Conditional Cross-Section:

Belief Difference and Characteristic Explanations

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Current Draft: November, 2016

ABSTRACT

We study the cross-sectional stock return variation when returns are conditioned on investors' belief difference and book-to-market ratio. In the conditional cross-section, returns decrease from low to high belief difference for all book-to-market ratios, and increase from low to high book-to-market ratio for all belief differences. Conventional risk-based model struggles to explain the given conditonal return variation. We offer plausible economic explanations based on characteristics that are related to investors' belief differences and market frictions. While the short-sale constraints and the max-return can help explain the decrease in returns across belief difference, cash flow risk can help explain the value premium across book-to-market ratios. Our study calls for a new theoretical framework that can account for investors' belief differences in explaining the cross-sectional return variation.

Keywords: Belief Difference, Value Premium, Short-Sale Constraint, Institutional Ownership, Max-Return, Lottery-Demand, Expected Idiosyncratic Skewness, Cross-Section of Stock Returns

JEL classification: G00, G02, G10, G11, G12

We would like to thank Hagen Kim, Michael Gallmeyer, Marcelo Ochoa, Jungwook Kim, Kyungsik Eom, Heungju Park and other seminar participants at Econometric Society European Meeting, Sungkyunkwan University, University of Seoul, and Chungnam National University. Part of this research has been undertaken when Jhang was at the Scheller College of Business at Georgia Institute of Technology.

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I. Introduction

It has long been recognized that investors' different beliefs can be an important pricing component in both rational and behavioral asset pricing literature. Some notable existing studies show that both in the cross-section and the time-series of stock returns, investors' belief difference can play a significant instrumental role in pricing assets in equilibrium.[1]

Based on Miller (1977) and Harrison and Kreps (1978), Diether, Malloy, and Scherbina (2002) show that there is negative relation between belief difference and future stock returns as belief difference induces overpricing particularly when short-sale constraints bind. Yu (2011) shows that aggregate belief difference seems to predict market return in the time-series just like other well known return predicting variables such as price-dividend ratio. Also he shows that the belief difference is negatively related to ex-post expected market return. Gallmeyer, Jhang, and Kim (2015) show that the value premium can arise in a general equilibrium model with investors' belief difference. Specifically, idiosyncratic cash flow risk is priced in equilibrium via belief difference in their model. As a result, the value premium arises as value (growth) stocks have higher (lower) belief difference and higher (lower) idiosyncratic cash flow risk. Yu (2011) also shows that the value effect is more pronounced in stocks with higher belief difference.[2]

Despite its importance, there are few empirical studies investigating the role of individual (portfolio) belief difference in the cross-sectional stock return variation. In this study, we try to fill this

[1] Investors' heterogeneous beliefs has been emphasized early by Lintner (1969), Miller (1977), Harrison and Kreps (1978), and among others. Subsequent works have studied impacts of economic agents' belief differences about underlying fundamental processes on equilibrium. Detemple and Murthy (1994) study the equilibrium effect of difference of beliefs in a production economy. Zapatero (1998), Basak (2000), Basak (2005), Buraschi and Jiltsov (2006), Jouini and Napp (2007), Gallmeyer and Hollifield (2008), David (2008), Dumas, Kurshev, and Uppal (2009), Weinbaum (2009) and Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2014) have studied the effect of investors' belief difference in an exchange economy.

[2] Yu (2011) differs from Gallmeyer, Jhang, and Kim (2015) in his explanation on the the value premium in regard to belief difference. He mainly focuses on time-series aspects of stock returns. He argues that growth stocks are more sensitive to aggregate (not its own) belief difference so that they are overpriced since they are subject to more optimism. Thus his conclusion on low future returns of growth stocks are based on overpricing. However he does not provide proper explanations on why value stocks enjoy higher returns than growth stocks. Thus his argument is more in line with behavioral mispricing. Also he uses a measure of belief difference that is different from Diether, Malloy, and Scherbina (2002) and Gallmeyer, Jhang, and Kim (2015), albeit similar.

gap. Specifically, we first investigate the conditional cross-section of stock return variation when asset returns are sorted by investors' belief differences and book-to-market ratios, and then, try to find economically plausible characteristics that may explain the given return variation. In doing so, we hope to shed new light on understanding the cross-section of stock return variation in association with investors' different beliefs.

We begin our investigation by sorting assets based on belief difference and book-to-market ratio. Specifically, we independently sort assets by 5 degrees of belief difference and 5 degrees of book-to-market ratio such that we have total of 25 (5×5) portfolios. With this asset sorting procedure, we obtain two underpinning empirical findings. First, returns decrease from low to high belief difference for all groups of book-to-market ratios.And the decrease in returns is most pronounced in the group of lowest book-to-market ratio. Second, the value effect exists as returns increase from low to high book-to-market ratio for all groups of belief difference. And the value effect is most pronounced in the group of highest belief difference. The former empirical finding has been shown in several previous studies especially when the return variation is investigated only along the belief difference dimension. The latter finding, however, has been shown only recently in a couple of studies such as Gallmeyer, Jhang, and Kim (2015).

To investigate plausible sources for the given conditional cross-sectional return variation, we first apply Fama-French three factor model to 25 portfolios to see whether a typical risk-based model has a significant explanatory power. We find that factor loadings in the high belief difference group are lower than ones in other belief difference groups when average returns are decreasing from low belief difference group to high belief difference group, which is quite opposite to what risk-based story argues. This observation is similar to Griffin and Lemmon (2002) that investigates the cross-sectional return variation by sorting assets by credit risk and book-to-market ratio. They find that assets in high credit risk group have particularly lower returns but higher (or similar) values of factor loadings than assets in other lower credit risk groups. Similar to their results, our seemingly counter-intuitive empirical finding along the belief difference dimension is most pronounced in assets with lowest book-to-market ratio.[3] This result implies that conventional risk-based model might not be appropriate for explaining the cross-section when stocks are conditioned by belief difference and book-to-market ratio. Under the current circumstances where a conventional risk factor model fails to capture the given return variations, we need an economically plausible alternative explanations. Our study guides us to find some risk-reward type explanations that are not exactly the same as risk-based stories. In this sense, we take characteristic-based approach following the philosophy of Daniel and Titman (1997). Specifically, we search characteristic pricing arguments for between-group and within-group respectively depending on theoretical backgrounds that we rely on. The between-group corresponds to the group by belief difference when book-to-market ratio is controlled and the within-group corresponds to the group by book-to-market ratios when the degree of belief difference is controlled. We keep this notion of between-group and within-group for the rest of the paper.

For between-group analysis, we focus on two characteristics in explaining the decrease in returns from low to high belief difference when book-to-market ratio is controlled. First characteristic is derived from the overpricing arguments by the combination of belief difference and the short-sale constraints. This argument has long been around since Miller (1977) and Harrison and Kreps (1978). When pessimistic investors are sidelined by the short-sale constraints, the most optimistic investors tend to drive up the price. As a result, stocks with higher belief difference and stronger short-sale constraints will likely be overpriced, hence have lower returns. Following existing studies such as Chen, Hong, and Stein (2002), Nagel (2005), Choi, Jin, and Yan (2013), and etc., we measure the degree of short-sale constraint using the reciprocal of institutional stock ownership. Higher institutional ownership of a stock implies more stock supply for trading, which leads to less degree of short-sale constraints. We find that the between-group return decrease is well in line with the degree of short-sale constraints as the short-sale constraints are usually stronger in stocks with

[3] In Griffin and Lemmon (2002), groups are sorted by the proxy for the distress risk, i.e., Ohlson's O-Score. In comparison, belief difference in our study plays the similar role of O-Score in grouping assets. There is positive correlation between O-Score and belief difference as both are closely related by size and profitability.

higher belief difference in association with the short-sale constraint. Also the difference between degrees of the short-sale constraints is most pronounced in the portfolio of lowest book-to-market ratio, which is consistent with our empirical finding.

For the second alternative characteristic, we rely on the max-return that was introduced in the literature by Bali, Cakici, and Whitelaw (2011). They measure the maximal daily return for the past one month for a given stock as the max-return. They show that higher the max-return is, the lower the future return of a stock is and this relationship is highly significant. They argue that the max-return represents investors' lottery-demand or expected idiosyncratic skewness that is consistent with explanations of Barberis and Huang (2008), Brunnermeier and Parker (2005), and Brunnermeier, Gollier, and Parker (2007). We measure monthly max-returns for our portfolios. We find that portfolios with higher belief difference have, in general, higher max-returns and lower returns. And the difference of max-returns between portfolios with low and high belief differences is most pronounced in the portfolio of lowest book-to-market ratio. Furthermore, the max-return explanation parallels very well with the quantitative magnitude of return decreases along the belief difference dimension. Relying on a existing theoretical argument such as Xu (2007), we show that the max-return can also be related to investors' belief differences and short-sale constraints in association with return skewness. Especially, similar to the case of the institutional ownership, expected (idiosyncratic) skewness is directly related to belief difference and the short-sale constraints.

Next, we investigate the within-group return variation - a conditional value premium: returns increase from low to high book-to-market ratio for all belief differences. To find an appropriate characteristic for this case, we follow Gallmeyer, Jhang, and Kim (2015). They show that the book-to-market ratio is positively associated with (idiosyncratic) cash flow risk such that the value premium arises. In their model, idiosyncratic cash flow risk can be priced in equilibrium via belief difference. And value (growth) stocks happen to have higher cash flow risk and be exposed to more diverse investors' opinions. Hence, the value premium arises as value stocks are rewarded for much higher cash flow risk than growth stocks. One of key implications in their study is that the value effect is most pronounced for assets with high belief difference. Thus their argument is appropriate for within-group analysis as we investigate book-to-market dimension when belief difference is controlled. Our investigation shows that the proxy for the cash flow risk in association with belief difference increases from low to high book-to-market assets across all belief differences. And the difference between assets with high and low book-to-market in terms of cash flow risk is most pronounced in the highest belief difference group. This confirms our theoretically motivated characteristic argument that the conditional value premium crucially depends on cash flow risk via belief difference.

In summary, while traditional risk-based model struggles to explain the conditional cross-section of stock return variations, we could provide economically plausible explanations for the given return variation by focusing on characteristics. More importantly, characteristics that we focus on are closely related to investors' different beliefs and the market friction such as the short-sale constraint. Main characteristics for return differentials in between groups are the degree of short-sale constraints and the max-return that proxy investors' lottery-demand or expected idiosyncratic return skewness. On the other hand, the characteristic for within group return variation is (idiosyncratic) cash flow risk with belief difference. Our analysis shows that asset pricing models may have to account for the effect of investors' belief differences when explaining the cross-sectional stock return variation.

The remainder of the paper is as follows. In section 2, we introduce main empirical findings and how conventional risk-based explanation works. In section 3, we investigate conditional return differences for between-group and within-group approaches respectively. In so doing, we introduce theoretical arguments that we rely on. In section 4, we discuss pros and cons of our approaches in detail. We describe details of the data and some key quantities in the appendix.

II. Empirical Background

A. Underpinning Empirical Findings

In this section, we show our underpinning empirical findings for the conditional cross-section of stock return variation when asset returns are sorted by investors' belief difference and book-to-market ratio. Our empirical approach is based on the observation that investors' belief difference can play an important instrumental role for pricing assets, especially in the cross-section. In particular, we are most motivated by Gallmeyer, Jhang, and Kim (2015) who show that investors' belief difference can be an important pricing component in the cross-section.[4] Gallmeyer, Jhang, and Kim (2015) theoretically show that stocks with high belief difference would be more exposed to non-systematic cash flow risk such that value stocks will have higher expected returns than growth stocks since value stocks happen to have higher belief difference and higher non-systematic cash flow fluctuations. Gallmeyer, Jhang, and Kim (2015) provide empirical supports for their theoretical results. We begin our investigation of the conditional cross-section by independently sorting assets into total of 25 (5×5) portfolios by 5 degrees of belief differences and 5 degrees of book-to-market ratios. To construct investors' belief difference measure, we use analysts' earnings forecasts following Diether, Malloy, and Scherbina (2002) and Gallmeyer, Jhang, and Kim (2015). Analysts' earnings forecasts data come from the I/B/E/S data set (the Institutional Brokers' Estimate System). For the reliability of the data set such as stock coverage, we use I/B/E/S from 1983 to 2014. Though I/B/E/S dataset does not cover the entire stock universe, it has been shown in some studies that well known cross-sectional stock return patterns based on characteristics such as book-to-market, size, and etc., are well preserved.[5] We compute the ratio of standard deviation to the absolute mean

[4]Also, we are partly motivated by Yu (2011). However, arguments in Yu (2011) are largely about aggregated belief difference whereas Gallmeyer, Jhang, and Kim (2015) focus on individual belief difference at the portfolio level. Specifically, Yu (2011) analyzes the time-series return predictability based on aggregated belief differences. He empirically shows that the value premium is most pronounced in the stocks that have high sensitivities to belief difference.

[5]Gallmeyer, Jhang, and Kim (2015) show that the value premium is well represented in the CRSP-COMPUSTAT-I/B/E/S merged data. They show that the magnitude and the pattern of the value premium are very similar to the value premium that can be found in CRSP-COMPUSTAT merged data.

of earnings per share forecasts at each month as an investors' monthly belief difference measure.[6] Since we use I/B/E/S data, our stock universe is CRSP-COMPUSTAT-I/B/E/S merged data set. We compute three month average of belief difference to sort assets into 5 belief difference group each month. More specifically, for an individual stock, we compute the average of belief differences of the stock for the latest three months including the current month t, i.e., t, $t-1$, and $t-2$ months. We use this measure for sorting assets at time t. Thus, the measure of belief difference has overlapping components for the next two months for each asset. With this smoothing, we may be able to rule out some outliers that can exist in any month.[7] For the monthly book-to-market ratio, we use the usual monthly book-to-market ratio following the conventional method in Ken French's website. We described more details of the data sorting procedure in the appendix. As was mentioned above, we independently sort stocks based on 3-month average belief difference measure and ordinary monthly book-to-market ratio. Thus, we have total of 25 (5×5) portfolios are constructed every month and we record their next month (at time $t+1$) return as a future stock return. We report monthly value weighted average of belief difference and book-to-market ratio of our portfolios in tables below.[8]

[Insert Table I]

[Insert Table II]

Table I and II show the monthly average belief difference and the book-to-market ratio for independently sorted portfolios. Some key features of conditioning variables in the given cross-section are 1) the difference between high and low belief differences across book-to-market groups is most pronounced in the group of highest book-to-market ratio, and 2) the difference between high and low book-to-market ratios along belief difference groups is most pronounced in the highest belief difference group. First feature is the between-group (belief difference) variation and the latter is

[6] We provide details in the appendix.

[7] Besides, one month belief difference measure is, to some extent, subject to the issue of number of available analysts forecasts. Three months moving average can mitigate this issue.

[8] Note that monthly average value of belief difference for each portfolio is overlapping as we use the current and the past two months when computing monthly belief difference of individual stocks.

the within-group (book-to-market) variation. To check the validity of our data, we compute the belief difference measure along the book-to-market ratio only. We compare this with Gallmeyer, Jhang, and Kim (2015).

[Insert Table III]

Table III shows 1-dimensional belief difference along the book-to-market ratio. As is shown, the belief difference almost uniformly increases from low to high book-to-market ratio, which is consistent with Gallmeyer, Jhang, and Kim (2015).[9] The most interesting feature in Table I would probably be that the difference of belief difference measures between low and high book-to-market ratio is most pronounced in the highest belief difference group. This is closely related to the value premium that most prevails in the highest belief difference group. We discuss more details with the conditional cross-sectional return variations below.

Table IV below shows value weighted monthly average returns for independently sorted portfolios.

[Insert Table IV]

Value weighted returns in the conditional cross-section reveal several key features. We first observe returns in between-groups, i.e., portfolio returns across belief difference. Returns almost uniformly decrease from low to high belief difference for all the groups of book-to-market ratio, and the decreasing magnitudes in returns are different across book-to-market ratios. In particular, the return difference between high and low belief difference is most pronounced in the portfolios with the lowest book-to-market ratio. The long - short strategy based on belief difference, i.e., long a portfolio with highest belief difference and short a portfolio with lowest belief difference, yields a significantly negative return only in the lowest book-to-market group as seen in the t-stats of the long - short strategy.

[9] Note that Gallmeyer, Jhang, and Kim (2015) use belief difference measure of individual stock for each month, not the smoothed one from the latest three months as is the case above. Thus the magnitude of the cross-sectional variation of belief difference along the book-to-market ratio would look a bit different from our case.

Next, we observe returns in within-groups, i.e., portfolios across book-to-market ratio. Within-group returns increase from low to high book-to-market ratio such that we have the well known value premium. Though the value premium exists across all the groups of belief difference, it is most pronounced in the portfolios with highest belief difference. The long - short strategy based on book-to-market ratio, i.e., long a portfolio with highest book-to-market ratio and short a portfolio with lowest book-to-market ratio, yields significantly positive return differentials only in the two highest belief difference groups.

What would the conditional cross-section look like if we use the equal weighted average returns for independently sorted portfolios? Table V below shows equal weighted monthly average returns for independently sorted portfolios.

[Insert Table V]

The pattern of equal weighted returns is, in general, similar to the one in value weighted returns. We observe that returns decrease from low to high belief difference across all the groups of book-to-market ratios and returns increase from low to high book-to-market across all the groups of belief difference. However, the quantitative magnitudes of return decrease (increase) in between (within) groups are somewhat different from the value-weighted case. In particular, both long - short strategies based on belief difference and book-to-market ratio yield significantly negative and significantly positive return differentials at all cases. Thus, we have consistent return patterns in the equal weighted case, but only in stronger magnitude. It should, however, be noted that the overall return patterns in both value and equal weight cases are similar.

For more robustness check for the conditional cross-section of stock return variation when returns are conditioned on belief difference and book-to-market ratio, we further investigate the cross-section when returns are dependently sorted.

[Insert Table VI]

[Insert Table VII]

Both Table VI and VII show the conditional cross-section of dependently sorted portfolio returns. Since the key conditioning variable in our investigation of the cross-section is investors' belief difference, we first sort individual stocks into 5 different groups by the degree of belief difference. And then we sort assets in each group of belief difference into 3 groups by the degree of book-to-market ratio. As is seen in the table, we have the similar return patterns. We observe decrease in returns along belief difference and increases in returns along book-to-market ratios. Also return patterns from the between and within groups in both the value and equal weight cases are very similar to the case when returns are independently sorted. Robustness checks show that key empirical findings in the conditional cross-section of stock returns when returns are sorted by belief difference and book-to-market ratio seem stable. We focus mainly on the independently sorted portfolios below unless otherwise.

In short, key empirical findings in the conditional cross-section of stock return variation are summarized as follows. Average returns decrease from low to high belief difference when book-to-market ratio is controlled, 2) The magnitude of return decrease in average returns is strongest in the group of lowest book-to-market ratio, 3) average returns increase from low to high book-to-market ratio (the value effect) when belief difference is controlled, and 4) the value effect is most pronounced in the group of highest belief difference.

B. Does Conventional Risk-Based Model Work?

Now we investigate whether the given conditional cross-section of stock return variation when returns are sorted by belief difference and book-to-market can be explained by a conventional risk-based model. To this end, we take Fama-French three factor model (see Fama and French (1993)) as the benchmark risk-based model. Thus we apply three-factor model to the conditional cross-section, i.e., total of 25 (5×5) portfolios to see if it can explain the given return variation.

[Insert Table VIII]

[Insert Table IX]

Table VIII and IX show the regression results when Fama-French 3-factor model is applied to all 25 (5×5) independently sorted portfolios for the entire monthly time-series sample. We focus on the factor loadings in the regression results to see whether the model can properly capture the risk-return relationships. As Tables show, loadings on SMB (size factor) and MKT (market factor) are almost uniformly increasing from low belief difference to high belief difference across all book-to-market ratios. Also loadings on HML factor have larger values in groups with higher belief difference than lower belief difference. Pricing errors become negative from low to high belief difference in all book-to-market groups and statistically significant in most cases. Also we observe that negative alpha (or overpricing effect in the view point of a 3-factor model) is more pronounced in the low book-to-market group. All these observations on factor loadings indicate that traditional risk-return relationships represented by a 3-factor model do not seem to hold. In fact, portfolios with higher risk (higher factor loadings) do seem to have lower average returns.[10]

C. Comparison with Previous Studies

Interestingly, results from the 3-factor model are similar to Griffin and Lemmon (2002) that investigate the conditional cross-section when returns are sorted by credit risk that is represented by O-score and book-to-market ratio. They find that returns significantly decrease from low to high credit risk (low O-score to high O-score) across all book-to-market groups. And the value premium still exists as returns increase from low to high book-to-market across all credit risk groups. Besides, returns decrease most in the lowest book-to-market group and the value effect is most pronounced in the highest credit risk group, which is in line with our empirical findings. They apply Fama-French 3-factor model to their 15 (5×3) portfolios to confirm that traditional risk-return relationship does not hold to account for the given cross-section as risk-return relationship seems to hold oppositely

[10]This observation is also inconsistent with the idea that investors' belief difference may stand for uncertainty risk. For robustness check, we also apply Fama-French three factor model to equal-weighted portfolios. The result is largely the same both qualitatively and quantitatively.

as factor loadings increase from low to high credit risk groups.[11] Thus the main similarity between ours and Griffin and Lemmon (2002) is that portfolios for high belief difference (high credit risk) with low book-to-market have low returns not because they are particularly risky in terms of conventional risk-reward relationship. In fact, factor loadings are increasing from low to high belief difference groups in the low book-to-market portfolios. Finally we note that the annual pricing error from (high - low) strategy based on belief difference in the group of lowest book-to-market ratio is large and significant at -13.548% which dominates -8.76% and -10.44% for small stocks and big stocks along the credit risk dimension in Griffin and Lemmon (2002).[12]

We make further comparisons with other studies in terms of within-group (book-to-market) return differences. For within-group return variations, we note that the the small return difference between low and high book-to-market stocks in the low belief difference group is consistent with Kothari, Shanken, and Sloan (1995) and Griffin and Lemmon (2002). In Griffin and Lemmon (2002), the value effect is the smallest in the low O-Score group where larger stocks usually exist. In Kothari, Shanken, and Sloan (1995), the value effect decreases by about 40% when returns are restricted to larger firms on the NYSE. Large firms are usually exposed to low degree of belief difference as was shown in Diether, Malloy, and Scherbina (2002). Thus, smaller value effect in the low belief difference group is consistent with existing studies. On the other hand, the large value effect in high belief difference group is similar to Gallmeyer, Jhang, and Kim (2015) and Yu (2011). Both studies emphasize the role of belief difference. However their explanations on why the value premium is large in the group of high belief difference is somewhat different. While Yu (2011) emphasize the overpricing of the growth stocks, Gallmeyer, Jhang, and Kim (2015) emphasize the role of higher cash flow risk amplified via high belief difference in value stocks.

[11] While they sort assets based on 5 credit risk groups, and 3 book-to-market ratios, they also split the whole sample into two subsamples based on firm size. Thus total number of portfolios in their investigation is 30.

[12] The compatibility between our study and Griffin and Lemmon (2002) should not be very surprising as investors' belief difference seems to play similar role as O-score. O-score captures financial distress risk and is highly related to credit ratings as the higher the financial distress, the lower the credit ratings. Results that long-short strategy based on belief difference is mostly captured by long-short credit rating strategy in Avramov, Chordia, Jostova, and Philipov (2009) implies that sorting assets based on investors' belief difference is positively correlated with sorting assets based on credit ratings.

In sum, conventional 3-factor model struggles to explain the given conditional cross-section of stock return variation. Our results show that the conditional return variation is not related to differences in factor loadings such that risk-return relationship does not hold. This result leads us to think about some alternative economic explanations on the given cross-sectional return variation. We follow Daniel and Titman (1997) in that if risk-return relationship does not hold, then characteristics may account for the return variation.

III. Characteristic-Based Explanation

We now search economically plausible characteristic-based explanations in the fashion of Daniel and Titman (1997). We search characteristics for the given cross-section in each dimension in the cross-section so that we investigate the characteristics that help explain the return variation in the between-group (across belief difference) and the within-group (across book-to-market ratio) respectively.

A. Between-Group Return Variation

A.1. Overpricing by Short-Sale Constraints and Belief Difference

Between-group return variation implies that average monthly returns decrease from stocks with low belief difference to stocks with high belief difference. It is not new to the literature that stocks with high belief difference have lower average returns than ones with low belief difference. This has long been documented in the literature since seminal works by Miller (1977) and Harrison and Kreps (1978).[13]

[13]While Miller (1977) studies a static model with investors' belief difference, Harrison and Kreps (1978) study the dynamic version of the similar sort of model with investors' having resale options. Thus in the model of Harrison and Kreps (1978), resulting equilibrium price can be higher than the one in a static model due to investors' speculation about each other. Other notable theoretical studies following the two are Scheinkman and Xiong (2003) and Hong, Scheinkman, and Xiong (2006) and many other studies that we mentioned earlier. Recently Hong and Stein (2007) summarize the role of belief difference in the literature in the perspective of behavioral finance. However, as they argue, the belief difference can be easily nested into rational framework. Detemple and Murthy (1994), Basak (2000),

In this section, we search for a characteristic that governs the decreasing average returns across belief difference groups. In so doing, we also investigate whether the characteristic can help explain the quantitative magnitude of decrease in average returns, i.e., returns decrease most in the low book-to-market group. Based on many existing studies in the belief difference literature, we first investigate the overpricing effect of belief difference in association with the short-sale constraints. Miller (1977) and Harrison and Kreps (1978) argue that lower equilibrium returns for stocks with higher belief difference is related to the existence of short-sale constraint. When the short-sale constraints likely bind, returns tend to be low since the most optimistic investors drive up the price and vice versa. Following this argument, Diether, Malloy, and Scherbina (2002) empirically test the overpricing hypothesis using investors' belief difference. In so doing, they focus on the role of the short-sale constraints. They use the notion that the degree of the short-sale constraints is inversely related to the firm size, which has been argued in the literature.[14] They find the significantly negative relationship between the degree of investors' belief difference and subsequent stock returns especially when the short-sale constraints bind. Nagel (2005) shows that the short-sale constraints are closely related to limits-to-arbitrage. In particular, the higher the short-sale constraints are, the stronger the degree of limits-to-arbitrage is. His argument is that many financial anomalies are more pronounced in stocks that are subject to higher short-sale constraints since those stocks are more affected by limits-to-arbitrage effect, for which he provides empirical supports. Besides, other existing studies provide strong evidences on the overpricing effect when the short-sale constraints and belief difference are combined in experimental settings. Haruvy and Noussair (2006) and Palfrey and Wang (2012) show that, in well designed experiments, participants induce strong overpricing effect on assets especially when short-sale constraints bind. They also confirm that with less short-sale constraints overpricing is mitigated to the large extent.[15]

Basak (2005), and some others incorporate belief difference into otherwise standard rational asset pricing models.

[14] For more details about this, see Diether, Malloy, and Scherbina (2002).

[15] Haruvy and Noussair (2006) is interesting in the sense that lifting the short-sale constraints doesn't make the price go back to the exact level of the value. Rather the price will become less than the exact value due to excessive short-selling behavior. Palfrey and Wang (2012) confirms the speculative overpricing part that is similar to Harrison and Kreps (1978).

On top of those studies above, Chen, Hong, and Stein (2002) might provide an attractive equilibrium argument that provides the most close rationale for our analysis for the between-group return variation. Chen, Hong, and Stein (2002) develop an equilibrium stock market model in which investors have different opinions and are subject to the short-sale constraints. Their main theoretical result is that equilibrium stock return is significantly affected by short-sale constraints when investors have more diverse opinions about the underlying fundamentals. Using mutual fund holdings data, they construct the stock ownership breadth measure to proxy the short-sale constraints. It is widely accepted in the literature that wider the ownership breadth is, higher the short-sale constraints are. Thus, ownership breadth measure plays like a characteristic pricing argument in their model and it induces the negative relationship between stock returns and belief difference.

There have been studies that use similar measures as the proxy for the short-sale constraints. Nagel (2005) uses the residual institutional ownership as the proxy for the inverse of the short-sale constraints. Also in many other studies such as Chen, Hong, and Stein (2002), Choi, Jin, and Yan (2013), and among others, institutional investor ownership has been shown as a good inverse proxy for the short-sale constraints. D'avolio (2002) shows that institutional investors are major stock suppliers such that the higher the institutional ownership is, the lower the short-sale constraints is.

Following those studies, we take the institutional ownership as the inverse proxy for the degree of the short-sale constraints as well as a main characteristic that drives the between-group return variation.

[Insert Table X]

Table X shows the institutional stock ownership measure for independently sorted 25 portfolios. Institutional stock ownership is measured as the percentage of a portfolio's shares held by institutional investors out of total outstanding number of shares of the portfolio at time t. For institutional holdings data, we utilize Thompson-Reuter Spectrum 13(f) in WRDS. Since 13(f) in WRDS is available only at quarterly frequency, we take overlapping data at every month using current and previously available quarterly data. Fortunately, this does not compromise the reliability of data since the variability of the institutional ownership is quite low and stable over time as we can see in the following Table.

[Insert Table XI]

As the Table X shows, the pattern of Institutional ownership is well in line with the given return variation in between groups. Note that institutional ownership is the inverse proxy for the degree of the short-sale constraints. Our background overpricing theory implies that when investors' belief difference is high and the institutional ownership is low, the subsequent asset return will likely be low. Consistent with this implication, inverse institutional ownership captures the decrease in average returns in the between-group as there is a positive relation between institutional stock ownership and future stock returns. Furthermore, the variation of the institutional ownership across belief difference is more pronounced in the low book-to-market group as institutional stock ownership measure decreases most in the low book-to-market group. According to the theory, if the variation in short-sale constraints is high, then the return variation should be high. This implies that the average return in the low book-to-market group would decrease the most from low to high belief difference, which is confirmed in our cross-section. Interestingly, on the other hand, in the high book-to-market group, institutional ownership measures do not fluctuate much. This in turn yields a conjecture that the decrease in return along belief difference will be mild, if any. Indeed, we confirm the least quantitative magnitude of the decrease in returns along belief difference in the high book-to-market group. One caveat of the current observation is that the combination of the short-sale constraints and belief difference does not perfectly work. As we can see in the highest book-to-market group, belief difference is quantitatively very large in the high belief difference group. Average returns, however, do not differ much across belief difference.[16]

In short, our findings on the relation between institutional ownership and between-group return variation, suggest that our baseline overpricing theory is, to some extent, consistent with

[16] This observation may indicate that the short-sale constraints might have first-order importance when combined with belief difference. However, we do not argue further on this issue as it would be out of what we study here.

between-group return variation. This implies that institutional ownership as a characteristic plays an instrumental role in generating cross sectional return variation when investors' belief difference is taken into consideration for the cross-section.

A.2. Max-Return: Lottery Demand

We now search an alternative characteristic that is functionally similar to the short-sale constraints Thus it is negatively related to future stock returns along belief difference dimension.To this end, we focus on the max-return that has recently been suggested as an important pricing characteristic by Bali, Cakici, and Whitelaw (2011). They define the max return of a stock as the maximal daily return of a stock in the previous month. They show that there is a significantly negative relation between the max-return and the future stock return both at the level of individual stocks and portfolios. They argue that the max-return represents investors' demand for lottery-like stocks or expected skewness. They base their arguments off of the model of errors in probability of Barberis and Huang (2008) that uses cumulative prospect theory by Tversky and Kahneman (1992) and the optimal expectation theory of Brunnermeier and Parker (2005) that uses belief distortion for the purpose of increasing expected future utility flows. The common ground for both models is that investors tilt their preferences to the state with small probability but with large positive gain (or utility). Hence, investors tend to invest more on assets with positively skewed returns. As a result, the price of these assets will be pushed up such that the subsequent returns will become low. Bali, Cakici, and Whitelaw (2011) find compelling empirical evidences for their interpretation by showing that the max-return relation tends to persist over time.

Table XII shows the monthly average of the max-returns of independently sorted portfolios.

[Insert Table XII]

Interestingly, quantitative variation of the max-returns in the between-group is surprisingly well in line with the return variation in the between-group. The higher the max-return is, the lower the average return of a portfolio in each book-to-market group is. Furthermore, the difference of max-returns between low and high belief difference group is most (least) pronounced in the lowest (highest) book-to-market group. Interestingly, in our conditional cross-section, the variation of the max-return is quantitatively more compatible with the between-group return variation than the variation of institutional ownership measure as a inverse proxy for the short-sale constraints. However, we need to go little further for the max-return argument. One of key arguments in the background theories such as Barberis and Huang (2008) and Brunnermeier and Parker (2005) is that investors prefer the state of large positive gain even with small probability in determining their investment weight. Also this preference likely occur especially when their portfolios are under-diversified. Accordingly, it is plausible that investors would like to put more money into a portfolio for which they observe higher max-return as a signal for high positive gain when the portfolio is particularly under-diversified. Table XIV shows that number of stock observations in portfolios of the conditional cross-section.

[Insert Table XIV]

We reasonably assume that portfolios with less number of stock observations might be less diversified. First three low book-to-market groups show that low probability arguments hold since the max-return increases as portfolios become more under-diversified from low to high belief difference groups. However, for the remaining two high book-to-market groups, this relation does not hold. In particular, in those high book-to-market groups, while the max-returns mildly increase from low to high belief difference group, we observe that portfolios become more diversified as the number of stock increases. Thus the direction of the max-return relation does not seem to be entirely compatible with under-diversification arguments in our background theory. In sum, the max-return is not a panacea for analyzing between-group return variation as there are some cases where relatively less (more) diversified portfolios have lower (higher) max-returns. However, it explains the between-group return variation quantitatively very well.

A.3. Max-Return: Expected Idiosyncratic Skewness

Thus far, the max-return argument has not been related to investors' belief difference or the short-sale constraints while it has been discussed as a proxy for investors' lottery demand. Since investors' belief difference is a pillar of our conditioning information in the cross-section, it is natural to ask whether we can connect the max-return arguments to belief difference. It appears that Xu (2007) provides a theoretical background for this. He develops a competitive equilibrium model where investors are prohibited from short-selling and agree to disagree about the signal on underlying fundamentals.[17] As a result, the functional form of equilibrium price shows convexity with respect to the signal (see Proposition 2 and Figure 1 in Xu (2007)). Specifically, equilibrium price as a function of the signal becomes more convex if an asset is subject to stronger short-sale constraints and investors have more diverse opinions about the signal.

[Insert Figure 1]

Figure 1 shows the modified version of Figure 1 of Xu (2007). If an asset is subject to higher short-sale constraints, then its equilibrium price would be more convex with respect to the signal from fundamentals (see the dashed line). In Xu (2007), the signal comes from asset's payoff. By regarding the book-to-market as a signal for a portfolio's payoff (or profitability), we can apply above logics to our case. Following conventional wisdom, we take the notion that high (low) book-to-market ratio signals low (high) profitability of a firm. When investors disagree about the observed signal, the degree of the short-sale constraints determines the convexity of equilibrium price. As we see in Table II, portfolios in each book-to-market group are exposed to the signal (book-to-market ratio) of the same magnitude despite that they are subject to different degrees of belief difference. As mentions above, at each signal, stronger short-sale constraint leads to more convex the equilibrium price function. Thus we expect to observe that a portfolio will have higher equilibrium price when it is subject to stronger short-sale constraints at the same signal. Note that the max-return can be

[17] This is quite a common assumption in the belief difference literature. See Gallmeyer, Jhang, and Kim (2015) for more details and references therein.

a proxy for higher contemporaneous price at the time of previous month. Also we observe that in the data the max-return and the short-sale constraints are positively correlated. All told, the logic in Xu (2007) can be well applied to our case: when a stock is short-constrained and is exposed to higher belief difference, then it would have high contemporaneous price. High contemporaneous price would be proxied by high max-return, which in turn leads to lower future stock return.

The model in Xu (2007) implies that higher price convexity leads to higher return skewness in equilibrium. Thus, we now investigate the relation between the max-return and idiosyncratic return skewness. There are many studies studying the relationship between return skewness and future stock returns. A general consensus in the literature is that return skewness and future stock returns are negatively related.[18] The reason that we investigate idiosyncratic skewness, not ordinary return skewness, is because the max-return is, by nature, exclusively an idiosyncratic measure. Bali, Cakici, and Whitelaw (2011) argue that the max-return might represent expected idiosyncratic skewness. Moreover they show that the max-return, to some extent, absorbs the effect of idiosyncratic skewness by showing that the significant negative relationship between the max-return and future stock return still exists even after idiosyncratic skewness is controlled in the cross-section.[19] Following them, we examine whether idiosyncratic skewness can help explain the return variation in the between-group and whether it can be quantitatively comparable to the max-return. Table XIII shows that idiosyncratic return skewness. To compute the idiosyncratic skewness, we follow Bali, Cakici, and Whitelaw (2011) and Mitton and Vorkink (2007). In particular, we use

[18] Asset pricing with more than second moments has long been studied in the literature. Arditti (1967), Kraus and Litzenberger (1976), Scott and Horvath (1980), and Kane (1982) are the first generation and they focus on the systematic pricing effect of skewness. Harvey and Siddique (1999), Bakshi, Kapadia, and Madan (2003), Smith (2007), and Chang, Christoffersen, and Jacobs (2013) provide empirical supports by showing that coskewness or loadings on market skewness has pricing power. However, another strand of study focus on the importance of individual or idiosyncratic skewness. Simkowitz and Beedles (1978), Conine Jr. and Tamarkin (1981), Mitton and Vorkink (2007), Boyer, Mitton, and Vorkink (2010), Zhang (2005), Amaya, Christoffersen, Jacobs, and Vasquez (2015), Conrad, Dittmar, and Ghysels (2013), and etc. show that there is significant pricing effect of idiosyncratic (or individual) skewness. It is worth noting that all the studies has a common pricing implication: higher the skewness, lower the future stock returns.

[19] In their study, Bali, Cakici, and Whitelaw (2011) also investigate the relationship between the max-return and future stock returns after controlling total skewness and systematic skewness (co-skewness). The effect of the max-return still exists at the significant level.

residuals from the following regression to obtain idiosyncratic skewness:

$$r_{i,d} = \alpha_i + \beta_i(r_{m,d} - r_{f,d}) + \gamma_i(r_{m,d} - r_{f,d})^2 + \epsilon_{i,d}, \tag{III.1}$$

where $r_{i,d}, r_{m,d}$ and $r_{f,d}$ are daily returns of portfolio i, market index, and the riskless asset at day d. As the Table XIII shows, a well established negative relation between idiosyncratic skewness and future portfolio return is observed across belief difference in each group of book-to-market ratio. In the group of lowest book-to-market ratio, idiosyncratic skewness increases a lot (from negative to positive). Thus the most decrease in return is consistent with the most increase in idiosyncratic skewness. On the other hand, in the group of highest book-to-market ratio, idiosyncratic skewness does not fluctuate much across belief difference. This is consistent with mild return variation in the high book-to-market group. While the negative relation between idiosyncratic skewness and future portfolio return seems to explain the return variation in the between-group to some extent, the overall magnitude of the variation is less satisfactory compared to the return variation induced by the max-return (see the return variation in the group of second lowest book-to-market ratio). This is partially consistent with Bali, Cakici, and Whitelaw (2011) in that the max-return seems to absorb the effect of idiosyncratic skewness in the cross-sectional return variation.

In this section, we investigate the relationship between the max-return and future stock return in view of the equilibrium price convexity, by interpreting book-to-market ratio as the signal for profitability (or payoffs) of a portfolio. Portfolios with varying short-sale constraints and varying belief difference would likely exhibit different convexities in equilibrium price function. Relying on our interpretation of the theory that higher the price convexity is, higher the max-return is, we empirically confirm that the higher max-returns leads to lower future stock returns across belief difference as the short-sale constraints and belief difference are positively related. Also, we further investigate the possibility of the max-return as an idiosyncratic return skewness based on existing theory where higher price convexity leads to higher skewness in returns. We note that the skewness under our consideration idiosyncratic in its nature since it is closely associated with the max-return that is idiosyncratic. In sum, we have connected the max-return to investors' belief difference and short-sale constraints.

B. Within-Group Return Variation

B.1. Book-to-Market Effect

We now turn to the investigation of the within-group return variation, i.e., the conditional value premium. We first note that traditional risk-based factor model is still capable of capturing the value premium.As Table IX shows, loadings in HML factor in each belief difference group increase from portfolio with low book-to-market ratio to the one with high book-to-market ratio, and the magnitude of increase in HML loadings is well in line with the the magnitude of the increase in returns. Thus, one can still argue that the value premium is well captured in terms of risk-return trade-off. This is not surprising. We note that HML is truly a powerful cross-sectional pricing factor. Although numerous return predicting variables have been suggested in the literature over the years (see Green, Hand, and Zhang (2013) and Harvey, Liu, and Zhu (2016)), B/M or HML have been shown to be one of most powerful cross-sectional pricing factors that survive strong statistical tests (see Harvey, Liu, and Zhu (2016) and Kim and Skoulakis (2014)). And yet, Fama-French factor model fails to explain the risk-return trade-off in our conditional cross-section especially when it comes to the between-group return variation. Therefore, we would like to find a characteristic other than HML. It should be closely related to B/M, and also should be capable of generating the within-group return variation, i.e., the value premium.

B.2. Cash Flow Share Ratio and Belief Difference

Some recent structural asset pricing models explain the value premium by endogenously connecting cash flow risk (or cash flow duration) to B/M and expected returns.[20] While many empirical

[20]For studies that connect cash flow risk and cash flow duration to the value premium via B/M, see Croce, Lettau, and Ludvigson (2014), Da (2009), Cornell (1999), Cornell (2000), Dechow, Sloan, and Soliman (2004), Binsbergen, Brandt, and Koijen (2012), Lettau and Wachter (2007), Santos and Veronesi (2010), Gallmeyer, Jhang, and Kim

studies such as Campbell and Vuolteenaho (2004) show that cash flow risk is crucial in explaining the value premium, some studies have shown that the systematic cash flow risk might not be able to generate enough return variation to justify the value effect. In the cross-section, it is well known that the cash flow risk premium and the discount rate risk premium move opposite to each other along the book-to-market dimension. According to conventional wisdom and equilibrium argument, discount rate (cash flow) risk premium prevails in growth (value) stocks due to the cash flow duration effect. Unless the cash flow risk premium sufficiently dominates the discount-rate risk premium in the book-to-market dimension, a counterfactual growth premium will eventually prevail. Santos and Veronesi (2010) address this issue in their typical general equilibrium asset pricing model by showing that the discount rate risk premium dominates the cash flow risk premium such that the growth premium arises as a result. To resolve this issue, they counterfactually amplify the (systematic) cash flow risk to generate enough return variation to induce the value premium. Lettau and Wachter (2007) also recognize this issue. They set up the stochastic discount factor to highlight the cash flow risk in association with investors sentiment that has nothing to do with systematic risk. Thus non-systematic risk component is priced in equilibrium, and overall cash flow risk dominates the discount rate risk effect such that the value premium arises.[21]

Following those studies (especially by adopting arguments of Lettau and Wachter (2007) and Santos and Veronesi (2010)), we focus on a characteristic that 1) can connect to B/M or HML effect, 2) is closely related to (possibly non-systematic) cash flow risk component and 3) is related to investors' sentiment. To this end, we find that the model of Gallmeyer, Jhang, and Kim (2015)

(2015), Bansal and Yaron (2004), Bansal, Dittmar, and Lundblad (2005), Campbell and Vuolteenaho (2004), Hansen, Heaton, and Li (2008), and Lettau and Wachter (2007).

[21] Understanding the term-structure of equity risk is very important as it allows us to better understand risk-return trade-off in asset pricing. Recently Binsbergen, Brandt, and Koijen (2012) show that the term-structure of equity is downward sloping as assets with short-term cash flow duration has higher risk premium than assets with long-term cash flow duration. As value stocks have shorter cash flow duration than growth stocks, the value premium represents the downward sloping equity term structure. Binsbergen, Brandt, and Koijen (2012) further show that typical asset pricing models such as Bansal, Dittmar, and Lundblad (2005) and Campbell and Cochrane (1999) generate the counterfactual growth premium. This has been further confirmed in other studies such as Weber (2015). Studies such as Gallmeyer, Jhang, and Kim (2015), Belo, Collin-Dufresne, and Goldstein (2015), Croce, Ai, Diercks, and Li (2013), Croce, Lettau, and Ludvigson (2014), and Jermann (2013) are the ones that resolve this issue in a various different contexts.

best suits our purpose. They study a continuous-time asset pricing model in an exchange economy with habit formation, where investors agree to disagree about individual cash flows via idiosyncratic shocks.[22] Since the stochastic discount factor in the model becomes similar to Lettau and Wachter (2007), idiosyncratic cash flow risk that doesn't covary with aggregate fundamentals, gets priced in equilibrium via investors' belief difference. Interestingly, value stocks happen to have higher belief difference and higher idiosyncratic cash flow risk than growth stocks such that the value premium arises. One of key theoretical results in their study is that the expected excess return can be expressed as

$$E_t\left[r_{s,t}\right] = \# \frac{\bar{s}}{s_t} + \# \frac{\bar{s}}{s_t} \frac{\bar{H}}{H_t} + \cdots , \tag{III.2}$$

where s_t is the cash flow share for a portfolio under investigation, H_t is the habit, $\bar{s}$ is the long-run mean of the cash flow share s_t, and $\bar{H}$ is the long-run mean of the habit H_t.[23] Key observation from this equation is that the share ratio $\bar{s}/s_t$ proxies the cash flow risk including idiosyncratic part, and the coefficient that is expressed in # has investors' belief difference term. Thus overall cash flow risk is captured by the interaction between the cash flow share ratio $\bar{s}/s_t$ and investors' belief difference.

[Insert Table XV]

Table XV shows the cash flow share ratio for independently sorted portfolios and the interaction between the cash flow share ratio and belief difference. Interaction terms increase from low to high book-to-market ratios across all belief difference groups. Furthermore, the magnitude of increase in interaction terms is most (least) pronounced in the highest (lowest) belief difference group, which is highly consistent with the within-group return variation. In order to confirm that systematic cash flow risk might not be able to properly capture the book-to-market effect or the value premium, we show the systematic cash flow risk for independently sorted 25 portfolios below.

[22] Thus the model is very similar to Santos and Veronesi (2010) other than that there exists investors' belief difference through idiosyncratic risk.

[23] For more details, see Gallmeyer, Jhang, and Kim (2015). Also see the appendix for the details of the cash flow share, s_t.

[Insert Table XVI]

The notion of conditional systematic cash flow risk, $\theta_{i,t}$ comes from Menzly, Santos, and Veronesi (2004) and Gallmeyer, Jhang, and Kim (2015). It measures the covariance between the cash flow share ratio and aggregate cash flow growth at time t. We use cash flow data for the past 24 months to construct the conditional systematic cash flow risk at each time t.[24] And then we take their time-series average to compare across book-to-market ratios. Table XVI shows that systematic cash flow risk does not seem to capture the value effect. Based on the statistical significance of the value effect in our conditional cross-section (see Table IV), we are particularly interested in the highest belief difference group where the value premium is most and significantly pronounced. In fact, systematic cash flow risk is actually lower in the group of high book-to-market ratio. This confirms aforementioned argument that systematic cash flow risk might not be able to capture the book-to-market effect properly in terms of risk-reward trade-off.

IV. Discussion

In this section, we discuss caveats of our two-way approach - between-group and within-group analyses - in explaining the conditional cross-section of return variation. The first characteristic that governs the return variation in the between-group is the institutional ownership as the inverse of the short-sale constraints. Higher the short-sale constraints (i.e., lower the institutional ownership) is, lower the future stock return is. For the first 3 belief difference groups, institutional ownership measures in the groups of high book-to-market ratios are lower than ones in the groups of low book-to-market ratios. Thus, in this case, it captures the value effect. However, it should be noted that the difference of the short-sale constraints between low and high book-to-market ratio decreases from low to high belief difference in those three groups, while the value effect gets stronger.[25] Moreover, for the remaining two high belief difference groups, the difference

[24] Thus conditional systematic cash flow risk measure $\theta_{i,t}$ uses overlapping cash flow data.

[25] Though it is not statistically significant, we can still see the value effect.

of the short-sale constraints between low and high book-to-market ratio is actually negative, which runs counter to the value premium relation. Thus the short-sale constraints doesn't seem to be appropriate for the analyzing the within-group return variation.

The second characteristic in explaining the between-group return variation is the max-return. The max-return shows some promising properties in explaining the value effect in the within-group return variation. We first note that lower the max-return is, higher the future stock return is. As shown in Table XII, the max-returns are lower in portfolios with high book-to-market ratio in groups with four high belief difference. Thus the value effect seems to be captured. However, the quantitative magnitude becomes bit of an issue. Although the difference between max-returns in low and high book-to-market group is most pronounced in the highest belief difference group, it is also well pronounced in low belief difference groups either. For low belief difference groups other than the highest belief difference group, the value premium is not significantly pronounced. However, the max-return variation in low belief difference groups seem quite significant. This implies that the max-return variation might over-capture the value premium when it comes to portfolios with low belief difference.

Idiosyncratic cash flow risk with belief difference is proxied by the interaction of the cash flow share ratio and belief difference. A caveat for this measure is as follows. As Table XV shows, the differences of interaction terms between low and high belief difference are all negative. If the interaction terms effectively represents the cash flow risk, then the order of the magnitude of the interaction terms across belief difference runs counter to the intuition. Thus (idiosyncratic) cash flow risk measure with belief difference is not capable of capturing the between-group return variation. In short, while our endeavor to explain the return variation in the between-group and in the within-group respectively shows some promising results, both approaches have their own caveats in explaining the return variation in other dimension.

Pros and cons of our two-way approach lead us to conclude that we need a new unified equilibrium model in order to successfully explain the currently given conditional cross-section of stock return variation. Conventional risk-based model has limited explanatory power in accounting for the given conditional cross-section. Empirical findings and discussions so far indicate that we need an equilibrium model where investors' belief difference and the market friction such as the short-sale constraints play instrumental roles in pricing assets. As Gallmeyer, Jhang, and Kim (2015) show, investors' belief difference can be imposed in a non-conventional way such that idiosyncratic sort of risks can be priced. This, in turn, would lead us to look at traditionally ignored unpriced components in studying the cross-section of stock return variation. As there have been piling empirical evidences indicating that idiosyncratic type of risk might not be dissipated away in optimal portfolio choices either because of under-diversification or of belief difference (i.e., Ang, Hodrick, Xing, and Zhang (2006), Basak (2005), Gallmeyer, Jhang, and Kim (2015), and etc.), it would be better for the model to allow the pricing of non-systematic (cash flow) risk in equilibrium. For instance, when searching equilibrium characteristics that can help explain the given conditional cross-section, we can adopt arguments suggested by Lettau and Wachter (2007), Santos and Veronesi (2010), and Gallmeyer, Jhang, and Kim (2015) for pricing beyond the systematic cash flow risk.

V. Concluding Remarks

In this study, we investigate the conditional cross-section of stock return variation when returns are sorted by investors' different beliefs and book-to-market ratio. Conventional risk-based model struggles to explain the given return variations. Following the arguments of Daniel and Titman (1997) who emphasize the characteristic-based risk-return relation, we try to find characteristics that are mainly related to belief difference. We explain the given cross-section by investigating the the between-group (across belief difference) return variation and the within-group (across book-to-market ratio) return variation respectively. For the between-group return variation, we find two characteristics - inverse institutional ownership and the max-return - by depending on 1) the overpricing argument by the combination of belief difference and the short-sale constraints and 2) theories based on investors' preference for lottery-like assets or expected idiosyncratic skewness. For the within-group return variation, we find a characteristic -the interaction term between the cash flow share ratio and belief difference - that reflects the (idiosyncratic) cash flow risk associated with belief difference. In the data, each characteristic well explains the given cross sectional return variation in the corresponding dimension. However, characteristics should be carefully taken into consideration as they lose their explaining power outside own dimension. Our empirical findings calls for a new equilibrium model where investors' different beliefs the basic cornerstone of the model. And the short-sale constraints can be imposed as restrictions on investors' trading behaviors, or it can be directly modeled into the stochastic discount factor.[26] In addition, non-traditional types of risk can be allowed to be priced in equilibrium, especially along the value dimension.

[26]For a direct modeling of the short-sale constraint into the stochastic discount factor, see Gallmeyer and Hollifield (2008).

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Appendix A : Data Description

We construct investors' belief disagreement measure following Diether, Malloy, and Scherbina (2002). This method was also adopted in Gallmeyer, Jhang, and Kim (2015). We retrieve Earnings-Per-Share (EPS) forecasts data from the I/B/E/S dataset. More specifically, we utilize I/B/E/S STATSUM dataset.[27] From STATSUM, We use monthly EPS forecasts with the most recent forecasts value. Each month, we find the latest analysts forecasts available. If the value is missing, we use the previously available forecast values. At the end of each forecast period, we take the standard deviation of EPS forecasts. If there is more than one report on the standard deviation, we take average of them. We then compute the coefficient of variation of EPS forecasts, i.e., the standard deviation of EPS forecasts decided by the absolute mean value of EPS forecasts.[28] We use this quantity as an empirical measure for investors belief difference, denoted as BD. With this way, we construct investors' belief dispersion on an individual stock at monthly frequency. For a portfolio, we take weighted average of individual dispersions based on a firm's market capitalization.

For the measure of short-sale constraint, we would like to use a simple measure that can be widely accepted in the short-sale constraint literature. Specifically, we use institutional ownership as proxy for short-sale constraint. We retrieve institutional holdings data from Thompson-Reuter Spectrum 13(f) in WRDS. One easy way to define institutional ownership is to compute the percentage of all holdings reporting institutions that report a positive stock position, where reporting institutions are all institutions that report any positions in the quarter. This definition is the same as Xu (2007). However, we define institutional ownership as the percentage share holdings. Thus we compute the percentage of institutions' holding shares out of total outstanding shares. This definition is consistent with many studies in the literature of short-sale constraints. Nagel (2005) and Chen, Hong, and Stein (2002) use a similar definition of institutional ownership though their definitions are slightly different. One caveat of this measure is the data frequency. In our empirical analysis, we use the monthly frequency for the most part. Unfortunately, Thompson-Reuter Spectrum 13(f) give us only quarterly data. Thus we use the most recently available 13(f) data at any point in time, which implies that we use overlapping data at the quarterly window.

For the measure of (idiosyncratic) cash flow, we follow Gallmeyer, Jhang, and Kim (2015), Stephens and Weisbach (1998), and Grullon and Michaely (2002). We measure the cash flow of a stock by summing dividends and stock repurchases. We construct time-series of cash flows of each stock on a monthly basis. For dividends, we use returns with and without dividends to compute the dividend yield. Dividend then is computed by multiplying the dividend yield with the market capitalization of a stock. To compute the market capitalization, we multiply the

[27]For more details of the dataset such as the difference between the detail file and summary file of the I/B/E/S dataset, readers refer to Diether, Malloy, and Scherbina (2002).

[28]Coefficient of variation is a popular measure that is widely used in empirical research area. This measure is popular since it enables us to compare the the quantity under scrutiny without the size effect.

mid-point of stock prices within a month with the outstanding number of shares of a stock. For stock repurchases, we compute the decrease in numbers of the outstanding shares of a stock every month by using accounting information in COMPUSTAT. If this decrease is positive (number of shares decreases), then we multiply the decrease in number of shares with the mid-price of a stock in a month. If decreases in the outstanding number of shares are negative (numbers of shares increase), then we take zero as a stock repurchase. Finally, we take the moving sum of current and past two months of dividends and share repurchases as the monthly cash flow in order to mitigate the missing value problem in cash flow data. The aggregate cash flow is computed by summing individual cash flows across all individual firms.

At each month t, we use cash flows of independently sorted 25 portfolios. In particular, we use a portfolios cash flows for the past 24 months and take the time-series average to obtain the long-run mean $\bar{s}$. Thus at every month t, we have 24 cash flow share ratio by taking the ratio of $\bar{s}$ at each time t to 24 s_t's. We compute the average of these cash flow share ratios to obtain one representative number for the cash flow share ratio at each time t. Finally, we show the time-series average of these share ratios to express the representative number of the cash flow share ratio for a proxy of cash flow risk.

Tables

Table I Conditional Cross-Sectional Variation in Belief Disagreement – This table shows average belief difference for independently sorted portfolios. We sort assets from CRSP-Compustat-I/B/E/S merged data into total of 25 (5×5) portfolios by belief difference and book-to-market ratio. We compute belief difference for each portfolio using value weighted average of monthly belief difference of individual stocks.

	B/M1 (lowest)	B/M2	B/M3	B/M4	B/M5 (highest)
BD1 (lowest)	0.009	0.010	0.010	0.010	0.009
BD2	0.027	0.027	0.027	0.027	0.028
BD3	0.050	0.050	0.050	0.051	0.051
BD4	0.102	0.102	0.103	0.104	0.106
BD5 (highest)	0.775	0.785	0.722	0.838	1.075

Table II Conditional Cross-Sectional Variation in Book-To-Market Ratio – This table shows average book-to-market ratio for independently sorted portfolios. We sort assets from CRSP-Compustat-I/B/E/S merged data set into total of 25 (5×5) portfolios by belief difference and book-to-market ratio. We compute average book-to-market ratio for each portfolio as value weighted average book-to-market ratio of individual stocks.

	B/M1 (Low)	B/M2	B/M3	B/M4	B/M5 (High)
BD1 (Low)	0.244	0.476	0.688	0.941	1.560
BD2	0.250	0.478	0.690	0.947	1.591
BD3	0.247	0.480	0.691	0.949	1.593
BD4	0.240	0.482	0.693	0.952	1.696
BD5 (High)	0.230	0.481	0.697	0.957	1.906

Table III Cross-Sectional Variation in Belief Dispersion – This table shows average belief difference for portfolios sorted by book-to-market ratio (value decile). We sort assets from CRSP-Compustat-I/B/E/S merged data set 10 portfolios by book-to-market ratio. Then we compute value weighted average belief difference of each portfolio using monthly belief difference of individual stocks in each portfolio. For the monthly belief difference of individual stocks, we use monthly belief differences in the latest three months, i.e., time t, $t-1$ and $t-2$, and average them. B/M1 and B/M10 are lowest and highest book-to-market ratios.

	B/M1	B/M2	B/M3	B/M4	B/M5	B/M6	B/M7	B/M8	B/M9	B/M10
BD	0.158	0.139	0.154	0.165	0.159	0.184	0.206	0.253	0.312	0.561

Table IV Conditional Cross Sectional Return Variation: Value Weighted– This table shows cross-sectional return variation when belief difference and book-to-market ratio are given as conditions for sorting assets. We use CRSP-Compustat-I/B/E/S merged data set as our stock universe. We independently sort stock returns such that assets are sorted by 5 different degrees of belief difference and 5 different degrees of book-to-market ratios. Thus total of 25 (5×5) portfolios are constructed every month. We report value weighted monthly average percentage returns for each portfolio. We also report average returns for long-short strategy for each book-to-market group and belief difference group. ** and * indicate statistical significance at the 5% level and 10% level respectively for the long-short strategy. Blank indicates the t-stat for each monthly average returns. B/M and BD indicate book-to-market ratio and belief difference respectively.

	Value Weighted					
	B/M1 (lowest)	B/M2	B/M3	B/M4	B/M5 (highest)	Value-Growth
BD1 (lowest)	1.080	1.122	1.199	1.383	1.476	0.395
	(4.643)	(4.627)	(4.847)	(5.794)	(5.418)	(1.692)
BD2	0.876	0.874	1.023	1.086	1.332	0.456
	(3.061)	(3.307)	(4.052)	(4.254)	(4.685)	(1.773)
BD3	0.755	1.119	0.808	1.239	1.181	0.426
	(2.163)	(3.776)	(2.883)	(4.426)	(3.563)	(1.415)
BD4	0.647	0.773	0.919	1.072	1.206	0.559
	(1.678)	(2.424)	(2.991)	(3.437)	(3.452)	(1.770)*
BD5 (highest)	0.324	0.841	0.678	0.820	1.179	0.855
	(0.664)	(2.183)	(1.849)	(2.065)	(2.927)	(2.063)**
High BD-Low BD	-0.756	-0.281	-0.521	-0.563	-0.296	
	(−1.908)*	(−0.953)	(−1.922)	(−1.806)	(−0.937)	

Table V Conditional Cross Sectional Return Variation: Equal Weighted– This table shows cross-sectional return variation when belief difference and book-to-market ratio are given as conditions for sorting assets. We use CRSP-Compustat-I/B/E/S merged data set as our stock universe. We independently sort stock returns such that assets are sorted by 5 different degrees of belief difference and 5 different degrees of book-to-market ratios. Thus total of 25 (5×5) portfolios are constructed every month. We report equal weighted monthly average percentage returns for each portfolio. We also report average returns for long-short strategy for each book-to-market group and belief difference group. ** and * indicate statistical significance at the 5% level and 10% level respectively for the long-short strategy. Blank indicates the t-stat for each monthly average returns. B/M and BD indicate book-to-market ratio and belief difference respectively.

	Equal Weighted					
	B/M1 (lowest)	B/M2	B/M3	B/M4	B/M5 (highest)	Value-Growth
BD1 (lowest)	1.313	1.407	1.517	1.630	2.004	0.690**
	(4.773)	(5.409)	(6.196)	(6.740)	(6.166)	(2.901)
BD2	0.795	1.193	1.346	1.390	1.866	1.072**
	(2.553)	(4.369)	(5.345)	(5.333)	(5.785)	(4.128)
BD3	0.898	1.099	1.224	1.413	1.655	0.757**
	(2.466)	(3.695)	(4.400)	(5.316)	(5.020)	(2.748)
BD4	0.815	0.854	1.100	1.204	1.613	0.798**
	(2.008)	(2.490)	(3.564)	(3.841)	(4.349)	(2.639)
BD5 (highest)	0.367	0.790	0.845	0.891	1.064	0.697**
	(0.782)	(1.923)	(2.244)	(2.449)	(2.574)	(2.244)
High BD-Low BD	-0.946**	-0.617**	-0.672**	-0.738**	-0.939**	
	-(3.201)	-(2.382)	-(3.032)	-(3.417)	-(4.266)	

Table VI Dependently Sorted Conditional Cross-Section: Value Weighted– This table shows cross sectional return variation when belief difference plays a conditioning role. We use CRSP-Compustat-I/B/E/S merged data set as our stock universe. We dependently sort stock returns. We first sort assets into 5 groups based on the degree of belief difference. Second, we sort assets in each group of belief difference into 3 groups based on the magnitude of book-to-market ratio. Thus we have 15 portfolios. We report value weighted average percentage returns for each portfolio. We also report average returns for long-short strategy for each book-to-market group and belief difference group. * indicates statistically significant at the 5% level.

	Value Weighted			
	Low B/M	Medium B/M	High B/M	Long-Short Value Strategy
BD1 (Low)	1.083	1.128	1.331	0.248*
BD2	0.880	0.848	1.220	0.340*
BD3	0.783	0.958	1.173	0.390*
BD4	0.607	0.878	1.202	0.595*
BD5 (High)	0.503	0.686	1.159	0.656*
Long-Short Belief Strategy	-0.580*	-0.442*	-0.173*	

Table VII Dependently Sorted Conditional Cross-Section: Equal Weighted– This table shows cross sectional return variation when belief difference plays a conditioning role. We use CRSP-Compustat-I/B/E/S merged data set as our stock universe. We dependently sort stock returns. We first sort assets into 5 groups based on the degree of belief difference. Second, we sort assets in each group of belief difference into 3 groups based on the magnitude of book-to-market ratio. Thus we have 15 portfolios. We report equal weighted average returns for each portfolio. We also report average percentage returns for long-short strategy for each book-to-market group and belief difference group. * indicates statistically significant at the 5% level.

	Equal Weighted			
	Low B/M	Medium B/M	High B/M	Long-Short Value Strategy
BD1 (Low)	1.316	1.401	1.675	0.359
BD2	0.834	1.195	1.550	0.716*
BD3	0.905	1.161	1.533	0.629*
BD4	0.837	1.035	1.445	0.608*
BD5 (High)	0.537	0.831	1.134	0.597*
Long-Short Belief Strategy	-0.779*	-0.570*	-0.541*	

Table VIII Alpha and Market Beta for Value Weighted Returns – This table shows alpha and market beta when Fama-French 3-factor model is applied to value weighted returns for independently sorted portfolios : assets from CRSP-Compustat-I/B/E/S merged data set are independently sorted into total of 25 (5×5) portfolios by belief difference and book-to-market ratio. Fame-French three-factor time series regressions are estimated over the entire monthly time periods for each portfolio as follows: $r_t = \alpha + \beta_m MKT_t + \beta_s SMB_t + \beta_h HML_t + \epsilon_t$, where MKT is the value-weighted market portfolios excess return, SMB is the factor returns on small minus big stocks, and HML is the factor returns on high minus low B/M. The coefficients α and MKT from these regressions and their corresponding t-stat values are reported below.

	α and $t(\alpha)$					
	B/M1 (Low)	B/M2	B/M3	B/M4	B/M5 (High)	High - Low
BD1 (Low)	0.345	0.131	0.149	0.340	0.341	-0.005
	(3.635)	(1.097)	(1.136)	(2.280)	(1.856)	(-0.025)
BD2	0.038	−0.174	−0.083	−0.042	0.179	0.142
	(0.305)	(−1.562)	(−0.642)	(−0.299)	(0.924)	(0.667)
BD3	−0.147	0.056	−0.324	0.020	−0.163	-0.016
	(−1.046)	(0.366)	(−2.426)	(0.152)	(−0.796)	(-0.070)
BD4	−0.238	−0.289	−0.236	−0.253	−0.178	0.061
	(−1.418)	(−1.747)	(−1.545)	(−1.769)	(−0.917)	(0.249)
BD5 (High)	−0.784	−0.370	−0.694	−0.634	−0.367	0.416
	(−2.977)	(−1.764)	(−3.398)	(−2.704)	(−1.558)	(1.193)
High - Low	-1.129	-0.501	-0.843	-0.974	-0.708	
	(-3.654)	(-1.922)	(-3.372)	(-3.400)	(-2.386)	

	β_m and $t(\beta_m)$					
	B/M1 (Low)	B/M2	B/M3	B/M4	B/M5 (High)	High - Low
BD1 (Low)	0.874	0.945	0.943	0.829	0.892	0.018
	(40.510)	(33.978)	(30.847)	(23.891)	(21.345)	(0.397)
BD2	1.081	1.070	1.004	0.972	0.932	-0.149
	(44.554)	(40.810)	(36.686)	(31.272)	(21.107)	(-3.079)
BD3	1.222	1.151	1.104	1.100	1.153	-0.069
	(38.157)	(36.246)	(35.395)	(35.483)	(24.485)	(-1.320)
BD4	1.230	1.160	1.182	1.224	1.253	0.023
	(30.880)	(30.504)	(33.024)	(37.310)	(26.888)	(0.414)
BD5 (High)	1.435	1.311	1.331	1.409	1.438	0.002
	(23.620)	(27.380)	(28.927)	(25.925)	(26.686)	(0.030)
High - Low	0.561	0.366	0.387	0.581	0.545	
	(7.955)	(6.161)	(6.783)	(8.881)	(8.056)	

Table IX Coefficients of SMB and HML for Value Weighted Returns – This table shows coefficients on SMB and HML when Fama-French 3-factor model is applied to value weighted returns for independently sorted portfolios : assets from CRSP-Compustat-I/B/E/S merged data set are independently sorted into total of 25 (5×5) portfolios by belief difference and book-to-market ratio. Fame-French three-factor time series regressions are estimated over the entire monthly time periods for each portfolio as follows: $r_t = \alpha + \beta_m MKT_t + \beta_s SMB_t + \beta_h HML_t + \epsilon_t$, where MKT is the value-weighted market portfolios excess return, SMB is the factor returns on small minus big stocks, and HML is the factor returns on high minus low B/M. The coefficients β_s and β_h from these regressions and their corresponding t-stat values are reported below.

	β_s and $t(\beta_s)$					
	B/M1 (Low)	B/M2	B/M3	B/M4	B/M5 (High)	High - Low
BD1 (Low)	-0.348	-0.116	-0.097	0.043	0.112	0.460
	(-11.126)	(-2.888)	(-2.180)	(0.862)	(1.843)	(7.005)
BD2	-0.194	-0.111	-0.033	-0.037	0.072	0.267
	(-5.520)	(-2.927)	(-0.838)	(-0.810)	(1.128)	(3.794)
BD3	0.091	-0.042	-0.017	0.033	0.118	0.027
	(1.950)	(-0.919)	(-0.383)	(0.744)	(1.726)	(0.362)
BD4	0.315	0.119	0.010	0.192	0.254	-0.061
	(5.445)	(2.157)	(0.185)	(4.024)	(3.752)	(-0.757)
BD5 (High)	0.752	0.461	0.337	0.269	0.311	-0.441
	(8.535)	(6.635)	(5.050)	(3.406)	(3.977)	(-3.818)
High - Low	1.100	0.577	0.434	0.225	0.199	
	(10.755)	(6.694)	(5.241)	(2.366)	(2.027)	

	β_h and $t(\beta_h)$					
	B/M1 (Low)	B/M2	B/M3	B/M4	B/M5 (High)	High - Low
BD1 (Low)	-0.360	0.095	0.305	0.533	0.698	1.058
	(-9.421)	(1.844)	(5.097)	(7.539)	(8.172)	(11.739)
BD2	-0.275	0.175	0.295	0.622	0.719	0.995
	(-5.745)	(3.408)	(5.637)	(9.739)	(7.967)	(10.081)
BD3	-0.320	0.149	0.432	0.668	0.962	1.282
	(-4.891)	(2.300)	(6.749)	(10.522)	(10.013)	12.036
BD4	-0.335	0.028	0.324	0.688	0.948	1.283
	(-4.230)	(0.358)	(4.414)	(10.267)	(10.088)	(11.265)
BD5 (High)	-0.207	0.208	0.427	0.703	1.180	1.387
	(-1.807)	(2.157)	(4.620)	(6.331)	(10.975)	(8.690)
High - Low	0.153	0.113	0.122	0.170	0.482	
	(1.212)	(0.987)	(1.086)	(1.274)	(3.526)	

Table X Institutional Ownership – This table shows institutional stock ownership for independently sorted portfolios : stocks in CRSP-Compustat-I/B/E/S merged data set are independently sorted into total of 25 (5×5) portfolios by belief difference and book-to-market ratio. For each portfolio, we compute monthly time-series average of institutional ownership using the percentage of institutions' share holdings over total outstanding shares from Thompson-Reuter Spectrum 13(f) in WRDS. Since 13(f) in WRDS is given quarterly, we take (overlapping) ownership data at every month using the previously available quarterly data.

	Institutional Stock Ownership				
	B/M1 (Low)	B/M2	B/M3	B/M4	B/M5 (High)
BD1 (Low)	0.542	0.503	0.446	0.422	0.415
BD2	0.544	0.521	0.486	0.456	0.444
BD3	0.507	0.505	0.489	0.463	0.446
BD4	0.444	0.469	0.479	0.471	0.446
BD5 (High)	0.371	0.405	0.441	0.442	0.426

Table XI Variation in Institutional Ownership – This table shows the standard deviation of time-series of institutional stock ownership for independently sorted portfolios : stocks in CRSP-Compustat-I/B/E/S merged data set are independently sorted into total of 25 (5×5) portfolios by belief difference and book-to-market ratio. For each portfolio, we compute quarterly institutional ownership using the percentage of institutions' share holdings over total outstanding shares from Thompson-Reuter Spectrum 13(f) in WRDS. We compute the standard deviation of quarterly time-series of institutional ownership.

	Variations in Institutional Stock Ownership				
	B/M1 (Low)	B/M2	B/M3	B/M4	B/M5 (High)
BD1 (Low)	0.137	0.135	0.136	0.153	0.146
BD2	0.136	0.127	0.124	0.144	0.148
BD3	0.127	0.124	0.118	0.127	0.133
BD4	0.118	0.119	0.123	0.121	0.127
BD5 (High)	0.130	0.131	0.141	0.140	0.133

Table XII The Max-Return for Portfolio – This table shows the max-return of independently sorted portfolios: stocks in CRSP-Compustat-I/B/E/S merged data set are independently sorted into total of 25 (5×5) portfolios by belief difference and book-to-market ratio. For each portfolio, we record maximum daily percentage return for the previous month. In particular, we pick a maximal daily percentage return of a portfolio at each month over time and take the average.

The Max-Return					
	B/M1 (Low)	B/M2	B/M3	B/M4	B/M5 (High)
BD1 (Low)	1.929	1.753	1.670	1.624	1.958
BD2	2.117	1.853	1.744	1.714	1.894
BD3	2.399	1.969	1.827	1.766	1.911
BD4	2.492	2.115	1.907	1.899	1.974
BD5 (High)	2.661	2.367	2.246	2.122	2.166

Table XIII Idiosyncratic Skewness for Portfolio – This table shows idiosyncratic skewness of independently sorted portfolios: stocks in CRSP-Compustat-I/B/E/S merged data set are independently sorted into total of 25 (5×5) portfolios by belief difference and book-to-market ratio. For each portfolio, we run the following regression using the daily returns of the portfolio: $r_{i,t} - r_{f,d} = \alpha_i + \beta_i(r_{m,d} - r_{f,d}) + \gamma_i(r_{m,d} - r_{f,d})^2 + \epsilon_{i,d}$, where $r_{i,d}$, $r_{m,d}$, and $r_{f,d}$ are daily returns of portfolio i, market index, and the risk free asset at day d. We plug the residual $\hat{\epsilon}_{i,d}$ into $r_{i,d}$ in a skewness equation $\frac{n(n-1)^{3/2}\sum_d r_{i,d}^3}{(n-1)(n-2)(\sum_d r_{i,d}^2)^{3/2}}$ to compute idiosyncratic skewness.

Idiosyncratic Skewness					
	B/M1 (Low)	B/M2	B/M3	B/M4	B/M5 (High)
BD1 (Low)	-0.076	0.049	-0.021	0.083	0.135
BD2	-0.035	-0.019	0.012	0.080	0.151
BD3	-0.045	0.020	0.061	0.116	0.138
BD4	-0.062	-0.025	0.001	0.042	0.094
BD5 (High)	0.056	0.022	0.100	0.102	0.139

Table XIV Number of Stocks in Portfolios – This table shows average number of stocks in independently sorted portfolios: stocks in CRSP-Compustat-I/B/E/S merged data set are independently sorted into total of 25 (5×5) portfolios by belief difference and book-to-market ratio. We record number of stock observations in each portfolio every month and take the time-series average.

Number of Stock Observations					
	B/M1 (Low)	B/M2	B/M3	B/M4	B/M5 (High)
BD1 (Low)	145.239	98.796	72.978	52.366	28.199
BD2	122.753	95.046	80.320	63.462	35.605
BD3	105.728	89.704	83.024	71.710	47.011
BD4	92.481	79.387	81.040	78.239	66.038
BD5 (High)	86.548	66.890	69.188	76.202	97.933

Table XV Cash Flow Share Ratio: Proxy for (Idiosyncratic) Cash Flow Risk – This table shows time-series average of the cash flow share ratio and the interaction of it with investors' belief difference. Cash flow share s_t is defined as the cash flow of a portfolio divided by aggregate cash flow. To compute the long-run mean of the cash flow share, $\bar{s}$, we use the cash flow share for the past 24 months. We thus have 24 cash flow share ratio each time for each independently sorted portfolio. We report time-series average of the cash flow share ratio, $\bar{s}/s_t$ for independently sorted portfolios from CRSP-Compustat-I/B/E/S merged data set. Interaction indicates the cross product of the average cash flow share ratio and average belief difference for each portfolio.

Average Share Ratio					
	B/M1 (Low)	B/M2	B/M3	B/M4	B/M5 (High)
BD1 (Low)	1.028	1.126	1.277	1.346	1.690
BD2	1.170	1.126	1.214	1.171	1.448
BD3	1.210	1.277	1.260	1.221	1.338
BD4	1.430	1.281	1.327	1.285	1.280
BD5 (High)	1.393	1.363	1.466	1.442	1.243
Interaction between Share Ratio and Belief Dispersion					
	B/M1 (Low)	B/M2	B/M3	B/M4	B/M5 (High)
BD1 (Low)	0.009252	0.01126	0.01277	0.01346	0.01521
BD2	0.03159	0.032832	0.032778	0.031617	0.040544
BD3	0.0605	0.06385	0.063	0.062271	0.068238
BD4	0.14586	0.130662	0.136681	0.13364	0.13568
BD5 (High)	1.079575	1.069955	1.058452	1.208396	1.336225

Table XVI Systematic Cash Flow Risk – We compute the systematic cash flow risk for independently sorted 25 portfolios from CRSP-Compustat-I/B/E/S merged data set. Systematic cash flow risk is defined as the covariance between the share ratio and aggregate cash flow growth such that we estimate $\theta_{i,t} \equiv Cov_t\left(ds_t/s_t, dD_t/D_t\right)$, where i is portfolio i, $\theta_{i,t}$ is the symbol of systematic cash flow risk of portfolio i with respect to aggregate cash flows, D_t is the aggregate cash flow at t. The notion of systematic cash flow risk $\theta_{i,t}$ comes from Menzly, Santos, and Veronesi (2004) and Gallmeyer, Jhang, and Kim (2015). We compute the conditional cash flow risk by using the past 24 months of cash flow data at each month t. And then, we take the time-series average of them.

Conditional Systematic Cash Flow Risk, θ_i					
	B/M1 (Low)	B/M2	B/M3	B/M4	B/M5 (High)
BD1 (Low)	-0.001	-0.003	-0.0004	-0.003	0.006
BD2	0.0004	-0.002	-0.002	0.001	-0.001
BD3	-0.003	-0.0004	-0.004	-0.004	-0.002
BD4	-0.00000	-0.006	-0.004	-0.001	-0.004
BD5 (High)	0.001	-0.003	0.001	-0.004	-0.002

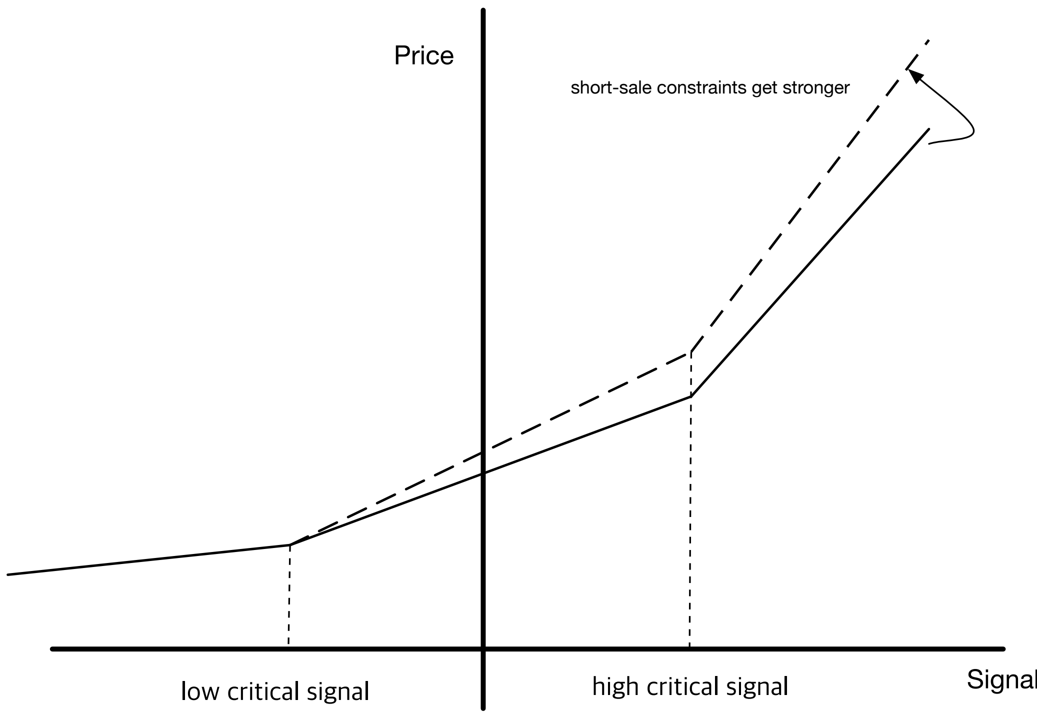

Figure 1. Convexity of Equilibrium Price – This Figure shows the convexity of equilibrium price in the Figure 1 of Xu (2007). A dashed line is added over to the original Figure 1 of Xu (2007). Proposition 2 in his theoretical results implies that the dashed line represents the response of the price for an asset that is subject to stronger short-sale constraint.