A New Method for Better Portfolio Investment

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ABSTRACT:

In this study, a method is devised to estimate a correlation matrix capable of constructing a well-diversified portfolio by the Markowitz mean-variance (MV) optimization function (MVOF), after which evidence is presented to empirically prove that the proposed method effectively reduces the degree of sensitivity of portfolio output caused by the error of input variables, such as the mean and standard deviation of stocks. The proposed method removes the property of the market factor included in the sample correlation matrix. The results demonstrate the comparative advantage of the proposed method in effectively reducing the degree of the sensitivity on both the estimation error in the past period and the prediction error in the future period from the mean and standard deviation of stocks. In particular, this comparative advantage is strongly dependent on the striking reduction of portfolio risk gained by constructing the well-diversified portfolio. The proposed method also contributes to high investment performance in the domain of risk-return relation, and especially, to be much stronger in the unstable situation of a market crash and a portfolio having higher risk. The contribution in this study is a new insight on how to enhance the practical applicability of the Markowitz optimum theory by controlling the property of the market factor in the sample correlation matrix.

Keywords: Mean-variance portfolio optimization, Correlation matrix, Random matrix theory, Sensitivity test, Simulation experiment.

JEL classification: G11; G17; C10

1. INTRODUCTION

The Markowitz (1952) mean-variance optimization function (MVOF) provides a method that can quantitatively determine the investment weight of stocks constructing a portfolio in the domain of risk-return relation. The MVOF can be used to construct an efficient portfolio satisfying both the risk-return investment rule and well-diversified investment weights. The risk-return investment rule provides an investment opportunity set composed of various combinations between portfolio risk and portfolio return for investors. The portfolio generated by the MVOF is an efficient portfolio having the minimum risk for given return or the maximum return for given risk. Markowitz (1952) emphasized that an efficient portfolio is strongly dependent on evenly distributing investment weights for stocks in a portfolio, rather than the increasing number of stocks in a portfolio. On the other hand, many practical investors have hesitated to utilize the MVOF as a tool for determining the allocation of investment weight for stocks in a portfolio, in spite of the theoretical and the scientific robustness. This is because of the practical problem of the MVOF, which biasedly distributes most investment weight into certain stocks in a portfolio. Therefore, the recent studies related to the Markowitz optimum theory have focused on uncovering an alternative method capable of overcoming the practical problem of the MVOF. The purpose of this study follows from these research efforts.

Michaud (1989), Best & Grauer (1991), and Jorion (1992) argued that the MVOF tends to ignore the error of input variables, and further amplifies the influence of the error into the investment weight of stocks in a portfolio as the output. The MVOF determines the amount to be invested in each stock during the future period using estimates of input variables in the past period, such as the mean and standard deviation of stocks and the correlation matrix between stocks. The input variables that are estimated in the past period cannot avoid the possibility of the error because it is not a true value. Michaud (1989) mentioned that the MVOF may produce a harmful allocation decision rather than a good allocation decision if the error of input variables is not adjusted. The MVOF has a tendency to excessively distribute investment weight into stocks having higher mean, lower standard deviation, and lower correlation matrix with other stocks, due to the objectives function for minimizing portfolio risk or maximizing portfolio return. Hence, the investment weight that is biasedly distributed by the preference of the MVOF may amplify the impact of the error of input variables on the portfolio output in the future period. Michaud (1989) described the MVOF as an error maximizer, based on the financial instabilities and defects that cause excessively big changes of output on the small error of input variables. Best & Grauer (1991) and Jorion (1992) empirically examined the sensitivity of the MVOF on the stock mean. In here, the sensitivity represents the magnitude of changes in portfolio output generated from the MVOF, such as portfolio return and risk as well as investment weight for each stock. The stock mean indicates the estimate in the past period as a role of the expected return of stock in the future period. According to their results, the portfolio output from the MVOF is very sensitive to the error of the stock mean estimated in the past period, in particular, when allowing the condition of short-sale, which is evidence to support the error maximizer asserted by Michaud (1989). On the other hand, the practical problem mentioned in the previous studies might be explained based on the characteristic of the sample covariance matrix, which is composed of both the standard deviation of stocks and the correlation matrix between stocks, as one of the input variables in the MVOF. The sample covariance matrix having a component of the correlation matrix among stocks must be the measurement to quantify the degree of relationship on the return changes of stocks. The pricing model in the field of finance explains the stock return changes as a compensation for the systematic risk of the common factor; i.e., the stock return changes can be explained by the common factor so that the sample correlation matrix contains the information of stock return changes related with the common factor. The stocks that are strongly influenced by the common factor contain many properties of the common factor, and so the correlation between these stocks has higher value, whereas the stocks that are dependent on the firm-specific property, rather than the property of common factor, have lower correlation with each other. The preferential tendency of the MVOF excessively distributes most investment weight into the stocks having a lower correlation with other stocks. After all, the biased distribution of investment weight for stocks generated from using the sample correlation (covariance) matrix may be the main source that amplifies the sensitivity of the MVOF on the error of the input variables.

Chan et al. (1999) and Ledoit & Wolf (2003, 2004) focused on the covariance matrix in order to improve the practical problem of the MVOF. In particular, the portfolio output from the perspective of out-of-sample in the future is empirically investigated. The method of these studies determines the investment weight for stocks by focusing on the role of the covariance matrix having sufficient properties of common factors in the MVOF. Chan et al. (1999) utilize the covariance matrix estimated from the factor models using the statistical factors and the fundamental factors. That is, the covariance matrix is calculated using regression coefficients (factor loadings) from the factor models. According to the results, the portfolio output of the MVOF using the covariance matrix that strongly reflects the properties of common factors has a lower degree of sensitivity on the prediction error of the input variables in the future period, compared to using the sample covariance matrix. Ledoit & Wolf (2003, 2004) proposed the shrinkage method that generates the weight-averaged covariance matrix between the sample covariance matrix and the estimated covariance matrix based on the research goal. The shrinkage constant as a weighting ratio is determined by minimizing the difference between estimated value and the true value of the covariance matrix. By the shrinkage method in Ledoit & Wolf (2003), a weighing of 70~80% is given to the covariance matrix that reflects the property of the common factor, on average. Therefore, the shrinkage method adjusts the weight in order to limit the impact of firm-specific properties of stocks in the covariance matrix and to simultaneously increase the influence of the property of the common factor in the covariance matrix. Ledoit & Wolf (2003, 2004) presented empirical evidence revealing that the sensitivity of the MVOF from the prediction error is reduced when using the shrinkage method based on the covariance matrixes estimated by the single-factor model of Sharpe (1963) and the constant correlation method of Elton & Gruber (1973).

This study aims to devise a method to estimate a correlation matrix capable of effectively reducing the influence of the error from the mean and standard deviation of stocks on the MVOF by constructing a well-diversified portfolio, and then to empirically investigate whether the proposed method substantially overcomes the practical problem of the MVOF. Generally, the practical applicability of the MVOF is expected in terms of the stability of portfolio performance from the passive investment strategy by asset allocation, rather than the profitability from the active investment strategy by prediction and selection. Hence, investors expect to be able to construct a well-diversified portfolio by using the investment weight for stocks generated by the MVOF. However, as mentioned in Michaud (1989), the MVOF using the sample correlation matrix induces a practical problem that biasedly distributes most investment weight into some stocks in a portfolio. In other words, the traditional MVOF using the sample correlation matrix cannot construct a well-diversified portfolio, contrary to our expectation. Therefore, establishing a method capable of constructing a well-diversified portfolio will give new insight into research efforts for overcoming the practical problem of the MVOF.

The method for estimating the correlation matrix proposed by this study removes the property of the market factor included in the sample correlation matrix. The role of the market factor in this study is defined as follows. The market factor is a representative common factor that plays a crucial role in explaining the stock return changes in both the covariance matrix and the pricing models, while simultaneously acting as a common factor that exerts a dominant influence preventing the MVOF from constructing the well-diversified portfolio. In addition, the rationale to devise the method removing the property of the market factor included in the sample correlation matrix is as follows. First, Elton & Gruber (1973) showed that the method that adjusts the magnitude of the correlation between stocks overcomes the practical problem occurring when determining the investment weight of stocks using the sample correlation matrix. The constant correlation method proposed by Elton & Gruber (1973) is a typical method that adjusts the magnitude of the correlation between stocks. That is, the method that adjusts the magnitude of the correlation matrix by constant correlation method prevents the defect from biasedly distributing most investment weight into several stocks having lower correlation with other stocks. Second, the general consensus in portfolio theory, i.e., the portfolio efficiency increases through the well-diversified investment of stocks such that the correlation among stocks in a portfolio decreases, may be realized by removing the property of the market factor from the sample correlation matrix. The market factor is an important factor that commonly explains the changes of stock return in the pricing model. Chan et al. (1999) mentioned that the covariance matrix estimated by the multi-factor model having many common factors cannot much further reduce the degree of sensitivity of the MVOF caused from the error of input variables, compared to using the covariance matrix estimated by the single-factor model with the market factor. Eom et al. (2015) presented empirical evidence that the method of removing the property of market factor from the sample correlation matrix can improve the defect of the portfolio diversification effect when market crashes occur in Korea and Japan. Recent researches of Eom & Park (2017) and Eom (2017) show evidence that common factors in the factor models significantly affect the determination of the investment weight's distribution for stocks from the MVOF, and a market factor plays a dominant role in constructing the well-diversified portfolio with better out-of-sample performance in the future period. Accordingly, based on previous studies, the dominant influence of market factor in the sample correlation matrix is deeply related to the practical problem that biasedly distributes most investment weight to some stocks in a portfolio from the MVOF.[1] On the basis of the aforementioned rationale, we expect that the method of adjusting the magnitude of correlation matrix by removing the property of the market factor from the sample correlation matrix will construct a further well-diversified portfolio, and thus will be able to reduce the degree of the sensitivity of the portfolio output caused by the error of the mean and standard deviation of stocks.

This study establishes two cases of the estimation error in the past period and the prediction error in the future period from the mean and standard deviation of stocks in a portfolio, in order to obtain robust evidence supporting the improvement gained by the proposed method on the practical problem of the MVOF. That is, by focusing on the method of the correlation matrix, the error from the mean and standard deviation of stocks is utilized as the error of the input variables. The impact of the estimation error of the input variables in the past period on the portfolio output from the MVOF is designed based on Best & Grauer (1991) and Jorion (1992). The influence of the prediction error of the input variables in the future period on the portfolio output is designed based on Chan et al. (1999) and Ledoit & Wolf (2003, 2004). We devise an error generator that creates the data having errors in each of the mean and standard deviation of stocks based on Best & Grauer (1991) and Jorion (1992), for the purpose of observing the sensitivity from the error of input variables through the simulation experiment. Best and Grauer (1991) investigated the sensitivity of portfolio output from the MVOF from the viewpoint of the number of stocks selected randomly in a portfolio. In here, the randomly selected stock indicates the stock having an error in the stock mean, and the number of stocks ranges from 1 to 50% among all stocks in a portfolio. Jorion (1992) utilized the difference between portfolio outputs from using the true value and the false value of stock mean as the degree of the sensitivity from the error of input variables. In here, the true value is the stock mean calculated within the given past period, and the false value is the stock mean calculated from the period having a different length in the given past period. Therefore, this study utilizes the number of stocks in a portfolio based on Best & Grauer (1991) as the range of the error, and the mean and standard deviation of stocks created based on Jorion (1992) as the true value and the false value. On the other hand, based on the method proposed herein, the main comparative method is the correlation matrix estimated using the single-factor model with the market factor by Sharpe (1963), the usefulness of which is proved by Chan et al. (1999) and Ledoit & Wolf (2003, 2004). The correlation matrix from the single-factor model has only the property of the market factor. Hence, this method contrasts decidedly with the proposed method of removing the property of the market factor from the sample correlation based on Eom et al. (2015), Eom & Park (2017), and Eom (2017). Through these opposite methods, the effect of the market factor on the construction of the well-diversified portfolio as well as the sensitivity from the input variables' error in the MVOF can be determined. In addition, to obtain robust results, we employ the 3-type stock group (all stocks, large stocks, and small stocks group) and 2-type portfolio (global minimum variance portfolio (GMVP), and tangency portfolio (TP)), along with various methods for estimating correlation matrix based on previous studies.

The results are summarized as follows. Regardless of portfolio types and stock group types, the proposed method of removing the property of the market factor from the sample correlation matrix clearly much further reduces the degree of sensitivity of the MVOF caused by the estimation error of the mean and standard deviation of stocks in the past period, compared to the other method based on previous studies. In addition, as in Michaud (1989), the MVOF using the sample correlation matrix produces much higher

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[1] The additional explanation is as follows. The stock returns are certainly affected by changes in the market situations. Hence, the property of the market factor must be a common part for all stocks. The stocks that are strongly affected by the market factor have a higher correlation with other stocks, while the stocks that are weakly affected by the market factor have a lower correlation with other stocks. Because of the preferential tendency of the MVOF for the input variables, the MVOF tends to excessively distribute investment weight into stocks having a lower correlation with other stocks, while distributing small or no investment weight into stocks having a higher correlation with other stocks. Therefore, the correlation matrix must be one of the main causes of the practical problem of the MVOF, and also the market factor having the dominant influence on the sample correlation matrix may be deeply related to the practical problem of the MVOF.

sensitivity of the portfolio output on the input variables' error. Under a certain condition without any prediction error from the mean and standard deviation of stocks in the future period, the proposed method has a higher degree of diversification and a higher magnitude of investment performance by the Sharpe ratio, compared to the other correlation matrixes. In particular, these results show that the diversification degree for stocks in a portfolio obviously has a positive relation with the magnitude of investment performance in the domain of risk-return relation. On the other hand, under condition of uncertainty with prediction error from the mean and standard deviation of stocks in the future period, the proposed method has a lower degree of the sensitivity of the MVOF caused by the prediction error of input variables. In particular, the small stocks group has a greater comparative advantage using the proposed method, compared to the large stocks group. These results are evidence supporting that the method of removing the property of the market factor from the sample correlation improves the ability of allocation to construct a well-diversified portfolio, and through this improvement, the proposed method generates a lower degree of sensitivity caused by input variables' error, compared to the other method for estimating the correlation matrix. As a result, the proposed method must be an alternative method capable of enhancing the practical applicability when applying the MVOF into the practice. Therefore, this study differs from previous studies based on academic and practical contributions to give new insight into research efforts for improving the practical applicability of the Markowitz optimum theory.

The rest of this paper is organized as follows. Section 2 introduces the data and main methods. Section 3 presents the results of the sensitivity test of the MVOF caused by the estimation error in the past period and the prediction error in the future period using the proposed method and the existing methods. The main findings are also discussed. Section 4 summarizes and concludes.

2. EMPIRICAL DESIGN

2.1. Data and Periods

This study utilizes stocks traded in the Korean stock market during the period from July 2003 to June 2014. Period setting and stock selection are based on Fama & French (1993). Hence, stocks are selected according to two conditions: the stocks belonging to the financial industry are excluded, and the stocks having only the fiscal month of December are included. Market data and accounting data are utilized in the test procedure. The market data are stock returns, return of market index (KOSPI), and risk-free rate (interest rate of monetary stabilization bond). The accounting data are selected for generating the common factors based on Fama & French (1993). We utilize the following 7 common factors to generate the correlation matrixes: a market factor, a size factor, and a value factor by Sharpe(1964) and Fama & French(1993), a momentum factor by Jagedeesh & Titman(1993) and Carhart(1997), short- and long-term reversal factors by Jegadeesh(1991) and DeBondt & Thaler(1985), and a liquidity factor by Amihud(2002). The method to generate each common factor is based on the Kenneth R. French's data library. In addition, according to the research goal, we divide the type of the sensitivity into a case of using the data of past period and a case of using out-of-sample data of future period. The first sensitivity test on the estimation error from the mean and standard deviation of stocks in the past period utilizes N=389 stocks having all price information over period from July 2003 to June 2014, based on Best & Grauer (1991) and Jorion (1992). On the basis of Chan et al. (1999) and Ledoit & Wolf (2003, 2004), the second sensitivity test on the prediction error from the mean and standard deviation of stocks in the future period uses stocks having all price data in each of 22 sub-periods established by roll-sampling method over period from January 2009 to June 2014. The sub-period is composed of 60 months as the period estimating input variables of the MVOF and 3 months as the future investment period to apply the investment weight of stocks generated in the past period. The length of the moving period is 3 months for non-overlapping with future investment period. For example, the first sub-period is from January 2004 to March 2009, the second sub-period from April 2004 to June 2009, and the last sub-period from April 2009 to June 2014. The number of stocks in each sub-period ranges from 395 to 488. According to Banz (1981), the firm size is an important influence factor for the changes of stock return. Therefore, we utilize the 3-type stock group divided by the market capitalization of firms, which is calculate by multiplying the stock price and the number of outstanding shares, i.e., all stocks group, large stocks group (top 40%), and small stocks group (bottom 40%).

2.2. Methods

This section introduces the three main methods of the MVOF, the estimation of the correlation matrix, and the error generator. The first method is the traditional MVOF proposed by Markowitz (1952). The MVOF utilizes the input variables of the expected return of stocks ($\bar{R}_i$) and covariance matrix between stocks ($\sigma_{i,j}$) in a portfolio, and the covariance is composed of the standard deviation of stock returns (σ_i, σ_j) and a correlation matrix between stocks ($\rho_{i,j}$).

$$\sigma_p = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{i,j}} = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_i \sigma_j \rho_{i,j}} \quad (1)$$

condition 1: $R_p = \sum_{i=1}^{N} w_i \bar{R}_i$

condition 2: $\sum_{i=1}^{N} w_i = 1.0$

condition 3: $w_i \geq 0$

where w_i indicates the investment weight of stock i in a portfolio. We generate the investment weight for each stock under the condition of short-sale not allowed for the practical perspective. Therefore, the MVOF in equation (1) does not include additional constraint conditions to adjust the investment weight for stocks.[2] The portfolio output generated by the MVOF considers the following two points. In here, portfolio output means the portfolio return, the portfolio risk and investment weight for each stock in a portfolio. First, we observe the results from the two types of portfolio, that is, the GMVP and the TP in the investment opportunity set extracted by the MVOF.[3] Second, we report the average values of portfolio output observed through simulation experiment. The portfolio is constructed by 50 stocks. The MVOF generates the portfolio output from the simulation experiment using 100 cases of portfolio constructed by the 50 stocks randomly selected by the sampling without replacement. Then, the average values of all portfolio outputs are reported as the result in the tables and figures. In addition, this study utilizes the 2-type measurement to quantify the distribution level of investment weight of stocks in portfolio, i.e., the Herfindahl index (HI) and the number of stocks having non-zero investment weight (NZ) utilized by Lvkovic et al.(2008) and Eom et al. (2015), as follows.

[2] As additional constraints in the MVOF, Chan et al. (1999) and Ledoit & Wolf (2004) set the upper limitation of investment weight of 2% and 10%, respectively. However, as confirmed in Table 4 of Chan et al. (1999), the diversification degree for stocks having non-zero investment weight in a portfolio is only 18~24%. This means that it is difficult to improve the degree of diversification for stocks through the additional constraint conditions for investment weight based on the subjective criterion.

[3] The characteristics of GMVP and TP are described as follows. GMVP is a portfolio generated by the MVOF having the objectives function of minimizing risk using N stocks as a risk asset, and has the highest level of diversification and the lowest magnitude of risk in the portfolio investment opportunity set. On the other hand, TP is a portfolio generated by the MVOF having the objectives function of maximizing portfolio performance (Sharpe ratio) using N stocks and a risk-free asset, and has the highest degree of portfolio performance and the highest magnitude of risk under the condition of short-sale not being allowed.

$$HI = \sum_{j=1}^{N} w_j^2 \quad (2)$$

$$\text{NZ} = \sum_{j=1}^{N} nz_j \begin{cases} nz_j = 1, w_j > 0 \\ nz_j = 0, w_j = 0 \end{cases} \quad (3)$$

The HI in equation (2) represents the degree of concentration of investment weighting into stocks in a portfolio. For example, if the portfolio is constructed by method of equal-weighting scheme ($w_i = 1/N$), the degree of diversification is $HI = (1/50)^2 = 0.0004$. The evaluation criterion is that a smaller HI means a lower degree of concentration of investment weight for stocks in a portfolio and a higher level of diversification. The NZ in equation (3) is a measurement that can intuitively verify the distribution level of investment weight for stocks in a portfolio under the condition of short-sale not allowed. For example if the portfolio is constructed by the method of equal-weighting scheme, the NZ has the same number of stocks in a portfolio, that is, NZ =50. The evaluation is that the degree of diversification will be higher as the NZ increases.

Next is a method to estimate the correlation matrix among stocks. Our proposed method estimates the correlation matrix by removing the property of the market factor from the sample correlation matrix. Based on Plerou et al. (2002), Tola et al.(2008), Bai et al.(2010), and Eom (2017), we employ the random matrix theory (RMT, Metha (1991)) that can mathematically control the properties included in the correlation matrix, in particular, in order to control the largest eigenvalue having the property of the market factor, as reported in previous studies.[4] According to Sengupta and Mitra (1999), as the number of stocks increases ($N \rightarrow \infty$) and the length of time series data increases ($L \rightarrow \infty$) in eigenvalue distribution on the correlation matrix among stocks, the probability density function ($P_{RM}(\lambda)$) of eigenvalue (λ) in random correlation matrix is as follows.

$$P_{RM}(\lambda) = \frac{Q}{2\pi} \frac{\sqrt{(\lambda_{+}^{RM} - \lambda)(\lambda - \lambda_{-}^{RM})}}{\lambda}, \quad (\lambda_{\pm}^{RM} = 1 + \frac{1}{Q} \pm 2\sqrt{\frac{1}{Q}},\ Q \equiv \frac{L}{N}) \quad (5)$$

In this equation, the eigenvalues belonging in the random correlation matrix is within the range of $\lambda_{+}^{RM} \geq \lambda \geq \lambda_{-}^{RM}$. As confirmed in Plerou et al. (2002) and Eom et al. (2009), the eigenvalues ($\lambda > \lambda_{+}^{RM}$) deviating from the maximum eigenvalue (λ_{+}^{RM}) have the economic meanings of the common factor that may explain the stock return changes. In particular, the largest eigenvalue has the property of the market factor regardless of the sample size selected in the stock market, based on King(1966), Brown(1989), Chen et al. (1986), and Eom et al. (2015). Therefore, through the RMT, the correlation matrix removing the property of the market factor from the sample correlation matrix is calculated using the remaining eigenvalues (λ_m, $m = 2,3,4, \dots N$) and their eigenvectors (EV_m) after excluding the largest eigenvalue ($\lambda_{m=1}$) and its eigenvector ($EV_{m=1}$), as follows.

$$\rho_{m=1} = \lambda_m\, EV_m EV_m', \quad (\text{m} = 2,3,4, \dots \text{N}) \quad (6)$$

In here, the correlation matrix of equation (6) is utilized as an input variable of the correlation matrix ($\rho_{m=1} \equiv \rho_{i,j}$) in the MVOF of equation (1). In order to save space in this paper, we henceforth name this correlation matrix to the non-market correlation matrix. Meanwhile, the existing methods of correlation matrix that are compared with the proposed method are as follows. First, the sample correlation matrix that causes the practical problem of the MVOF reported by Best & Grauer (1991) and Jorion (1992) is utilized. Second, the correlation matrix calculated from the single-factor model with the market factor based on

[4] Another method to remove the property of the market factor in correlation matrix is simple regression that has a dependent variable of stock return and an independent variable of a market factor. That is, the correlation matrix without the market factor's property is calculated using the residuals from the simple regression. However, this method using the simple regression may introduce an error in the middle of the estimating regression coefficients. Therefore, the RMT has the advantage of mathematically generating the correlation matrix without the possibility of error that occurs with the simple regression.

Sharpe (1963) is utilized, i.e., the correlation matrix having only the property of the market factor (hereafter, the market correlation matrix), Third, the correlation matrix calculated by the constant correlation method of Elton & Gruber (1973) that adjusts the magnitude of the correlation between stocks with the overall average value of all off-diagonal elements in the sample correlations matrix is utilized (hereafter, the constant correlation matrix). Fourth, the correlation matrix that is calculated by the three- and seven-factor models based on Chan et al. (1999) is utilized, i.e., the correlation matrix having the properties of market, size and value factors from Fama & French (1993) (hereafter, 3-factor correlation matrix); and the correlation matrix having the properties of momentum, short-term reversal, long-term reversal, and liquidity factors, along with three-factors of Fama & French (1993) (hereafter, 7-factor correlation matrix). Fifth, based on the shrinkage method proposed by Ledoit and Wolf (2003, 2004), we utilize the shrinkage correlation matrix calculated by weighted averaging between the sample correlation matrix and the estimated correlation matrix mentioned above. Hence, we investigate the research goal using an 11-type correlation matrix.

Finally, we introduce the error generator that artificially includes the error in the mean and standard deviation of stocks as input variables of the MVOF. The error generator considers the two points from the viewpoint of range and value of the error based on Best & Grauer (1991) and Jorion (1992). The first consideration is the range of the error. Best & Grauer (1991) used the number of stocks in a portfolio within the range from one stock to 50% of all stocks. Accordingly, the error range in this study is employed by 4-type number of 1, 5, 10, and 25 stocks that are randomly selected among 50 stocks in a portfolio. The second consideration is the value of the error. Jorion (1992) used the mean and standard deviation of stocks calculated from the given period as the true value, and the mean and standard deviation of stocks calculated from the period that has a different length within the given period as the false value having the error. In addition, we separately utilize a case of the estimation error in the past period and a case of the prediction error in the future period. In the case of the estimation error in the past period, the true value is the mean and standard deviation that are calculated using the monthly stock return in the whole period (120 months) from July 2003 to June 2014, while the mean and standard deviation that are calculated using the monthly stock return in the period having different lengths (84 months, 96 months, and 108 months) within the given whole period is the false value having the estimation error. In here, the total types of estimation error in accordance with periods having the different length are 9. The 11-type correlation matrix has no estimation error because of using the data in the given whole past period. In the case of the prediction error in the future period from January 2009 to June 2014, the true value is the mean and standard deviation that are calculated using daily stock returns in the 3-month future investment period in each sub-period, while the mean and standard deviation calculated using daily stock returns of 75 trading days in the past period (60 months) of each sub-period is the false value having the prediction error. We utilize the daily stock returns to obtain the stable input values from a statistical perspective. The mean and standard deviation of stocks calculated using daily return data are converted into the periodic values for 3 months before applying into the MVOF. The total types of prediction error using 75 trading days in the past period are 12. The 11-type correlation matrix has a prediction error because of using the data in past period of 60 months, not in the future period. Therefore, the sensitivity for the 9-type estimation error iterates simulations of 9,900 times (=11×100×9), and the sensitivity for the 12-type prediction error iterates simulations of 12,000 times (=11×100×12) in each 22-type sub-period. The estimation error and the prediction error created by the error generator are applied to the simulation experiment according to correlation matrix methods in each case of 2-type portfolio (GMVP and TP) and 3-type stock group (all stocks, large stocks and small stocks).

3. RESULTS

This section presents the results for empirically examining the sensitivity on the portfolio output of the MVOF caused by the error of input variables from the mean and standard deviation of stocks, in accordance with methods for estimating the correlation matrix. The results for the sensitivity are divided into the two cases of the estimation error in the past period and the prediction error in the future period. Then, the results are discussed.

3.1. Sensitivity on the estimation error in the past period

The results for the sensitivity from the estimation error of input variables in the past period according to 11 methods of the correlation matrix are presented in this section. First of all, the results of the preliminary test on the 11-type method of correlation matrix are presented in **Figure 1**. The distribution on the average value and standard deviation of all off-diagonal elements in each correlation matrix through the simulation experiment is shown in **Figure 1(a)** and **Figure 1(b)** using the box-plot method, respectively. The X-axis indicates the 11-type correlation matrix, as follows. First, the sample correlation matrix (SM), second, the market correlation matrix and its shrinkage correlation matrix (MC, ShMC), third, the non-market correlation matrix and its shrinkage correlation matrix (NC, ShNC), fourth, the constant correlation matrix and its shrinkage correlation matrix (CC, ShCC), fifth, the 3-factor correlation matrix and its shrinkage correlation matrix (3FC, Sh3FC), and finally, the 7-factor correlation matrix and its shrinkage correlation matrix (7FC, Sh7FC).

[Insert **FIGURE 1** about here.]

From **Figure 1(a)**, the average value from the non-market correlation matrix shows the smallest value among all correlation matrixes. The difference between the average values of the non-market correlation matrix and the market correlation matrix has a high value of 18.82%. This difference indicates that the market factor must be a common factor that exerts a dominant influence on all stocks. The dominant influence of the market factor is also verified by comparison with the 3-factor and 7-factor correlation matrixes. In other words, even though the multi-factor models are more reflective of the property of additional common factors compared to the single-factor model of the market factor, the average value from these correlation matrixes has only value of 4.7% higher than the market correlation matrix. The shrinkage correlation matrix has a higher value than the original correlation matrix. This is because the shrinkage method calculates the weight-averaged correlation matrix using the sample correlation matrix having a higher average value. Next, according to the results of **Figure 1(b)**, the standard deviation from the non-market correlation matrix has a much narrower distribution compared to other correlation matrixes. The constant correlation matrix has a zero standard deviation because all off-diagonal elements of the correlation matrix have the same value by the overall average value. The shrinkage correlation matrix has a higher value and a wider distribution, compared to the original correlation matrix. This is because that the shrinkage method calculates the weight-averaged correlation matrix using the sample correlation matrix having a higher standard deviation. **Figure 1** clearly verifies the existence of the difference in the average value and the standard deviation according to the 11-type correlation matrix. The magnitude of the correlation matrix directly affects the portfolio risk measured by the covariance matrix among stocks. Therefore, we report the standardized portfolio risk, i.e., the result reported in the tables and figures can be interpreted as evidence of the method's own comparative advantage. In here, the standardized portfolio risk is calculated by subtracting the average value and dividing it by the standard deviation.

For the methods of the correlation matrix, the results of the sensitivity on the portfolio output of the MVOF caused by the estimation error from the mean and standard deviation of stocks in a portfolio are presented in **Tables 1** and **2**. The main empirical design is briefly as follows. We utilize the monthly return for each of the 3-type stock group such as all stocks (N=398), large stocks (top 40%, N=155), and small stocks (bottom 40%, N=155) during the past period from July 2003 to June 2014. The 2-type portfolio of the GMVP and the TP is utilized. The true value without estimation error is the mean and standard deviation that are calculated using monthly stock returns in the past period of 120 months from July 2003 to June 2014. The false value with estimation error is the mean and standard deviation that are calculated using monthly stock returns in the period having different lengths (84, 96 and 108 months) within the given whole past period. The sensitivity is calculated based on the difference between portfolio outputs generated from using the true value and the false value, and the measurement of the sensitivity is the mean absolute error (MAE) and the root mean square error (RMSE). **Table 1** presents a comparison of the main methods such as the sample correlation matrix, the market correlation matrix, the non-market correlation matrix and their shrinkage correlation matrixes. In the table, the left-column part (*Error Mean*) shows the sensitivity due to the estimation error from the stock mean, while the right-column part (*Error St.Dev.*) shows the sensitivity due to the estimation error from the stock standard deviation. The results of portfolio output (*return, risk, and weight*) are divided by the two types of the GMVP and the TP. The sensitivity of the portfolio output on the investment weight is reported by the MAE from the HI measurement. The three types of stock group are divisively presented in all stocks group of Panel A, large stocks group of Panel B, and small stocks group of Panel C. Each panel in the table contains the results for each error range of 1, 5, 10, and 25 stocks.

[Insert **TABLE 1** about here.]

From **Table 1**, regardless of portfolio types and stock group types, as the error range increases from 1 stock to 25 stocks, the degree of sensitivity of the portfolio output caused by the estimation error from the mean and standard deviation of stocks clearly increases. The sensitivity on the estimation error from the stock mean is higher than the case of estimation error from the stock standard deviation. The case of using the sample correlation matrix shows much higher sensitivity, compared to the other correlation matrixes. That is evidence to support Michaud (1989), who named the MVOF as the error maximizer. On the other hand, the sensitivity from the non-market correlation matrix has a smaller value in the estimation error from input variables, compared to the sample correlation matrix and the market correlation matrix. The results in detail are as follows. In the results for the GMVP of **Table 1**, all the values of the MAE for the portfolio risk and the investment weight of stocks due to the estimation error from the stock mean are zero. This is because the GMVP for calculating the investment weight of stocks is not affected by the changes of the stock mean at all. In the results of the GMVP due to estimation error from the stock standard deviation, the non-market correlation matrix has a smaller MAE in the portfolio return and the investment weight, whereas the case of portfolio risk shows a smaller MAE in the shrinkage correlation matrix using the non-market correlation matrix. For example, in the comparison of the sensitivity on the portfolio return of the GMVP due to the estimation error from the stock standard deviation in the case of the highest range of error (25 stocks), the non-market correlation matrix has a very small value of 25.29% of the MAE from the sample correlation matrix, and 28.18% of the MAE from the market correlation matrix. In the results on the TP due to the estimation error from the mean and standard deviation of stocks, the non-market correlation matrix shows a smaller value of the sensitivity in all cases of portfolio output. For example, in the comparison of the sensitivity on the portfolio return of the TP in the case of the highest error range (25 stocks), the sensitivity from the non-market correlation matrix has a the smaller value of 55.82% and 23.20% of the MAE from the sample correlation matrix, and 56.15% and 25.91% of the MAE from the market correlation matrix in cases of estimation error from the mean and standard deviation of stocks, respectively. In the results from large stocks group of Panel B, the GMVP shows that the sensitivity from the market correlation matrix has a smaller value of the MAE in the lower level of the error range, compared to the results of Panel A. However, as the error range increases to 25 stocks, the non-market correlation matrix shows a smaller value of the MAE, compared to the market correlation matrix. The small stocks group in Panel C, compared to Panel A and B, shows strong evidence that the non-market correlation matrix has the smallest sensitivity, compared to the other correlation matrixes.

To check the robustness of the results of **Table 1** that the non-market correlation matrix has a comparative advantage in effectively reducing the degree of the sensitivity on the estimation error from the input variables in the past period, **Table 2** presents the results based on the additional methods of the correlation matrix, such as the constant correlation matrix, the 3-factor correlation matrix, the 7-factor correlation matrix, and their shrinkage correlation matrixes, along with results from the non-market correlation matrix of **Table 1**. This table shows only the results for all stocks group in the case of the highest error range (25 stocks). In the table, the results are divided according to the 2-type portfolio (*GMVP, TP*), the 2-type input variable error (*Error Mean, Error St.Dev.*), and the 3-type portfolio output (*return, risk, weight*).

[Insert **TABLE 2** about here.]

According to the results, the non-market correlation matrix also has a smaller degree of sensitivity from input variables' error when comparing with the additional methods to estimate the correlation matrix. The results for the larger stocks group and the small stocks group that are not reported in this paper are not qualitatively different with results observed in **Table 2**. The results of **Tables 1** and **2** present evidence supporting that the proposed method of the non-market correlation matrix has a comparative advantage of effectively reducing the degree of sensitivity of the MVOF caused by the estimation error from input variables in the past, compared to the other correlation matrixes.

3.2. Sensitivity on the prediction error in the future period

Next, the results for the sensitivity on the prediction error from input variables in the future period in accordance with the 11-type correlation matrix are presented. The main empirical design is briefly as follows. The 22 sub-periods established within the given future period from January 2009 to June 2014 are utilized. Each sub-period is composed of the 60-month past period to predict input variables and to apply them into the MVOF and the 3-month future investment period to apply the investment weight for each stock generated in the past period into the actual data in the future period. In all sub-periods, all stocks group includes the number of stocks within the range from 395 to 488, and the large stocks group (top 40%) and the small stocks group (bottom 40%) include 158~195 stocks, respectively. The sensitivity test in the future period is divided into a case of a certain condition without prediction error from the input variables and a case of an uncertain condition with prediction error from the input variables. The certain condition indicates no prediction error in the mean and standard deviation of stocks in the future investment period, that is, $\mathrm{E}(R_j) = R_j$ and $\mathrm{E}(\sigma_j) = \sigma_j$, whereas the uncertain condition indicates that the mean and standard deviation of stocks have the prediction error, that is, $\mathrm{E}(R_j) \neq R_j$ and $\mathrm{E}(\sigma_j) \neq \sigma_j$. The test procedure in each case is as follows. The true value without prediction error under certainty is the mean and standard deviation that are calculated using stock returns in the future investment period of 3 months. The correlation matrix is estimated in the past period of 60 months. The investment weight for each stock through the MVOF using the true value of the mean and standard deviation of stocks as well as the correlation matrix is generated. And finally, the portfolio output in the future period is calculated by applying the investment weight for stocks in the past period into the actual stock means and the actual covariance matrix between stocks that are calculated in the future investment period of 3 months.[5] On the other hand, the false value with the

[5] Using the investment weight ($\boldsymbol{w_j}$) of stocks from the MVOF in the past period, the method to calculate the portfolio output in the

prediction error under uncertainty is the mean and standard deviation that are repeatedly calculated using stock returns in the past period of 75 trading days within 60 months. The correlation matrix is estimated in the past period of 60 months. The investment weight from each stock through the MVOF using the false value of the mean and standard deviation of stocks, as well as the correlation matrix, is generated. In here, we create the 12-type prediction error of the mean and standard deviation of stocks in the past period. Finally, the portfolio output in the future period is calculated by applying the investment weight for each stock generated in the past period into the actual stock means and the actual covariance matrix between stocks that are calculated in the future investment period of 3 months.[6] Based on the aforementioned test procedure, the sensitivity is measured by the difference between portfolio outputs generated using the true value and using the false value.

First of all, the results under a certain condition without prediction error from the mean and standard deviation of stocks according to the methods estimating the correlation matrix are presented in **Table 3**. The table reports the results from the sample correlation matrix, the market correlation matrix, the non-market correlation matrix, and their shrinkage correlation matrixes.[7] The results from the two-type portfolio of the GMVP and the TP are divisively presented. Each panel has the results of the 3-type stock group, i.e., all stocks, large stocks, and small stocks groups. The portfolio output is composed of the 5-type measurement, i.e., portfolio risk, portfolio return, Sharpe ratio, the HI and the NZ. The table reports the average values of results observed from each 22 sub-period.

[Insert **TABLE 3** about here.]

According to the results under a certain condition without prediction error from the mean and standard deviation of stocks, regardless of portfolio types and stock group types, the non-market correlation matrix shows a larger Sharpe ratio and a higher degree of diversification by the HI and the NZ, compared to the other correlation matrixes. The portfolio return and risk observed from each correlation matrix definitely maintain the positive risk-return relation, and the TP clearly has a higher portfolio risk and a higher portfolio return than the GMVP. The results in detail are as follows. The results of the GMVP show that the non-market correlation matrix has a significant portfolio output, compared to the other correlation matrixes. The non-market correlation matrix has a positive Sharpe ratio in contrast to the other correlation matrixes having a negative value, and further, has many more stocks having non-zero investment weight compared to the

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future investment period is as follows. First, using daily stock returns, we calculate the mean ($\overline{r_{j,t}}$) and standard deviation (s_j) of stocks and the correlation matrix between stocks ($\rho_{i,j}$) in the future investment period of 3 months. Second, the calculated values are converted into periodic values of 3 month, such as $\bar{R}_j = \prod_{t=1}^{T}(1+\overline{r_{j,t}})$ and $\sigma_j = s_j \times \sqrt{T}$, where T is the number of trading days in 3 months. Finally, we calculate the portfolio return ($R_P = \sum_{j=1}^{N} \boldsymbol{w_j} \bar{R}_j$) and risk ($\sigma_P = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} \boldsymbol{w_i} \boldsymbol{w_j} \sigma_i \sigma_j \rho_{i,j}}$), and Sharpe ratio (SR $= (R_p - R_F)/\sigma_p$, where R_F indicates the rate of a risk-free asset).

[6] The portfolio output under uncertain condition is calculated as follows. First, the mean ($\overline{r_{j,t(H)}}$) and standard deviation ($s_{j(H)}$) are calculated using daily stock returns during the 75 trading days within the given past period. Second, these values are converted into the periodic values for the T_H=75 trading days, such as $\overline{R_{j(H)}} = \prod_{t=1}^{T_H}(1+\overline{r_{j,t(H)}})$ and $\sigma_{j(H)} = s_{j(H)} \times \sqrt{T_H}$. The correlation matrix between stocks ($\rho_{i,j(H)}$) in the past period of 60 months is calculated. Third, using all input variables, the investment weight ($\boldsymbol{w_j}$) for each stock in a portfolio is generated through the MVOF. Fourth, in the future investment period of 3 months, we calculate the mean ($\overline{r_{j,t}}$) and standard deviation (s_j) of stocks and the correlation matrix between stocks ($\rho_{i,j}$) using daily stock returns. Fifth, these values are converted into periodic values of 3 month, such as $\bar{R}_j = \prod_{t=1}^{T}(1+\overline{r_{j,t}})$ and $\sigma_j = s_j \times \sqrt{T}$, where T is the number of trading days in 3 months. Finally, the portfolio return ($R_P = \sum_{j=1}^{N} \boldsymbol{w_j} \bar{R}_j$), portfolio risk ($\sigma_P = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} \boldsymbol{w_i} \boldsymbol{w_j} \sigma_i \sigma_j \rho_{i,j}}$), and Sharpe ratio (SR $= (R_p - R_F)/\sigma_p$, where R_F indicates the rate of a risk-free asset) are calculated.

[7] The shrinkage constant is utilized to calculate the weight-averaged correlation matrix as the weight ratio. The average values from the shrinkage constant in 22 sub-periods are on average within the range from 57.82% to 72.42%. Therefore, the shrinkage method assigns smaller weight to the sample correlation matrix, like Ledoit & Wolf (2003,2004).

other correlation matrixes. These results suggest that the non-market correlation matrix may substantially help the construction of a well-diversified portfolio and the attainment of a higher Sharpe ratio in the future period. The small stocks group from using the non-market correlation matrix has a significantly higher portfolio return and higher Sharpe ratio than the large stocks group. The results of the TP in Panel B also show that the non-market correlation matrix has the significantly highest Sharpe ratio and the most well-diversified portfolio among all the correlation matrixes. For example, in the case of the all stocks group, the non-market correlation matrix has a Sharpe ratio of 1.36 times greater than the sample correlation matrix, and a Sharpe ratio of 1.19 times greater than the market correlation matrix. Also, from the diversification viewpoint, the non-market correlation matrix has a high NZ of 4.9 times greater than the sample correlation matrix, and 3.97 times greater than the market correlation matrix. In particular, this comparative advantage from the non-market correlation matrix is much stronger in the case of the small stocks group, compared to the large stocks group.

From the results of **Table 3**, we can determine the possible explanation from the portfolio risk as a denominator in the equation of the Sharpe ratio. In the case of the all stocks group of the TP using the non-market correlation matrix, the magnitude of portfolio risk is a small value of 30.39% of the portfolio risk from the sample correlation matrix, and 33.92% of the portfolio risk from the market correlation matrix. This means that the portfolio from the non-market correlation matrix has a smaller risk compared to the other correlation matrixes. In addition, another possible explanation may be found in the well-distributed investment weight for stocks in a portfolio, as confirmed in **Table 3**. This means that the portfolio from the non-market correlation matrix is more well-diversified compared to other correlation matrixes. Through these possible explanations, we may consider the positive relationship between the portfolio diversification and the investment performance. Therefore, we empirically investigate this relationship in all methods estimating the correlation matrix, and the results are presented in **Figure 2**. The figure utilizes the average values of the results of investment performance (Sharpe ratio) and the distribution level (HI) calculated in 22 sub-periods according to the 11-type correlation matrix. For a robust result, we divided into **Figure 2(a)** from the average values of all results and **Figure 2(b)** from the average values of results excluding the extreme values. That is, **Figure 2(b)** shows the relation using the results of the 50% remaining after excluding the upper 25% and the lower 25% among all results. The X-axis indicates the HI which has a higher level of diversification as moving further into the left-side of the figure. The Y-axis denotes the Sharpe ratio which has the higher investment performance as moving further into the top-side of the figure. Hence, the left-top position means a higher degree of diversification and a higher investment performance, while the right-bottom position means a lower degree of diversification and a lower investment performance. The results of both the GMVP and the TP are displayed in the figure together.

[Insert **FIGURE 2** about here.]

Figure 2 clearly shows the positive relation between the Sharpe ratio and the HI. In other words, the magnitude of the portfolio investment performance and the degree of well-diversified portfolio are positively related to each other. In the figure, the non-market correlation matrix is located in the left-top of the figure, and the sample correlation matrix and the constant correlation matrix tend to be positioned in the right-bottom. These results suggest that the method of the non-market correlation matrix must be helpful for constructing a well-diversified portfolio and maximizing the higher investment performance in the domain of risk-return relation, and in particular, is capable of overcoming the practical problem from the sample correlation matrix through this comparative advantage.

Next, the results under the condition of uncertainty with prediction error from the mean and standard deviation of stocks according to the methods for estimating the correlation matrix are presented in **Tables 4** and **5**. In here, the uncertainty means that the mean and standard deviation of stocks in a portfolio are calculated using return data in 75 trading days within the given past period of each sub-period, i.e., false value of input variables. The true value that is needed for quantifying the degree of sensitivity utilizes the results in 22 sub-periods observed from **Table 3** under a certain condition without prediction error from the input variables. **Table 4** shows results from the sample correlation matrix, the market correlation matrix, the non-market correlation matrix, and their shrinkage correlation matrixes. In the table, the left-column part (*Error Mean*) indicates the sensitivity due to the prediction error from the stock mean, and the right-column part (*Error St.Dev.*) displays the sensitivity due to the prediction error of the stock standard deviation. The results of the portfolio output (*return, risk, weight*) are divisively presented for all stocks group (Panel A), large stocks group (Panel B), and small stocks group (Panel C) in each case of the GMVP and the TP. The sensitivity on the diversification is shown by the MAE between the HI measurements.

[Insert **TABLE 4** about here.]

From **Table 4** under uncertainty with prediction error from input variables, regardless of portfolio types and stock group types, the sample correlation matrix shows the highest value of the MAE, while the non-market correlation matrix shows the lowest MAE. The prediction error from the stock mean has a higher impact on the portfolio output than the prediction error from the stock standard deviation. In addition, the MAE of the portfolio output increases as the number of stocks having prediction error increases from 1 stock to 25 stocks. The results in detail are as follows. In the case of all stocks group of the GMVP, the non-market correlation matrix shows the lowest sensitivity of portfolio output due to the prediction error of the stock standard deviation. In the case of the portfolio return of the GMVP when comparing in the highest prediction error of 25 stocks, the degree of sensitivity for the portfolio return from the non-market correlation matrix has a small value of 54.13% of the sensitivity from the sample correlation matrix, and 53.18% of the sensitivity from the market correlation matrix. In comparison of the portfolio risk of the GMVP, the degree of sensitivity from the non-market correlation matrix has a small value of 39.62% and 46.42% of the sensitivity from the sample correlation matrix and the market correlation matrix, respectively. These findings from the GMVP are identically verified in the results for the sensitivity of portfolio output on the TP. In the case of the highest prediction error of 25 stocks, the sensitivity in the portfolio return of the TP shows that the MAE of the non-market correlation matrix has a small value of 34.55% and 46.97% of the MAE from using the sample correlation matrix, and the 35.72% and 44.37% of the MAE from using the market correlation matrix in the case of the prediction error from each of the mean and standard deviation of stocks, respectively. In addition, in comparison to the sensitivity in the portfolio risk of the TP due to the prediction error of each of the mean and standard deviation of stocks, the MAE from the non-market correlation matrix has a smaller value of 13.75% and 12.55% of the MAE from the sample correlation matrix, and 14.08% and 13.34% of the MAE from the market correlation matrix. In these results of the TP on the prediction error, the non-market correlation matrix shows a much lower degree of sensitivity in the portfolio risk than the case of the portfolio return. The results from both the large stocks group and the small stocks group do not differ from the results confirmed by the all stocks group and, in particular, the small stocks group shows a much stronger comparative advantage of the non-market correlation matrix.

To ensure robust results, **Table 5** shows compares the additional 6-type correlation matrix, such as the constant correlation matrix, the 3-factor correlation matrix, the 7-factor correlation matrix and their shrinkage correlation matrixes, along with results from the non-market correlation matrix in **Table 4**. The table presents the results for all stocks group in the case of the highest error range (25 stocks). In addition, the results are divided into the 2-type of portfolio (*GMVP, TP*), the 2-type input variable error (*Error Mean, Error St.Dev.*), and the 3-type portfolio output (*return, risk, weight*).

[Insert **Table 5** about here.]

Table 5 robustly verifies the comparative advantage of the non-market correlation matrix in the sensitivity on the prediction error from the mean and standard deviation of stocks by comparing with 6-type correlation matrix. In other words, the degree of sensitivity of the portfolio output caused by prediction error from input variables is the smallest in the case of using the non-market correlation matrix, regardless of the portfolio type. The results for large stocks group and small stocks group that are not reported in this paper are not qualitatively different from the results of **Table 5**. Consequently, these results under uncertainty with the prediction error from input variables represent evidence supporting the comparative advantage of the proposed method of the non-market correlation matrix in effectively reducing the degree of sensitivity of the MVOF caused by the prediction error of input variables compared to the other correlation matrixes.

3.3. Discussion

This study has empirically presented robust evidence supporting that the proposed method of the non-market correlation matrix, which is capable of constructing a well-diversified portfolio, effectively reduces the degree of the sensitivity of the MVOF caused by both the estimation error and prediction error from the mean and standard deviation of stocks. Also, the proposed method is verified to achieve the higher investment performance in the domain of risk-return relation, compared to other methods to estimate the correlation matrix. In this section, we present further discussion about the proposed method's substantial enhancement of the practical applicability of the MVOF, from the viewpoint of similarities and differences with the existing methods of the correlation matrix.

First of all, the proposed method has the similarity of improving the direction of the existing methods focusing on the correlation matrix. As in the previous studies, when using the sample correlation matrix, the preference of the MVOF on the input variables introduces the practical problem that biasedly distributes most investment weight into some stocks and amplifies the impact of the input variables' error into the portfolio output due to the biased distribution of investment weight for stocks. Hence, the allocation decision of investment weight for stocks in a portfolio using the sample correlation matrix tends to strongly depend on the firm-specific characteristics of several stocks. In order to enhance the practical applicability of the MVOF, Chan et al. (1999) suggests the method of determining the investment weight for stocks using the correlation matrix that strongly reflects the properties of common factors estimated from the pricing models. Ledoit & Wolf (2003, 2004) suggest the shrinkage method that estimates the weight-averaged correlation matrix having characteristics of both the sample correlation matrix and the correlation matrix that reflects the properties of the common factors. The proposed non-market correlation matrix does not differ from the improving direction of the existing studies. As verified in **Figure 1**, the dominant influence of the market factor on all stocks makes it difficult to evenly distribute the investment weight for stocks based on firm characteristics such as the industry and the firm specific information. That is, the strong commonality of the market factor on all stocks prevents the MVOF from constructing a well-diversified portfolio based on the firm characteristics. Accordingly, by removing the property of the market factor from the sample correlation matrix, the magnitude of the correlation matrix decreases, and the investment weight for stocks from the MVOF can be determined based on the firm characteristics. As a result, the improvement gained by the MVOF through the proposed method is not different from previous research efforts to try to avoid the biased distribution of investment weight for several stocks.

Next, the proposed method has a difference from the perspective of the comparative advantage compared to the existing methods for estimating the correlation matrix. The previous studies improve the practical problem of the MVOF by strongly reflecting the properties of the common factors into the correlation matrix. Nevertheless, as verified in **Table 3**, determining the investment weight for stocks through the existing methods still lacks the construction of a well-diversified portfolio. On the other hand, the proposed method provides an allocation decision that evenly distributes the investment weight into all stocks in a portfolio. Therefore, a well-diversified portfolio can be constructed by using the proposed method of the non-market correlation matrix. In addition, through the well-diversified portfolio from the proposed method, the degree of sensitivity of portfolio output caused by the estimation error and prediction error from input variables' error can be greatly reduced. This improvement by the proposed method is stronger in the case of the TP than in that of the GMVP. Moreover, the method of the non-market correlation matrix significantly achieves higher investment performance under the domain of risk-return relation by effectively reducing the portfolio risk through the improvement gained by constructing a well-diversified portfolio. These results present evidence of the comparative advantage from the non-market correlation matrix compared to the existing methods for estimating the correlation matrix. Accordingly, for the reliability of the comparative advantage from the proposed method, the behavior of portfolio risk on the GMVP and the TP during 22 sub-periods is additionally compared in **Figure 3** among the sample correlation matrix, the market correlation matrix, and the non-market correlation matrix.

[Insert **FIGURE 3** about here.]

Figure 3 verifies that the magnitude of the reduction of portfolio risk through the proposed method is much higher in the TP, compared to the GMVP. The difference between portfolio risks of the GMVP and the TP is in the range of 0.46%~5.77% when using the sample correlation matrix, and 0.33%~5.72% when using the market correlation matrix. However, the difference when using the non-market correlation matrix is in the range of only 0.03%~1.13%. In particular, the result shows that the portfolio risk in the GMVP and the TP has stable behavior on the changes of market situation when using the non-market correlation matrix. In the case of the market crash in the figure, such as the European debt crisis in 2011-12, the non-market correlation matrix shows a stable trend and small difference between portfolio risks of the GMVP and the TP, in contrast to the cases of the sample correlation matrix and the market correlation matrix that show a sharply increasing trend and large difference. Consequently, these results suggest that the comparative advantage of the proposed method is dependent on the low magnitude of portfolio risk and shows the stable behavior of portfolio risk according to the changes of the market situation, in particular, the event of a market crash.

In addition, a further comparative advantage offered by the proposed method is confirmed by comparison with the equal-weighted portfolio (EWP). This study empirically investigates the sensitivity of portfolio output from the investment weight generated through the MVOF by estimating the correlation matrix among stocks. The EWP is a well-diversified portfolio that in advance distributes the same weight into all stocks in a portfolio. Hence, the EWP is not suitable for the this study's aim, because determining the risk and return of the EWP is perfectly independent of the correlation matrix and the MVOF. However, research topics of portfolio selection are most commonly compared against the EWP. Therefore, we compare the risk and return of the proposed portfolio with those of the EWP, based on **Table 3,** with portfolio output in the future investment period. The risk and return of EWP are determined according to the same testing process of **Table 3** in each of 22 sub-periods over the test period from January 2009 to June 2014. According to the result of the all stocks (large stocks, small stocks) group, the EWP has a statistically significant portfolio return of **0.051541** (0.034289, 0.069804), and a statistically significant portfolio risk of **0.006852** (0.007134, 0.007653). The comparison is based on each GMVP and TP generated by the MVOF using the non-market correlation matrix in **Table 3**, as follows. First, we compare with the GMVP from the proposed method. For the all stocks (large stocks, small stocks) group and compared to the GMVP, the EWP has both a higher portfolio return of **0.030426** (0.023567, 0.030426) and a higher portfolio risk of **0.004365** (0.004980, 0.004669). In the domain of risk-return relation and compared to the GMVP, the EWP has a higher portfolio risk and hence a higher portfolio return. As a result, it is difficult to assess the comparative advantages between the EWP and the GMVP from the proposed method. Second, we compare with the TP with from the proposed method. For the all stocks (large stocks, small stocks) group and compared to the TP, the EWP has a higher portfolio risk of **0.0.06577** (0.006655, 0.007734, *whereas the EWP has a lower portfolio risk for the small stocks group*), but a much smaller portfolio return of **0.192338** (0.172884, 0.228099). That is, the TP's portfolio return is **3.73** (5.04, 3.27) times that of the EWP's portfolio return for the all stocks (large stocks, small stocks) group. Therefore, the EWP does not substantially compensate for the portfolio return that suffers from the higher portfolio risk. Moreover, although the TP from the proposed method has a smaller portfolio risk than the EWP, the TP compensates with a much higher portfolio return, compared to the EWP. Accordingly, the portfolio risk and return achieved by using the proposed method show clear evidence supporting the proposed method's comparative advantage against the EWP in the domain of risk-relation relation.

4. CONCLUSIONS

This study aims to devise a method to estimate a correlation matrix capable of reducing the sensitivity of the MVOF caused by the input variables' error from the mean and standard deviation of stocks by constructing a well-diversified portfolio, and then to produce empirical evidence demonstrating that the proposed method improves the practical problem of the MVOF. The proposed method generates a correlation matrix without the property of the market factor included in the sample correlation matrix. Our rationale is based on the dominant influence of the market factor on all stocks that prevents the MVOF from constructing a well-diversified portfolio through the lower correlation among stocks in a portfolio. The main results are summarized as follows. The proposed method of the non-market correlation matrix significantly reduces the degree of the sensitivity of the MVOF caused by both the estimation error and the prediction error from the mean and standard deviation of stocks, compared to the other correlation matrixes. Moreover, the proposed method substantially helps the construction of a well-diversified portfolio and the attainment of higher investment performance in the domain of risk-return relation. This improvement is strongly related to the striking reduction of the portfolio risk. On the basis of results, we uncover evidence revealing that the practical applicability of the Markowitz optimum theory can be enhanced through the method to control the property of the market factor in the sample correlation matrix. Therefore, we expect future research to systematically examine the usefulness of the non-market correlation matrix in the practice of the MVOF.

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Figure 1. Distributions of averages and standard deviations for 11-type correlation matrix method

This figure shows the distribution on the average values and standard deviations of all off-diagonal elements in each 11-type correlation matrix through the simulation experiment, in the case of all stocks group over the whole period. Figure 1(a) shows the average values and Figure 1(b) the standard deviation by box-plot method. The X-axis indicate the 11 types of correlation matrixes, such as the sample correlation matrix (SM), the market correlation matrix (MC), the non-market correlation matrix (NC), the constant correlation matrix (CC), the 3-factor correlation matrix (3FC), the 7-factor correlation matrix (7FC), and their shrinkage correlation matrixes (ShMC, ShNC, ShCC, Sh3FC, Sh7FC).

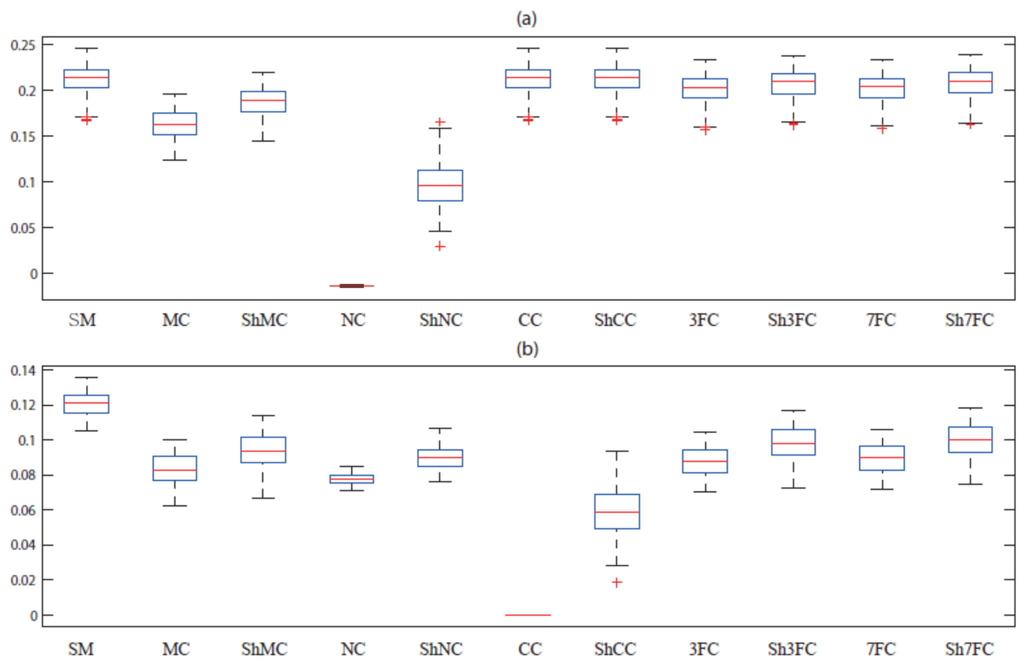

Table 1. Sensitivity on the estimation error from the mean and standard deviation of stocks

This table shows the results for the sensitivity of portfolio output caused by the estimation error in the past period from the mean and standard deviation of stocks according to the correlation matrix methods such as the sample correlation matrix (SC), the market correlation matrix (MC), the non-market correlation matrix (NC) and their shrinkage correlation matrix (ShMC, ShNC). In the table, the left-part shows the sensitivity due to the estimation error from the stock mean (Error Mean), and the right-part the sensitivity due to the estimation error from the stock standard deviation (Error St.Dev.). The portfolio output is divided into portfolio return (return), portfolio risk (risk) and investment weight for stocks (weight), and these results are divided by the 2-type portfolio of the GMVP and the TP. The portfolio risk reported in this table is the standardized portfolio risk in order to control the influence from the magnitude of each correlation matrix method. In this table, the sensitivity is measured using the MAE. The three types of stock group in the table are all stocks (Panel A), large stocks (Panel B) and small stocks (Panel C) groups. Each panel contains the results for each error range of 1, 5, 10, and 25 stocks.

	Error Mean						**Error St.Dev.**					
	GMVP			**TP**			**GMVP**			**TP**		
	return	risk	weight	return	risk	weight	return	risk	weight	return	risk	weight
Panel A. using all stocks group												
Case 1. the number of stocks having estimation error = 1												
SC	0.000294	0.000000	0.000000	0.000460	0.065644	0.009299	0.000074	0.022037	0.001837	0.000085	0.032162	0.001976
MC	0.000128	0.000000	0.000000	0.000284	0.038324	0.004136	0.000034	0.007750	0.000653	0.000023	0.009315	0.000735
ShMC	0.000227	0.000000	0.000000	0.000377	0.047222	0.006741	0.000054	0.015102	0.001169	0.000053	0.018515	0.001422
NC	0.000136	0.000000	0.000000	0.000141	0.037261	0.000095	0.000013	0.021559	0.000044	0.000012	0.023277	0.000041
ShNC	0.000261	0.000000	0.000000	0.000367	0.029141	0.004866	0.000055	0.010695	0.001083	0.000060	0.014706	0.001092
Case 2. the number of stocks having estimation error = 5 (10%)												
SC	0.000893	0.000000	0.000000	0.001666	0.228101	0.023226	0.000216	0.072347	0.007262	0.000310	0.114052	0.009663
MC	0.000584	0.000000	0.000000	0.001219	0.166728	0.014182	0.000144	0.050735	0.004869	0.000197	0.073979	0.006044
ShMC	0.000766	0.000000	0.000000	0.001447	0.188386	0.019065	0.000179	0.061023	0.006041	0.000257	0.093018	0.007863
NC	0.000487	0.000000	0.000000	0.000499	0.095353	0.000226	0.000032	0.077743	0.000125	0.000032	0.085142	0.000119
ShNC	0.000769	0.000000	0.000000	0.001278	0.097903	0.011992	0.000146	0.035760	0.004723	0.000208	0.053038	0.005104
Case 3. the number of stocks having estimation error = 10 (20%)												
SC	0.001144	0.000000	0.000000	0.002740	0.270483	0.024610	0.000309	0.091054	0.008207	0.000838	0.150849	0.011392
MC	0.000803	0.000000	0.000000	0.002313	0.233717	0.016780	0.000264	0.065785	0.005413	0.000620	0.100624	0.007416
ShMC	0.001000	0.000000	0.000000	0.002546	0.233871	0.020960	0.000287	0.077730	0.006734	0.000720	0.125044	0.009386
NC	0.000944	0.000000	0.000000	0.001000	0.127873	0.000279	0.000074	0.139078	0.000150	0.000102	0.142440	0.000144
ShNC	0.001013	0.000000	0.000000	0.002164	0.114160	0.012387	0.000230	0.045610	0.005233	0.000571	0.072123	0.006051
Case 4. the number of stocks having estimation error = 25 (50%)												
SC	0.001640	0.000000	0.000000	0.003916	0.295256	0.028775	0.000517	0.119396	0.011116	0.001276	0.191071	0.015619
MC	0.001532	0.000000	0.000000	0.003547	0.263971	0.021893	0.000497	0.099019	0.007420	0.001002	0.144777	0.011213
ShMC	0.001572	0.000000	0.000000	0.003762	0.265716	0.025734	0.000497	0.107364	0.009102	0.001128	0.165256	0.013269
NC	0.001792	0.000000	0.000000	0.001784	0.239748	0.000386	0.000129	0.275991	0.000230	0.000181	0.279166	0.000221
ShNC	0.001520	0.000000	0.000000	0.003148	0.125662	0.014455	0.000397	0.064016	0.006647	0.000889	0.095495	0.008020

	Error Mean						Error St.Dev.					
	GMVP			TP			GMVP			TP		
	return	risk	weight	return	risk	weight	return	risk	weight	return	risk	weight
Panel B. using large stocks group (top 40%)												
Case 1. the number of stocks having estimation error = 1												
SC	0.000234	0.000000	0.000000	0.000145	0.039315	0.003107	0.000064	0.018712	0.001269	0.000058	0.026168	0.001028
MC	0.000073	0.000000	0.000000	0.000048	0.007063	0.000756	0.000025	0.004290	0.000321	0.000006	0.002561	0.000267
ShMC	0.000174	0.000000	0.000000	0.000099	0.024052	0.001811	0.000048	0.012033	0.000773	0.000041	0.016051	0.000775
NC	0.000095	0.000000	0.000000	0.000091	0.025211	0.000056	0.000010	0.020702	0.000033	0.000010	0.020847	0.000031
ShNC	0.000218	0.000000	0.000000	0.000156	0.023110	0.002205	0.000053	0.012027	0.000811	0.000046	0.016056	0.000653
Case 2. the number of stocks having estimation error = 5 (10%)												
SC	0.000978	0.000000	0.000000	0.001502	0.273125	0.021851	0.000161	0.080734	0.010266	0.000257	0.105728	0.007198
MC	0.000647	0.000000	0.000000	0.001139	0.165658	0.015748	0.000101	0.050810	0.007111	0.000171	0.063527	0.006234
ShMC	0.000856	0.000000	0.000000	0.001359	0.225203	0.018826	0.000137	0.066152	0.008868	0.000221	0.085363	0.007012
NC	0.000447	0.000000	0.000000	0.000451	0.093317	0.000249	0.000026	0.085080	0.000127	0.000024	0.086299	0.000100
ShNC	0.000881	0.000000	0.000000	0.001223	0.155203	0.012659	0.000122	0.051297	0.006934	0.000206	0.067491	0.004561
Case 3. the number of stocks having estimation error = 10 (20%)												
SC	0.001126	0.000000	0.000000	0.002264	0.270483	0.024610	0.000262	0.099167	0.011284	0.000312	0.151038	0.010450
MC	0.000835	0.000000	0.000000	0.001851	0.233717	0.016780	0.000191	0.067851	0.008007	0.000227	0.101140	0.008871
ShMC	0.001019	0.000000	0.000000	0.002119	0.233871	0.020960	0.000228	0.083841	0.009865	0.000274	0.126269	0.010038
NC	0.000810	0.000000	0.000000	0.000812	0.127873	0.000279	0.000041	0.133668	0.000138	0.000037	0.136559	0.000140
ShNC	0.001045	0.000000	0.000000	0.001860	0.114160	0.012387	0.000190	0.064972	0.007638	0.000230	0.097504	0.006846
Case 4. the number of stocks having estimation error = 25 (50%)												
SC	0.001559	0.000000	0.000000	0.003952	0.305341	0.027073	0.000408	0.143216	0.013753	0.000431	0.194103	0.011352
MC	0.001415	0.000000	0.000000	0.003929	0.264346	0.021089	0.000369	0.109491	0.009426	0.000386	0.149014	0.009834
ShMC	0.001516	0.000000	0.000000	0.003997	0.281509	0.024060	0.000394	0.125988	0.011817	0.000394	0.171190	0.011288
NC	0.002149	0.000000	0.000000	0.002206	0.246472	0.000429	0.000104	0.299784	0.000180	0.000100	0.299037	0.000173
ShNC	0.001520	0.000000	0.000000	0.003511	0.172439	0.015650	0.000328	0.096789	0.008959	0.000340	0.130339	0.007927

	Error Mean						Error St.Dev.					
	GMVP			TP			GMVP			TP		
	return	risk	weight	return	risk	weight	return	risk	weight	return	risk	weight
Panel C. using small stocks group (bottom 40%)												
Case 1. the number of stocks having estimation error = 1												
SC	0.000031	0.000000	0.000000	0.000411	0.066180	0.006891	0.000124	0.010488	0.001101	0.000210	0.051124	0.004522
MC	0.000194	0.000000	0.000000	0.000509	0.046572	0.003400	0.000184	0.029123	0.000524	0.000155	0.049521	0.003696
ShMC	0.000102	0.000000	0.000000	0.000453	0.055272	0.004553	0.000187	0.022025	0.001018	0.000169	0.053963	0.004032
NC	0.000124	0.000000	0.000000	0.000160	0.021394	0.000063	0.000034	0.041591	0.000026	0.000042	0.036214	0.000043
ShNC	0.000059	0.000000	0.000000	0.000332	0.020677	0.002837	0.000117	0.005921	0.000686	0.000136	0.015971	0.001662
Case 2. the number of stocks having estimation error = 5 (10%)												
SC	0.000435	0.000000	0.000000	0.001210	0.224200	0.020464	0.000361	0.045278	0.003912	0.000459	0.108077	0.009421
MC	0.000607	0.000000	0.000000	0.001024	0.135578	0.010304	0.000329	0.045332	0.001809	0.000354	0.073462	0.006787
ShMC	0.000491	0.000000	0.000000	0.001078	0.175403	0.013734	0.000369	0.044517	0.002580	0.000387	0.089175	0.007942
NC	0.000395	0.000000	0.000000	0.000444	0.057150	0.000123	0.000070	0.079048	0.000066	0.000089	0.070208	0.000111
ShNC	0.000425	0.000000	0.000000	0.000847	0.060974	0.007472	0.000269	0.017393	0.001778	0.000295	0.033118	0.003718
Case 3. the number of stocks having estimation error = 10 (20%)												
SC	0.000599	0.000000	0.000000	0.001665	0.287698	0.026833	0.000508	0.067650	0.005420	0.000640	0.133856	0.011245
MC	0.000791	0.000000	0.000000	0.001294	0.195129	0.019491	0.000456	0.057627	0.002665	0.000479	0.096898	0.008322
ShMC	0.000647	0.000000	0.000000	0.001480	0.241967	0.023179	0.000501	0.059553	0.003381	0.000543	0.115384	0.009442
NC	0.000608	0.000000	0.000000	0.000594	0.121716	0.000289	0.000092	0.126833	0.000100	0.000117	0.117099	0.000150
ShNC	0.000578	0.000000	0.000000	0.001146	0.164060	0.015149	0.000372	0.025489	0.002376	0.000417	0.043207	0.004630
Case 4. the number of stocks having estimation error = 25 (50%)												
SC	0.001160	0.000000	0.000000	0.003265	0.380587	0.025559	0.001437	0.229511	0.014944	0.002261	0.279975	0.029573
MC	0.001229	0.000000	0.000000	0.002439	0.260036	0.013262	0.001047	0.155784	0.008679	0.001786	0.163308	0.016756
ShMC	0.001138	0.000000	0.000000	0.002859	0.320806	0.017529	0.001226	0.185327	0.010774	0.002037	0.209721	0.022655
NC	0.001119	0.000000	0.000000	0.001006	0.233765	0.000358	0.000216	0.320515	0.000325	0.000360	0.264268	0.000382
ShNC	0.001033	0.000000	0.000000	0.002289	0.114759	0.010337	0.001013	0.095905	0.007879	0.001575	0.094564	0.014737

Table 2. Robustness of the Sensitivity on the estimation error from input variables

This table shows the results using another method to estimate the correlation matrix, such as the constant correlation matrix (CC), the 3-factor correlation matrix (3FC), 7-factor correlation matrix (7FC) and their shrinkage correlation matrixes (ShCC, Sh3FC, Sh7FC), in the case of all stocks group having the highest error range (25 stocks). In addition, the results from the non-market correlation matrix (NC) and its shrinkage correlation matrix (ShNC) of Table 1 are also presented for comparison. In the table, the results are divisively presented in accordance with 2-type portfolio (GMVP, TP) and 3-type portfolio output (return, risk, weight) on the 2-type input variable error (Error MEAN, Error ST.Dev.).

	GMVP			TP		
	return	risk	weight	return	risk	weight
Panel A: Error MEAN						
NC	0.001792	0.000000	0.000000	0.001784	0.239748	0.000386
ShNC	0.001520	0.000000	0.000000	0.003148	0.125662	0.014455
CC	0.001589	0.000000	0.000000	0.004697	0.593368	0.037494
ShCC	0.001640	0.000000	0.000000	0.004436	0.127993	0.034211
3FC	0.001718	0.000000	0.000000	0.004013	0.264138	0.028112
Sh3FC	0.001666	0.000000	0.000000	0.003963	0.258027	0.028249
7FC	0.001735	0.000000	0.000000	0.003989	0.258391	0.028064
Sh7FC	0.001673	0.000000	0.000000	0.003951	0.256729	0.028162
Panel B: Error St.Dev.						
NC	0.000129	0.275991	0.000230	0.000181	0.279166	0.000221
ShNC	0.000397	0.064016	0.006647	0.000889	0.095495	0.008020
CC	0.000408	0.237299	0.012262	0.001430	0.496253	0.016126
ShCC	0.000452	0.051069	0.011922	0.001369	0.098278	0.016768
3FC	0.000473	0.102959	0.009254	0.001177	0.161911	0.014407
Sh3FC	0.000491	0.106291	0.009989	0.001219	0.166393	0.014815
7FC	0.000463	0.102713	0.009168	0.001172	0.161068	0.014175
Sh7FC	0.000487	0.106829	0.009998	0.001215	0.166704	0.014659

Table 3. Portfolio output under a certain condition without prediction error of input variables

This table shows the results under a certain condition without prediction error from the mean and standard deviation of stocks according to the correlation matrix methods such as the sample correlation matrix (SC), the market correlation matrix (MC), the non-market correlation matrix (NC), and their shrinkage correlation matrixes (ShMC, ShNC) in 22 sub-periods over the period from January 2009 to June 2014. The results are presented by the 2-type portfolio (GMVP, TP) and 3-type stock group (all stocks, large stocks, small stocks). The portfolio output from the MVOF is divisively presented by portfolio return (return), portfolio risk (risk), the Sharpe ratio (SR), the Herfindahl index (NI), and the number of stocks having non-zero investment weight (NZ). The markers 'a', 'b', and 'c' indicate the statistical significance at the level of '1%', '5%', and '10%', respectively.

	return	risk	SR	HI	NZ
Panel A. global minimum Variance Portfolio, GMVP					
case 1: All stocks group					
SC	-0.003106	0.002729[a]	-5.029721	0.248222[a]	11.36[a]
MC	-0.001668	0.002404[a]	-3.802452	0.192497[a]	15.89[a]
ShMC	-0.003167	0.002440[a]	-5.000007	0.211990[a]	14.36[a]
NC	0.023567[c]	0.004365[a]	7.710174[c]	0.033368[a]	49.75[a]
ShNC	0.001665	0.002825[a]	-2.064281	0.132651[a]	24.29[a]
case 2: Large stocks group (top 40%)					
SC	-0.005724	0.002830[a]	-4.877169[c]	0.229288[a]	11.29[a]
MC	-0.005669	0.002503[a]	-4.070203	0.182584[a]	14.74[a]
ShMC	-0.006572	0.002542[a]	-4.942296	0.200460[a]	13.65[a]
NC	0.017326	0.004980[a]	4.179306	0.030323[a]	49.79[a]
ShNC	0.000155	0.002898[a]	-2.295752	0.132185[a]	22.20[a]
case 3. Small stocks group (bottom 40%)					
SC	-0.004261	0.003045[a]	-5.686257	0.255397[a]	11.38[a]
MC	-0.001427	0.002724[a]	-5.152585	0.194300[a]	17.00[a]
ShMC	-0.003241	0.002762[a]	-5.868606	0.214580[a]	14.88[a]
NC	0.030426[b]	0.004669[a]	9.792658[c]	0.036171[a]	49.75[a]
ShNC	0.000677	0.003149[a]	-3.159943	0.136040[a]	23.86[a]
Panel B. Tangency Portfolio, TP					
case 1: All stocks group					
SC	0.484856[a]	0.021639[a]	34.881627[a]	0.281254[a]	6.78[a]
MC	0.470011[a]	0.019391[a]	39.849963[a]	0.238434[a]	8.36[a]
ShMC	0.483929[a]	0.020733[a]	38.120684[a]	0.254977[a]	7.76[a]
NC	0.192338[a]	0.006577[a]	47.265140[a]	0.060752[a]	33.21[a]
ShNC	0.360453[a]	0.011571[a]	46.067563[a]	0.157426[a]	13.25[a]
case 2: Large stocks group					
SC	0.345739[a]	0.014161[a]	34.926378[a]	0.280499[a]	6.49[a]
MC	0.339612[a]	0.013103[a]	38.034739[a]	0.250452[a]	7.54[a]
ShMC	0.344892[a]	0.013520[a]	37.075399[a]	0.261587[a]	7.17[a]
NC	0.172884[a]	0.006655[a]	38.395875[a]	0.061754[a]	31.86[a]
ShNC	0.294015[a]	0.009974[a]	42.297843[a]	0.165405[a]	11.91[a]
case 3: Small stocks group					
SC	0.670135[a]	0.035943[a]	31.142380[a]	0.293896[a]	6.72[a]
MC	0.620983[a]	0.028690[a]	37.747480[a]	0.236199[a]	8.90[a]
ShMC	0.650268[a]	0.032106[a]	35.271338[a]	0.258913[a]	7.98[a]
NC	0.228099[a]	0.007734[a]	50.500431[a]	0.058953[a]	33.87[a]
ShNC	0.468945[a]	0.016508[a]	44.054950[a]	0.158812[a]	12.98[a]

Figure 2. Relationship between investment performance and degree of diversification

This figure shows the results of investigating the relationship between the investment performance and the diversification degree measured from each 11-type correlation matrix method in 22 sub-periods over the period from January 2009 to June 2014. The investment performance indicates the Sharpe ratio (SR) in the Y-axis, and the degree of diversification denotes the Herfindahl index (HI) in the X-axis. Figure 2(a) shows the result from the average values using all results and Figure 2(bthose from the average values using results after removing the upper 25% and the lower 25% from all results in order to control the potential influence of outliers. The results of both the GMVP and the TP are presented in the figure together.

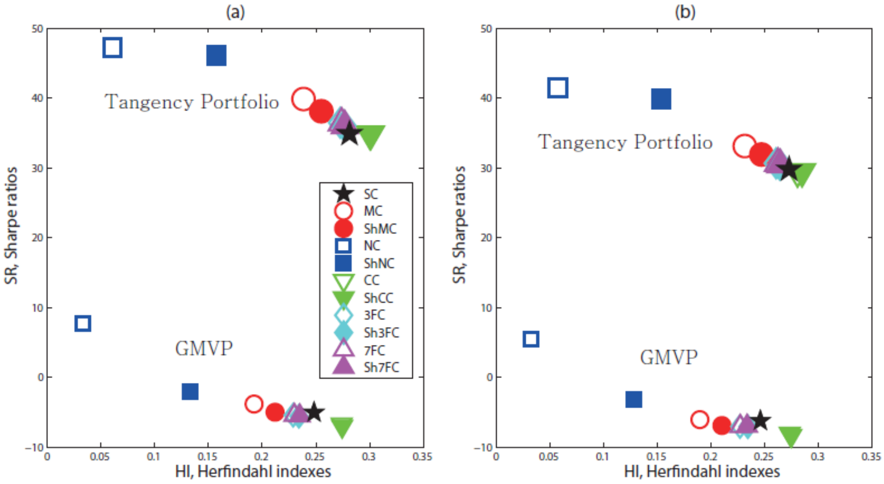

Table 4. Sensitivity on the prediction error from the mean and standard deviation of stocks under an uncertain condition

This table shows the results for the sensitivity of portfolio output under an uncertain condition with the prediction error in the future period from the mean and standard deviation of stocks according to the correlation matrix methods such as the sample correlation matrix (SC), the market correlation matrix (MC), the non-market correlation matrix (NC) and their shrinkage correlation matrix (ShMC, ShNC). In the table, the left-part shows the sensitivity due to the prediction error from the stock mean (Error Mean), and the right-part shows the sensitivity due to the prediction error from the stock standard deviation (Error St.Dev.). The portfolio output is divided into portfolio return (return), portfolio risk (risk) and investment weight for stocks (weight), and these results are divided by the 2-type portfolio (GMVP, TP). The portfolio risk reported in this table is the standardized portfolio risk. The sensitivity is measured using the MAE. The three types of stock group in the table are all stocks (Panel A), large stocks (Panel B) and small stocks (Panel C) groups. Each panel contains the results for each error range of 1, 5, 10, and 25 stocks.

	Error Mean						**Error St.Dev.**					
	GMVP			**TP**			**GMVP**			**TP**		
	return	risk	weight	return	risk	weight	return	risk	weight	return	risk	weight
Panel A. using all stocks group												
Case 1. the number of stocks having prediction error = 1												
SC	0.000000	0.000000	0.000000	0.019308	0.000829	0.004773	0.001062	0.000045	0.002249	0.003263	0.000494	0.002706
MC	0.000000	0.000000	0.000000	0.013555	0.000296	0.004103	0.000385	0.000016	0.000577	0.002596	0.000497	0.002331
ShMC	0.000000	0.000000	0.000000	0.015611	0.000442	0.004308	0.000689	0.000027	0.001101	0.003022	0.000543	0.002692
NC	0.000000	0.000000	0.000000	0.006312	0.000102	0.001309	0.000327	0.000014	0.000140	0.000553	0.000032	0.000314
ShNC	0.000000	0.000000	0.000000	0.012916	0.000277	0.002585	0.000844	0.000036	0.001130	0.001397	0.000191	0.001598
Case 2. the number of stocks having prediction error = 5 (10%)												
SC	0.000000	0.000000	0.000000	0.065572	0.001313	0.010494	0.002528	0.000144	0.007091	0.009504	0.001497	0.009298
MC	0.000000	0.000000	0.000000	0.057573	0.000881	0.011106	0.001538	0.000062	0.002580	0.007584	0.001293	0.009470
ShMC	0.000000	0.000000	0.000000	0.061955	0.001084	0.010495	0.001959	0.000090	0.004016	0.008272	0.001379	0.009885
NC	0.000000	0.000000	0.000000	0.024547	0.000161	0.004394	0.001403	0.000069	0.000735	0.002093	0.000133	0.001443
ShNC	0.000000	0.000000	0.000000	0.048664	0.000471	0.006411	0.002384	0.000118	0.003838	0.005654	0.000651	0.005286
Case 3. the number of stocks having prediction error = 10 (20%)												
SC	0.000000	0.000000	0.000000	0.115911	0.002216	0.020048	0.004783	0.000283	0.012559	0.018840	0.003205	0.021927
MC	0.000000	0.000000	0.000000	0.107103	0.001788	0.020183	0.003453	0.000173	0.006195	0.016141	0.002838	0.021199
ShMC	0.000000	0.000000	0.000000	0.113112	0.002123	0.020121	0.003933	0.000214	0.008295	0.017011	0.003019	0.022200
NC	0.000000	0.000000	0.000000	0.042105	0.000283	0.007270	0.002521	0.000118	0.001464	0.004689	0.000263	0.003076
ShNC	0.000000	0.000000	0.000000	0.084920	0.000763	0.011422	0.004376	0.000217	0.006788	0.011794	0.001405	0.012499
Case 4. the number of stocks having prediction error = 25 (50%)												
SC	0.000000	0.000000	0.000000	0.244135	0.004416	0.046742	0.015089	0.001522	0.029542	0.038624	0.010103	0.071085
MC	0.000000	0.000000	0.000000	0.236146	0.004310	0.046455	0.015357	0.001299	0.022478	0.040884	0.009506	0.069144
ShMC	0.000000	0.000000	0.000000	0.244907	0.004738	0.046833	0.015377	0.001389	0.025901	0.040319	0.009940	0.072295
NC	0.000000	0.000000	0.000000	0.084350	0.000607	0.017077	0.008167	0.000603	0.004687	0.018140	0.001268	0.010140
ShNC	0.000000	0.000000	0.000000	0.178980	0.001461	0.029582	0.014048	0.001138	0.018548	0.032641	0.005506	0.041425

	Error Mean						Error St.Dev.					
	GMVP			TP			GMVP			TP		
	return	risk	weight	return	risk	weight	return	risk	weight	return	risk	weight
Panel B. using large stocks group (top 40%)												
Case 1. the number of stocks having prediction error = 1												
SC	0.000000	0.000000	0.000000	0.019456	0.000536	0.006821	0.001466	0.000060	0.002445	0.001914	0.000344	0.003411
MC	0.000000	0.000000	0.000000	0.014830	0.000312	0.006799	0.000461	0.000022	0.000710	0.001074	0.000301	0.003087
ShMC	0.000000	0.000000	0.000000	0.016857	0.000399	0.007307	0.000980	0.000037	0.001339	0.001466	0.000323	0.003337
NC	0.000000	0.000000	0.000000	0.006112	0.000061	0.001356	0.000424	0.000014	0.000137	0.000611	0.000033	0.000382
ShNC	0.000000	0.000000	0.000000	0.013413	0.000208	0.004228	0.001186	0.000049	0.001283	0.001154	0.000170	0.001942
Case 2. the number of stocks having prediction error = 5 (10%)												
SC	0.000000	0.000000	0.000000	0.056987	0.001268	0.010763	0.003216	0.000142	0.007828	0.004296	0.000890	0.007447
MC	0.000000	0.000000	0.000000	0.050398	0.001093	0.011015	0.001680	0.000064	0.003126	0.002965	0.000813	0.007586
ShMC	0.000000	0.000000	0.000000	0.053348	0.001149	0.010999	0.002469	0.000100	0.005026	0.003613	0.000864	0.007897
NC	0.000000	0.000000	0.000000	0.021731	0.000180	0.004133	0.001084	0.000064	0.000651	0.001732	0.000123	0.001205
ShNC	0.000000	0.000000	0.000000	0.043205	0.000574	0.006726	0.002668	0.000133	0.004408	0.003414	0.000487	0.004831
Case 3. the number of stocks having prediction error = 10 (20%)												
SC	0.000000	0.000000	0.000000	0.094850	0.001966	0.017199	0.004770	0.000243	0.012467	0.008012	0.001849	0.018440
MC	0.000000	0.000000	0.000000	0.088252	0.001708	0.018220	0.002716	0.000122	0.005644	0.006360	0.001728	0.019266
ShMC	0.000000	0.000000	0.000000	0.091736	0.001773	0.018281	0.003887	0.000181	0.008563	0.007124	0.001813	0.019791
NC	0.000000	0.000000	0.000000	0.037861	0.000384	0.008163	0.002213	0.000164	0.001279	0.003603	0.000268	0.002723
ShNC	0.000000	0.000000	0.000000	0.074456	0.000958	0.010165	0.004312	0.000243	0.007662	0.006605	0.001051	0.011521
Case 4. the number of stocks having prediction error = 25 (50%)												
SC	0.000000	0.000000	0.000000	0.184575	0.003072	0.033488	0.011342	0.000557	0.026613	0.019648	0.005004	0.052395
MC	0.000000	0.000000	0.000000	0.180473	0.002856	0.034568	0.009240	0.000398	0.017538	0.018548	0.004850	0.051709
ShMC	0.000000	0.000000	0.000000	0.184043	0.002921	0.035416	0.010235	0.000473	0.022106	0.019400	0.005032	0.054082
NC	0.000000	0.000000	0.000000	0.076191	0.000659	0.017237	0.004819	0.000331	0.003480	0.008370	0.000587	0.007186
ShNC	0.000000	0.000000	0.000000	0.149855	0.001748	0.020821	0.009814	0.000485	0.017580	0.015420	0.002753	0.031867

	Error Mean						Error St.Dev.					
	GMVP			TP			GMVP			TP		
	return	risk	weight	return	risk	weight	return	risk	weight	return	risk	weight
Panel C. using small stocks group (bottom 40%)												
Case 1. the number of stocks having prediction error = 1												
SC	0.000000	0.000000	0.000000	0.030763	0.001294	0.017724	0.001210	0.000099	0.002748	0.005096	0.000936	0.012040
MC	0.000000	0.000000	0.000000	0.028181	0.001050	0.016024	0.001358	0.000078	0.002178	0.005566	0.000967	0.011067
ShMC	0.000000	0.000000	0.000000	0.030101	0.001307	0.017274	0.001245	0.000086	0.002407	0.005593	0.000982	0.011977
NC	0.000000	0.000000	0.000000	0.007165	0.000154	0.001669	0.000950	0.000022	0.000179	0.002125	0.000146	0.001354
ShNC	0.000000	0.000000	0.000000	0.019416	0.000603	0.009560	0.001292	0.000058	0.001290	0.004065	0.000531	0.005972
Case 2. the number of stocks having prediction error = 5 (10%)												
SC	0.000000	0.000000	0.000000	0.129935	0.005014	0.043704	0.007245	0.000578	0.008654	0.021747	0.004645	0.040070
MC	0.000000	0.000000	0.000000	0.120346	0.004318	0.040718	0.006470	0.000474	0.005955	0.027076	0.005303	0.039824
ShMC	0.000000	0.000000	0.000000	0.128092	0.004954	0.042578	0.006629	0.000512	0.006829	0.025891	0.005288	0.042190
NC	0.000000	0.000000	0.000000	0.031888	0.000611	0.004810	0.003527	0.000120	0.000643	0.009486	0.000696	0.004887
ShNC	0.000000	0.000000	0.000000	0.080702	0.001880	0.026817	0.005982	0.000367	0.003457	0.018959	0.002772	0.021745
Case 3. the number of stocks having prediction error = 10 (20%)												
SC	0.000000	0.000000	0.000000	0.213381	0.007033	0.051861	0.012361	0.001024	0.011260	0.034695	0.007397	0.057258
MC	0.000000	0.000000	0.000000	0.199670	0.006235	0.048164	0.011571	0.000844	0.008847	0.040152	0.008301	0.057869
ShMC	0.000000	0.000000	0.000000	0.211516	0.007219	0.050331	0.011802	0.000914	0.009773	0.038782	0.008288	0.061138
NC	0.000000	0.000000	0.000000	0.056078	0.000769	0.006364	0.005778	0.000250	0.001206	0.015654	0.001056	0.006532
ShNC	0.000000	0.000000	0.000000	0.140206	0.002472	0.031142	0.010503	0.000684	0.005481	0.024493	0.004734	0.032035
Case 4. the number of stocks having prediction error = 25 (50%)												
SC	0.000000	0.000000	0.000000	0.383987	0.010500	0.070214	0.028482	0.002661	0.025919	0.078590	0.023910	0.107714
MC	0.000000	0.000000	0.000000	0.359727	0.008560	0.067426	0.028107	0.002171	0.020449	0.086815	0.023977	0.107994
ShMC	0.000000	0.000000	0.000000	0.377838	0.009861	0.071818	0.028185	0.002350	0.023290	0.086697	0.024445	0.112373
NC	0.000000	0.000000	0.000000	0.106801	0.001374	0.016207	0.016274	0.000736	0.002951	0.039019	0.003363	0.011527
ShNC	0.000000	0.000000	0.000000	0.262231	0.003752	0.044778	0.026245	0.001828	0.015293	0.069127	0.015248	0.065043

Table 5. Robustness of the Sensitivity on the prediction error from input variables under uncertainty

This table shows the results using another method to estimate the correlation matrix, such as the constant correlation matrix (CC), the 3-factor correlation matrix (3FC), 7-factor correlation matrix (7FC) and their shrinkage correlation matrixes (ShCC, Sh3FC, Sh7FC), in the case of all stocks group having the highest error range (25 stocks). In addition, the results from the non-market correlation matrix (NC) and its shrinkage correlation matrix (ShNC) of Table 4 are also presented for comparison. In the table, the results are divisively presented in accordance with 2-type portfolio (GMVP, TP) and 3-type portfolio output (return, risk, weight) on the 2-type input variable error (Error MEAN, Error ST.Dev.).

	GMVP			TP		
	return	risk	weight	return	risk	weight
Panel A: Error Mean						
NC	0.000000	0.000000	0.000000	0.084350	0.000607	0.017077
ShNC	0.000000	0.000000	0.000000	0.178980	0.001461	0.029582
CC	0.000000	0.000000	0.000000	0.277411	0.006315	0.058994
ShCC	0.000000	0.000000	0.000000	0.275245	0.006168	0.054970
3FC	0.000000	0.000000	0.000000	0.255747	0.005271	0.049211
Sh3FC	0.000000	0.000000	0.000000	0.254829	0.005250	0.048936
7FC	0.000000	0.000000	0.000000	0.255301	0.005408	0.049704
Sh7FC	0.000000	0.000000	0.000000	0.254508	0.005368	0.049293
Panel B: Error St.Dev.						
NC	0.008167	0.000603	0.004687	0.018140	0.001268	0.010140
ShNC	0.014048	0.001138	0.018548	0.032641	0.005506	0.041425
CC	0.017320	0.001535	0.031055	0.036634	0.010987	0.084926
ShCC	0.016821	0.001635	0.032398	0.038409	0.010928	0.083224
3FC	0.015900	0.001465	0.030296	0.043262	0.010871	0.077783
Sh3FC	0.015632	0.001484	0.030755	0.041659	0.010658	0.077232
7FC	0.015721	0.001464	0.029989	0.041070	0.010460	0.076873
Sh7FC	0.015558	0.001484	0.030417	0.039957	0.010347	0.076385

Figure 3. Trend of the difference between the portfolio risks of the GMVP and the TP in 22 sub-periods

This figure shows the behavior of portfolio risks of the GMVP and the TP during the 22 sub-periods in the whole period from January 2009 to June 2014 according the correlation matrix methods such as the sample correlation matrix (○, SC), the market correlation matrix (▽, MC) and the non-market correlation matrix (□, NC) in the case of all stocks group. In the figure, the black-bar indicates the returns of market index (KOSPI). The X-axis indicates the periods and the dual Y-axis presents the market returns (left-Y) and the difference between the portfolio risks of the GMVP and the TP (right-Y), together.