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Inflation Risk Premium and Foreign Exchange Rate

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Abstracts

Engel (2016) highlights puzzling patterns regarding interest rate differentials and foreign exchange rates. When the interest rate of country A is higher than that of country B, currency A earns positive excess returns in the short run; at the same time, currency A earns negative excess returns in the long run. We present an explanation of these patterns based on inflation risk premium. Short-term interest rate is negatively correlated with long-term inflation risk premium, which generates the long-term pattern of FX excess return. This is due to long-term money neutrality. On the other hand, due to short-term price stickiness, short-term inflation risk premium does not respond to short-term interest rate. This allows real risk premium to generate the short-term pattern of FX excess return.

1. Introduction

Engel (2016) highlights puzzling patterns regarding interest rate differentials and foreign exchange rates. When the interest rate of country A is higher than that of country B, currency A earns positive excess returns in the short run (let us call this "the short-run pattern"); at the same time, currency A earns negative excess returns in the long run ("the long-run pattern"). Engel argues that these two patterns are hard to explain with existing theories. Models of FX risk premium may explain the short-run pattern, but not the long-run pattern. Models of overshooting may explain the long-run pattern, but not the short-run pattern. So these two patterns together remain a puzzle to explain.

We seek an explanation of this puzzle by focusing on inflation risk premium. Inflation risk premium has not received its due attention in the FX literature so far; our research aims to fill this gap in the literature.

Our explanation is based on three observations: (i) Inflation risk premium differential is a component of FX risk premium. (ii) Long-term inflation risk premium responds negatively to interest rate movement, generating the long-run pattern of the FX excess returns. (iii) Short-term inflation risk premium does not respond to interest rate movement, thus allowing the other components of FX risk premium to generate the short-run pattern of the FX excess returns.

We derive (i) from the definition of inflation risk premium and the international version of Fisher equation. We can also understand this more intuitively. Inflation risk premium is what bond market participants demand as compensation for being exposed to inflation risk (i.e., for having short exposure to inflation rate). FX market participants are exposed to inflation risk in the same way as bond market participants, thus they demand the same compensation for this exposure. Looking at it this day, it is only natural that inflation risk premium is a part of FX risk premium. So, 1% increase in inflation risk premium results in 1% increase in FX risk premium.

We motivate (ii) and (iii) from the phenomenon of price stickiness. Suppose that the foreign central bank tightens monetary policy.[1] Then this is what happens in the foreign country: Short-tern nominal interest rate goes up (high i_S^*). Short-term inflation expectation is not affected—due to price stickiness—and, thus, short-term *real* rate rises instead (high r_S^*). Price is not sticky for the long term, so people expect inflation rate to become lower eventually, after some delay. Thus, long-term inflation expectation becomes lower, which in turn leads to lower long-term inflation risk premium (low irp_L^*).[2] In the FX market, the following happens: Higher short-term real rate in the foreign country leads to higher FX excess return (high r_S^* → high ρ_S). This is due to positive correlation between real rate differential and FX risk premium, which is a well established fact in the FX risk premium literature. Lower long-term inflation risk premium in the foreign country leads to lower long-term FX excess return (low irp_L^* → low ρ_L). This follows from the fact that inflation risk premium differential is a component of FX risk premium. Thus, we obtain both higher FX excess return in the short-run and the lower FX excess return in the long-run.

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[1] We motivate our idea by assuming that the changes in short rate mostly reflect the changes in monetary policy. In reality, short rate is influenced by many other factors. So our motivation is just a motivation, not a complete theory.

[2] Kliesen and Schmid (2004) report that monetary policy shock affects the quantity of inflation risk.

Note that short-term inflation premium in the foreign country (irp_S^*) does not change when short-term interest rate changes. This is due to price stickiness. Note also that long-term real interest rate in the foreign country (r_L^*) does not change. This reflects the limited ability of monetary policy to influence long-term real rate.

We can express our idea in terms of how the term structure of nominal rate differential ($i^* - i$) affects the term structure of FX risk premium (frp). One can think of frp as the sum of $i^* - i$ and the expected change in the FX rate. Thus, $i^* - i$ is a part of frp, and any change in the term structure of $i^* - i$ is likely to be reflected in the term structure of frp. When the foreign central bank tightens monetary policy, the short end of $i^* - i$ goes up, and the long end of $i^* - i$ goes down. The short end of $i^* - i$ goes up because short-term real rate responds to monetary tightening due to price stickiness. The long end of $i^* - i$ goes down because long-term real rate does not respond to monetary tightening ("money neutrality") and long-term inflation expectation responds to monetary tightening. The term structure of frp changes in the same manner, generating positive short-term FX excess return and negative long-term FX excess return.

What if money is not neutral in the long run? Even in such case, the long end of frp may still go down if the other component--the expected change in the FX rate—declines sufficiently. The expected change in the FX rate *should* decline in response to real interest rate rise and inflation expectation rise according to the real interest parity and the purchasing power parity ideas. Thus, even without money neutrality, we may still obtain the downward term structure of frp.

How does term premium in FX excess return or interest rate change the story? If there exists positive term premium, our measure of long-term FX risk premium (which include interest rate) may overstate the expected value of long-term FX excess return. The consequence depends on the correlation between the short rate and the term premium. If they move in the same direction, our conclusion will be strengthened. (When short rate goes up, term premium goes up, and the expected value of long-term FX excess return is likely to be smaller than our calculation suggests.) However, if the short rate and the term premium move in the opposite directions, our conclusion will be weakened.

We confirm (ii) and (iii) empirically using the USD-GBP exchange rates. We use the real interest rates and the breakeven inflation rates implied by inflation-indexed bonds. Given the difficulty of measuring long-run expected returns, we follow Engel's (2016) VECM methodology to estimate the expected value of long-term FX excess returns.

The rest of this paper is organized as follows. In Section 2, we review related literature. In Section 3, we present a decomposition of FX risk premium. Sections 4 and 5 contain empirical analysis. Section 4 is a first look at the data, and Section 5 discuss VECM-based analysis. As this work is still on-going, there is no concluding section yet.

2. Related Literature

The starting point for our work is Engel (2016). We review the findings of Engel (2016), and also discuss other related literature.

The foreign exchange rate puzzle highlighted by Engel (2016) can be restated as follows: There are two strands of literature dealing with foreign exchange rate. The first is the finance view of foreign exchange rate (Bilson, 1981; Fama, 1984; Engel 2014), and it emphasizes the deviation from the interest rate parity, i.e., the existence of carry profit. When a country has a higher interest rate, the interest rate parity requires that its currency depreciate over time. In reality, such currency tends to depreciate, but not as much as required by the interest rate parity. Thus, relative to interest rate parity, high-interest currency is under-valued.

The second strand of literature is the economic view of foreign exchange rate, and it emphasizes the deviation from the purchasing power parity (Rogoff, 1996; Taylor and Taylor, 2004). One of the stylized facts in this literature is the overshooting of the FX rate. The standard models of overshooting (e.g. Dornbusch, 1976) predict (and such prediction has been confirmed empirically) that, when a country has a higher interest rate, its currency becomes over-valued relative to the equilibrium PPP-consistent level. This is essentially the same phenomenon as the excess volatility emphasized by Rogoff (1996). The FX rate exhibits excess volatility as it often overshoots above the equilibrium level. Without overshooting, the volatility would have been smaller than the volatility of the equilibrium level.

Thus, the finance view and the economic view say the opposite things: the currency of a high-interest country is under-valued in one measure and over-valued in another measure. That the exchange rate seems under-valued and over-valued at the same time is not a puzzle in its own right. We have a puzzle only because the existing theories and models cannot explain two views simultaneously.

The finance view explains the under-valuation via FX risk premium. If the FX risk premium for a high-interest currency is positive, then the value of the currency should be lower than interest-rate-parity level, to reward investors for taking extra risk. This line of thinking cannot explain over-valuation relative to long-run equilibrium level.

The economic view explains the over-valuation via sticky prices. When there is monetary tightening, price level does not adjust due to sticky prices, and the real interest rate goes up. The higher real interest rate pushes up the real value of the currency more than the equilibrium level since the current real interest rate is higher than the equilibrium real interest rate. (The equilibrium real interest rate is lower since the real interest rate will eventually come down.) This line of thinking cannot explain the under-valuation in the short-run.

In our research, we start from the finance view, and introduce price stickiness and inflation risk premium. Then, the underadjustment of FX rates to short rate changes is the result of the real-exchange-rate risk premium; at the same time, the overadjustment occurs as a result of the expectation of the delayed response of inflation rate.

Let us take the finance view of the FX rate, and note the fact that the FX risk premium has two components: (1) real FX risk premium and (2) inflation risk premium. If a country has a higher interest rate, and if a higher interest rate is associated with higher FX risk (as is typically assumed), then the value of the currency should be lower than the interest-rate-parity level, to reward investors for taking FX risk. This is what the standard finance view says. Note that we can replace "interest rate" with "real interest rate," and also "FX risk" with "real FX risk" since short-term nominal interest rate is likely to be close to real interest rate. For example, monetary shocks may not affect short-term inflation expectation, making short-term nominal interest rate identical to real interest rate.

Then how about long-run inflation expectation? Monetary shocks will affect long-run inflation expectation; investors will believe that inflation rate will be affected at some point in the future. If the monetary shock is in the form of tightening, then investors will expect lower inflation rate in the future, and inflation risk premium may be negative. This will be translated into negative expected FX return for some future periods. Such negative expected return is consistent with the over-valuation relative to the long-run equilibrium level.

Engel (2016) emphasized a possible role of liquidity premium behind the puzzle. Inflation risk premium emphasized in our research is conceptually distinct from, but empirically related to liquidity premium. Inflation risk premium is what people are demanding for being exposed to inflation; the size of the premium is independent from the liquidity of a particular instrument that people choose to trade. To measure inflation risk premium in an empirical work, however, particular instruments are selected, and the liquidity premium is compounded in the measurement of inflation risk premium. It has been noted by many authors (e.g., Gurkaynak, Sack, Wright, 2008) that the yield on the inflation protected bonds include illiquidity premium, but its precise measurement is often difficult, so a measure of inflation risk premium may include some of the liquidity premium.

Our research will contribute to the uncovered interest parity puzzle literature (Bilson, 1981; Fama, 1984; Engel 2014) by investigating the following questions: How much of FX risk premium is attributable to inflation risk premium? How much of time variation in FX risk premium is attributable to time variation in inflation risk premium?

Sarno, Schneider, and Wagner (2012), using a structural model of yield curve, showed that FX risk premium can explain the uncovered interest parity puzzle (i.e. the short-term pattern we described in 1). Our contribution is to identify components of FX risk premium and show that FX risk premium drives the long-term pattern as well as short-term pattern.

Our research will contribute to the purchasing power parity puzzle literature (Rogoff, 1996; Taylor and Taylor, 2004) by investigating the following questions: Does the inflation risk premium generate mean-reversion in real exchange rate? Do we find stronger co-integrating relation between real exchange rate and real interest differential (Meese and Rogoff, 1988; Nakagawa, 2002; Cushman and Michael, 2011; Camarero and Ordonez, 2012) once we account for FX risk premium and inflation risk premium?

Our research will contribute to the growing body of literature on inflation risk premium (Ang, Bekaert, and Wei, 2008; Gurkaynak, Sack, Wright, 2008). A number of papers reported that inflation risk premium is significant, sometimes as high as 1% per annum. A simple algebra based on Fisher equation suggests that inflation risk premium is an important component of FX risk premium. However, no one has investigated the role of inflation risk premium in explaining FX returns. So it seems worthwhile to investigate whether inflation risk premium can help explain the movement of FX returns.

Balducci and Chiang (2016) show that real exchange rate predicts FX excess return. Our research complements their paper by showing that inflation risk premium differential predicts many-year ahead FX excess returns. Our decomposition formula shows that inflation risk premium differential and the change in real exchange rate are components of FX excess returns; not accounting for inflation risk premium may attribute some of the predictability of inflation risk premium to real exchange rates.

3. Decomposition of FX Risk Premium

In this section, we present the decomposition of FX risk premium, i.e. the expected value of FX excess return.

Let us start with the uncovered interest rate parity relation:

(1) $i^* + \mathrm{E}(\log S_1/S_0) = i$

The left-hand-side is what an investor expect to earn from foreign currency deposit: i^* is foreign-currency log-deposit rate, S_1 is one-period ahead exchange rate, and S_0 is current exchange rate. The exchange rates are expressed as the number of dollars equivalent to one unit of foreign currency. The right-hand-side i is the U.S. dollar short-term log-deposit rate. FX risk premium is defined as the deviation from the uncovered interest rate parity, i.e. expected excess return to the foreign currency deposit:

(2) $frp \equiv \mathrm{E}(\rho) \equiv i^* - i + \mathrm{E}(\log S_1/S_0)$

To introduce inflation risk premium, we now turn to Fisher equation:

(3) $i = r + \pi^e$

r is real interest rate, and π^{e} is the expected inflation rate. Inflation risk premium is defined as the deviation from Fisher equation:

(4) $irp = i - r - \pi^e$

It is the reward to nominal bond holders for being exposed to inflation risk. Nominal bond holders have long exposure to inflation, and to fix the idea, we will consider positive inflation risk premium as typical. (As Bekaert and Wang (2010) explain, the inflation risk premium may well be negative, if marginal utility of investors tends to high when inflation rate is high.)

Note that we treat real interest rate r as observable quantity. r is defined as the real yield on an inflation-indexed bond (TIPS). Regardless of whether TIPS are actually traded or not (they are not traded in many countries in the world), conceptually, r can be treated as an observable.

Combining inflation risk premium definition with FX risk premium definition, we get:

(5) $frp = \mathrm{E}(\log S_1/S_0) + (\pi^{e*} - \pi^e) + (r^* - r) + irp^* - irp$

While the above formulation is new, similar ideas have been examined in different contexts. For example, international economics textbooks (e.g. Krugman and Obsfeld, 1994) talk about international Fisher equation, from which the above formulation can be easily derived.

The first two terms in the right-hand side is the deviation from *purchasing power parity*. It indicates the increase in the purchasing power of the foreign currency. The first three terms in the right-hand side is the deviation from *real interest rate parity*. Under real interest rate parity, the increase in the purchasing power of the foreign currency, $\mathrm{E}(\log S_1/S_0) + (\pi^{e^*} - \pi^e)$ would be completely offset by real interest rate differential, $(r^* - r)$, and the sum of the two would be zero. When the sum of the two is not zero, it indicates FX risk premium in real term. So we call it real FX risk premium. (Real FX risk premium can be also described as the expected return to inflation-hedged FX investors.)

(6) $$rfrp \equiv \mathrm{E}(\log S_1/S_0) + (\pi^{e^*} - \pi^e) + (r^* - r)$$

With this definition of real FX risk premium, we may write FX risk premium as the sum of two components, real FX risk premium and inflation risk premium differential:

(7) $$frp = rfrp + (irp^* - irp)$$

The above decomposition of FX risk premium can be motivated by money neutrality idea as well. If money were neutral, then inflation risk premium would be zero, and all the FX risk premium can be thought of as real FX risk premium. That is, real FX risk premium is the part of FX risk premium that is not attributable to the violation of money neutrality. Inflation risk premium differential picks up the consequence of the violation of money neutrality.

4. A First Look at the Data

In this section, we examine whether the basic implications of our hypothesis are supported by USD-GBP data.

The empirical pattern than we would like to explain—Engel's (2016) puzzle—is the following: foreign short-term nominal rate differential $i_{s,t}^* - i_{s,t}$ is positively correlated with short-term FX excess return ρ_{t+1} (one month ahead return), and negatively correlated with long-term FX excess return $\rho_{t+1} + \cdots + \rho_{t+h}$ (h being 60 or so). In terms of covariances, we may write:

(8) C1. $\mathrm{cov_t}(i_{s,t}^* - i_{s,t}, \rho_{t+1}) = \mathrm{cov_t}\left(i_{s,t}^* - i_{s,t}, E_t(\rho_{t+1})\right) > 0$

(9) C2. $\mathrm{cov_t}(i_{s,t}^* - i_{s,t}, \sum_{j=1}^{h} \rho_{t+j}) = \mathrm{cov_t}\left(i_{s,t}^* - i_{s,t}, E_t(\sum_{j=1}^{h} \rho_{t+j})\right) < 0$, h being 60 or so

Our explanation is the following: When foreign short-term interest rate $i_{s,t}^*$ goes up (reflecting monetary tightening in the foreign country), foreign short-term real rate $r_{s,t}^*$ rises due to price stickiness. In the long-end of the term structure, foreign long-term real rate $r_{L,t}^*$ does not move much due to long-term money neutrality, and foreign long-term inflation expectation ${\pi_{L,t}^{e}}^*$ and inflation risk premium $irp_{L,t}^*$ goes down. Short-term FX excess return ρ_{t+1} covaries with real rate differential $r_{s,t}^* - r_{s,t}$, and h-period FX excess return $\sum_{j=1}^{h} \rho_{t+j}$ covaries with long-term inflation risk premium differential $irp_{L,t}^* - irp_{L,t}$. In terms of covariances, we expect:

(10) C3. $\mathrm{cov_t}(i_{s,t}^* - i_{s,t}, r_{s,t}^* - r_{s,t}) > 0$

(11) C4. $\text{cov}_t\left(r_{s,t}^{*}-r_{s,t}, E_t(\rho_{t+1})\right)>0$

(12) C5. $\text{cov}\left(i_{s,t}^{*}-i_{s,t}, irp_{L,t}^{*}-irp_{L,t}\right)<0$

(13) C6. $\text{cov}_t\left(irp_{L,t}^{*}-irp_{L,t}, E_t\left(\sum_{j=1}^{h}\rho_{t+j}\right)\right)>0$

We will check C1 and C2, but they are not our main concern as these covariances are reported by Engel (2016). C3 is a well established fact, and C4 is almost immediate from C1. So, we will not concern ourselves with these two. Ang et al. (2008) report what amounts to C5 for the U.S. All we need to do is to check that these patterns survive when we consider a pair of countries at the same time. So, what is the most critical is C6.

Regarding C5, Evans and Marshall (1998), Drakos (2001), and Berument and Froyen (2006) find that monetary tightening does not affect long-term nominal rate much. Such findings are not inconsistent with C5; long-term real rate may go up mildly and inflation compensation may go down. Stronger support for C5 comes from Romer and Romer (2000) and Ellingsen and Soderstrom (2001, 2003), who suggest that monetary tightening lowers long-term nominal rates once the endogeneity (why central bank decides to tighten money supply in the first place) is properly taken care of. That is, when inflation expectation is high, monetary tightening follows. The net effect of the policy is a lower long-term rate, but it may appear to untrained eye that the effect is opposite. Kliesen and Schmid (2004) and Kiley (2008) report findings consistent with this idea.

For the preliminary analysis, we will not estimate inflation risk premium. (This requires fitting the yield curve via a structural model). For now, we will use breakeven inflation rate $\pi_{L,t}^{B}$ (also known as inflation compensation), which is the sum of inflation expectation and inflation risk premium. Given that inflation expectation is not very volatile, most of the flunctuation in breakeven inflation is in fact the fluctuation in inflation risk premium (Gurkaynak, Sack, Wright, 2008). Thus, we check:

(14) C5′. $\text{cov}\left(i_{s,t}^{*}-i_{s,t}, \pi_{L,t}^{B}{}^{*}-\pi_{L,t}^{B}\right)<0$

(15) C6′. $\text{cov}_t\left(\pi_{L,t}^{B}{}^{*}-\pi_{L,t}^{B}, E_t\left(\sum_{j=1}^{h}\rho_{t+j}\right)\right)>0$

In calculating covariances involving h-period excess returns, Engel (2016) has adopted the VECM methodology. Working with monthly data, when h is 60, direct computation of covariances leads to extremely imprecise estimates, so adopting VECM becomes useful. We present the results based on direct calculation as well as VECM.

The approach taken by Ang et al. (2008) allows us to estimate inflation risk premium without inflation-protected bond prices. For the preliminary analysis, we only consider the period for which inflation-protected bond prices are available. (For US-UK pair, after requiring 60 months lagged data, our sample period becomes Feb 2004 to Jun 2016. This is perhaps too short for serious analysis; anyway, this is what we have for now.)

We have collected data for U.S. and U.K. since real yield curves for these countries are readily available in the web sites of the central banks. We have also collected data for Australia, but the quality of data is rather questionable (Finlay and Wende, 2011). Breakeven inflation rate is reported at the quarterly frequency (for US and UK, this number is available at the daily frequency). Also, the market for indexed bonds is apparently not very large (Finlay and Wende, 2011). So perhaps it is better to ignore the results for Australia for now.

For the short rate $i_{s,t}$, we have collected federal funds rates/official bank rates as well as one month and three month interbank rates. It turned out that it does not really matter which one to use. So, below we discuss the results based on three month interbank rates. For the long-term real rate and breakeven inflation rates, we considered 5, 10, and 20 year rates, and obtained mostly comparable results. So, below we focus on the results based on 10 year rates. The U.S. real yield curve is described in Gurkaynak, Sack, Wright (2008). The U.K. real yield curve is described in the web site of Bank of England. We use zero-coupon yields rather than par yields.

The results reported in Table 1 are consistent with C1 and C2. Short-term nominal rate differentials are positively correlated with short-term FX excess returns, but negatively correlated with 5-year FX excess returns. Correlations are not statistically significant, but as mentioned above, this is to be expected. Engel (2016) also report similar results when VECM is not adopted.

[Table 1 about here]

Table 2 shows C5′. Short-term nominal rate differentials are negatively correlated with 10-year breakeven inflation rates. This correlation is statistically significant.

[Table 2 about here]

The last row of Table 3 shows C6′. 10-year breakeven inflation rates are positively correlated with long-term FX excess returns. Again, statistical significance is low, but that is to be expected without VECM.

[Table 3 about here]

5. VECM Analysis

To improve the precision of the regression analysis reported in the previous section, we estimate the VECM for exchange rates, from which we obtain estimates of long-term FX excess returns.

We focus on the regression reported in the last row of Table 3. The regression is meant to confirm

$$(16)\qquad \text{C6}'.\ \operatorname{cov}_t\left({\pi_{L,t}^{B}}^{*}-\pi_{L,t}^{B}, E_t\left(\textstyle\sum_{j=1}^{h}\rho_{t+j}\right)\right)>0$$

For $E_t\left(\sum_{j=1}^{h}\rho_{t+j}\right)$, we have used the realized excess return, which is at best a poor estimate. We follow Engel (2016), and adopt VECM to improve the precision of the calculation.

Our calculation of $E_t\left(\sum_{j=1}^{h}\rho_{t+j}\right)$ differs from Engel's in two respects. First, since we have values for long-term real interest rate and breakeven inflation rate from the indexed bond prices, we use these values in place of VECM-computed values. That is, instead of using

$$(17)\qquad \hat{E}_t\left(\textstyle\sum_{j=1}^{h}\rho_{t+j}\right)=\hat{E}_t^{VECM}\left(\log\frac{S_{t+h}}{S_{t+1}}\right)+\hat{E}_t^{VECM}\left(\textstyle\sum_{j=1}^{h}r_{S,t+j-1}^{*}-r_{S,t+j-1}+{\pi_{S,t+j}}^{*}-\pi_{S,t+j}\right)$$

we use

$$(18)\qquad \hat{E}_t\left(\textstyle\sum_{j=1}^{h}\rho_{t+j}\right)=\hat{E}_t^{VECM}\left(\log\frac{S_{t+h}}{S_{t+1}}\right)+r_{L,t+j-1}^{*}-r_{L,t+j-1}+{\pi_{L,t+j}^{B}}^{*}-\pi_{L,t+j}^{B}$$

Engel has noted that the sum of expected short rates is not identical to the expected long rate. Given the availability of the latter, it makes more sense to use the latter.

The second departure from Engel's calculation is that we compute 60-month excess return $E_t\left(\sum_{j=1}^{60}\rho_{t+j}\right)$ instead of infinite horizon excess return $E_t\left(\sum_{j=1}^{\infty}\rho_{t+j}\right)$. This allows us to use the observed values of long-term real interest rate and breakeven inflation rate.

Our VECM implementation is identical to Engel's. The VECM can be written in terms of

$$(19)\qquad x_t=\begin{pmatrix} s_t \\ p_t^{*}-p_t \\ i_t^{*}-i_t \end{pmatrix}$$

where s_t is the log exchange rate and p_t^{*} and p_t are price levels in the UK and in the US, respectively. The purchasing power parity is assumed to hold in the long run so that the log real exchange rate $q_t=s_t+p_t^{*}-p_t$ approaches its long-term mean in the limit. Under this assumption, the VECM can be restated as a VAR of the following variables:

$$(20)\qquad y_t=\begin{pmatrix} q_t \\ \pi_t^{*}-\pi_t \\ i_t^{*}-i_t \end{pmatrix}$$

The VAR formulation is easier to implement as it allows us to calculate h-period ahead forecast in a simple way.

In Table 4 below, we report the regression of $\hat{E}_t\left(\sum_{j=1}^{60}\rho_{t+j}\right)$ on ${\pi_{L,t+j}^{B}}^{*}-\pi_{L,t+j}^{B}$. We use two variants of

$\hat{E}_t\left(\sum_{j=1}^{60}\rho_{t+j}\right)$. $E_t^B\left(\sum_{j=1}^{60}\rho_{t+j}\right)$ is as described above. $E_t^A\left(\sum_{j=1}^{60}\rho_{t+j}\right)$ uses the realized FX rates instead of VECM estimate. That is,

(21) $$E_t^A\left(\sum_{j=1}^{60}\rho_{t+j}\right) = \log\frac{S_{t+60}}{S_{t+1}} + r_{L,t+j-1}^{*} - r_{L,t+j-1} + {\pi_{L,t+j}^{B}}^{*} - \pi_{L,t+j}^{B}$$

(22) $$E_t^B\left(\sum_{j=1}^{60}\rho_{t+j}\right) = \hat{\mathrm{E}}_{\mathrm{t}}^{VECM}\left(\log\frac{S_{t+60}}{S_{t+1}}\right) + r_{L,t+j-1}^{*} - r_{L,t+j-1} + {\pi_{L,t+j}^{B}}^{*} - \pi_{L,t+j}^{B}$$

This allows us to see the improvement in precision due to VECM more clearly.

[Table 4 about here]

Adopting VECM, the correlation between the long-term breakeven inflation rate and the long-term FX excess return became significant. We can (and will) adjust t statistic using a bootstrap method.

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Table 1. Short-term nominal rate differential vs. FX excess returns

The table reports the coefficient estimate (b) and its t statistic in the regression of Y on X. No adjustment is made in calculation of t statistic.

Y	X	t	N	b	t
ρ_{t+1}	$i_{S,t}^* - i_{S,t}$	1999m1 ~ 2011m6	150	1.16	0.55
$\rho_{t+1}+\cdots+\rho_{t+12}$	$i_{S,t}^* - i_{S,t}$	1999m1 ~ 2011m6	150	8.34	0.98
$\rho_{t+1}+\cdots+\rho_{t+24}$	$i_{S,t}^* - i_{S,t}$	1999m1 ~ 2011m6	150	30.08	2.53
$\rho_{t+1}+\cdots+\rho_{t+36}$	$i_{S,t}^* - i_{S,t}$	1999m1 ~ 2011m6	150	54.24	4.26
$\rho_{t+1}+\cdots+\rho_{t+48}$	$i_{S,t}^* - i_{S,t}$	1999m1 ~ 2011m6	150	26.48	1.74
$\rho_{t+1}+\cdots+\rho_{t+60}$	$i_{S,t}^* - i_{S,t}$	1999m1 ~ 2011m6	150	-6.50	-0.38

Table 2. Short-term nominal rate differential vs. long-term breakeven inflation rate differential

The table reports the coefficient estimate (b) and its t statistic in the regression of Y on X. No adjustment is made in calculation of t statistic.

Y	X	t	N	b	t
$\pi_{L,t}^* - \pi_{L,t}$	$i_{S,t}^* - i_{S,t}$	1999m2 ~ 2016m6	210	-0.12	-6.00

Table 3. Long-term breakeven inflation rate differential vs. FX excess returns

The table reports the coefficient estimate (b) and its t statistic in the regression of Y on X. No adjustment is made in calculation of t statistic.

Y	X	t	N	b	t
ρ_{t+1}	$\pi_{L,t}^{B*} - \pi_{L,t}^{B}$	1999m1 ~ 2011m6	150	-17.21	-2.15
$\rho_{t+1}+\cdots+\rho_{t+12}$	$\pi_{L,t}^{B*} - \pi_{L,t}^{B}$	1999m1 ~ 2011m6	150	-152.76	-5.05
$\rho_{t+1}+\cdots+\rho_{t+24}$	$\pi_{L,t}^{B*} - \pi_{L,t}^{B}$	1999m1 ~ 2011m6	150	-265.49	-6.44
$\rho_{t+1}+\cdots+\rho_{t+36}$	$\pi_{L,t}^{B*} - \pi_{L,t}^{B}$	1999m1 ~ 2011m6	150	-185.37	-3.74
$\rho_{t+1}+\cdots+\rho_{t+48}$	$\pi_{L,t}^{B*} - \pi_{L,t}^{B}$	1999m1 ~ 2011m6	150	-33.18	-0.56
$\rho_{t+1}+\cdots+\rho_{t+60}$	$\pi_{L,t}^{B*} - \pi_{L,t}^{B}$	1999m1 ~ 2011m6	150	42.82	0.66

Table 4. Long-term breakeven inflation rate differential vs. VECM-based FX excess returns

The table reports the coefficient estimate (b) and its t statistic in the regression of Y on X. No adjustment is made in calculation of t statistic.

Y	X	t	N	b	t
$E_t^A(\rho_{t+1}+\cdots+\rho_{t+60})$	$\pi_{L,t}^{B*}$ - $\pi_{L,t}^{B}$	1999m1 ~ 2011m6	150	59.73	1.02
$E_t^B(\rho_{t+1}+\cdots+\rho_{t+60})$	$\pi_{L,t}^{B*}$ - $\pi_{L,t}^{B}$	1999m8 ~ 2015m3	188	48.09	2.16

Appendix. Details of VECM

Engel's Eq. (6) is

$$\Delta x_t = Gx_{t-1} + C_0 + C_1\Delta x_{t-1} + C_2\Delta x_{t-2} + C_3\Delta x_{t-3} + u_t$$

where

$$x_t = \begin{pmatrix} s_t \\ p_t^R \\ i_t^R \end{pmatrix}$$

and

$$G = \begin{pmatrix} g_{11} & -g_{11} & g_{13} \\ g_{21} & -g_{21} & g_{23} \\ g_{31} & -g_{32} & g_{33} \end{pmatrix}$$

The actual estimation is done via the following equation:

$$y_t = D_0 + D_1 y_{t-1} + D_2 y_{t-2} + D_3 y_{t-3} + D_4 y_{t-4} + v_t$$

where

$$y_t = \begin{pmatrix} q_t \\ \pi_t^R \\ i_t^R \end{pmatrix}$$

and

$$D_4 = \begin{pmatrix} d_{4,11} & 0 & d_{4,13} \\ d_{4,21} & 0 & d_{4,23} \\ d_{4,31} & 0 & d_{4,33} \end{pmatrix}$$

The relationship between (G, C_0, C_1, C_2, C_3) and $(D_0, D_1, D_2, D_3, D_4)$ are as follows. Define matrices F and H as

$$F = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and

$$H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Note that

$$F^{-1} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

After some tedious algebra, one can verify

$$\begin{aligned} D_0 &= FC_0 \\ D_1 &= F(C_1F_1^{-1} + GH + H) \\ D_2 &= F(C_2F^{-1} - C_1H) \\ D_3 &= F(C_3F^{-1} - C_2H) \\ D_4 &= -FC_3H \end{aligned}$$

Or

$$\begin{aligned} F^{-1}D_0 &= C_0 \\ F^{-1}D_1 &= C_1F_1^{-1} + GH + H \end{aligned}$$

$$F^{-1}\mathrm{D}_2 = C_2F^{-1} - C_1H$$
$$F^{-1}\mathrm{D}_3 = C_3F^{-1} - C_2H$$
$$\mathrm{F}^{-1}\mathrm{D}_4 = -\mathrm{C}_3H$$

To recover C_1, C_2, C_3, one requires a bit more algebra because H is nonsingular. Since

$$\mathrm{F}^{-1}\mathrm{H} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = H$$

and

$$\mathrm{HH} = \mathrm{H}$$

by pre-multiplying the three equations in the middle by H,

$$F^{-1}\mathrm{D}_1H = (C_1 + G + I)H = (C_1 + G + I)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$F^{-1}\mathrm{D}_2H = (C_2 - C_1)H = (C_2 - C_1)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$F^{-1}\mathrm{D}_3H = (C_3 - C_2)H = (C_3 - C_2)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

From the last equation,

$$\mathrm{F}^{-1}\mathrm{D}_4 = -\mathrm{C}_3H = -C_3\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The above equations determine the first and the third columns of $\mathrm{C}_1, C_2, C_3, G$.

Since

$$H(I - H) = \mathrm{H}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = 0$$

and

$$\mathrm{F}^{-1}(I - H) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = F^{-1} - H$$

when we post-multiply the same three equations by $\mathrm{I} - \mathrm{H}$,

$$F^{-1}\mathrm{D}_1(I - H) = \mathrm{C}_1(F^{-1} - H) = \mathrm{C}_1\begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$F^{-1}\mathrm{D}_2(I - H) = \mathrm{C}_2(F^{-1} - H) = \mathrm{C}_2\begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$F^{-1}\mathrm{D}_3(I - H) = \mathrm{C}_3(F^{-1} - H) = \mathrm{C}_3\begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Also, from the restriction imposed on G,

$$0 = \mathrm{G}(\mathrm{F}^{-1} - H) = G\begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The above equation determines the second column of C_1, C_2, C_3 once the first column has been determined.

Now, to obtain C_3, subtract the equation starting with $F^{-1}D_4$ from the equation starting with $F^{-1}D_3(I-H)$:

$$F^{-1}D_3(I-H) - F^{-1}D_4 = C_3F^{-1}$$

which implies

$$C_3 = F^{-1}[D_3(I-H) - D_4]F$$

To obtain C_2, add the equations starting with $F^{-1}D_3H$ and $F^{-1}D_4$, and subtract the sum from the equation starting with $F^{-1}D_2(I-H)$:

$$F^{-1}D_2(I-H) - (F^{-1}D_3H + F^{-1}D_4) = C_2(F^{-1}-H) + C_2H$$

which implies

$$C_2 = F^{-1}[D_2(I-H) - D_3H - D_4]F$$

To obtain C_1, add the equations starting with $F^{-1}D_4$, $F^{-1}D_3H$, and $F^{-1}D_2H$, and subtract the sum from the equation starting with $F^{-1}D_1(I-H)$:

$$F^{-1}D_1(I-H) - (F^{-1}D_2H + F^{-1}D_3H + F^{-1}D_4) = C_1(F^{-1}-H) + C_1H$$

which implies

$$C_1 = F^{-1}[D_1(I-H) - D_2H - D_3H - D_4]F$$

To obtain G, add the equations starting with $F^{-1}D_4$, $F^{-1}D_3H$, $F^{-1}D_2H$, and $F^{-1}D_1H$, and add the sum from the equation starting with 0:

$$0 + (F^{-1}D_1H + F^{-1}D_2H + F^{-1}D_3H + F^{-1}D_4) = G(F^{-1}-H) + (G+I)H$$

which implies

$$G = F^{-1}[D_1H + D_2H + D_3H + D_4 - I]F$$

Calculation of infinite sum is easier with this formulation. Let us re-write the equation using "big matrices":

$$\begin{pmatrix} y_t \\ y_{t-1} \\ y_{t-2} \\ y_{t-3} \end{pmatrix} = \begin{pmatrix} D_0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} D_1 & D_2 & D_3 & D_4 \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{pmatrix} \begin{pmatrix} y_{t-1} \\ y_{t-2} \\ y_{t-3} \\ y_{t-4} \end{pmatrix} + \begin{pmatrix} v_t \\ v_{t-1} \\ v_{t-2} \\ v_{t-3} \end{pmatrix}$$

Or

$$z_t = A_0 + Az_{t-1} + \omega_t$$

After "de-meaning" the variable:

$$\tilde{z}_t = A\tilde{z}_{t-1} + \omega_t$$

Then

$$\tilde{z}_{t+k} = A^k z_t + A^{k-1}\omega_{t+1} + \cdots + A\omega_{t+k-1} + \omega_{t+k}$$

and

$$E_t[\tilde{z}_{t+k}] = A^k z_t$$

$$E[\tilde{z}_t + \cdots + \tilde{z}_{t+k}] = (I + \cdots + A^k)z_t$$

$$E_t\left[\sum_{j=0}^{\infty} \tilde{z}_{t+j}\right] = (I-A)^{-1}\tilde{z}_t$$

Consider Engel's Eq. (9):

$$\widehat{\mathrm{E}}_t\left[\sum_{j=0}^{\infty}\left(\rho_{t+1+j}-\bar{\rho}\right)\right]=\zeta_\rho+\beta_\rho(\hat{r}_t^*-\hat{r}_t)+u_{\rho t}$$

From Eq. (3), the left-hand-side of the above equation is:

$$\widehat{\mathrm{E}}_t\left[\sum_{j=0}^{\infty}\left(\rho_{t+1+j}-\bar{\rho}\right)\right]=\widehat{\mathrm{E}}_t\left[\sum_{j=0}^{\infty}\left(r_{t+j}^*-r_{t+j}-\overline{r^*-r}\right)\right]-\left(q_t-\lim_{k\to\infty}E_t q_{t+k}\right)$$

Given the stationarity assumption for $\mathrm{q_t}$,

$$\widehat{\mathrm{E}}_t\left[\sum_{j=0}^{\infty}\left(\rho_{t+1+j}-\bar{\rho}\right)\right]=\widehat{\mathrm{E}}_t\left[\sum_{j=0}^{\infty}\left(r_{t+j}^*-r_{t+j}-\overline{r^*-r}\right)\right]-(q_t-\bar{q})$$

The first term in the right hand side can be obtained as follows:

$$\begin{aligned}\widehat{\mathrm{E}}_t\left[\sum_{j=0}^{\infty}\left(r_{t+j}^*-r_{t+j}-\overline{r^*-r}\right)\right]&=\widehat{\mathrm{E}}_t\left[\sum_{j=0}^{\infty}\left(i_{t+j}^*-i_{t+j}-\overline{\iota^*-\iota}\right)\right]-\widehat{\mathrm{E}}_t\left[\sum_{j=0}^{\infty}\left(\pi_{t+1+j}^*-\pi_{t+1+j}-\overline{\pi^*-\pi}\right)\right]\\&=\mathrm{e}_3{}'\left(I-\hat{A}\right)^{-1}\tilde{z}_t-e_2'\widehat{\mathrm{E}}_\mathrm{t}\left[\left(I-\hat{A}\right)^{-1}\tilde{z}_{t+1}\right]\\&=\mathrm{e}_3{}'\left(I-\hat{A}\right)^{-1}\tilde{z}_t-e_2'\left(I-\hat{A}\right)^{-1}A\tilde{z}_t\end{aligned}$$

Let us now consider the formula again paying attention to the actual data to be used in the estimation. Note first that given the restriction on D_4, we do not need the value for π_1^R. Thus, no data is lost from the re-formulation.

Let Y be the matrix of data. Let us denote the time period by $1,\cdots,\mathrm{T}$. We need matrices to represent the data excluding the first and last several rows. Let us use the notation $\mathrm{Y}_{1:\mathrm{T}-4}, Y_{2:T-3}, \mathrm{Y}_{3:\mathrm{T}-2}, Y_{4:T-1}, Y_{5:T}$ to indicate data from row 1 to row $\mathrm{T}-4$, etc. Similarly, $\mathrm{Y}_{1:\mathrm{T}-3}, Y_{2:T-2}, Y_{3:T-1}, Y_{4:T}$ to indicate data from row 1 to row $\mathrm{T}-3$, etc.

The VECM estimation is done from the following equation:

$$\mathrm{Y}_{5:\mathrm{T}}=D_0+D_1Y_{4:T-1}+D_2Y_{3:T-2}+D_3Y_{2:T-3}+D_4Y_{1:T-4}+v$$

From the estimated coefficients, we form the big matrix A:

$$\widehat{\mathrm{A}}=\begin{pmatrix}\widehat{D}_1 & \widehat{D}_2 & \widehat{D}_3 & \widehat{D}_4\\ I & 0 & 0 & 0\\ 0 & I & 0 & 0\\ 0 & 0 & I & 0\end{pmatrix}$$

Vector $\mathrm{z_t}$ can be assembled only from $\mathrm{t}=4$. So we form matrix Z as

$$\mathrm{Z}=[Y_{4:T}, Y_{3:T-1}, Y_{2:T-2}, Y_{1:T-3}]$$

Then time-series of $\widehat{\mathrm{E}}_t\left[\sum_{j=0}^{\infty}\left(r_{t+j}^*-r_{t+j}-\overline{r^*-r}\right)\right]$ for $\mathrm{t}=4,\cdots,\mathrm{T}$ can be assembled into a row vector as follows:

$$\mathrm{e}_3'\left(I-\hat{A}\right)^{-1}\tilde{Z}'-e_2'\left(I-\hat{A}\right)^{-1}A\tilde{Z}'$$

where $\tilde{Z}$ is obtained after subtracting means from Z. The time series of $(q_t-\bar{q})$ can be assembled into the following row vector

$$\mathrm{e}_1'\tilde{Z}'$$

Thus, the time-series of $\widehat{\mathrm{E}}_t\left[\sum_{j=0}^{\infty}\left(\rho_{t+1+j}-\bar{\rho}\right)\right]$ is

$$\mathrm{e}_3'\left(I-\hat{A}\right)^{-1}\tilde{Z}'-e_2'\left(I-\hat{A}\right)^{-1}A\tilde{Z}'-\mathrm{e}_1'\tilde{Z}'$$

The time-series of $(\hat{r}_t^*-\hat{r}_t)$ is

$$\mathrm{e}_3'\tilde{Z}'-e_2'A\tilde{Z}$$