traders, can smooth their consumption but non-Ricardians, or non-traders, cannot smooth their consumption. They, however, cannot generate the liquidity effect because their economy does not have money. Nakajima (2006), however, produces only the liquidity effect. In his model, since there are no capital stocks in the closed economy, both traders and non-traders cannot smooth their consumption over time. This difference between these models produces contrasting results with regard to the Taylor rule. Gali et al. (2004) argue that the presence of non-Ricardians, or non-traders, calls for a more aggressive interest rule for determinacy than the Taylor principle suggests. Nakajima (2006), however, argues that if the liquidity effect exists, the Taylor rule should be passive for determinacy.

From the mid-1980s to early 2000s, the U.S. economy experienced remarkably reduced fluctuations in terms of both output and inflation. Many economists have called this striking feature of the economy "the Great Moderation." Bernanke (2004) addressed that improved performance of the monetary policy might have dampened the fluctuations.[6] Many economists have argued that the change in the monetary policy after Volcker was appointed as the chairman of the Federal Reserve Board had improved the economic performance and had stabilized the variability in both output and inflation.[7] [8] Given the historical lessens learned, we want to avoid fluctuations in some variables such as inflation and output from self-fulfilling expectations. The issue of the stabilization of interesting economic variables can be reduced to the issue of seeking unique solution to the macroeconomic models. Therefore, when the monetary authority implements mon-

[5]Gali et al. (2007) study the fiscal policy under Gali et al. (2004). Lama and Medina (2007) study the monetary policy in an open economy by introducing the two features of the exogenous market segmentation into the New Keynesian model; their approach, however, follows the Ramsey approach and hence they do not study the issue of determinacy.

[6]Bernanke, Ben S., 2004, "The Great Moderation," Speech at the meeting of the Eastern Economic Association, Washington, DC, February 20, 2004.

[7]For details, see Clarida, Gali and Gertler (2000).

[8]Taylor (2010) argues that there is a deviation from the monetary policy of the Great Moderation, 2003-2005. Taylor, John B., 2010, "Macroeconomic Lessons from the Great Deviation," Remarks at the 25th NBER Macro Annual Meeting, May, 2010.

etary policies to stabilize the macroeconomic variables, it should significantly consider determinacy.

Woodford (2001) discusses that the Taylor principle satisfies determinacy and eliminates the self-fulfilling spiral of inflation in the neo-Wicksellian model, and shows that the Taylor rule can be a proper tool to stabilize inflation and output gap. Carlstrom and Fuerst (2005) discuss how the introduction of capitals and investment affects determinacy in a discrete-time model. In the Calvo sticky price model, if the monetary authority uses forward-looking rules, the investment results in a very tight condition for determinacy. In their calibration, the existence of investment may yield determinacy that was originally deemed impossible in a forward-looking rule. In a current-looking Taylor rule, the introduction of investment does not affect the condition for determinacy, as in an aggressive Taylor rule. Dupor (2001), however, shows that the introduction of capital may yield a passive Taylor rule in the continuous-time model. Clarida et al. (2000) argue that the Taylor rule satisfied the Taylor principle in the Great Moderation period, and Lubik and Schorfheide (2004) show that the Taylor rule satisfying the Taylor principle increases determinacy in the Great Moderation period. Davig and Leeper (2007) argue that the macroeconomic fluctuations can be more frequent when the Taylor principle is not satisfied since monetary policy results in more the impact of fundamental shocks.

This paper starts by asking the following question: is the Taylor principle still a good criterion for uniqueness in the New Keynesian model with the market segmentation having the liquidity effect and the difference in consumption smoothing between traders and non-traders? This paper's key results are as follows. First, the Taylor rule does not need to be too aggressive or passive. The Taylor rule can satisfy the Taylor principle in the New Keynesian model with market segmentation. Second, the threshold of the coefficient of inflation in the Taylor rule that is the minimum for satisfying determinacy is constant regardless of the proportion of traders. These mean that the contrasting influences of the liquidity effect and the difference in consumption smoothing may be offset. Third, the behaviors of non-traders in response to exogenous shocks are very different from those of traders. This can result in impulse response of output and consumption to exogenous shocks being hump shaped.

This paper is organized as follows. We set the New Keynesian model with the market segmentation in section 2. We analyze the equilibrium and the linearization of the economy in section 3. We discuss the main results of this paper in section 4. We conclude in section 5.

2 Model

We combine two models, the New Keynesian model in the supply side and the market segmentation model in the demand side. In terms of demand, households are assumed to be divided into two types, traders, who can participate into the financial market and can accumulate the capital, and non-traders, who cannot participate into the financial market and cannot accumulate the capital. Therefore, the asset market is exogenously segmented as in Alvarez, Lucas and Weber (2001).[9] For consumption, both traders and non-traders are restricted by the cash-in-advance (CIA) constraint. The market segmentation can yield the liquidity effect. In this economy, only traders can accumulate the capital and this distinguishes traders from non-traders in that only the traders can smooth their consumption over time. Money is supplied through the open market operation, and hence the money injection is absorbed only by traders. Traders and non-traders do not choose their labor supply. In terms of supply, we assumed that there are two types of firms, the intermediate goods firms and final goods firms. The intermediate goods firm produces differentiated intermediate goods in a monopolistic competition market and follows the Calvo (1983) pricing setting. The final goods firms produce one final good in a perfect competition market.

2.1 Households

2.1.1 Traders

We assume that there are two types households, traders and non-traders, who live infinitely, as mentioned above. A fraction of households λ can accumulate their own

[9] We adopt the exogenous market segmentation due to simplicity, for comparison with previous papers, and because as stated in the second footnote, Mulligan and i Martin (2000) and Mankiw (2000) support the existence of non-traders.

physical capital and can participate in the asset market to trade bonds issued by the government, and hence can smooth their consumption over time. They receive profits from the intermediate goods firms and a lump-sum transfer from the government, and provide their labor to intermediate goods firms. We refer to such households as traders. Since only traders can participate in the asset markets, they absorb all of the money injected by the monetary authority. Let a superscript "T" denote the traders' variables. A representative trader maximizes the lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t^T)^{1-\sigma}}{1-\sigma} - \frac{(N_t^T)^{1+\varphi}}{1+\varphi} \right),$$

subject to the CIA constraint[10] [11]

$$P_t C_t^T + R_t^{-1} B_{t+1} = M_{t-1}^T + B_t + T_t, \tag{1}$$

where B_t denotes the risk-free one-period bonds issued by the government that are carried from period $t-1$ to period t, P_t is the price level in period t, C_t^T is the traders' consumption in period t, R_t is the nominal return on the bonds in period t, M_{t-1}^T is the nominal money balance of traders carried from period $t-1$ to period t, and T_t is the lump-sum transfer from the government in period t; and subjected to the budget constraint

$$M_t^T + P_t I_t^T = W_t N_t^T + R_t^k K_t^T + \frac{\Pi_t}{\lambda}, \tag{2}$$

where I_t^T is the investment in period t, K_t^T is the capital stock in period t, N_t^T is the work hours supplied by the traders, W_t is the nominal wage, R_t^k is the nominal rental, and Π_t is the return from the immediate goods firms.[12] The traders' capital accumulation equation is

$$K_{t+1}^T = (1-\delta) K_t^T + \phi \left(\frac{I_t^T}{K_t^T} \right) K_t^T. \tag{3}$$

[10]We assume that the equality in the CIA condition always holds.

[11]In general, in the market segmentation model, the velocity shock is introduced to capture the relation between money and prices. For simplicity, we assume that there is no velocity shock.

[12]In this model, the investment is not restricted by the CIA constraint. Wang and Wen (2006) argue that the output's response to monetary shocks becomes more persistent with the degree of capital constraint by the CIA condition. Instead, for output persistence, we introduce adjustment costs in this model. For more details on the relation between the capital restricted by the CIA condition and the output persistence, see Wang and Wen (2006).

$$\phi^{'} > 0, \quad \phi^{''} \leq 0, \quad \phi^{'}(\delta) = 1, \quad \phi(\delta) = \delta,$$

where $\phi(I_t^T/K_t^T)K_t^T$ denotes the capital adjustment cost and δ is the rate of depreciation. The first-order conditions for the traders' optimization problem are as following, Euler equation,[13]

$$\frac{1}{R_t} = \beta E_t \left\{ \left(\frac{C_{t+1}^T}{C_t^T} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right\}, \tag{4}$$

and Tobin q, the (real) shadow value of capital, given as

$$\begin{aligned} Q_t &= E_t \left\{ \frac{1}{R_{t+1}} \left(\frac{R_{t+1}^k}{P_{t+1}} + Q_{t+1} \left((1-\delta) + \phi_{t+1} - \left(\frac{I_{t+1}^T}{K_{t+1}^T} \right) \phi_{t+1}^{'} \right) \right) \frac{P_{t+1}}{P_t} \right\} \\ Q_t &= \frac{1}{\phi_t^{'}}. \end{aligned} \tag{5}$$

The elasticity of the investment-capital ratio with respect to Q is assumed as

$$-\frac{1}{\phi''(\delta)\delta} \equiv \eta.$$

We do not derive the equation of labor supply by traders because we will assume that labor hours are assumed to be determined by firms.[14]

2.1.2 Non-traders

The remaining fraction of households $1-\lambda$ can consume only with their wage and cannot participate in the asset market and cannot accumulate capital. Therefore, they cannot smooth their consumption over time and their existence violates the persistent income hypothesis. We will call them non-traders. A superscript "N" denotes the non-traders' variables. A representative non-trader maximizes the lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t^N)^{1-\sigma}}{1-\sigma} - \frac{(N_t^N)^{1+\varphi}}{1+\varphi} \right),$$

subject to the CIA constraint

$$P_t C_t^N = M_{t-1}^N,$$

[13]In this economy, only traders can obtain the Euler equation. Therefore, the real interest rate will be related only to traders' consumption.

[14]This assumption is from Gali et al. (2007) and is made for simplicity. Under this assumption, the real wage is determined to equal the marginal rate of substitution at all times, and it is optimal for households to supply labor as much as firms' demand.

where C_t^N is the non-traders' consumption in period t and M_{t-1}^n is the non-traders' money balance carried from period $t-1$ to period t; and subject to the budget constraint

$$M_t^N = W_t N_t^N,$$

where W_t is the nominal wage and N_t^N is the labor supply from non-traders. From the CIA constraint, the consumption of non-traders is

$$C_t^N = \frac{W_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} N_{t-1}^N. \tag{6}$$

This equation will provide us with a logic to explain the non-traders' behavior in response to exogenous shocks. Further, non-traders' labor will be determined by the firms' labor demand.

2.1.3 Wage Schedule

Let us assume that there exists a schedule for wage determination

$$E_t \frac{W_t}{P_{t+1}} = H(C_{t+1}, N_t), \tag{7}$$

where $H_C > 0$ and $H_N > 0$. Under this schedule, each firm will hire labor equally across households regardless of the household's type. Therefore, $N_t = N_t^T = N_t^N$ for all t. For consistency with balanced growth, H can be expressed as $E_t C_{t+1}^{\sigma} N_t^{\varphi}$.[15]

2.2 Firms

2.2.1 Final Goods Firms

We assume that final good firms produce one final good by using a continuum of intermediate goods as inputs in the perfect competitive markets. The final goods' price is flexible. The final good production function is a constant elasticity of substitution (CES) bundler, and is given as

$$Y_t = [\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di]^{\frac{\epsilon}{\epsilon-1}},$$

where $\epsilon > 1$ is the elasticity of substitution in production and $Y_t(i)$ is the amount of intermediate good i required as input.

[15] For more details, see Gali et al. (2007).

Given P_t and $P_t(i)$, we can obtain the demand for a good i from the zero-profit problem for the final goods firms, as follows;

$$Y_t(i) = Y_t \left(\frac{P_t}{P_t(i)} \right)^{\epsilon}.$$

By incorporating the demand for a good i into the function of Y_t, we can obtain the final good pricing rule as

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}}.$$

2.2.2 Intermediate Goods Firms

We assume that there is a continuum of intermediate goods firms indexed by $i \in [0, 1]$ producing differentiated goods and that these differentiated goods are used as inputs in the production of the final good. A firm i's production function follows the Cobb-Douglas production function as follows:

$$Y_t(i) = A_t K_t(i)^{\alpha} N_t(i)^{1-\alpha}. \tag{8}$$

Every intermediate goods firm wants to minimize its total cost in every period, and hence,

$$\min_{K_t(i), N_t(i)} \quad R_t^k K_t(i) + W_{t+j} N_t(i)$$

subject to $Y_t(i) = A_t K_t(i)^{\alpha} N_t(i)^{1-\alpha}$. The first-order condition for this problem is

$$\frac{(1-\alpha) R_t^k}{\alpha W_t} = \frac{N_t(i)}{K_t(i)}. \tag{9}$$

We can obtain the demand functions for the labor and capital of a firm i by incorporating the above equation into the production function as

$$N_t(i) = \frac{Y_t(i)}{A_t} \left(\frac{R_t^k (1-\alpha)}{W_t \alpha} \right)^{\alpha}, \tag{10}$$

$$K_t(i) = \frac{Y_t(i)}{A_t} \left(\frac{R_t^k (1-\alpha)}{W_t \alpha} \right)^{\alpha-1}. \tag{11}$$

Further, we can write the real marginal cost for this firm by incorporating these equations into the total cost function as

$$RMC_t = \frac{W_t/P_t}{(1-\alpha) A_t} \left(\frac{\left(R_t^k / P_t \right) (1-\alpha)}{(W_t/P_t)\, \alpha} \right)^{\alpha}. \tag{12}$$

In monopolistic competitive markets, the intermediate goods firms cannot change their price every period as in the Calvo (1983). A fraction of all intermediate goods firms θ, price stickiness, cannot adjust their price while the remaining fraction $1-\theta$ can optimally choose their price in every period. The firms that can adjust their price in period t are selected randomly. The optimization problem for the intermediate firms in period t is the choice of the optimal price P_t^* to maximize

$$\max_{P_t^*(i)} \quad E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} Y_{t+j} \left(\frac{P_{t+j}}{P_t^*(i)}\right)^{\epsilon} \left[P_t^*(j) - \frac{W_{t+j}}{(1-\alpha)A_{t+j}} \left(\frac{R_{t+j}^k(1-\alpha)}{W_{t+j}\alpha}\right)^{\alpha}\right],$$

subject to $Y_{t+j}(i) = A_{t+j}K_{t+j}(i)^{\alpha}N_{t+j}(i)^{1-\alpha}$.

We can obtain the optimal price P_t^* by solving this problem as

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j Y_{t+j} \frac{W_{t+j}}{(1-\alpha)A_{t+j}} \left(\frac{R_{t+j}^k(1-\alpha)}{W_{t+j}\alpha}\right)^{\alpha}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j Y_{t+j}},$$

where $\frac{\epsilon}{\epsilon-1}$ is the mark-up. We can express the price level by combining the final goods pricing rule and the Calvo pricing rule as

$$P_t = \left[\theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}.$$

2.3 Government

The government can inject money into the economy through the open-market operation, and hence, the government's budget constraint is

$$M_t + E_t R_t^{-1} \frac{B_{t+1}}{\lambda} = M_{t-1} + \frac{B_t}{\lambda} + \frac{T_t}{\lambda}. \tag{13}$$

From this budget equation, we can see that the injected money is absorbed only by traders through the trading of bonds in the asset market. The growth rate of money is

$$\log(M_t/M_{t-1}) = \mu_t.$$

The government can implement monetary policies. We assume that there are two alternative monetary policies, the simple Taylor rule and the controlling of the money growth rate, given as

$$r_t = r + \phi_\pi \pi_t + \phi_y \hat{y}_t + \upsilon_t, \tag{14}$$

$$\mu_t = \rho_\mu \mu_{t-1} + \varepsilon_t^{\mu}, \tag{15}$$

where $r_t(\equiv R_t - 1)$ is the nominal interest rate, r is the nominal interest rate in the steady state, $\hat{y}_t$ is the deviation from their steady state, and π_t is the inflation rate. We assume the *zero* steady state inflation rate.

3 Equilibrium

3.1 Clearing condition

The clearing condition for consumption is

$$C_t = \lambda C_t^T + (1-\lambda) C_t^N. \tag{16}$$

The clearing condition for the capital market is

$$\int_0^1 K_t(i)di = K_t = \lambda K_t^T, \quad I_t = \lambda I_t^T. \tag{17}$$

The clearing condition for the labor market is

$$\int_0^1 N_t(i)di = N_t = \lambda N_t^T + (1-\lambda) N_t^N = N_t^T = N_t^N. \tag{18}$$

The clearing condition for the money market is

$$M_t = \lambda M_t^T + (1-\lambda) M_t^N = P_t C_t, \tag{19}$$

The above implies that the quantity theory holds for this economy.

The clearing condition for the goods market is

$$Y_t = C_t + I_t \tag{20}$$

The profit is as follows:

$$\begin{aligned} \Pi_t = P_t \int_0^1 \Pi_t(i)di &= \int_0^1 P_t(i)Y_t(i)di - \frac{W_{t+j}}{(1-\alpha)A_{t+j}} \left(\frac{R_{t+j}^k (1-\alpha)}{W_{t+j}\alpha} \right)^\alpha \int_0^1 Y_t(i)di \\ &= P_t Y_t - \frac{W_{t+j}}{(1-\alpha)A_{t+j}} \left(\frac{R_{t+j}^k (1-\alpha)}{W_{t+j}\alpha} \right)^\alpha \int_0^1 Y_t(i)di. \end{aligned} \tag{21}$$

3.2 Linearization

First, we have to derive the steady state values of the variables. The appendix in the end of this paper will present the process for the steady state values of the variables. The small characters with a hat denote the variables' deviation from the steady state. After linearizing, we can derive a difference system to analyze the dynamics and determinacy of this economy.

From equation (3), we can get the following linearized form for the capital accumulation:

$$\hat{k}_{t+1} = (1-\delta)\hat{k}_t + \delta\hat{i}_t. \tag{22}$$

Equation (4) provides us with the Euler equation:

$$\hat{c}_t^T = E_t\hat{c}_{t+1}^T + \frac{1}{\sigma}\left(E_t\pi_{t+1} - \hat{r}_t\right). \tag{23}$$

In this economy, we have to note that the Euler equation holds only for the traders, and the IS curve in the New Keynesian model with the market segmentation will be different from that in the standard New Keynesian model. We can derive a linearized form for given Tobin q:

$$\hat{q}_t = \beta E_t\hat{q}_{t+1} + \chi_q E_t\hat{r}_{t+1}^k - E_t(\hat{r}_{t+1} - \pi_{t+1}), \tag{24}$$

where $\chi_q \equiv 1-\beta(1-\delta)$. Note that $\hat{q}_t$ depends on $\hat{r}_{t+1}$ and not $\hat{r}_t$. We can also obtain an alternate form for Tobin q from its definition:

$$\hat{q}_t = \frac{1}{\eta}\hat{i}_t - \frac{1}{\eta}\hat{k}_t.$$

From equation (6), the non-traders' current consumption is

$$\hat{c}_t^N = \hat{w}_{t-1} - \pi_t + \hat{n}_{t-1}. \tag{25}$$

We can obtain the labor demand for intermediate goods firms from equation (7) as[16]

$$\varphi\hat{n}_t + \sigma E_t\hat{c}_{t+1} = \hat{w}_t - E_t\pi_{t+1}. \tag{26}$$

[16] Actually, in this equation, the elasticity of labor supply should be replaced with the elasticity of wages with respect to hours. In the case of perfect competition in labor markets, however, the elasticity of labor supply is equal to the elasticity of wages with respect to hours because real wages should be the marginal rate of substitution.

A linearized form for the aggregate production function is

$$\hat{y}_t = \hat{a}_t + \alpha\hat{k}_t + (1-\alpha)\hat{n}_t. \tag{27}$$

We can derive a linearized form for the first-order condition for the intermediate goods firms from equation (9) as

$$\hat{r}_t^k - \hat{w}_t = \hat{n}_t - \hat{k}_t. \tag{28}$$

From equation (10) and (11), the demands for labor and capital are as follows:

$$\hat{n}_t = \hat{y}_t - \hat{a}_t + \alpha(\hat{r}_t^k - \hat{w}_t), \tag{29}$$

$$\hat{k}_t = \hat{y}_t - \hat{a}_t + (\alpha - 1)(\hat{r}_t^k - \hat{w}_t). \tag{30}$$

We note that since equation (29) and (30) can be derived from equation (27) and (28), we do not use these equations to derive the difference equation system.

A linearized form for the real marginal cost function can be derived from equation (12) as

$$\begin{aligned} \hat{mc}_t &= (1-\alpha)\hat{w}_t + \alpha\hat{r}_t^k - \hat{a}_t \\ &= -\hat{y}_t + \hat{n}_t + \hat{w}_t = -\hat{y}_t + \hat{k}_t + \hat{r}_t^k. \end{aligned} \tag{31}$$

The second equality is derived from equation (28) and (29). The Taylor rule is

$$\hat{r}_t = \phi_\pi \pi_t + \phi_y \hat{y}_t + \upsilon_t. \tag{32}$$

From the definition of μ_t, μ_t can be expressed as follows:

$$\mu_t = \hat{m}_t + \pi_t - \hat{m}_{t-1} = \hat{c}_t + \pi_t - \hat{c}_{t-1}. \tag{33}$$

The current consumption can be written as

$$\hat{c}_t = \lambda\tau_c\hat{c}_t^T + (1-\lambda\tau_c)\hat{c}_t^N, \tag{34}$$

where $\tau_c \equiv \frac{\bar{C}^T}{\bar{C}}$.

A linearized form for the aggregate capital stock is

$$\hat{k}_t = \hat{k}_t^T. \tag{35}$$

The aggregate output has two components, consumption and investment, given as

$$\hat{y}_t = \tau_y \hat{c}_t + (1 - \tau_y)\hat{i}_t, \tag{36}$$

where $\tau_y \equiv \frac{\bar{C}}{\bar{Y}}$.

The optimal price, given by equation (13), provides us with an equation to describe the dynamics of inflation:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{mc}_t, \tag{37}$$

where $\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$. The exogenous shocks are given as follows:

$$\begin{aligned} \hat{a}_t &= \rho_a \hat{a}_{t-1} + \varepsilon_t^a \\ \upsilon_t &= \rho_\upsilon \upsilon_{t-1} + \varepsilon_t^\upsilon \\ \mu_t &= \rho_\mu \mu_{t-1} + \varepsilon_t^\mu \end{aligned}$$

4 Dynamics

To study the determinacy and dynamics of this economy, we have to derive a difference equation system by combining equations (22)-(37). After a tedious arithmetic operation, we derived the following difference equation system for this economy:

$$\mathbf{A} E_t \mathbf{x}_{t+1} = \mathbf{B} \mathbf{x}_t, \tag{38}$$

where $\mathbf{x}_t \equiv [\hat{k}_t, \hat{n}_t, \hat{c}_t, \pi_t, \hat{h}_t]'$ and all elements in matrices $\mathbf{A}$ and $\mathbf{B}$ are functions of the structural parameters of this model.[17] To obtain a unique solution for the difference system, since there are two predetermined variables, $\hat{k}_t$ and $\hat{h}_t$, three eigenvalues of $[A^{-1}B]$ should lie outside and two eigenvalues of $[A^{-1}B]$ should lie inside the unit circle.[18] The appendix contains details about derivation of this system.

4.1 Calibration

For the baseline calibration, we will use the basic values from Gali et al. (2004). In this model, we assume that the time preference β is 0.99. This means that the real interest rate in the asset market in the steady state is about 4% annually. The elasticity

[17] We drop the terms relating to exogenous shocks since we only analyze determinacy.

[18] For more details, see Blanchard and Kahn (1980).

of substitution in production ϵ is assumed to be 6, which is the value consistent with 0.2 of the mark-up in the steady state. The elasticity of output with respect to capital α is assumed to be 1/3. The rate of depreciation is assumed to be 0.025, implying an annual rate of 10%. The parameter prescribing the stickiness of prices θ is set to 0.75, implying that the average duration is $(1-0.75)^{-1}$ quarters or one year. The elasticity of investment-capital ratio with respect to Q, given as η, is set as 1. The proportion of traders in the population λ is assumed to be 0.5 as in Campbell and Mankiw (1990). In the literature, 1 is set as the elasticity of substitution with regard to consumption, $1/\sigma$, and the elasticity of labor supply in general. If φ is set as 1, the elasticity of labor supply is the unit Frisch elasticity of labor supply. In the Taylor rule, we will assume that $\phi_\pi = 1.5$ and $\phi_y = 0.5$. ρ_a, ρ_v, and ρ_μ are set as 0.5.

4.2 Determinacy

The literature on the market segmentation has argued that the liquidity effect, the short run negative relationship between the money supply and the nominal interest rate, may arise because the injection of money is absorbed exclusively by only some agents in this economy.[19] If there exists only money, but not capital, in the New Keynesian model with market segmentation, the economy can generate the liquidity effect and in this economy the current non-traders' consumption will be affected by the last period labor income, $n_{t-1} + w_{t-1}$, and the current inflation, π_t. Suppose that a sunspot boosting current consumption happens, then the monetary authority will increase the interest rate to eliminate the sunspot shock. In the normal economy the authority's action will decrease the total consumption, but the action will force for non-traders to increase their consumption because of decrease in the current inflation in this economy. If the fraction of traders is sufficiently high, this economy will react similarly with the standard New

[19] The money supply has two effects. First, the expected inflation effect. This means that the positive money supply will increase the expected inflation. Second, the segmentation effect. This means that the positive money supply will decrease the real interest rate because the increase in the real balance of traders will simulate the consumption of traders. If the segmentation effect dominates the expected inflation effect, the liquidity effect can be happened. And as the fraction of traders decreases, the liquidity effect will increase.

Keynesian model to the policy. If, however, the fraction of non-traders is sufficiently high, the policy to increase the interest rate will amplify the effect of the sunspot shock since the increase in non-traders' consumption may exceed the decrease in traders' consumption. Therefore, if the fraction of non-traders is sufficiently high, the monetary authority may decrease the interest rate to eliminate the effect of the sunspot shock. So, the passive Taylor rule can guarantee the determinacy if there is only money in the New Keynesian model with market segmentation. [20]

Gali et al. (2004) discuss that the existence of non-traders, referred to as non-Ricardian or the rule-of-thumb consumers in their paper, in the standard New Keynesian model with capital may affect the Taylor rule for local determinacy. They insist that when the fraction of traders λ is sufficiently low, the Taylor rule becomes even more aggressive for the unique equilibrium. Their intuition is as follows. If there are no non-traders in the economy and sunspot shocks increase in labor hours, then the consumption of households increases. By increasing the interest rate, the initial consumption boom can be eliminated. If, however, the fraction of non-traders is sufficiently high and so is the increase in labor hours by sunspot shocks, the consumption of traders and non-traders will increase. The increase in the interest rate will reduce only the consumption of traders because non-traders' consumption is affected by labor income. If the increase in the consumption of non-traders is greater than the decrease in the consumption of traders, the initial consumption boom can be sustained. To eliminate the consumption boom, the monetary authority should increase the interest rate more than usual. Therefore, the Taylor rule becomes more aggressive.

There exists an interesting phenomenon. Both Nakajima (2006) and Gali et al. (2004)

[20] Nakajima (2006) argues that under the liquidity effect, a passive interest rate rule implies the unique equilibrium. His intuition is as follows: let $\hat{r}_t = \phi_\pi \pi_t$ be the Taylor rule and let the output be fixed. If the real interest rate is constant, the Euler equation, $\hat{r}_t = E_t \pi_{t+1}$, can be reduced to the Fisher effect, and then $E_t \pi_{t+1} = \phi_\pi \pi_t$. In this economy, as π_t is a non-predetermined variable, it requires $\pi_t > 1$ for local determinacy. This implies that $|\pi_{t+1}| > |\pi_t|$ is required for determinacy. When, however, the liquidity effect exists, the Fisher effect does not hold. Therefore, the active Taylor rule no longer guarantees $|\pi_{t+1}| > |\pi_t|$. The low interest rate can support the high expected inflation under the liquidity effect. Hence, the Taylor rule may become passive to guarantee $|\pi_{t+1}| > |\pi_t|$.

assume the New Keynesian model with the exogenous market segmentation. While Nakajima (2006) assumes the market segmentation with cash and without capital, Gali et al. (2004) study the market segmentation with capital and without cash. This implies that while Nakajima (2006) can obtain the liquidity effect but not the differences in consumption smoothing between traders and non-traders, Gali et al. (2004) can obtain the phenomenon that traders can smooth their consumption and non-traders cannot, but cannot obtain the liquidity effect. This difference results in the contrasting findings of these two papers. As non-traders increase, Nakajima (2006) argues that under the liquidity effect, the passive Taylor rule can generate determinacy, and while Gali et al. (2004) point out that the the presence of non-traders may require the Taylor rule to be more aggressive for determinacy. The exogenous market segmentation model, however, has both features, the liquidity effect and the difference in the consumption smoothing. As such, we can raise a question about determinacy: should the Taylor rule be too aggressive or too passive in the New Keynesian model with market segmentation? This paper will show that the Taylor rule can satisfy the Taylor principle.

Figure 1 shows the ranges for the parameters (ϕ_y, ϕ_π) pertaining to determinacy under the current Taylor rule with the values of the structural parameters assumed above. The range for determinacy from this paper shows that a too aggressive or passive Taylor rule may not be required in our model. However, this figure does not guarantee that the Taylor rule satisfies the Taylor principle at this stage. From Figure 1, we can expect that the contrasting influences on determinacy from the liquidity effect and the difference in consumption smoothing may be offset under our baseline calibration. This figure is similar to the one of the standard New Keynesian model. An interesting observation from Figure 1 is that as the coefficient of output in the Taylor rule increases, the coefficient of inflation rate in the Taylor rule increases. This is very different from the standard New Keynesian model in which ϕ_π and ϕ_y have a negative relationship because of the standard Phillips curve.[21] In this model, the real marginal cost is affected by the expected inflation

[21] In the standard New Keynesian model, the slope should be negative for the Taylor principle to be satisfied from the long-run positive relation between the inflation and the output. For more details, see Gali (2008).

because of the cash-in-advance constraint. In the long run, the permanent increase in inflation will increase the real marginal cost, this will increase prices then decrease the aggregate demand. Therefore, the inflation can be related negatively to the output in the long-run.

If there is an permanent increase in the inflation rate of size $d\pi$ from the steady state, then we can rewrite the Taylor rule as (the Appendix contains the details)

$$d\hat{r} = \phi_\pi d\pi + \phi_y d\hat{y} = \frac{c_1\phi_\pi - c_2\phi_y}{c_1 - \phi_y} d\pi.$$

If the Taylor principle is satisfied, the coefficient of $d\pi$, $\frac{c_1\phi_\pi - c_2\phi_y}{c_1-\phi_y}$, will be greater than 1 and can be rearranged as

$$\phi_\pi > 1 + \frac{c_2 - 1}{c_1}\phi_y.$$

The value of $\frac{c_2-1}{c_1}$ is about 0.37 from the baseline calibration, and this value exactly coincides with the slope of the relation between ϕ_π and ϕ_y in Figure 1. The ranges satisfying $\frac{c_1\phi_\pi - c_2\phi_y}{c_1-\phi_y} > 1$ are consistent with Figure 1. Therefore, we can say that the Taylor rule satisfies the Taylor principle in the New Keynesian model with market segmentation.

Figure 2 shows the ranges of the parameters (λ, ϕ_π) for determinacy under baseline calibration. As you can see in the Appendix, the fraction of traders λ does not affect the tangent of the relationship between ϕ_π and ϕ_y in Figure 1. This may mean that the threshold of the coefficient of inflation in the Taylor rule cannot be influenced by the proportion of traders. Figure 2 confirms this guess. The intuition behind the result is as follows. If λ, the proportion of traders, is sufficiently high, the liquidity effect in this economy will weaken or will be dominated by the Fisher effect, and this economy will become close to the New Keynesian model with capital accumulation and without the market segmentation. As papers on the determinacy in the New Keynesian model point out, the existence of capital stock will not have an effect on the determinacy condition of the current Taylor rule under discrete-time conditions. Therefore, an economy with a sufficiently large λ can have a constant value of the threshold of ϕ_π in response to the change in the proportion of traders λ. This finding is consistent with the one in Gali et al. (2004). If, however, the proportion of traders is low enough to make the liquidity effect strong and the consumption smoothing effect weak, an economy may experience conflicting influences from the liquidity effect that require the Taylor rule to be passive for determinacy and the weaken consumption smoothing effect that requires the Taylor rule to be more aggressive for determinacy. As λ is decreasing, the liquidity effect may be further strengthened to allow the Taylor rule to be passive and the consumption smoothing effect become more weak to allow the Taylor rule to be more aggressive. As a result, the effect of the increased liquidity may almost be dramatically offset by the effect of the decreased consumption smoothing. Therefore, the Taylor rule may not have to be too passive or aggressive to achieve a unique solution to the system from equation (38). The result is considerably different from that in Nakajima (2006) where the Taylor rule can be passive for determinacy and from that in Gali et al. (2004) where the Taylor rule can be too aggressive when the proportion of non-traders is increasing. The exogenous market segmentation model has both features, the liquidity effect and the difference in consumption smoothing. Hence, we can expect that the New Keynesian model with market segmentation including these features may yield the Taylor principle where the interest rate should respond more one for one to the movement of inflation rates. Figure 2 visually shows this intuition.

Figure 3 shows the ranges of the parameters (λ, ϕ_π) for determinacy when σ, the reciprocal of the intertemporal elasticity of substitution, changes from 2 to 4 and 5. When σ=2, the top panel in Figure 3, there is no difference from Figure 2. When, however, σ=4, the middle panel in Figure 3, the ranges for indeterminacy increase a little when the proportion of traders is too low. Moreover, you can see that when $\sigma = 5$, the bottom panel in Figure 3, the range for indeterminacy increases considerably for a sufficiently low proportion of traders, λ. From this, we can know that as the intertemporal elasticity of substitution of consumption decreases, indeterminacy increases.[22] The intuition behind this is as follows. If current consumption is increased by sunspot shocks, then this will increase the expected inflation. If the intertemporal elasticity of substitution of consumption is sufficiently low, we cannot expect that the next consumption is sharply

[22] Akay (2010) argues that the CIA model is not frequently used in the New Keynesian model because of this finding. Gali et al. (2004), however, show this finding without the CIA constraint. Therefore, we think that this finding cannot be the reason why the CIA model is not used in the Keynesian model.

reduced by the increased expected inflation. Therefore, the initial consumption boom can be sustained. If there exists capital, since capital plays the role of a consumption buffer, the initial consumption boom by sunspot shocks cannot be sustained. From this, we get that the capital can be a factor to eliminate indeterminacy from the low intertemporal elasticity of the substitution of consumption.

In Figure 4, the top panel on the left side shows the ranges of the parameters (φ, ϕ_π) for determinacy under the baseline calibration. As you can see in the Appendix, as the elasticity of labor supply φ increases, the tangent of the relationship between ϕ_π and ϕ_y in Figure 1 will decrease, and hence, the threshold of ϕ_π will also decrease. The first panel of Figure 4 shows this. The elasticity of labor supply, however, does not change the sign of the tangent. The top panel on the right side of Figure 4 shows the ranges of parameters (θ, ϕ_π) for determinacy under the baseline calibration. As you can see in the Appendix, as the price stickiness θ increases, the tangent of the relationship between ϕ_π and ϕ_y in Figure 1 remains constant around 1.25 and falls sharply after θ reaches 0.75. When θ is around 0.86 and 0.87, the sign of the slope changes from positive to negative. Then, the relation between ϕ_π and ϕ_y becomes negative. The bottom panel on the left side of Figure 4 shows the ranges of the parameters (α, ϕ_π) for determinacy under the baseline calibration. The threshold of ϕ_π will decrease slowly around α=0.7 because the change in the elasticity of output with respect to capital causes a moderate reduction in the tangent of the relationship between ϕ_π and ϕ_y in Figure 1. However, after around α=0.7, the tangent rapidly falls and the sign of the tangent changes at around α=0.77 or 0.78. The bottom panel on the right side of Figure 4 shows the ranges of parameters (β, ϕ_π) the determinacy under the baseline calibration. If the time preference is less than about 0.5, the threshold of ϕ_π is 0. Following this, as the time preference β increases, the threshold of ϕ_π will rise because the tangent increases. At about β=0.95 or 0.96, the sign of the tangent changes from negative to positive. From Figure 4, we can find that parameters θ, α, and β can change the sign of the slope. The values of θ, α, and β, however, that affect the sign of the tangent are unreasonable. Therefore, under reasonable these parameters space, we can deduce that the relationship between ϕ_π and ϕ_y will be positive and that this paper's results remain unchanged. From the above figures, we can see that the range for determinacy in this economy, the New Keynesian model with market segmentation including the two features, in response to the changes in the values of structural parameters is more stable than that from Gali et al. (2004).

4.3 Impulse response

In this section, we will show the impulse response functions of this model to analyze the dynamic properties of the economy. Figure 5 shows the impulse response function to the technology shock a_t under the baseline calibration with $\rho_a = 0.5$. A 1% technological improvement will increase the output, the first panel from the left, by about 0.7 or 0.8 % and will decrease the nominal interest rate, the fifth panel from the right, by about 0.6%. The consumption, the second panel from the right, will go up about 0.8 %. The traders' consumption, the third panel from the left, rises by about 1%. The non-traders' consumption, the third panel from the right, goes up by about 0.5 % because the positive technology shock reduces the inflation rate by about 0.4%. In the next period, the consumption of non-traders declines because of the decrease in the demand for labor, the second panel from the left, by about 0.5% and the real wages, the fourth panel from the right, by about 0.7% because of the positive technology shock. Since the technology shock sharply decreases non-traders' consumption, the output and consumption do not respond persistently in this economy. The behavior of non-traders can be confirmed by equation (25) in which the current consumption of non-traders depends on the last period's real wage, the last period's labor supply, and the current inflation.

Figure 6 shows the impulse response function to the positive interest shock υ_t under the baseline calibration with $\rho_\upsilon = 0.5$. If there is a contractionary monetary policy shock, $\varepsilon_t^\upsilon > 0$, of 1%, the nominal rate, the fifth panel from the right, will go up by about 0.3% and this will reduce directly only the consumption of traders, the third panel from the left, by about 0.7%. Their consumption, however, will rebound promptly because of a drop in the inflation rate and investment and the rise in the return on bonds. You can see that this contractionary monetary policy shock makes the impulse response of output, the first panel from the left, and consumption, the second panel from the right, hump-shaped. It seems that this is due to the hump-shaped non-traders' consumption impulse response, the third panel from the left. When there is a positive interest shock, the consumption of non-traders will increase immediately because of a decline in the inflation rate, the fifth panel from the left. The shock, however, will dwindle the demand for labor, the second panel from the left, and the real wage, the fourth panel from the right, and this will then sharply decrease the non-traders' consumption by about 1.7% in the next period; this situation will sustain for eight or nine quarters.

Figure 7 shows the impulse response function to the positive money growth shock μ_t under the baseline calibration with $\rho_\mu = 0.5$. The existence of the liquidity effect is an important aspect in this figure because the standard New Keynesian model does not generate the liquidity effect.[23] When there is an expansionary monetary policy shock, $\varepsilon_t^\mu > 0$, both output, the first panel from the left, and inflation, the fifth panel from the left, will increase by about 0.5%. The nominal rate, the fifth panel from the right, declines by about 0.6% when the shock appears, confirming the existence of the liquidity effect. As, however, time passes, the nominal rate overshoots a little. This implies that although the liquidity effect prevails against the Fisher effect in the short run, the Fisher effect will dominate the liquidity effect in the long run. The liquidity effect will raise the traders' consumption, the third panel from the left. Although the increase in the inflation rate owing to the expansionary monetary policy shock reduces the non-traders' consumption, the third panel from the right, the rise in the demand of labor, the second panel from the left, and real wages, the fourth panel from the right, allows the non-traders' consumption to increase in the future. Therefore, non-traders' consumption is hump-shaped. This will make the impulse responses of output and the consumption, the second panel from the right, hump-shaped. From Figure 6 and Figure 7, we can induce that the hump-shaped behavior of the non-traders can generate the hump-shaped response of output and consumption to exogenous shocks.

From these impulse response functions, we can say that monetary policies can asymmetrically affect the consumption of traders and non-traders. Therefore, when the monetary authority implements monetary policies, it should pay attention to the different behaviors of traders and non-traders in response to the policy. Further, this fact mo-

[23] See Gali (2008).

tivates us to study the monetary policy and fiscal policy in the New Keynesian model with market segmentation.

Finally, we will provide a sketch on the system and the ranges for determinacy in the forward-looking Taylor rule in the Appendix without any explanation. Figure 8 shows that the ranges for determinacy under the forward-looking Taylor rule are not different from the ones under the current Taylor rule. We can think that the logic in the current Taylor rule can be similarly applied to the forward-looking Taylor rule.

5 Conclusion

We study the Taylor rule's condition for determinacy in the New Keynesian model with market segmentation. The standard New Keynesian model supports the Taylor principle. Papers, however, introducing the market segmentation into the New Keynesian model insists that the market segmentation can result in a departure from the Taylor principle for the determinacy. Nakajima (2006) and Gali et al. (2004) adopt only one of the two features of the market segmentation model, the liquidity effect and the difference in the consumption smoothing, respectively. Nakajima (2006) suggests that under the liquidity effect in the New Keynesian model with market segmentation, the Taylor rule is required to be passive for determinacy, but Gali et al. (2004) propose that the difference in smoothing consumption between traders and non-traders calls for the Taylor rule to be more aggressive for determinacy if the proportion of traders is sufficiently low. The general exogenous market segmentation model, however, has both features that have contrasting influences. If we want to introduce the exogenous market segmentation into the New Keynesian model, we should consider the two features. We set the New Keynesian model with market segmentation including two features. The contrasting influences from the two features are offset in this model, as we predict. The following are the results of this paper. First, the Taylor rule can satisfy the Taylor principle in the New Keynesian model with market segmentation. Second, the coefficients of the output gap and inflation are positively related. Third, although the proportion of traders is sufficiently low, the threshold of the coefficient of inflation in the Taylor rule is constant regardless of the proportion of traders since influence of the liquidity effect is offset by that of the lessened consumption smoothing. Fourth, the intertemporal elasticity of substitution of consumption $1/\sigma$ is sufficient low, the range for indeterminacy in this economy will increase dramatically when the proportion of traders λ is sufficiently low. However, capital can eliminate indeterminacy from the low intertemporal elasticity of substitution of consumption. Fifth, asymmetric behaviors are exhibited by traders and non-traders. As such, the impulse response of output and consumption can be hump-shaped.

While there are many papers studying the fiscal policy in the New Keynesian model with the rule-of-thumb consumers, or non-traders, and studying monetary policies in the market segmentation model without the sticky price assumption and the endogenous production, the studies on the monetary policies using the New Keynesian model with market segmentation including the two features, the liquidity effect and the difference in consumption smoothing, are rare. As we have seen above, the behaviors of traders in response to a monetary policy are very different from those of non-traders. This finding yields that the monetary authority should consider the existence of non-traders when they implement monetary policies. Therefore, we may have motivation to study monetary or fiscal policies in the New Keynesian model with market segmentation. We can apply this model to study the monetary and fiscal policies in the near future. Further, this model can be applied to analyze the policies on economic polarization between the rich (traders) and the poor (non-traders).

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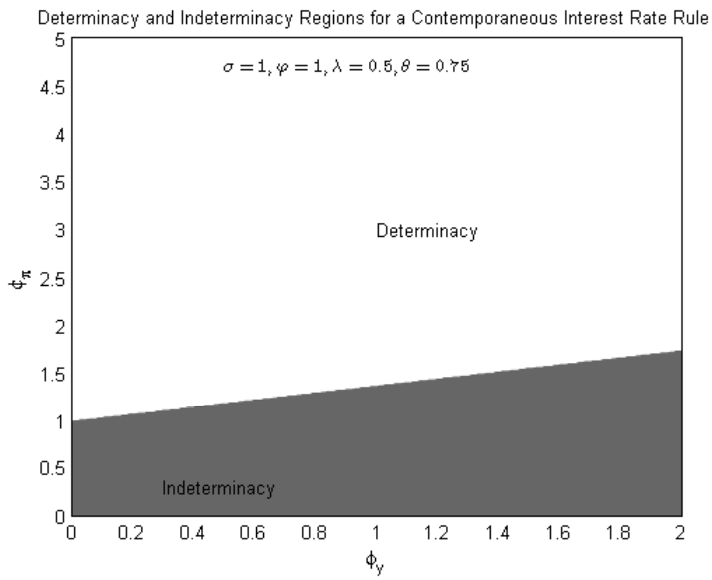

Figure 1: Determinacy Ranges for the Current Taylor Rule

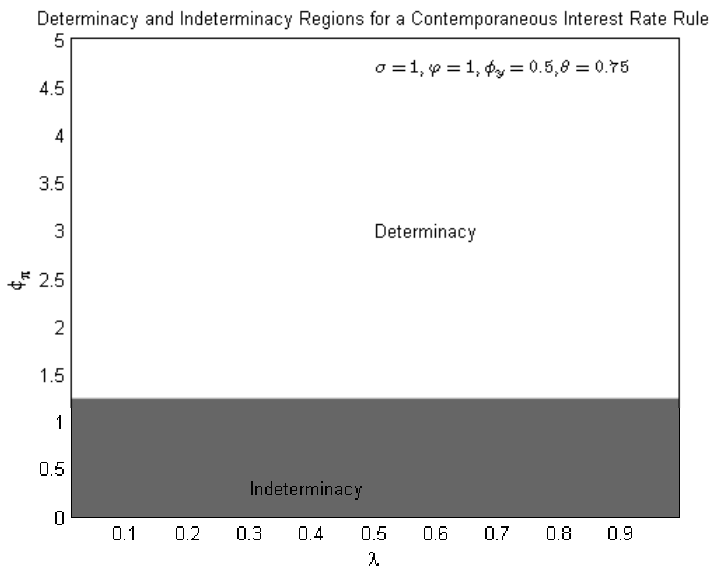

Figure 2: Determinacy Ranges for (λ, ϕ_π) under the current Taylor Rule

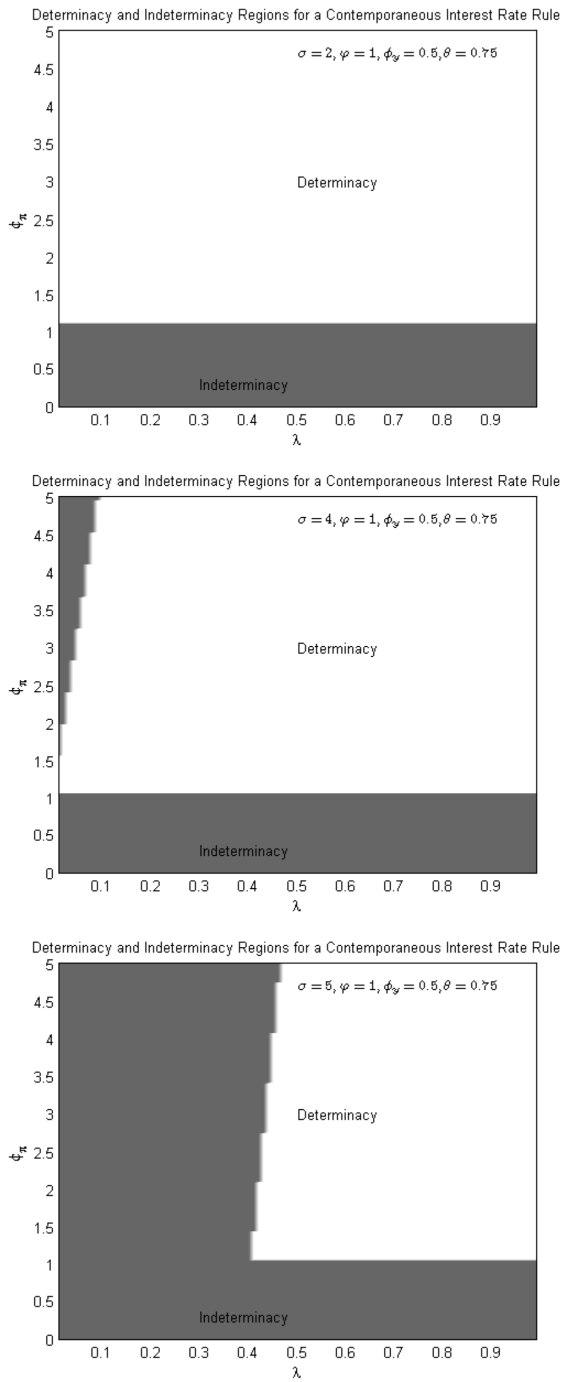

Figure 3: Determinacy Ranges for the current Taylor Rule:σ=2, 4, and 5

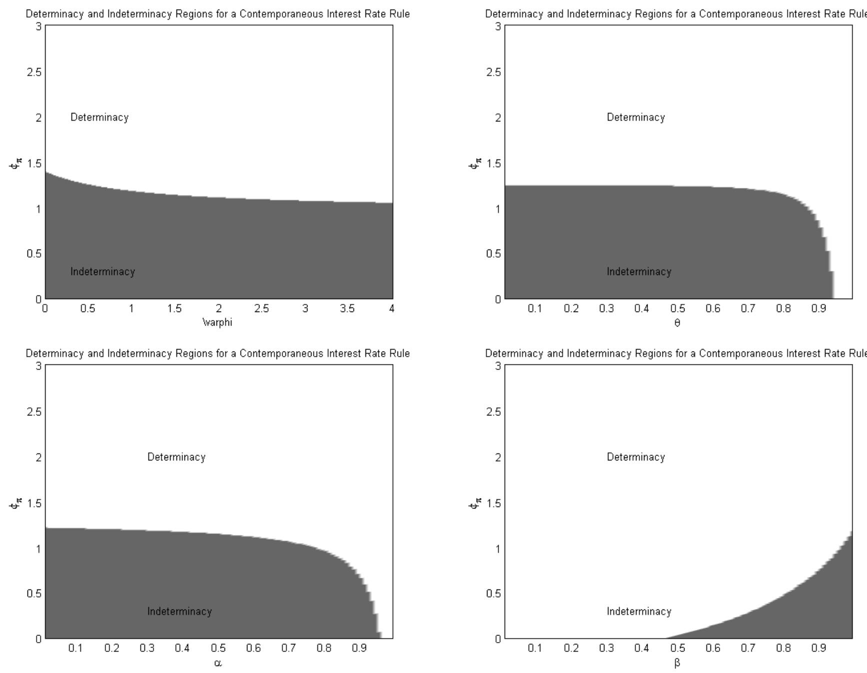

Figure 4: Determinacy Ranges under the current Taylor Rule

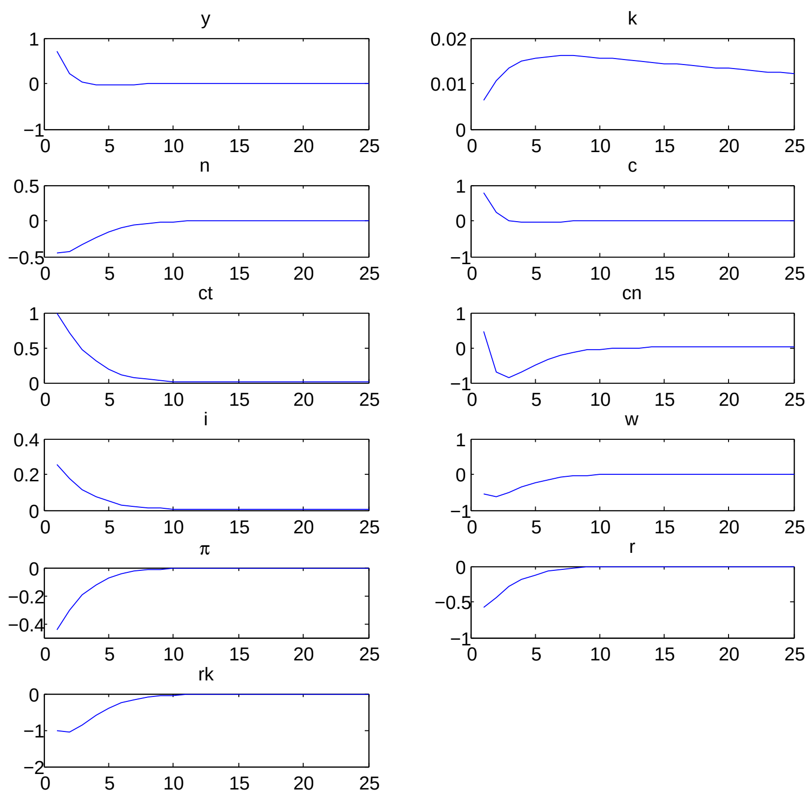

Figure 5: Impulse Response to the Technology Shock

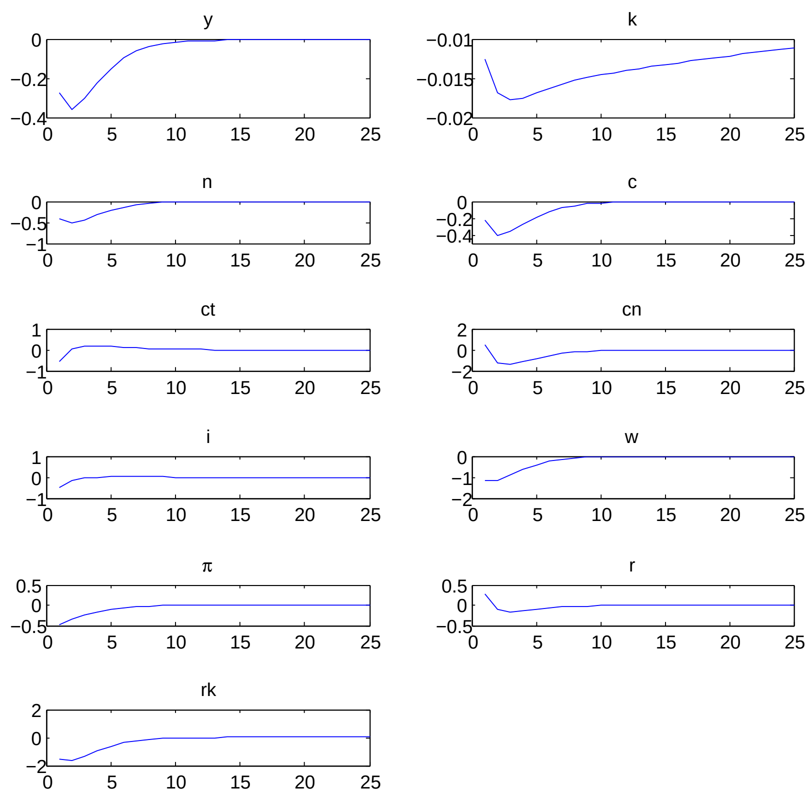

Figure 6: Impulse Response to the Interest Shock

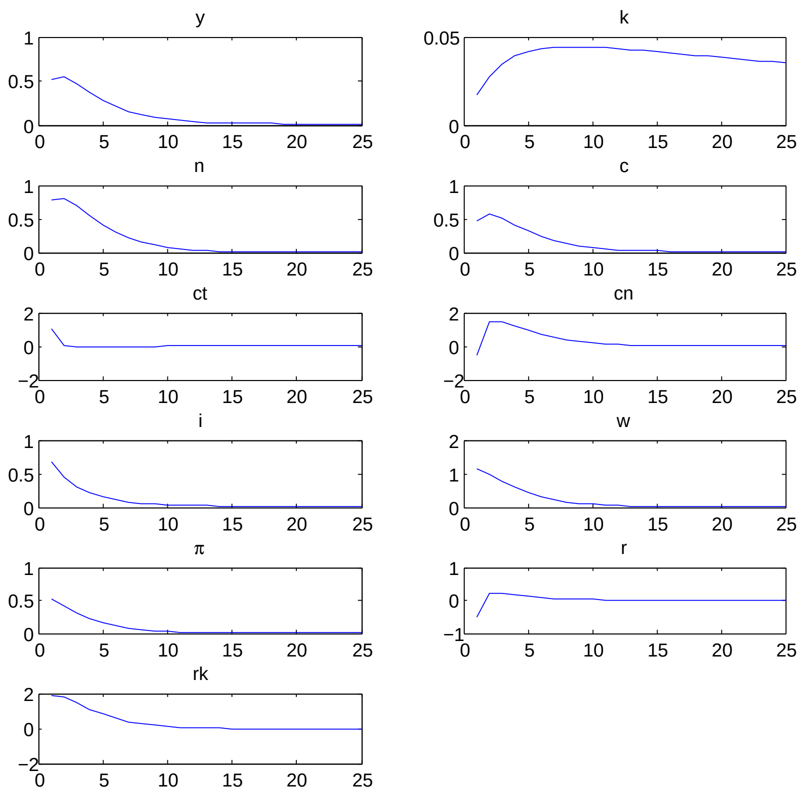

Figure 7: Impulse Response to the Money supply Shock

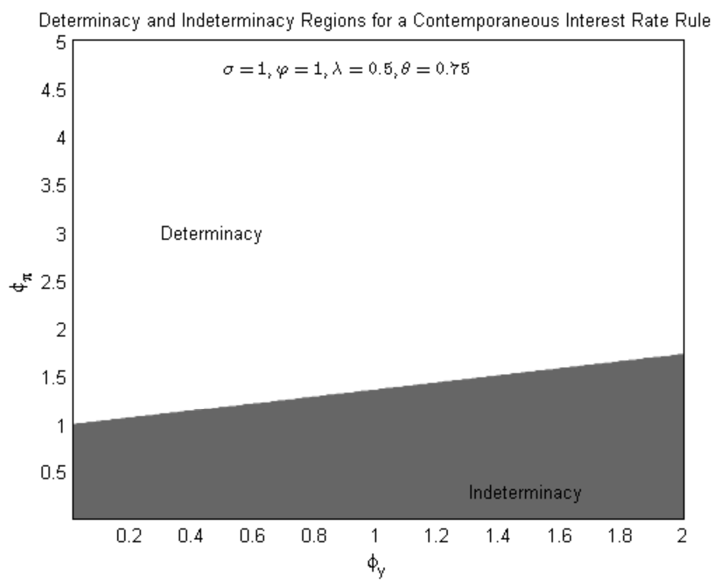

Figure 8: Determinacy Ranges for the Forward-looking Taylor Rule

A Steady state

$A_t = \bar{A} = 1$, $M_t/M_{t-1} = 1$, $P_t/P_{t-1} = 1$, $\bar{I} = \delta\bar{K}$.

Assuming *zero* inflation steday state.

From equation (4):

$$\bar{R} = \frac{1}{\beta}$$

From the definition of the Tobin q and $\bar{I} = \delta\bar{K}$:

$$\bar{Q} = 1$$

From equation (5), r^k is real capital rates:

$$\begin{aligned} 1 &= \beta(\bar{r}^k + (1-\delta)) \\ \bar{r}^k &= \tfrac{1}{\beta} - (1-\delta) \end{aligned}$$

From equation (6), w is real wage rates:

$$\bar{C}^N = \bar{w}\bar{N}$$

From Gali et al. (2007), the relationship between the intertemproral consumption marginal rate of substitution and the real wage is

$$\sigma\bar{C} + \varphi\bar{N} = \bar{w}.$$

From equation (9):

$$\frac{\bar{r}^k(1-\alpha)}{\bar{w}\alpha} = \gamma = \frac{\bar{N}}{\bar{K}} \Rightarrow \bar{K} = \frac{1}{\gamma}\bar{N}$$

From equation (12)

$$\begin{aligned} R\bar{M}C = \tfrac{\epsilon-1}{\epsilon} &= \tfrac{\bar{w}}{1-\alpha}\left(\tfrac{\bar{r}^k(1-\alpha)}{\bar{w}\alpha}\right)^{\alpha} \\ \bar{w} &= \left(\tfrac{(\epsilon-1)(1-\alpha)^{1-\alpha}\alpha^{\alpha}}{\epsilon(\bar{r}^k)^{\alpha}}\right)^{1/(1-\alpha)} \end{aligned}$$

From equation (8)

$$\bar{Y} = \bar{K}^{\alpha}\bar{N}^{1-\alpha} = \gamma^{1-\alpha}\bar{K}.$$

From equation (20):

$$\begin{aligned}
\bar{Y} &= \bar{C} + \bar{I} \\
\bar{C} &= \bar{Y} - \delta\bar{K} = \left(1 - \delta\gamma^{\alpha-1}\right)\bar{Y} \\
\bar{C} &= \left(1 - \delta\gamma^{\alpha-1}\right)\gamma^{-\alpha}\bar{N} \\
\bar{C} &= \left(1 - \delta\gamma^{\alpha-1}\right)\gamma^{-\alpha}\left(\beta\bar{w}\bar{C}^{-\sigma}\right)^{1/\varphi} \\
\bar{C} &= \left(\left(1 - \delta\gamma^{\alpha-1}\right)\gamma^{-\alpha}\left(\beta\bar{w}\right)^{1/\varphi}\right)^{\varphi/(\varphi+\sigma)}.
\end{aligned}$$

From the value of $\bar{C}, the value of \bar{N}$ is

$$\bar{N} = \frac{1}{\left(1 - \delta\gamma^{\alpha-1}\right)\gamma^{-\alpha}}\bar{C}.$$

Then, the value of $\bar{K}$, $\bar{Y}$, $\bar{C}^N$, and $\bar{C}^T$ are, respectively,

$$\begin{aligned}
\bar{K} &= \tfrac{1}{\gamma}\bar{N}, \\
\bar{Y} &= \bar{K}^{\alpha}\bar{N}^{1-\alpha}, \\
\bar{C}^N &= \bar{w}\bar{N}, \\
\bar{C}^T &= \tfrac{1}{\lambda}\left(\bar{C} - (1-\lambda)\bar{C}^N\right).
\end{aligned}$$

Also, we can obtain the steady state value of variables remained.

B Dynamics

[24] From equations (27) and (36):

$$\begin{aligned}
\hat{y}_t &= \alpha\hat{k}_t + (1-\alpha)\hat{n}_t = \tau_y\hat{c}_t + (1-\tau_y)\hat{i}_t \\
\Rightarrow \hat{i}_t &= \tfrac{\alpha}{1-\tau_y}\hat{k}_t + \tfrac{1-\alpha}{1-\tau_y}\hat{n}_t - \tfrac{\tau_y}{1-\tau_y}\hat{c}_t
\end{aligned}$$

Incorporating this equation into equation (22):

$$\begin{aligned}
\hat{k}_{t+1} &= (1-\delta)\hat{k}_t + \delta\left(\frac{\alpha}{1-\tau_y}\hat{k}_t + \frac{1-\alpha}{1-\tau_y}\hat{n}_t - \frac{\tau_y}{1-\tau_y}\hat{c}_t\right) \\
\hat{k}_{t+1} &= \left((1-\delta) + \frac{\delta\alpha}{1-\tau_y}\right)\hat{k}_t + \frac{\delta(1-\alpha)}{1-\tau_y}\hat{n}_t - \frac{\delta\tau_y}{1-\tau_y}\hat{c}_t \qquad (39)
\end{aligned}$$

[24] We drop terms of shocks because the term is not needed in solving the uniqueness of the system, equation (38).

From equation (31):

$$\begin{aligned}
\hat{mc}_t &= -\hat{y}_t + \hat{n}_t + \hat{w}_t \\
&= -(\alpha\hat{k}_t + (1-\alpha)\hat{n}_t) + \hat{n}_t + \hat{w}_t - E_t\pi_{t+1} + E_t\pi_{t+1} \\
&= -\alpha\hat{k}_t + \alpha\hat{n}_t + \varphi\hat{n}_t + \sigma E_t\hat{c}_{t+1} + E_t\pi_{t+1} \\
&= -\alpha\hat{k}_t + (\alpha+\varphi)\hat{n}_t + \sigma E_t\hat{c}_{t+1} + E_t\pi_{t+1}.
\end{aligned}$$

Incorporatin this equation into equation (37):

$$\begin{aligned}
\pi_t &= \beta E_t\pi_{t+1} + \kappa(-\alpha\hat{k}_t + (\alpha+\varphi)\hat{n}_t + \sigma E_t\hat{c}_{t+1} + E_t\pi_{t+1}) \\
\pi_t &= (\beta+\kappa)E_t\pi_{t+1} - \alpha\kappa\hat{k}_t + \kappa(\alpha+\varphi)\hat{n}_t + \kappa\sigma E_t\hat{c}_{t+1} \\
\kappa\sigma E_t\hat{c}_{t+1} + (\beta+\kappa)E_t\pi_{t+1} &= \alpha\kappa\hat{k}_t - \kappa(\alpha+\varphi)\hat{n}_t + \pi_t. \qquad (40)
\end{aligned}$$

From equation (34)

$$\begin{aligned}
\hat{c}_t &= \lambda\tau_c\hat{c}_t^T + (1-\lambda\tau_c)\hat{c}_t^N \\
\hat{c}_t^T &= \tfrac{1}{\lambda\tau_c}\hat{c}_t - \tfrac{1-\lambda\tau_c}{\lambda\tau_c}\hat{c}_t^N
\end{aligned}$$

From equation (25):

$$\begin{aligned}
\hat{c}_t^N &= \hat{w}_{t-1} - \pi_t + \hat{n}_{t-1} \\
&= \sigma\hat{c}_t + \varphi\hat{n}_{t-1} + \hat{n}_{t-1} \\
&= \sigma\hat{c}_t + (1+\varphi)\hat{n}_{t-1}.
\end{aligned}$$

This euqation into equation on c_t^T, (23):

$$\begin{aligned}
\hat{c}_t^T &= \tfrac{1}{\lambda\tau_c}\hat{c}_t - \tfrac{1-\lambda\tau_c}{\lambda\tau_c}\left(\sigma\hat{c}_t + (1+\varphi)\hat{n}_{t-1}\right) \\
&= \tfrac{1-\sigma(1-\lambda\tau_c)}{\lambda\tau_c}\hat{c}_t - \tfrac{(1-\lambda\tau_c)(1+\varphi)}{\lambda\tau_c}\hat{n}_{t-1} \\
&= \tfrac{\chi_c}{\lambda\tau_c}\hat{c}_t - \tfrac{\chi_n}{\lambda\tau_c}\hat{n}_{t-1},
\end{aligned}$$

where $\chi_c \equiv 1 - \sigma(1-\lambda\tau_c)$ and $\chi_n \equiv (1-\lambda\tau_c)(1+\varphi)$. Putting this equation into 23):

$$\begin{aligned}
\frac{\chi_c}{\lambda\tau_c}\hat{c}_t - \frac{\chi_n}{\lambda\tau_c}\hat{n}_{t-1} &= E_t\frac{\chi_c}{\lambda\tau_c}\hat{c}_{t+1} - \frac{\chi_n}{\lambda\tau_c}\hat{n}_t + \frac{1}{\sigma}(E_t\pi_{t+1} - \hat{r}_t) \\
\sigma\chi_c\hat{c}_t - \sigma\chi_n\hat{n}_{t-1} &= \sigma\chi_c E_t\hat{c}_{t+1} - \sigma\chi_n\hat{n}_t + \lambda\tau_c(E_t\pi_{t+1} - \hat{r}_t) \\
\sigma\chi_c E_t\hat{c}_{t+1} + \lambda\tau_c E_t\pi_{t+1} &= \sigma\chi_c\hat{c}_t + \sigma\chi_n\hat{n}_t + \lambda\tau_c\hat{r}_t - \sigma\chi_n\hat{n}_{t-1} \\
\sigma\chi_c E_t\hat{c}_{t+1} + \lambda\tau_c E_t\pi_{t+1} &= \sigma\chi_c\hat{c}_t + \sigma\chi_n\hat{n}_t + \lambda\tau_c(\phi_\pi\pi_t + \phi_y(\alpha\hat{k}_t + (1-\alpha)\hat{n}_t)) - \sigma\chi_n\hat{n}_{t-1} \\
\sigma\chi_c E_t\hat{c}_{t+1} + \lambda\tau_c E_t\pi_{t+1} &= \lambda\tau_c\phi_y\alpha\hat{k}_t + (\lambda\tau_c\phi_y(1-\alpha) + \sigma\chi_n)\hat{n}_t \\
&\quad + \sigma\chi_c\hat{c}_t + \lambda\tau_c\phi_\pi\pi_t - \sigma\chi_n\hat{n}_{t-1} \qquad (41)
\end{aligned}$$

From equation (31):

$$\begin{aligned}
\hat{mc}_t &= -\hat{y}_t + \hat{n}_t + \hat{w}_t = -\hat{y}_t + \hat{k}_t + \hat{r}_t^k \\
\Rightarrow \hat{n}_t - E_t\pi_{t+1} + E_t\pi_{t+1} &= \hat{k}_t + \hat{r}_t^k \\
\Rightarrow \hat{r}_t^k &= -\hat{k}_t + (1+\varphi)\hat{n}_t + \sigma E_t c_{t+1} + E_t\pi_{t+1}
\end{aligned}$$

$$\hat{r}_t = \phi_\pi \pi_t + \phi_y(\alpha \hat{k}_t + (1-\alpha)\hat{n}_t)$$

Putting $\hat{i}_t = \frac{\alpha}{1-\tau_y}\hat{k}_t + \frac{1-\alpha}{1-\tau_y}\hat{n}_t - \frac{\tau_y}{1-\tau_y}\hat{c}_t$ into the definition of Tobin q:

$$\begin{aligned}
\hat{q}_t &= \tfrac{1}{\eta}\left(\tfrac{\alpha}{1-\tau_y}\hat{k}_t + \tfrac{1-\alpha}{1-\tau_y}\hat{n}_t - \tfrac{\tau_y}{1-\tau_y}\hat{c}_t - \hat{k}_t\right) \\
&= \tfrac{1}{\eta}\left(\tfrac{\alpha-1+\tau_y}{1-\tau_y}\hat{k}_t + \tfrac{1-\alpha}{1-\tau_y}\hat{n}_t - \tfrac{\tau_y}{1-\tau_y}\hat{c}_t\right) \\
&= \psi_1\hat{k}_t + \psi_2\hat{n}_t - \psi_3\hat{c}_t
\end{aligned}$$

$$\psi_1 \equiv \frac{\alpha - 1 + \tau_y}{\eta(1-\tau_y)} \quad \psi_2 \equiv \frac{1-\alpha}{\eta(1-\tau_y)} \quad \psi_3 \equiv \frac{\tau_y}{\eta(1-\tau_y)}$$

Putting $\hat{r}_t^k$ and $\hat{q}_t$ into equation (24):

$$\begin{aligned}
\psi_1\hat{k}_t + \psi_2\hat{n}_t - \psi_3\hat{c}_t &= \beta E_t\left(\psi_1\hat{k}_{t+1} + \psi_2\hat{n}_{t+1} - \psi_3\hat{c}_{t+1}\right) \\
&\quad + \chi_q E_t\left(-\hat{k}_{t+1} + (1+\varphi)\hat{n}_{t+1} + \sigma E_t c_{t+2} + E_t\pi_{t+2}\right) - E_t\left(\hat{r}_{t+1} - \pi_{t+1}\right)
\end{aligned}$$

$$\begin{aligned}
&(\beta\psi_1 - \chi_q - \phi_y\alpha)\, E_t\hat{k}_{t+1} + (\beta\psi_2 + \chi_q(1+\varphi) - \phi_y(1-\alpha))\, E_t\hat{n}_{t+1} \\
&\quad -\beta\psi_3 E_t\hat{c}_{t+1} - (\phi_\pi - 1)E_t\pi_{t+1} + \sigma\chi_q E_t c_{t+2} + \chi_q E_t\pi_{t+2} \\
&\quad = \psi_1\hat{k}_t + \psi_2\hat{n}_t - \psi_3\hat{c}_t
\end{aligned}$$

From equations (40) and (41), we can get $E_t\hat{c}_{t+2}$ and $E_t\hat{\pi}_{t+2}$, respectively,

$$\begin{aligned}
E_t\pi_{t+2} &= d_k\hat{k}_{t+1} - d_n E_t\hat{n}_{t+1} - d_c E_t\hat{c}_{t+1} + d_\pi E_t\pi_{t+1} + d_l\hat{n}_t \\
E_t\hat{c}_{t+2} &= f_k\hat{k}_{t+1} - f_n E_t\hat{n}_{t+1} + f_c E_t\hat{c}_{t+1} + f_\pi E_t\pi_{t+1} - f_l\hat{n}_t
\end{aligned}$$

$$
\begin{array}{rclrcl}
d_k & = & \frac{\chi_c a_k - \kappa b_k}{\chi_c(\beta+\kappa)-\kappa\lambda\tau_c} & d_n & = & \frac{\chi_c a_n + \kappa b_n}{\chi_c(\beta+\kappa)-\kappa\lambda\tau_c} \\
d_c & = & \frac{\kappa b_c}{\chi_c(\beta+\kappa)-\kappa\lambda\tau_c} & d_\pi & = & \frac{\chi_c - \kappa b_\pi}{\chi_c(\beta+\kappa)-\kappa\lambda\tau_c} \\
d_l & = & \frac{\kappa b_l}{\chi_c(\beta+\kappa)-\kappa\lambda\tau_c} & & & \\
f_k & = & \frac{a_k-(\beta+\kappa)d_k}{\sigma\kappa} & f_n & = & \frac{a_n-(\beta+\kappa)d_n}{\sigma\kappa} \\
f_c & = & \frac{(\beta+\kappa)d_c}{\sigma\kappa} & f_\pi & = & \frac{1-(\beta+\kappa)d_\pi}{\sigma\kappa} \\
f_l & = & \frac{(\beta+\kappa)d_l}{\sigma\kappa} & & & \\
a_k & = & \alpha\kappa & a_n & = & \kappa(\alpha+\varphi) \\
b_k & = & \lambda\tau_c\phi_y\alpha & b_n & = & \lambda\tau_c\phi_y(1-\alpha)+\sigma\chi_n \\
b_c & = & \sigma\chi_c & b_\pi & = & \lambda\tau_c\phi_\pi \\
b_l & = & \sigma\chi_n & & &
\end{array}
$$

Incorporating thses equations into the above equation:

$$
\begin{aligned}
(\beta\psi_1 - \chi_q(1-\sigma f_k - d_k) - \phi_y\alpha)\, E_t\hat{k}_{t+1} + (\beta\psi_2 + \chi_q((1+\varphi) - \sigma f_n - d_n) - \phi_y(1-\alpha))\, E_t\hat{n}_{t+1} \\
-(\beta\psi_3 - \chi_q(\sigma f_c - d_c))E_t\hat{c}_{t+1} + (\chi_q(\sigma f_\pi + d_\pi) - (\phi_\pi - 1))E_t\pi_{t+1} \\
= \psi_1\hat{k}_t + (\psi_2 + \chi_q(\sigma f_l - d_l))\hat{n}_t - \psi_3\hat{c}_t (42)
\end{aligned}
$$

From equations (39)-(42), we can obtain a dynamic difference equation system for this model:

$$
\mathbf{A}\mathbf{x}_{t+1} = \mathbf{B}\mathbf{x}_t
$$

$$
\mathbf{x}_t \equiv [\hat{k}_t, \hat{n}_t, \hat{c}_t, \pi_t, \hat{h}_t]' \quad \hat{h}_t \equiv n_{t-1}
$$

$$
A = \begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{23} & a_{24} & 0 \\ 0 & 0 & a_{33} & a_{34} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 \\ 0 & 0 & 0 & 0 & a_{55} \end{bmatrix},
$$

$$
B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & 0 & 0 \\ b_{21} & b_{22} & 0 & b_{24} & 0 \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} \\ b_{41} & b_{42} & b_{43} & 0 & 0 \\ 0 & b_{52} & 0 & 0 & 0 \end{bmatrix},
$$

$a_{11} = a_{55} = 1$

$a_{23} = \sigma\kappa$

$a_{24} = \beta + \kappa$

$a_{33} = \sigma\chi_c$

$a_{34} = \lambda\tau_c$

$a_{41} = \beta\psi_1 - \chi_q(1 - \sigma f_k - d_k) - \phi_y\alpha$

$a_{42} = \beta\psi_2 + \chi_q((1+\varphi) - \sigma f_n - d_n) - \phi_y(1-\alpha)$

$a_{43} = -(\beta\psi_3 - \chi_q(\sigma f_c - d_c))$

$a_{44} = \chi_q(\sigma f_\pi + d_\pi) - (\phi_\pi - 1)$

$b_{11} = (1-\delta) + \frac{\delta\alpha}{1-\tau_y}$

$b_{12} = \frac{\delta(1-\alpha)}{1-\tau_y}$

$b_{13} = -\frac{\delta\tau_y}{1-\tau_y}$

$b_{21} = \alpha\kappa$

$b_{22} = -\kappa(\alpha+\varphi)$

$b_{24} = b_{52} = 1$

$b_{31} = \lambda\tau_c\phi_y\alpha$

$b_{32} = \lambda\tau_c\phi_y(1-\alpha) + \sigma\chi_n$

$b_{33} = \sigma\chi_c$

$b_{34} = \lambda\tau_c\phi_\pi$

$b_{35} = -\sigma\chi_n$

$b_{41} = \psi_1$

$b_{42} = \psi_2 + \chi_q(\sigma f_l - d_l)$

$b_{43} = -\psi_3$

To unique solution to this difference equation system, three eigenvalues of the matrix $[A^{-1}B]$ should be out of unit circle.

C The response of interest rates to inflation

From equation (32)

$$d\hat{r} = \phi_\pi d\pi + \phi_y d\hat{y}, \tag{43}$$

From equation (31)

$$d\hat{mc} = -d\hat{y} + d\hat{n} + d\hat{w} = -d\hat{y} + (1+\varphi)d\hat{n} + \sigma d\hat{c} + d\pi.$$

Putting this equation into the Phillipse curve equation, the equation 37),

$$(\beta + \kappa - 1)d\pi = \kappa d\hat{y} - \kappa(1+\varphi)d\hat{n} - \kappa\sigma d\hat{c}. \tag{44}$$

From equation (27)

$$d\hat{y} = \alpha d\hat{k} + (1-\alpha)d\hat{n} \tag{45}$$

From equation (36)

$$d\hat{y} = \tau_y d\hat{c} + (1-\tau_y)d\hat{k} \tag{46}$$

becuase $d\hat{i} = d\hat{k}$.

From equation (24)

$$\chi_q d\hat{r}^k - d\hat{r} + d\pi = 0$$

because $d\hat{q} = 0$. And from the equation 31)

$$d\hat{r}^k = -d\hat{k}_t + (1+\varphi)d\hat{n} + \sigma d\hat{c} + d\pi.$$

Combining two equations on $d\hat{r}^k$ above

$$\begin{aligned}
\chi_q(-d\hat{k} + (1+\varphi)d\hat{n} + \sigma d\hat{c} + d\pi) - d\hat{r} + d\pi &= 0 \\
\chi_q\left(-d\hat{k} + d\hat{y} - \frac{\beta+\kappa-1}{\kappa}d\pi + d\pi\right) - d\hat{r} + d\pi &= 0 \\
-\chi_q d\hat{k} + \chi_q d\hat{y} - \left(\chi_q\frac{\beta-1}{\kappa} - 1\right)d\pi - d\hat{r} &= 0 \qquad (47)
\end{aligned}$$

where the second equality is from equation (44).

From equation (45)

$$d\hat{k} = \frac{1}{\alpha}d\hat{y} - \frac{1-\alpha}{\alpha}d\hat{n}. \qquad (48)$$

Putting this equation into equation (46)

$$\begin{aligned}
d\hat{y} &= \tau_y d\hat{c} + (1-\tau_y)\left(\frac{1}{\alpha}d\hat{y} - \frac{1-\alpha}{\alpha}d\hat{n}\right) \\
\left(1 - \frac{1-\tau_y}{\alpha}\right)d\hat{y} &= \tau_y d\hat{c} - \frac{(1-\tau_y)(1-\alpha)}{\alpha}d\hat{n} \\
d\hat{c} &= a_1 d\hat{y} + a_2 d\hat{n}. \qquad (49)
\end{aligned}$$

where $a_1 \equiv \frac{\alpha-1+\tau_y}{\tau_y\alpha}$ and $a_2 \equiv \frac{(1-\tau_y)(1-\alpha)}{\tau_y\alpha}$. Putting this equation into the equation 44)

$$\begin{aligned}
(\beta+\kappa-1)d\pi &= \kappa d\hat{y} - \kappa(1+\varphi)d\hat{n} - \kappa\sigma(a_1 d\hat{y} + a_2 d\hat{n}) \\
(\beta+\kappa-1)d\pi &= \kappa(1-\sigma a_1)d\hat{y} - \kappa(1+\varphi+\sigma a_2)d\hat{n} \\
d\hat{n} &= b_1 d\hat{y} - b_2 d\pi, \qquad (50)
\end{aligned}$$

where $b_1 \equiv \frac{1-\sigma a_1}{1+\varphi+\sigma a_2}$ and $b_2 \equiv \frac{\beta+\kappa-1}{\kappa(1+\varphi+\sigma a_2)}$.

Putting equation (48) into the equation (47) after equation (50) is replaced into equation (48)

$$-\chi_q\left(\frac{1}{\alpha}d\hat{y} - \frac{1-\alpha}{\alpha}(b_1 d\hat{y} - b_2 d\pi)\right) + \chi_q d\hat{y} - \left(\chi_q\frac{\beta-1}{\kappa} - 1\right)d\pi - d\hat{r} = 0$$

$$d\hat{r} = c_1 d\hat{y} + c_2 d\pi, \qquad (51)$$

where $c_1 \equiv -\chi_q\left(\frac{1}{\alpha} - \frac{(1-\alpha)b_1}{\alpha} - 1\right)$ and $c_2 \equiv -\left(\chi_q\frac{(1-\alpha)b_2}{\alpha} + \frac{\chi_q(\beta-1)-\kappa}{\kappa}\right)$. Then $d\hat{y} = \frac{1}{c_1}d\hat{r} - \frac{c_2}{c_1}d\pi$ and put this into equation (43)

$$d\hat{r} = \frac{c_1\phi_\pi - c_2\phi_y}{c_1 - \phi_y}d\pi. \tag{52}$$

If $\frac{c_1\phi_\pi - c_2\phi_y}{c_1-\phi_y} > 1$, the Taylor rule in this model satisfies the Taylor Principle.

D The forward-looking interest rate rules.

Equation (41) can be rearranged as

$$\sigma\chi_c E_t\hat{c}_{t+1} + \lambda\tau_c E_t\pi_{t+1} = \sigma\chi_n\hat{n}_t + \sigma\chi_c\hat{c}_t + \lambda\tau_c\hat{r}_t - \sigma\chi_n\hat{n}_{t-1} \tag{53}$$

From equations (40) and (53), we can get $E_t\hat{c}_{t+2}$ and $E_t\hat{\pi}_{t+2}$, respectively,

$$\begin{aligned} E_t\pi_{t+2} &= h_k\hat{k}_{t+1} - h_nE_t\hat{n}_{t+1} - h_cE_t\hat{c}_{t+1} + h_\pi E_t\pi_{t+1} - h_r\hat{r}_{t+1} + h_l\hat{n}_t \\ E_t\hat{c}_{t+2} &= j_k\hat{k}_{t+1} - j_nE_t\hat{n}_{t+1} + j_cE_t\hat{c}_{t+1} + j_\pi E_t\pi_{t+1} + j_r\hat{r}_{t+1} - j_l\hat{n}_t \end{aligned}$$

$$\begin{aligned} h_k &= \tfrac{\chi_c a_k}{\chi_c(\beta+\kappa)-\kappa\lambda\tau_c} & h_n &= \tfrac{\chi_c a_n + \kappa g_n}{\chi_c(\beta+\kappa)-\kappa\lambda\tau_c} \\ h_c &= \tfrac{\kappa g_c}{\chi_c(\beta+\kappa)-\kappa\lambda\tau_c} & h_\pi &= \tfrac{\chi_c}{\chi_c(\beta+\kappa)-\kappa\lambda\tau_c} \\ h_r &= \tfrac{\kappa g_r}{\chi_c(\beta+\kappa)-\kappa\lambda\tau_c} & h_l &= \tfrac{\kappa g_l}{\chi_c(\beta+\kappa)-\kappa\lambda\tau_c} \\ j_k &= \tfrac{a_k-(\beta+\kappa)h_k}{\sigma\kappa} & j_n &= \tfrac{a_n-(\beta+\kappa)h_n}{\sigma\kappa} \\ j_c &= \tfrac{(\beta+\kappa)h_c}{\sigma\kappa} & j_\pi &= \tfrac{1-(\beta+\kappa)h_\pi}{\sigma\kappa} \\ j_r &= \tfrac{(\beta+\kappa)h_r}{\sigma\kappa} & j_l &= \tfrac{(\beta+\kappa)h_l}{\sigma\kappa} \\ a_k &= \alpha\kappa & a_n &= \kappa(\alpha+\varphi) \\ g_n &= \sigma\chi_n & g_c &= \sigma\chi_c \\ b_r &= \lambda\tau_c & b_l &= \sigma\chi_n \end{aligned}$$

Equation (42) can be expressed as

$$\begin{aligned} &(\beta\psi_1 - \chi_q(1-\sigma j_k - h_k))\,E_t\hat{k}_{t+1} + (\beta\psi_2 + \chi_q((1+\varphi) - \sigma j_n - h_n))\,E_t\hat{n}_{t+1} \\ &-(\beta\psi_3 - \chi_q(\sigma j_c - h_c))E_t\hat{c}_{t+1} + (\chi_q(\sigma j_\pi + h_\pi)+1)E_t\pi_{t+1} + (\chi_q(\sigma j_r - h_r)-1)\hat{r}_{t+1} \\ &= \psi_1\hat{k}_t + (\psi_2 + \chi_q(\sigma j_l - h_l))\hat{n}_t - \psi_3\hat{c}_t \end{aligned} \tag{54}$$

The forward-looking interest rate rule is

$$\hat{r}_t = \phi_\pi E_t\pi_{t+1} + \phi_y E_t\hat{y}_{t+1}. \tag{55}$$

We can derive a difference equation system from the equations (39), (40), (53), (54), (55), and $\hat{h}_t = \hat{n}_{t-1}$.